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Electromagnetic mass splitting of hadrons in the quark model*

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Assuming that electromagnetic interactions between quarks are Coulomb plus magnetic-moment interactions, we derive an equality and several inequalities among the hadron electromagnetic mass splittings, including two inequalities which relate mass splittings of strange baryons and mesons. None of the mass relations we derive is in contradiction with the present experimental data.

I. INTRODUCTION

A number of authors have obtained sum rules relating the electromagnetic mass splittings of hadrons, assuming SU(3) or SU(6) symmetry of the strong interactions.[1] Rubinstein and other authors[2-5] have obtained similar electromagnetic mass relations without explicitly using any symmetry other than isospin invariance. Instead these authors assumed that in baryons two-body quark-interaction energies were independent of the presence of the third quark. Lipkin[6] has discussed the connection between the approaches based on symmetry and those based on the independent quark model with additive two-body interaction energies.

In our paper, we reexamine the electromagnetic mass splittings of hadrons using the quark model. We assume, following a number of authors,[7,4] that the electromagnetic mass splittings of hadrons arise partly from the intrinsic mass differences of their constituent quarks, and partly from the Coulomb and magnetic interactions between the quarks. In Sec. II of our paper, in which the formalism is given, and in Sec. III, which contains our main new results, we do not assume SU(3) or SU(6) invariance of the strong interactions. We find we do not obtain the same results as Rubinstein *et al.*,[3] but instead obtain an equality and several inequalities among hadron masses, none of which is contradicted by the present experiments. In Sec. IV, we do assume SU(3) and SU(6) invariance to facilitate a comparison with previous work.

As in previous papers,[7,4] we neglect certain terms including relativistic terms, which contribute to the electromagnetic interaction. Our reason for neglecting these terms, aside from the fact that in positronium they are small, is pragmatic. With these terms present, the expressions for the hadron masses become so complicated that we have been unable to obtain any useful mass relations among the hadrons without assuming SU(3) invariance. De Rújula *et al.*[8] have obtained useful mass relations even though including additional terms in the interaction (although neglecting some relativistic effects). However, these authors have assumed SU(6) invariance for the unperturbed wave functions.

In our model, although the electromagnetic interactions are all two-body interactions, the two-body quark interaction *energies* are not independent of the presence of a third quark, as assumed by Rubinstein *et al.*[3] and by Franklin.[5] This is the reason we do not obtain all the sum rules of Rubinstein *et al.*[3] Instead, we obtain (among other inequalities relating electromagnetic mass differences) two new inequalities between baryon and meson mass splittings. By assuming that the strong interactions do not break SU(3) symmetry too badly, we obtain additional inequalities. None of these relations is in contradiction to the present experimental data. We also find that if we incorporate SU(6) invariance of the spatial wave functions, we indeed obtain the result that two-body interaction energies in baryons are independent of the third quark. We then recover the results of Rubinstein *et al.*[3] However, we also obtain from experiment a result which contradicts one of the original assumptions of our model. We therefore prefer to keep the weaker relations which follow from isospin invariance and mildly broken SU(3) invariance of the strong interactions of quarks.

In the usual quark model there are three kinds—or flavors—of quarks, which we denote by u, d, and s. There is some indirect evidence that the quark model should contain one or more heavy charmed quarks as additional flavors. In fact, a number of papers have already appeared which treat the electromagnetic mass splittings of charmed hadrons with a quark model.[9,10] Because, at present, charm is a speculative topic, and it is not clear how many charmed quarks the model should have, we prefer to confine ourselves to hadrons not containing any charmed quarks.

If quarks come in three colors, then it is possible to have a model in which the quarks have integral charge, as proposed by Han and Nambu,[11] rather than fractional charge. However, if the quarks have magnetic moments proportional to their charges, we shall see that the electromagnetic mass splittings of hadrons are independent of the color scheme. The basic reason for this is that, even if quarks have integral charge, the average charge of all quarks of different colors but a single flavor must be fractional. Therefore, we do not need to concern ourselves with quark color.

II. EXPRESSIONS FOR THE HADRON MASS DIFFERENCES

We shall confine ourselves to hadrons which are composed of quarks in states with zero orbital angular momenta, as these are the states for which the electromagnetic mass splittings are known experimentally. The experimental values, from the compilation of the Particle Data Group,[12] are given in Table I.

TABLE I. Experimental hadron mass splittings in MeV from the compilation of the Particle Data Group (Ref. 12). In this table, the symbol for a particle denotes its mass.

