Cluster model satisfying limited charge exchange

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The distribution function for the production of charged and neutral pions via isoscalar and isovector clusters is developed. The production of the clusters satisfies the limited-charge-exchange constraint and they decay into from one to four final-state pions. We find that isoscalar-cluster production is suppressed and that the cluster size must vary slowly with the total energy.

I. INTRODUCTION

The idea that multiparticle production in high-energy hadron collisions proceeds through an intermediate stage in which the final-state particles are grouped in clusters has been the subject of a number of investigations.[1] Until recently, support for intermediate clusters has been indirect, namely through comparisons of experimental multiplicity distributions, two-particle correlations, and charge transfer between c.m. hemispheres, to model-dependent phenomenological and theoretical predictions.[2] Recent experimental analysis of the rapidity difference between produced particles has, however, provided a new facet in these comparisons.[3,4] These new data show that for large rapidity gaps the density of produced particles as a function of rapidity-gap size falls exponentially. This dependence indicates that final-state particles separated by these large rapidity gaps are produced independently. For smaller rapidity gaps, however, an interdependence of production is indicated as the density is above the exponential decay line. This is easily explained in a cluster model since particles produced with large rapidity gaps would come from separate clusters with essentially independent production, while those separated by small gaps come from the same cluster. On this basis it is claimed that the actual production of clusters is exhibited.[4]

This recent experimental analysis has also provided data on the charge carried between clusters by an exchange mechanism.[5] The nonzero value of this exchanged charge ΔQ shows that cluster-production models must include charged clusters in addition to the isoscalar ones considered in the past.[6] Also, the suppression of $|\Delta Q| \geq 2$ for large rapidity gaps indicates that the exchanged charge is limited to the values $\Delta Q = 0, \pm 1$.

In this paper, we consider the production of isoscalar clusters and isovector clusters satisfying this limited-charge-exchange constraint. The subsequent decay of these clusters is assumed to result only in final-state pions and we consider different size clusters. The size of a cluster is specified by the resulting number of pions and we look at clusters decaying into from one to four pions.

Following Levy,[7] we note that there are three possible schemes within a cluster model to account for the energy dependence of the number of final-state particles produced:

(i) The average cluster size is constant but the average number of clusters increases with energy.

(ii) The average number of clusters is constant but the average cluster size increases.

(iii) Both the average number of clusters and average cluster size varies.

We have investigated both of the first two possibilities and find that the experimental two-particle-correlation data eliminates the case of a constant cluster size but allows that of a varying cluster size. We also investigated the admixture of isoscalar and isovector clusters, using the ratio of the number of charged links to the number of neutral links in the multiperipheral chain. The primary result of this paper is the development of the limited-charge-exchange distribution and the resulting analytical expressions for the average number of charged and neutral particles produced and the two-particle correlation coefficients.

The distribution function for describing cluster production and the subsequent decay into final-state particles and analytic expressions for the moments are developed in Sec. II. In Sec. III, various comparisons to data are used to limit the generality of the model and Sec. IV contains the conclusions of the paper.

II. THE LIMITED-CHARGE-EXCHANGE (LCEX) MODEL

In this section we shall develop an expression giving the probability of producing various final states according to a statistical model which satisfies the LCEX constraint. The model assumes that isoscalar and isovector clusters are produced independently (a Poisson distribution). A binomial distribution is used to describe the relative production of isoscalar and isovector clusters, the

LCEX constraint is imposed on the isovector part, and multinomial distributions are used to separate each of these into different sizes of clusters. Finally, the possible decays of each type of cluster into final-state pions via different decay modes are again described statistically with the binomial (or, where needed, the multinomial) distribution.