$\pi^+ - \pi^0$	4.6043 ± 0.0037
$\rho^+ - \rho^0$	-4.4 ± 2.4
$K^0 - K^+$	3.99 ± 0.13
$K^{*0} - K^{*+}$	6.1 ± 1.5
$n - p$	$1.293\,44 \pm 0.000\,07$
$\Sigma^- - \Sigma^+$	7.99 ± 0.08
$\Sigma^- - \Sigma^0$	4.87 ± 0.06
$\Xi^- - \Xi^0$	6.4 ± 0.6
$\Delta^0 - \Delta^{++}$	1.4 ± 0.4
$\Sigma^{*-} - \Sigma^{*+}$	4.1 ± 1.4
$\Xi^{*-} - \Xi^{*0}$	3.4 ± 0.6

As stated in the Introduction, we assume the electromagnetic interaction between quarks is given by a Coulomb term plus a magnetic term. This interaction H'_{ij} can be written[7]

$$H'_{ij} = Q_i Q_j / r_{ij} - \tfrac{8}{3}\pi\, \vec{\mu}_i \cdot \vec{\mu}_j \delta(\vec{r}_{ij}) , \tag{1}$$

where Q_i is the charge and $\vec{\mu}_i$ is the magnetic moment of the ith quark, and r_{ij} is the distance between ith and jth quarks. We shall evaluate H'_{ij} in perturbation theory by taking its expectation value with respect to eigenfunctions of the strong interaction. Because the electromagnetic interaction between quarks is small compared to the strong interaction between them, this should be a good approximation.

In order to proceed further, we need to obtain more detailed expressions for the quark magnetic moments appearing in Eq. (1). If one assumes that the quarks are members of an unbroken SU(3) triplet, then it is plausible that the magnetic moments are proportional to their charges.[13] This assumption enables one to calculate the ratio of the neutron magnetic moment μ_n to the proton moment μ_p, and the ratio of the Λ moment μ_Λ to the proton moment. The well-known results are

$$\mu_n/\mu_p = -\tfrac{2}{3}, \quad \mu_\Lambda/\mu_p = -\tfrac{1}{3}.$$

The calculated values agree very well with experiment for the magnetic moment of the neutron, but the calculated magnetic moment of the Λ comes out a little bigger than the experimental one. We therefore break SU(3) symmetry by writing the magnetic-moment operator of the ith quark as

$$\vec{\mu}_i = \tfrac{1}{2} a_i \vec{\sigma}_i Q_i , \tag{2}$$

where $\vec{\sigma}_i$ is the Pauli spin operator and the a_i are positive constants satisfying

$$a_u = a_d > a_s .$$

The factor $\frac{1}{2}$ in Eq. (2) is inserted only for convenience so that if the quarks had Dirac moments, the a_i would be their inverse masses. The constants a_u and a_s are chosen to give the known proton and Λ moments. Our Eq. (2) is a weaker condition on the quark magnetic moments than is usually made.[13] Without our assumption, we would not be able to understand the ratio of the neutron to proton magnetic moments, and one of the most important successes of the quark model would be lost.

In letting $a_u = a_d$ we are assuming isospin invariance. This approximation introduces only a small error in the perturbation expression for the expectation value of H'_{ij}, and therefore causes only a second-order error in the hadron masses.

An interesting consequence of Eq. (2) is that it allows us to obtain expressions for the hadron electromagnetic mass differences which are independent of the color scheme. A simple proof of this, due to Franklin,[14] is given in Appendix A.

We take the wave function ψ_h of a hadron ($h = M$ for meson or B for baryon) to be a product of a function ψ_h which depends on spatial coordinates, a function χ_h which depends on spin coordinates, and a function η_h which depends on flavor. We then can write the expectation value $\langle H'_{ij}\rangle_h$ in the form

$$\langle H'_{ij}\rangle_h = \langle \eta_h | Q_i Q_j | \eta_h \rangle \left[\left\langle \Psi_h \left| \frac{1}{r_{ij}} \right| \Psi_h \right\rangle - \frac{2\pi a_i a_j}{3} \langle \chi_h | \vec{\sigma}_i \cdot \vec{\sigma}_j | \chi_h \rangle \langle \Psi_h | \delta(\vec{r}_{ij}) \Psi_h \rangle \right] \tag{3}$$

We next discuss the symmetry of the baryon wave functions under the interchange of any two quarks. It is often assumed that the baryons belong to the symmetric 56-dimensional representation of SU(6), and that their wave functions are symmetric under the interchange of the coordinates (excluding color coordinates) of any two quarks. We shall make the less restrictive assumption that a baryon wave function is symmetric under the interchange of the coordinates (again excluding color) of any two quarks of the same flavor.[5] In addition, we assume, of course, that baryon wave functions are invariant under isospin rotations. We give the baryon wave functions in Appendix B. These wave functions differ from those of Franklin[5] in their spatial behavior. The meson wave functions are simpler than those for the baryons, but, for completeness, we also give them in Appendix B.

In our model the mass of a hadron is given by the sum of three terms. The first is the sum of the effective masses of the quarks within it. The second term is a strong-interaction energy V_h. The third term is the electromagnetic interaction energy calculated in perturbation theory.

Then we can write the mass of a meson composed of a quark i and antiquark j as

$$m_M = m_i + m_j + V_M + \langle H'_{ij}\rangle_M . \tag{4}$$

Likewise, we can write the mass of a baryon composed of quarks i, j, and k as

$$m_B = m_i + m_j + m_k + V_B + \langle H'_{ij}\rangle_B + \langle H'_{ik}\rangle_B + \langle H'_{jk}\rangle_B . \tag{5}$$

The strong-interaction energy V_h is independent of the z component of isospin and therefore is the same for all members of an isospin multiplet. This quantity will therefore not enter explicitly into any expression for electromagnetic mass differences of different hadrons belonging to the same isospin multiplet. Of course, there is an implicit dependence of the mass difference on the strong interaction because the hadron wave function Ψ_h depends on it.