In this investigation we consider the production of intermediate clusters which exchange only neutral or singly charged objects (i.e., particles or Reggeons) between them. Since we consider only isoscalar and isovector cluster production, each cluster is limited to being neutral or singly charged except for end clusters. The ends of the cluster chain are complicated by the need to consider the possible transfer of the incident particle's quantum numbers along the chain. We have circumvented this complication in our study by not considering charge-exchange processes so that each incident particle emerges undisturbed as a leading particle. However, a cluster may occur in conjunction with either incident particle and is termed an end cluster. Since we do not consider charge-exchange processes, an end cluster will decay into the incident particle plus pions and the charge of the incident particle must be added to that of the meson cluster. This is illustrated at the top end of the chain of clusters in Fig. 1(a), which shows an incident proton forming a doubly charged end cluster; the mesons in this cluster have a net positive charge.

The first stage of our statistical development gives the probability of producing k independent clusters, of which k_V are isovector and k_S are isoscalar:

$$P(k_V, k_S) = e^{-\lambda} \frac{\lambda^k}{k!} \binom{k}{k_V} V^{k_V} S^{k_S}, \tag{2.1}$$

with $k = k_V + k_S$ and $V + S = 1$. The admixture of isovector and isoscalar clusters must be determined from experiment in order to fix V and S. The average number of clusters produced λ is the sum of the average number of isovector clusters λ_V and isoscalar clusters λ_S:

$$\lambda = \lambda_V + \lambda_S, \tag{2.2}$$

where

$$\lambda_V = V\lambda \quad \text{and} \quad \lambda_S = S\lambda. \tag{2.3}$$

Equation (2.1) can thus be written as the product of two Poisson distributions in the form

$$e^{-\lambda_V} \frac{\lambda_V^{k_V}}{k_V!} \times e^{-\lambda_S} \frac{\lambda_S^{k_S}}{k_S!}. \tag{2.4}$$

The LCEX constraint which affects only isovector cluster production prevents certain sequences of charged clusters along the multiperipheral

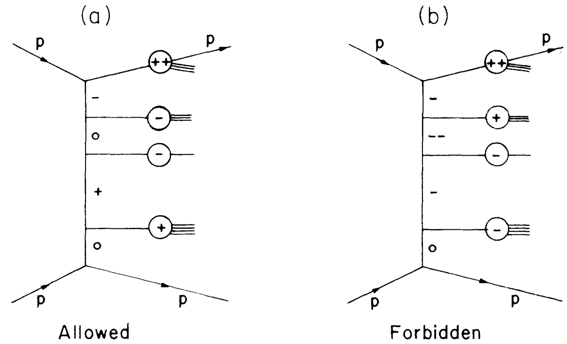

FIG. 1. Multiperipheral chains of clusters which are allowed and forbidden by the limited-charge-exchange constraint.

chain. Thus the combinatorial factor will be different from that of a multinomial in the number of positive, negative, and neutral clusters. In deriving the effects of LCEX, we first note that the suppression of $|\Delta Q| \geq 2$ exchanges requires all positive and negative clusters to be arranged in pairs along the chain.

When u pairs of charged clusters are produced, each pair can be arranged in two ways; thus the combinatorial factor will include a term 2^u. Also there are $(2u+1)$ "bins" between and at the ends of the $2u$ charged clusters in which neutral clusters can occur. Since there is no known constraint on the placement of these neutral clusters, there is no *a priori* reason for not assigning equal probability to each of the possible arrangements of a total of v neutral clusters into the possible "bins" and the number of distinguishable distributions is[8] the binomial coefficient

$$\binom{2u+v}{v}.$$

Thus, the overall probability that k_V clusters are independently produced and that these consist of u pairs of charged clusters and v neutral clusters is

$$P(u, v) = \frac{1}{N} \frac{\lambda_V^{k_V}}{k_V!} \binom{2u+v}{v} 2^u a^{2u} b^v, \tag{2.5}$$

where a and b are the relative probabilities that a cluster is charged or neutral so that

$$2a + b = 1 \tag{2.6}$$

and

$$2u + v = k_V. \tag{2.7}$$

The factor N is required to normalize the distribution function.

Equation (2.5) can be written as

$$P(u, v) = \left(\frac{1}{N} e^{-2a\lambda_V} \frac{(\sqrt{2}\, a\lambda_V)^{2u}}{(2u)!} \right) \left(e^{-b\lambda_V} \frac{(b\lambda_V)^v}{v!} \right). \tag{2.8}$$

N is now given by

$$N=e^{-2a\lambda_V}\sum_{u=0}^{\infty}\frac{(\sqrt{2}\,a\lambda_V)^{2u}}{(2u)!}=e^{-2a\lambda_V}\cosh(\sqrt{2}\,a\lambda_V) \tag{2.9}$$

and we have

$$P(u,v)=\frac{e^{-b\lambda_V}}{\cosh(\sqrt{2}\,a\lambda_V)}\,\frac{(\sqrt{2}\,a\lambda_V)^{2u}}{(2u)!}\,\frac{(b\lambda_V)^v}{v!}. \tag{2.10}$$

To determine a and b, we first note that by not considering charge-exchange reactions, the experimental[9] results $2\sigma_{\pi^0}=\sigma_{\pi^+}+\sigma_{\pi^-}$ reduces to equal production of negative and neutral pions. Further, it is easy to show that for a given cluster size, equal numbers of negative and neutral clusters are required to produce equal numbers of negative and neutral pions. We also assume that the size distributions of charged and neutral clusters are the same, thus assuring equal numbers of π^-'s and π^0's on average. From Eq. (2.8) we obtain for the average number of negative (or positive) and neutral clusters respectively

$$\langle u\rangle=\frac{a\lambda_V}{\sqrt{2}}\tanh(\sqrt{2}\,a\lambda_V)\,, \tag{2.11a}$$

$$\langle v\rangle=b\lambda_V\,. \tag{2.11b}$$

The equality of these two quantities together with Eq. (2.6) allows a to be found as the solution of the transcendental equation

$$1-2a=\frac{a}{\sqrt{2}}\tanh(\sqrt{2}\,a\lambda_V)\,. \tag{2.12}$$

If λ_V is large enough so that $\tanh(\sqrt{2}a\lambda_V)\approx 1$,

$$a\approx\frac{1}{2\sqrt{2}+1}\,, \tag{2.13a}$$

$$b\approx\frac{1}{2\sqrt{2}+1}\,. \tag{2.13b}$$

We note that

$$2\langle u\rangle+\langle v\rangle\approx\frac{3\lambda_V}{2\sqrt{2}+1}\,, \tag{2.14}$$

not λ_V as before the imposition of LCEX. By letting $\lambda_V\to\frac{1}{3}(2\sqrt{2}+1)\lambda_V$, we have $b\lambda_V\to\frac{1}{3}\lambda$ and $\sqrt{2}\,a\lambda_V\to\frac{2}{3}\lambda_V$. Equation (2.10) then becomes in this approximation

$$P(u,v)=\frac{e^{-\lambda_V/3}}{\cosh(\frac{2}{3}\lambda_V)}\,\frac{(\frac{2}{3}\lambda_V)^{2u}}{(2u)!}\,\frac{(\frac{1}{3}\lambda_V)^v}{v!}. \tag{2.15}$$

For $\lambda_V\gtrsim 3$ we have $\langle u\rangle\approx\langle v\rangle$ and $2\langle u\rangle+\langle v\rangle=\lambda_V$ to within a few percent. We can write the probability of producing u negative isovector clusters, v neutral isovector clusters, and k_S isoscalar clusters as

$$P(u,v,k_S)=\frac{e^{-\lambda_S}\lambda_S^{k_S}}{k_S!}\,\frac{e^{-\lambda_V/3}}{\cosh(\frac{2}{3}\lambda_V)}\,\frac{(\frac{2}{3}\lambda_V)^{2u}}{(2u)!}\,\frac{(\frac{1}{3}\lambda_V)^v}{v!}. \tag{2.16}$$