With the wave functions of Appendix B, we can calculate expressions for the hadron masses in a straightforward way using Eqs. (3), (4), and (5). We shall not write down all these formulas for the masses, but only formulas for the electromagnetic mass differences. We let $m_u = m$ and let $m_d - m = \epsilon$. The electromagnetic mass difference ϵ between the u and d quark is a parameter which we do not attempt to calculate from first principles. We introduce the notation

$$C_h^{ij} = \alpha \langle \Psi_h | 1/r_{ij} | \Psi_h \rangle , \tag{6}$$

$$\mathfrak{M}_h^{ij} = \tfrac{2}{3}\pi\alpha a_i a_j \langle \Psi_h | \delta(\vec{r}_{ij}) | \Psi_h \rangle . \tag{7}$$

We omit the superscripts i and j for mesons, because there are only two quarks involved. Then the electromagnetic mass splittings are

$$m_{\pi^+} - m_{\pi^0} = \tfrac{1}{2}C_\pi + \tfrac{3}{2}\mathfrak{M}_\pi , \tag{8a}$$

$$m_{K^0} - m_{K^+} = \epsilon - \tfrac{1}{3}C_K - \mathfrak{M}_K , \tag{8b}$$

$$m_{\rho^+} - m_{\rho^0} = \tfrac{1}{2}C_\rho - \tfrac{1}{2}\mathfrak{M}_\rho , \tag{8c}$$

$$m_{K^{*0}} - m_{K^{*+}} = \epsilon - \tfrac{1}{3}C_{K^*} + \tfrac{1}{3}\mathfrak{M}_{K^*} \tag{8d}$$

For the baryons, we can see from the symmetry of the wave functions of Appendix B that we have $C_B^{13} = C_B^{23}$ and $\mathfrak{M}_B^{13} = \mathfrak{M}_B^{23}$. Therefore, we can eliminate C_B^{23} and $\mathfrak{M}_B^{23}$ from the expressions for the baryon masses. Furthermore, because the Δ baryons have completely symmetric spatial wave functions, we can eliminate C_B^{13} and $\mathfrak{M}_B^{13}$ from the Δ mass formulas as well. We then obtain the following expressions for the baryon electromagnetic mass differences:

$$m_n - m_p = \epsilon - \tfrac{1}{3}(C_N^{12} - \mathfrak{M}_N^{12}) , \tag{9a}$$

$$m_{\Sigma^0} - m_{\Sigma^+} = \epsilon - \tfrac{2}{3}(C_\Sigma^{12} - \mathfrak{M}_\Sigma^{12}) + \tfrac{1}{3}(C_\Sigma^{13} + 2\mathfrak{M}_\Sigma^{13}) , \tag{9b}$$

$$m_{\Sigma^-} - m_{\Sigma^0} = \epsilon + \tfrac{1}{3}(C_\Sigma^{12} - \mathfrak{M}_\Sigma^{12}) + \tfrac{1}{3}(C_\Sigma^{13} + 2\mathfrak{M}_\Sigma^{13}) , \tag{9c}$$

$$m_{\Xi^-} - m_{\Xi^0} = \epsilon + \tfrac{2}{3}(C_\Xi^{13} + 2\mathfrak{M}_\Xi^{13}) , \tag{9d}$$

$$m_{\Delta^+} - m_{\Delta^{++}} = \epsilon - \tfrac{4}{3}(C_\Delta^{12} - \mathfrak{M}_\Delta^{12}) , \tag{9e}$$

$$m_{\Delta^0} - m_{\Delta^+} = \epsilon - \tfrac{1}{3}(C_\Delta^{12} - \mathfrak{M}_\Delta^{12}) , \tag{9f}$$

$$m_{\Delta^-} - m_{\Delta^0} = \epsilon + \tfrac{2}{3}(C_\Delta^{12} - \mathfrak{M}_\Delta^{12}) , \tag{9g}$$

$$m_{\Sigma^{*0}} - m_{\Sigma^{*+}} = \epsilon - \tfrac{2}{3}(C_{\Sigma^*}^{12} - \mathfrak{M}_{\Sigma^*}^{12}) + \tfrac{1}{3}(C_{\Sigma^*}^{13} - \mathfrak{M}_{\Sigma^*}^{13}) , \tag{9h}$$

$$m_{\Sigma^{*-}} - m_{\Sigma^{*0}} = \epsilon + \tfrac{1}{3}(C_{\Sigma^*}^{12} - \mathfrak{M}_{\Sigma^*}^{12}) + \tfrac{1}{3}(C_{\Sigma^*}^{13} - \mathfrak{M}_{\Sigma^*}^{13}) , \tag{9i}$$

$$m_{\Xi^{*-}} - m_{\Xi^{*0}} = \epsilon + \tfrac{2}{3}(C_{\Xi^*}^{13} - \mathfrak{M}_{\Xi^*}^{13}) . \tag{9j}$$

III. RELATIONS AMONG THE HADRON MASS DIFFERENCES

In obtaining the expression of Eqs. (8), and (9), we have not assumed SU(3) or SU(6) invariance of the strong-interaction wave functions. We have also not used any explicit assumptions about the spatial behavior of the strong-interaction wave functions other than that these wave functions have no orbital angular momentum, are symmetric under the interchange of identical quarks, and are invariant under isospin rotations. Therefore, we obtain a relatively small amount of information about the electromagnetic mass splittings.