Each type of cluster that we have considered can, in general, decay into different numbers of pions (i.e., have different sizes). The number of positive and negative isovector clusters decaying into i pions is denoted by $u_i^{\pm}$ and the number of neutral isovector clusters of size i is v_i. Likewise, the number of i-particle isoscalar clusters is k_{Si} ($i=1$ is not permitted because a one particle isoscalar cluster cannot decay into only pions). The relative probability for production of each size of isovector cluster is d_i and that for isoscalar clusters is e_i. The distribution of the various sizes of clusters produced is described by a multinomial distribution. Thus, the probability for producing u_1^+ positive one-particle isovector clusters, etc., is

$$P(u_1^+,\ldots,u_4^+,u_1^-,\ldots,u_4^-,v_1,\ldots,v_4,k_{S2},\ldots,k_{S4})$$
$$=P(u,v,k_S)\,\frac{u!\,d_1^{u_1^+}d_2^{u_2^+}d_3^{u_3^+}d_4^{u_4^+}}{u_1^+!\,u_2^+!\,u_3^+!\,u_4^+!}\,\frac{u!\,d_1^{u_1^-}d_2^{u_2^-}d_3^{u_3^-}d_4^{u_4^-}}{u_1^-!\,u_2^-!\,u_3^-!\,u_4^-!}\,\frac{v!\,d_1^{v_1}d_2^{v_2}d_3^{v_3}d_4^{v_4}}{v_1!\,v_2!\,v_3!\,v_4!}\,\frac{k_S!\,e_2^{k_{S2}}e_3^{k_{S3}}e_4^{k_{S4}}}{k_{S2}!\,k_{S3}!\,k_{S4}!}\,, \tag{2.17}$$

where $P(u,v,k_S)$ is given by Eq. (2.16).

Finally, we describe the decay of each type and size of cluster into final-state pions. The possible decays of these clusters along with the relative probabilities of decay into each final state and the variable describing the number of events for each final state are given in Table I. When more than one final state is available for the decay of a cluster, the actual number of decays is predicted with a binomial distribution. We illustrate this procedure with a three-particle positive isovector cluster. From Table I, we see that the probability for producing n_{31} $\pi^+\pi^+\pi^-$ events and n_{32} $\pi^+\pi^0\pi^0$ events from u_3^+ clusters is

$$P(n_{31}\,|\,u_3^+)=\frac{u_3^+}{n_{31}!\,n_{32}!}\,p_3^{n_{31}}q_3^{n_{32}}$$
$$=\binom{u_3^+}{n_{31}}p_3^{n_{31}}(1-p_3)^{u_3^+-n_{31}}\,. \tag{2.18}$$

This decay proceeds through three possible intermediate isospin states: isotensor plus isovector,

TABLE I. Decay of clusters.

Cluster	Final state	No. of events	Relative probability
		Isoscalar clusters	
$\bar{C}_2^0$	$\pi^+\pi^-$	$\bar{n}_{11}$	$\frac{2}{3}$
	$\pi^0\pi^0$	$\bar{n}_{12}$	$\frac{1}{3}$
$\bar{C}_3^0$	$\pi^+\pi^-\pi^0$		1
$\bar{C}_4^0$	$\pi^+\pi^+\pi^-\pi^-$	$\bar{n}_{41}$	$\bar{r}_4$
	$\pi^+\pi^-\pi^0\pi^0$	$\bar{n}_{42}$	$\bar{s}_4$
	$\pi^0\pi^0\pi^0\pi^0$	$\bar{n}_{43}$	$\bar{t}_4$
		Isovector clusters	
C_1^+	π^+	u_1^+	1
C_1^0	π^0	v_1	1
C_1^-	π^-	u_1^-	1
C_2^+	$\pi^+\pi^0$	u_2^+	1
C_2^0	$\pi^+\pi^-$	v_2	1
C_2^-	$\pi^-\pi^0$	u_2^-	1
C_3^+	$\pi^+\pi^+\pi^-$	n_{31}	p_3
	$\pi^+\pi^0\pi^0$	n_{32}	q_3
C_3^0	$\pi^+\pi^-\pi^0$	n_{35}	r_3
	$\pi^0\pi^0\pi^0$	n_{36}	s_3
C_3^-	$\pi^+\pi^-\pi^-$	n_{33}	p_3
	$\pi^-\pi^0\pi^0$	n_{34}	q_3
C_4^+	$\pi^+\pi^+\pi^-\pi^0$	n_{41}	p_4
	$\pi^+\pi^0\pi^0\pi^0$	n_{42}	q_4
C_4^0	$\pi^+\pi^+\pi^-\pi^-$	n_{45}	r_4
	$\pi^+\pi^-\pi^0\pi^0$	n_{46}	s_4
C_4^-	$\pi^+\pi^-\pi^-\pi^0$	n_{43}	p_3
	$\pi^-\pi^0\pi^0\pi^0$	n_{44}	q_4

two isovectors, and isovector plus isoscalar. For each intermediate state, the values of p_3 and q_3 are combinations of Clebsch-Gordan coefficients. The respective values of these relative probabilities for each intermediate isospin state of the different clusters considered are listed in Table II. We note that for isovector clusters (as expected from the Wigner-Eckart theorem) we have $2p_i + r_i = 2$. Thus only one independent variable describes the number of final-state events for these clusters. Also from Table II we find limits on these relative probabilities:

$$\tfrac{1}{2} \le p_3 \le \tfrac{2}{3}\,,$$

$$\tfrac{3}{5} \le p_4 \le 1\,,$$

$$\tfrac{1}{3} \le \bar{r}_4 \le \tfrac{4}{9}\,,$$

$$\tfrac{4}{9} \le \bar{s}_4 \le \tfrac{2}{3}\,,$$

$$0 \le \bar{t}_4 \le \tfrac{1}{9}\,.$$

The various probability functions are now assembled to yield the distribution function which gives the probability for producing each possible final state:

TABLE II. Dependence of relative decay probabilities on intermediate isospin states.

4-particle isoscalar clusters

Relative probability \ Intermediate isospin states	2+2	1+1	0+0
$\bar{r}_4$	$\frac{19}{45}$	$\frac{1}{3}$	$\frac{4}{9}$
$\bar{s}_4$	$\frac{22}{45}$	$\frac{2}{3}$	$\frac{4}{9}$
$\bar{t}_4$	$\frac{4}{45}$	0	$\frac{1}{9}$

3-particle isovector clusters

Relative probability \ Intermediate isospin states	2+1	1+1	1+0
p_3	$\frac{19}{30}$	$\frac{1}{2}$	$\frac{2}{3}$
q_3	$\frac{11}{30}$	$\frac{1}{2}$	$\frac{1}{3}$
r_3	$\frac{11}{15}$	1	$\frac{2}{3}$
s_3	$\frac{4}{15}$	0	$\frac{1}{3}$

4-particle isovector clusters

Relative probability \ Intermediate isospin states	2+2	2+1 and 1+2	1+1	1+0 and 0+1
p_4	$\frac{3}{5}$	$\frac{14}{15}$	1	$\frac{2}{3}$
q_4	$\frac{2}{5}$	$\frac{1}{15}$	0	$\frac{1}{3}$
r_4	$\frac{4}{5}$	$\frac{2}{15}$	0	$\frac{2}{3}$
s_4	$\frac{1}{5}$	$\frac{13}{15}$	1	$\frac{1}{3}$

$$P(u_1^+,\ldots,u_4^+,u_1^-,\ldots,u_4^-,v_1,\ldots,v_4,k_{S2},\ldots,k_{S4},n_{31},n_{33},n_{35},n_{41},n_{43},n_{45},\bar{n}_{21},\bar{n}_{41},\bar{n}_{42})$$
$$=P(u_1^+,\ldots,k_{S4})P(n_{31}|u_3^+)P(n_{33}|u_3^-)P(n_{35}|v_3)P(n_{41}|u_4^+)P(n_{43}|u_4^-)P(n_{45}|v_4)P(\bar{n}_{21}|k_{S2})P(\bar{n}_{41},\bar{n}_{42}|k_{S4})\,. \quad (2.19)$$

The functions $P(u_1^+,\ldots,k_{S4})$ and $P(n_{31}|u_3^+)$ are given in Eqs. (2.17) and (2.18), respectively.