But some things can be said. For example, the Coulomb energy C_h^{ij} and the magnetic energy $\mathfrak{M}_h^{ij}$ defined in Eqs. (6) and (7) are both positive-definite quantities. Therefore, we see from Eq.

(8a) that

$$m_{\pi^+} - m_{\pi^0} > 0 \,, \tag{10}$$

in agreement with experiment, as can be seen from Table I. This inequality has been obtained previously by Gal and Scheck.[4] Furthermore, by comparing Eqs. (8) and (9) we see that, independent of the value of ϵ, we obtain the following inequalities between strange-baryon electromagnetic mass differences and a strange-meson electromagnetic mass difference:

$$m_{\Xi^-} - m_{\Xi^0} > m_{K^0} - m_{K^+} \tag{11a}$$

$$\tfrac{1}{3}(2m_{\Sigma^-} - m_{\Sigma^0} - m_{\Sigma^+}) > m_{K^0} - m_{K^+} \,. \tag{11b}$$

Such relations between baryon and meson mass differences cannot be obtained from the usual SU(3) or SU(6) symmetries, but follow from the quark model (with our assumptions) without these symmetries. As far as we know, these inequalities have not been previously derived from the quark model with quarks of spin $\frac{1}{2}$. [Franklin[15] derived the inequality (11a) assuming that quarks have spin $\frac{3}{2}$.] From Table I we see that the inequalities (11) are in agreement with experiment.

From Table I and Eqs. (8) and (9), we can put limits on the parameter ϵ, which is the mass difference between the d and u quarks. We get

$$4.0 < \epsilon < 4.3 \text{ MeV} \,. \tag{12}$$

From Eq. (9a) we see that we must have $C_N^{12} > \mathfrak{M}_N^{12}$, or the n-p mass difference would be greater than 4 MeV. This says that, in the nucleon, the spatial wave function of the three quarks is such as to make the Coulomb energy greater than the magnetic energy. We may ask what the consequences are if this same condition holds for the other baryons. [We do not need to assume that SU(3) and SU(6) are good symmetries, only that they are not broken so badly as to preclude a qualitative similarity of the baryon spatial wave functions.] If we have $C_B^{ij} > \mathfrak{M}_B^{ij}$ for all baryons, then we obtain from Eqs. (9) the following additional inequalities:

$$\begin{aligned} &m_{\Sigma^-} - m_{\Sigma^0} > m_{\Sigma^0} - m_{\Sigma^+} \,, \\ &m_{\Delta^-} - m_{\Delta^0} > m_{\Delta^0} - m_{\Delta^+} > m_{\Delta^+} - m_{\Delta^{++}} \,, \\ &m_{\Sigma^{*-}} - m_{\Sigma^{*0}} > m_{\Sigma^{*0}} - m_{\Sigma^{*+}} \,. \end{aligned} \tag{13}$$

We also see that

$$\begin{aligned} &m_{\Sigma^-} - m_{\Sigma^0} > \epsilon \,, \\ &m_{\Delta^-} - m_{\Delta^0} > \epsilon \,, \\ &m_{\Sigma^{*-}} - m_{\Sigma^{*0}} > \epsilon \,, \\ &m_{\Xi^{*-}} - m_{\Xi^{*0}} > \epsilon \,, \end{aligned} \tag{14}$$

where $\epsilon > 4.0$ MeV. Within the experimental errors, the inequalities (13) and (14) are all satisfied, although some have not yet been tested (see Table I).

From Eqs. (9e)–(9g), we see that the three Δ electromagnetic mass differences are given in terms of only two parameters: ϵ and the combination $C_\Delta^{12} - \mathfrak{M}_\Delta^{12}$. We can therefore eliminate these parameters to obtain the relation

$$m_{\Delta^-} - m_{\Delta^{++}} = 3(m_{\Delta^0} - m_{\Delta^+}) \,. \tag{15}$$

This relation has been previously obtained by Rubinstein *et al.*[3] Unfortunately, the experimental values of the Δ masses are not known sufficiently well at present to test this relation. Likewise, we can obtain expressions for ϵ and $C_\Delta^{12} - \mathfrak{M}_\Delta^{12}$,

$$\epsilon = m_{\Delta^-} + m_{\Delta^0} - 2m_{\Delta^+} \,, \tag{16}$$

$$C_\Delta^{12} - \mathfrak{M}_\Delta^{12} = m_{\Delta^0} + m_{\Delta^{++}} - 2m_{\Delta^+} \,. \tag{17}$$

Again, the experimental mass differences are not known sufficiently well to obtain the values of ϵ and $C_\Delta^{12} - \mathfrak{M}_\Delta^{12}$.