The goal of this development is, of course, the distribution function giving the probability for producing n negative pions and m neutral pions. This function is obtained by summing Eq. (2.19) over all values of final-state events subject to the constraints that the proper numbers of negative and neutral pions are produced. By considering the decays listed in Table I, we see that these numbers are given by

$$n=u+v_2+v_4+n_{31}+n_{33}+n_{35}+n_{41}+n_{43}+n_{45}+\bar{n}_{21}+2\bar{n}_{41}+\bar{n}_{42}\,, \quad (2.20a)$$

$$m=u_2^++u_2^-+2(u_3^++u_3^-)+3(u_4^++u_4^-)+v_1+3v_3+2v_4-2(n_{31}+n_{33}+n_{35}+n_{41}+n_{43}+n_{45})+2k_{S2}+k_{S3}+4k_{S4}-2\bar{n}_{21}-4\bar{n}_{41}-2\bar{n}_{42}\,. \quad (2.20b)$$

Thus, the probability for producing n negative and m neutral pions is

$$P(n,m)=\sum_{\text{final states}} P(u_1^+,\ldots,\bar{n}_{42})\Delta\,, \quad (2.21)$$

where Δ is a product of Kronecker δ's:

$$\Delta=\delta_{n,\,u+v_2+\cdots+2\bar{n}_{41}+\bar{n}_{42}}\,\delta_{m,\,u_2^++u_2^-+\cdots-4\bar{n}_{41}-2\bar{n}_{42}}\times\delta_{u,\,\Sigma u_i^+}\delta_{u,\,\Sigma u_i^-}\delta_{v,\,\Sigma v_i}\delta_{k_S,\,k_{S2}+k_{S3}+k_{S4}}\,. \quad (2.22)$$

From the distribution function (2.19) and the relations (2.20a) and (2.20b), we have derived analytic expressions for the average numbers of negative and neutral pions produced and the two-particle correlation coefficients. These general expressions are listed in Table III and will be used for data comparisons in the next section.

TABLE III. Moments of multiplicity distributions in the limited-charge-exchange model.

$$\langle n\rangle = [(1+2d_3p_3+2d_4p_4)\tanh(\tfrac{2}{3}\lambda_V)+d_2+d_3r_3+d_4(1+r_4)]\tfrac{1}{3}\lambda_V+(2e_2+3e_3+4e_4)\tfrac{1}{3}\lambda_S\ ,$$

$$\langle m\rangle = [(2d_2+4d_3(1-p_3)+2d_4(3-2p_4)\tanh(\tfrac{2}{3}\lambda_V)+d_1+d_3(3-2r_3)+2d_4(1-r_4)]\tfrac{1}{3}\lambda_V+(2e_2+3e_3+4e_4)\tfrac{1}{3}\lambda_S\ ,$$

$$f_2^{--} = [1+2(d_3p_3+d_4p_4)]^2[1-\tanh^2(\tfrac{2}{3}\lambda_V)]\tfrac{1}{9}\lambda_V^2+\{[4(d_3p_3+d_4p_4)-1]\tanh(\tfrac{2}{3}\lambda_V)+4d_4r_4\}\tfrac{1}{6}\lambda_V+2e_4\bar{r}_4\lambda_S$$

$$f_2^{00} = [d_2+2d_3(1-p_3)+d_4(3-2p_4)]^2[1-\tanh^2(\tfrac{2}{3}\lambda_V)]\tfrac{4}{9}\lambda_V^2$$
$$+\{[4d_3(1-p_3)+12d_4(1-p_4)]\tanh(\tfrac{2}{3}\lambda_V)+6d_3(1-r_3)+2d_4(1-r_4)\}\tfrac{1}{3}\lambda_V+[2e_2+6e_4(\bar{s}_4+6\bar{t}_4)]\tfrac{1}{3}\lambda_S\ ,$$