From the inequality (12), we can take $\epsilon = 4.15 \pm 0.15$ MeV. Using this value in Eqs. (9), we obtain the following values for certain Coulomb and magnetic energies:

$$C_N^{12} - \mathfrak{M}_N^{12} = 8.6 \pm 0.5 \text{ MeV} \,, \tag{18a}$$

$$C_\Sigma^{13} + 2\mathfrak{M}_\Sigma^{13} = 0.5 \pm 0.5 \text{ MeV} \,, \tag{18b}$$

$$C_\Xi^{13} + 2\mathfrak{M}_\Xi^{13} = 3.2 \pm 0.9 \text{ MeV} \,. \tag{18c}$$

According to these equations, the Coulomb energy C_N^{12} has to be considerably larger than either C_Σ^{13} or C_Ξ^{13}. This is not the case in some specific models in which the baryon wave function can be calculated. Therefore, some doubt is cast on our initial assumptions. Nevertheless, in view of the agreement of our inequalitites with experiment, these assumptions may not be too badly in error.

IV. ADDITIONAL MASS RELATIONS

Further mass relations were obtained by Rubinstein *et al.*[3] using the assumption that hadron electromagnetic mass shifts are caused by two-body quark-quark (or quark-antiquark) interaction energies which depend only on the flavor and spin configuration of the two quarks. Furthermore, Gal and Scheck[4] obtained relations among the meson masses by assuming SU(6) invariance of the strong interactions. We do not necessarily believe that these are realistic assumptions. Nevertheless, it is instructive to see what restrictions we must place on the spatial wave functions to obtain the results of Rubinstein *et al.* and of Gal and Scheck.

If we impose SU(3) invariance on the strong-interaction wave functions of Appendix A, then the quantities C_h^{ij} are the same for all members of an

SU(3) multiplet. The Coulomb energies C_h^{ij} still depend on the indices i and j for the baryon octet, but not for the decuplet. The reason is that in the octet the relative spatial wave function of the first and third quarks will, in general, be different from the relative wave function of the first and second quarks, because their relative spin wave functions are different.[16] Later, when we impose SU(6) invariance, we can drop the indices i and j. For the magnetic energies, we explicitly take into account the difference in the constants a_i for s and u quarks in the expression of Eq. (7) for $\mathfrak{M}_h^{ij}$ by multiplying it by a factor $R=a_s/a_u<1$ if one of the quarks is strange and by R^2 if both quarks are strange. The dependence of $\mathfrak{M}_h^{ij}$ on the indices i and j is the same as for C_h^{ij}.

Then Eqs. (8) become

$$m_{\pi^+}-m_{\pi^0}=\tfrac{1}{2}C_P+\tfrac{3}{2}\mathfrak{M}_P\,, \tag{19a}$$

$$m_{K^0}-m_{K^+}=\epsilon-\tfrac{1}{3}C_P-R\mathfrak{M}_P\,, \tag{19b}$$

$$m_{\rho^+}-m_{\rho^0}=\tfrac{1}{2}C_V-\tfrac{1}{2}\mathfrak{M}_V, \tag{19c}$$

$$m_{K^{*0}}-m_{K^{*+}}=\epsilon-\tfrac{1}{3}C_V+\tfrac{1}{3}R\mathfrak{M}_V\,, \tag{19d}$$

where we have used the subscripts P and V to denote members of the pseudoscalar- and vector-meson nonets, respectively. Likewise Eqs. (9) become

$$m_n-m_p=\epsilon-\tfrac{1}{3}(C_B^{12}-\mathfrak{M}_B^{12})\,, \tag{20a}$$

$$m_{\Sigma^0}-m_{\Sigma^+}=\epsilon-\tfrac{2}{3}(C_B^{12}-\mathfrak{M}_B^{12})+\tfrac{1}{3}(C_B^{13}+2R\mathfrak{M}_B^{13})\,, \tag{20b}$$

$$m_{\Sigma^-}-m_{\Sigma^0}=\epsilon+\tfrac{1}{3}(C_B^{12}-\mathfrak{M}_B^{12})+\tfrac{1}{3}(C_B^{13}+2R\mathfrak{M}_B^{13})\,, \tag{20c}$$

$$m_{\Xi^-}-m_{\Xi^0}=\epsilon+\tfrac{2}{3}(C_B^{13}+2R\mathfrak{M}_B^{13})\,, \tag{20d}$$

$$m_{\Delta^+}-m_{\Delta^{++}}=\epsilon-\tfrac{4}{3}(C_D-\mathfrak{M}_D)\,, \tag{20e}$$

$$m_{\Delta^0}-m_{\Delta^+}=\epsilon-\tfrac{1}{3}(C_D-\mathfrak{M}_D)\,, \tag{20f}$$

$$m_{\Delta^-}-m_{\Delta^0}=\epsilon+\tfrac{2}{3}(C_D-\mathfrak{M}_D)\,, \tag{20g}$$

$$m_{\Sigma^{*0}}-m_{\Sigma^{*+}}=\epsilon-\tfrac{1}{3}C_D+\tfrac{1}{3}(2-R)\mathfrak{M}_D\,, \tag{20h}$$

$$m_{\Sigma^{*-}}-m_{\Sigma^{*0}}=\epsilon+\tfrac{2}{3}C_D-\tfrac{1}{3}(1+R)\mathfrak{M}_D\,, \tag{20i}$$

$$m_{\Xi^{*-}}-m_{\Xi^{*0}}=\epsilon+\tfrac{2}{3}C_D-\tfrac{2}{3}R\mathfrak{M}_D\,, \tag{20j}$$

where we have used the subscript B to denote a member of the baryon octet and D to denote a member of the baryon decuplet.