$$f_2^{0-} = \{(1+2d_3p_3+2d_4p_4)[d_2+d_3(1-p_3)+d_4(3-2p_4)]\}[1-\tanh^2(\tfrac{2}{3}\lambda_V)]\tfrac{2}{9}\lambda_V^2$$
$$+\{[d_2+2d_3(1-p_3)+3d_4]\tanh(\tfrac{2}{3}\lambda_V)+d_3r_3+2d_4)\}\tfrac{1}{3}\lambda_V+(e_3+2e_4\bar{s}_4)\lambda_S\ .$$

III. COMPARISON TO DATA

The limited-charge-exchange model developed in Sec. II is quite general and has a large number of parameters, each of which may have an energy dependence. In this section, we consider various submodels which are defined by assuming a particular energy dependence for certain parameters. We then make crude comparisons to data in the sense that we do not adjust parameters to achieve cosmetic fits but instead consider the general trends of the data. We observe in Sec. III A that experimental data on charge transferred between clusters suggests a strong suppression of isoscalar relative to isovector cluster production. Thus, we look only at isovector clusters for the remainder of Sec. III. In Sec. III B, we consider the model which has a fixed average cluster size (i.e., the d_i are assumed constant) so that results of varying the average number of clusters are developed. The reverse situation is in Sec. III C, where we report results of an energy-dependent cluster size and a constant average number of clusters. The more general (and probably more realistic) case where both cluster size and average number of clusters have an energy dependence is discussed in Sec. III D.

A. Isospin of clusters

The general expression for producing n negative pions and m neutral pions [Eq. (2.21)] contains contributions from both isoscalar and isovector clusters. In order to determine the relative production of each type of cluster, we have investigated the behavior of a parameter introduced by Pirilä, Thomas, and Quigg[5] in their work with LCEX models. They identify the ratio R of the number of charged exchanges or links between clusters to the number of neutral exchanges as being an important parameter for exchange models. To calculate this ratio in our model, we note that intra-charged-pair exchanges are charged while inter-charged-pair exchanges are neutral [see Fig. 1(a)]. Also each neutral cluster that is added to a link causes another link of the same charge. Thus the number of charged links is equal to the number of pairs of charged clusters plus the number of neutral clusters located within a pair of charged ones. Since the probability that a neutral cluster falls inside a charged pair is $u/(2u+1)$, the average number of such neutral clusters produced is $(v+k_S)u/(2u+1)$ and the average number of charged exchanges is

$$l_{\rm ch} = u + \frac{u(v+k_S)}{2u+1}\ . \tag{3.1}$$

The total number of exchanges[10] is $2u+v+k_S-1$, therefore the number of neutral exchanges is

$$l_{\rm neutral} = u + (v+k_S)\left(1-\frac{u}{2u+1}\right) - 1. \tag{3.2}$$

The average value of the ratio $R = l_{\rm ch}/l_{\rm neutral}$ was calculated as a function of the average number of clusters produced, λ, and the ratio of the average number of isoscalar clusters to isovector clusters, λ_S/λ_V. The results which are plotted in Fig. 2 show that for a small number of clusters (i.e.,

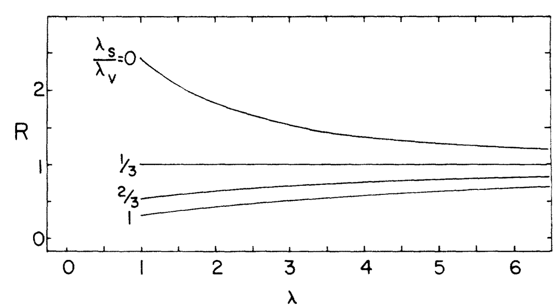

FIG. 2. The ratio of charged to neutral exchanges between clusters as a function of the average number of clusters produced.