It is interesting that Eqs. (19) for the meson mass differences still contain so many free parameters that we obtain no additional mass relations.

Turning to the baryons, we obtain from Eqs. (20) the well-known Coleman-Glashow[17] relation

$$m_n-m_p=m_{\Sigma^-}-m_{\Sigma^+}+m_{\Xi^0}-m_{\Xi^-} \tag{21}$$

as well as the relations[3]

$$\begin{aligned} &m_{\Delta^0}-m_{\Delta^+}=m_{\Sigma^{*-}}-m_{\Sigma^{*+}}-m_{\Xi^{*0}}-m_{\Xi^{*-}}\,,\\ &m_{\Sigma^{*-}}+m_{\Sigma^{*+}}-2m_{\Sigma^{*0}}=m_{\Delta^-}+m_{\Delta^+}-2m_{\Delta^0}\;. \end{aligned} \tag{22}$$

The Coleman-Glashow relation is in disagreement with experiment by only 0.3 MeV. The Δ masses are not known well enough to test Eqs. (22).

There are troubles, however. We can solve Eqs. (20a)–(20c) for ϵ in terms of the nucleon and Σ mass differences. We get

$$\begin{aligned}\epsilon&=m_n-m_p+\tfrac{1}{3}(m_{\Sigma^+}+m_{\Sigma^-}-2m_{\Sigma^0})\\ &=1.9\,\text{MeV}\,,\end{aligned} \tag{23}$$

a result previously obtained by Franklin.[5] But this value of ϵ is incompatible with the inequality (12) which we previously derived. Thus, we must either give up SU(3) invariance of the strong-interaction hadron wave functions or assume that the effective mass difference ϵ between the d and u quarks is different in mesons and baryons. An experimental test is possible of whether ϵ is the same in mesons and baryons. We obtained the value $\epsilon>4.0$ MeV from the measured K^0-K^+ meson mass difference. But it is possible to obtain the value of ϵ from Eq. (16) from accurate measurements of the masses of the Δ^+, Δ^0, and Δ^- baryons. Unfortunately, because of the large width of the Δ, sufficiently precise measurements will be difficult to make. In any case, it is known that SU(3) is a broken symmetry in strong interactions, and thus we need not expect the hadron wave functions to be invariant under SU(3).

Despite the fact that we cannot even maintain SU(3) invariance for the baryons, we shall go on to assume SU(6) invariance of the meson and baryon strong-interaction wave functions. We do this to facilitate comparison with the work of Rubinstein *et al.*[3] and of Gal and Scheck.[4] For the mesons we set $C_P=C_V$ and $\mathfrak{M}_P=\mathfrak{M}_V$ in Eqs. (19) and obtain the inequalities

$$m_{\pi^+}-m_{\pi^0}>m_{\rho^+}-m_{\rho^0}\,, \tag{24}$$

$$m_{K^{*0}}-m_{K^{*+}}>m_{K^0}-m_{K^+}\,, \tag{25}$$

$$\begin{aligned} m_{\rho^+}-m_{\rho^0}<m_{\pi^+}-m_{\pi^0}-\tfrac{3}{2}(m_{K^{*0}}-m_{K^{*+}})\\ +\tfrac{3}{2}(m_{K^0}-m_{K^+})\,.\end{aligned} \tag{26}$$

As can be seen from Table I, all these inequalities are satisfied by the experimental meson masses. In previous calculations assuming SU(6) invariance of the meson wave functions, the mass difference between the strange and nonstrange quarks was neglected.[4,18] This is a consistent approximation because if the masses are different, one would expect that the dynamics would lead to SU(6) breaking in the wave functions. If this mass difference is neglected, R becomes unity. Then the inequalities (24) and (25) remain, but (26) becomes an equality. This equality, which was implied by the work of Gal and Scheck,[4] although not explicitly written down by them, is not satisfied very well by experiment. The left-hand side is -4.4 ± 2.3 MeV and the right-hand side is 1.4 ± 2.3 MeV. However, the experimental errors are too large to say that there is a definite contradiction.

For the baryons, we drop the indices i and j and set $C_B=C_D$ and $\mathfrak{M}_B=\mathfrak{M}_D$. We do not need to set $R=1$ to obtain the following additional mass relations previously obtained by Rubinstein *et al.*[3]:

$$m_n-m_p=m_{\Delta^0}-m_{\Delta^+}\,, \tag{27}$$

$$m_{\Sigma^-}+m_{\Sigma^+}-2m_{\Sigma^0}=m_{\Sigma^{*-}}+m_{\Sigma^{*+}}-2m_{\Sigma^{*0}}\,. \tag{28}$$

We also obtain the following inequality previously found by Franklin[5]:

$$4(m_{\Xi^-}-m_{\Xi^0})>m_{\Xi^{*-}}-m_{\Xi^{*0}}+3(m_n-m_p)+m_{\Sigma^-}+m_{\Sigma^+}-2m_{\Sigma^0}\,. \tag{29}$$

Of these relations, only the inequality (29) can be tested at present, and it agrees with experiment.

Rubinstein *et al.*[2,3] and Franklin[5] have emphasized that they obtained their results using the assumption of the additivity of two-body quark interaction energies without the use of SU(3) or SU(6) invariance. The electromagnetic interactions between quarks which we have used is a special case of two-body quark-quark interactions. However, this interaction does not give rise to additive two-body interaction energies. Therefore, we were not able to obtain all the results of Rubinstein *et al.* and of Franklin without the additional assumption of SU(6) invariance of the baryon spatial wave functions.

We do not know how to construct a model using conventional forces in which the two-body interaction energies are independent of the flavor or configuration of the third quark without using SU(6)-invariant wave functions. Certainly, our simple model of two-body Coulomb and magnetic interactions between quarks is not a model of the kind that Rubinstein *et al.* and Franklin envision.

In summary, we have assumed that the electromagnetic mass splittings of hadrons arise from the intrinsic mass splittings of the u and d quarks plus Coulomb and magnetic-moment interactions between the quarks. We have also assumed that the quark strong-interaction wave functions are invariant under isospin rotations and are symmetric under the interchange of identical quarks. We have derived a small number of relations between hadron mass splitting, including two inequalities relating meson and baryon mass splittings. Assuming SU(3) is not too badly broken, we have obtained additional inequalities among baryon mass splittings. None of these inequalities is in contradiction to the present experimental data. Going on to assume SU(3) and SU(6) invariance of the strong-interaction wave functions, we have obtained additional mass relations previously obtained by Gal and Scheck, Rubinstein *et al.*, and Franklin. Most of these relations are satisfied by experiment. However, we found that the assumption of SU(3) invariance of the baryon wave functions leads us to the conclusion that the effective mass difference ϵ between the d and u quark is not the same in mesons and baryons. Whether we may take ϵ to be a single constant for both mesons and baryons even without SU(3) invariance could be decided by furture measurements of the masses of the Δ baryons.

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APPENDIX A

We here give a modified version of Franklin's proof[14] of the known result that the electromagnetic mass differences of hadrons are the same for fractionally charged quarks and for integrally-charged Han-Nambu quarks.

From Eqs. (3), (4) and (5) we see that each term in the expression for the mass of a hadron is either independent of the charges of the quarks or depends on the charges of two quarks as a factor Q_iQ_j. By the Gell-Mann–Nishijima formula, each charge can be written

$$Q_i=I_{zi}+y_i\,, \tag{A1}$$

where I_z is the z component of isospin and y_i is an operator which is independent of isospin. In the usual quark model $y_i=\frac{1}{2}Y_i$, where Y_i is the hypercharge of the quark.

In the Han-Nambu scheme, each type of quark comes in 3 colors, and the different colors may have different charges, or equivalently, different values of y_i. If we denote the color index by α, we have

$$Q_{i\alpha}=I_{zi}+y_{i\alpha},\quad \alpha=1,2,3\,. \tag{A2}$$

We do not use a subscript α on I_{zi} because the isospin of a quark is independent of its color. For any such scheme to work, the average value of $Q_{i\alpha}$ must be equal to the charge of the usual Gell-Mann–Zweig fractionally charged quark, i.e.,

$$\langle Q_{i\alpha}\rangle\equiv\frac{1}{3}\sum_{\alpha=1}^{3}Q_{i\alpha}=Q_i\,, \tag{A3}$$

or equivalently,

$$\langle y_{i\alpha}\rangle=\tfrac{1}{3}\sum_{\alpha} y_{i\alpha}=y_i\,. \tag{A4}$$

Now in any color scheme, the mesons and baryons are color singlets. This implies for mesons that we must make the replacement

$$Q_iQ_j\to\tfrac{1}{3}\sum_{\alpha} Q_{i\alpha}Q_{j\alpha}\,. \tag{A5}$$

Then, using Eq. (A2), we obtain

$$\tfrac{1}{3}\sum_{\alpha} Q_{i\alpha}Q_{j\alpha}=\tfrac{1}{3}\sum_{\alpha} I_{zi}I_{zj}+\tfrac{1}{3}\sum_{\alpha}(I_{zi}y_{j\alpha}+I_{zj}y_{i\alpha}) +\tfrac{1}{3}\sum_{\alpha} y_{i\alpha}y_{j\alpha}\,. \tag{A6}$$

Now the first term on the right-hand side of Eq. (A6) is independent of α, and therefore of the color scheme. Likewise, in view of Eq. (A4), the second term on the right is also independent of the color. The third term of Eq. (A6) does depend on color, but it does not depend on I_z. Therefore, it must cancel out of any equation describing the mass difference of two memebers of the same isospin multiplet. Thus, we have proved that the electromagnetic mass splittings of mesons are independent of the color index α and consequently, independent of the quark charges.

The proof for baryons is very similar. In this case we must make the replacement

$$Q_iQ_j\to\tfrac{1}{6}\sum_{\alpha,\beta}{}' Q_{i\alpha}Q_{j\beta}\,, \tag{A7}$$

where the prime on the sum indicates that we must omit the term $\alpha=\beta$. Using Eq. (A2) in Eq. (A7) we find that the proof goes through just as for the mesons. We omit the details.

APPENDIX B

In this appendix, the symbol for a hadron stands for its wave function. We denote a baryon spatial wave function which has no orbital angular momentum and which is symmetric in all three spatial coordinates of the quarks by Ψ_B^S. If the wave function is symmetric only under the interchange of the spatial coordinates of the first and second quark, we denote the wave function by $\Psi_B^S(12)$. We denote a baryon spin-$\frac{3}{2}$ wave function by χ_3 and two linearly independent spin-$\frac{1}{2}$ wave functions by χ_1^S and χ_1^A, the S and A denoting whether the wave function is symmetric or antisymmetric under the interchange of the spins of the first and second quarks.

Using this notation, and incorporating isospin invariance, we obtain the following expressions for the quark-model functions of the baryon SU(3) decimet:

$$\begin{aligned}
\Delta^{++}&=uuu\chi_3\Psi_\Delta^S\,,\\
\Delta^{+}&=uud\chi_3\Psi_\Delta^S\,,\\
\Delta^{0}&=ddu\chi_3\Psi_\Delta^S\,,\\
\Delta^{-}&=ddd\chi_3\Psi_\Delta^S\,,\\
\Sigma^{*+}&=uus\,\chi_3\Psi_{\Sigma^*}^S(12)\,,\\
\Sigma^{*0}&=uds\,\chi_3\Psi_{\Sigma^*}^S(12)\,,\\
\Sigma^{*-}&=dds\,\chi_3\Psi_{\Sigma^*}^S(12)\,,\\
\Xi^{*0}&=ssu\chi_3\Psi_{\Xi^*}^S(12)\,,\\
\Xi^{*-}&=ssd\chi_3\Psi_{\Xi^*}^S(12)\,,\\
\Omega^{-}&=sss\,\chi_S\Psi_\Omega^S\,.
\end{aligned} \tag{B1}$$

Similarly, the wave functions of the baryon SU(3) octet are

$$\begin{aligned}
p&=uud\chi_1^S\Psi_N^S(12)\,,\\
n&=ddu\chi_1^S\Psi_N^S(12)\,,\\
\Sigma^{+}&=uus\,\chi_1^S\Psi_\Sigma^S(12)\,,\\
\Sigma^{0}&=uds\,\chi_1^S\Psi_\Sigma^S(12)\,,\\
\Sigma^{-}&=dds\,\chi_1^S\Psi_\Sigma^S(12)\,,\\
\Lambda&=uds\,\chi_1^A\Psi_\Lambda^S(12)\,,\\
\Xi^{0}&=ssu\chi_1^S\Psi_\Xi^S(12)\,,\\
\Xi^{-}&=ssd\chi_1^S\Psi_\Xi^S(12)\,.
\end{aligned} \tag{B2}$$

For the mesons, we let the spatial wave functions be denoted by Ψ_M and use the notation χ_1 and χ_0 for spin wave functions corresponding to spin 1 and 0, respectively. We can then write the wave functions of the vector-meson nonet as

$$\begin{aligned}
\rho^{+}&=u\bar{d}\chi_1\Psi_\rho\\
\rho^{0}&=(u\bar{u}-d\bar{d})\chi_1\Psi_\rho/\sqrt{2}\,,\\
\rho^{-}&=d\bar{u}\chi_1\Psi_\rho\,,\\
K^{*+}&=u\bar{s}\chi_1\Psi_{K^*}\,,\\
K^{*0}&=d\bar{s}\chi_1\Psi_{K^*}\,,\\
\bar{K}^{*0}&=s\bar{d}\chi_1\Psi_{K^*}\,,\\
K^{*-}&=s\bar{u}\chi_1\Psi_{K^*}\,,\\
\omega&=(u\bar{u}+d\bar{d})\chi_1\Psi_\omega/\sqrt{2}\,,\\
\varphi&=s\bar{s}\chi_1\Psi_\chi\,.
\end{aligned} \tag{B3}$$

In the above expressions, we have assumed that the φ is composed purely of strange quarks and the ω purely of nonstrange quarks. Actually, both the ω and φ will contain small admixtures of the other quarks. However, since the ω and φ are isospin singlets, their properties are irrelevant for calculating electromagnetic mass splittings.

Likewise, the wave functions of the pseudoscalar nonet are

$$\pi^+ = u\bar{d}\chi_0\Psi_\pi\,, \quad \pi^0 = (u\bar{u} - d\bar{d})\chi_0\Psi_\pi/\sqrt{2}\,,$$
$$\pi^- = d\bar{u}\chi_0\Psi_\pi\,, \quad K^+ = u\bar{s}\chi_0\Psi_K\,, \quad K^0 = d\bar{s}\chi_0\Psi_K\,,$$
$$K^0 = s\bar{d}\chi_0\Psi_K\,, \quad K^- = s\bar{u}\chi_0\Psi_K\,, \tag{B4}$$
$$\eta = [a(u\bar{u} + d\bar{d})/\sqrt{2} + bs\bar{s}]\chi_0\Psi_\eta\,, \quad a^2 + b^2 = 1$$
$$\eta' = [b(uu + dd)/\sqrt{2} - as\bar{s}]\chi_0\Psi_{\eta'}\,.$$

Here again, because the η and η' are isosinglets, we do not need to know the values of the mixing parameters a and b.

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