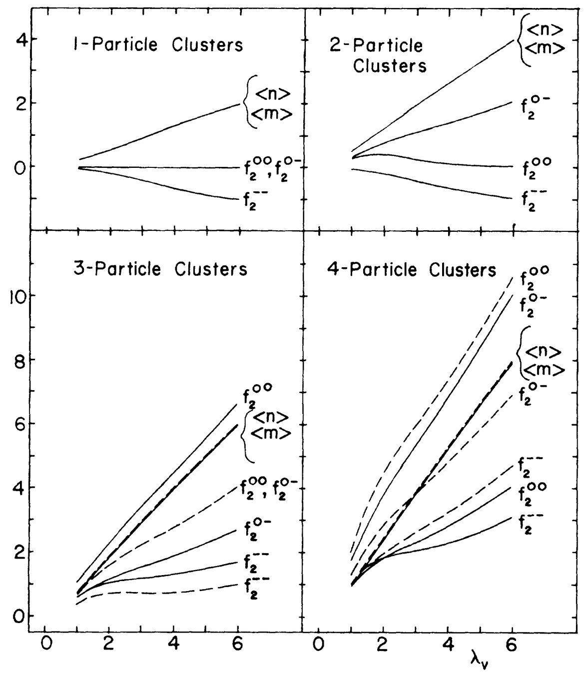

FIG. 3. Topological cross-section moments for the fixed-cluster-size LCEX model. The solid lines are for isovector cluster production with $p_3 = 0.67$ or $p_4 = 1$. The dashed lines are for isovector cluster production with $p_3 = 0.5$ or $p_4 = 0.6$.

$\lambda \lesssim 3$), R is quite sensitive to both the total number of clusters produced and to their isospin admixture. Pirilä *et al.*[5] observed that the 205-GeV/c p-p data suggest approximately twice as many charged exchanges occur as neutral ones; that is $R \simeq 2$. Therefore unless the total number of clusters produced is very small ($\lambda \lesssim 1.5$), the ratio of isoscalar to isovector cluster production must be approximately zero. Such a small number of clusters can only occur if they become unreasonably large. For example, the production of an average number of 1.5 clusters in a 405-GeV/c p-p interaction would require an average cluster size of about 7 particles. Thus, we conclude that isoscalar cluster production is strongly suppressed and for the rest of our data comparisons, we assume it is zero.

B. Fixed cluster size

In order to fix the cluster size and type, we take the general expression for the probability of producing n negative pions and m neutral pions [Eq. (2.21)] and set the probability for producing each size and type of cluster equal to 1 for the desired clusters and equal to zero for all others. Since we are assuming that no isoscalar clusters are produced, we set $\lambda_S = 0$. Then studying clusters decaying into three pions, for example, we set $d_3 = 1$ and all other d_i equal to zero. The resulting expressions for the average numbers of negative and neutral pions produced and the two-particle integrated correlation coefficients for each cluster size considered are plotted as a function of λ_V in Fig. 3.

Looking first at the average number of negative particles produced, we note that for large values of λ_V [i.e., for λ_V such that $\tanh(\sqrt{2}a\lambda_V) \simeq 1$], $\langle n \rangle$ depends linearly on λ_V. Thus by giving λ_V a $\ln s$ dependence, we can match the apparent experimental energy dependence of $\langle n \rangle$. However, the two-particle integrated correlation coefficients also depend linearly on λ_V (again, for large λ_V). Thus we are led to a contradiction with experiment since a $\ln^2 s$ term is apparently needed[9, 11] to explain the experimental f_2^{--} data. The necessity of a $\ln^2 s$ term is clearly demonstrated in Fig. 4, where Whitmore's empirical expression,[9]

$$f_2^{--} = (2.47 \pm 0.19) - (1.51 \pm 0.09)\ln s + (0.21 \pm 0.01)\ln^2 s, \qquad (3.3)$$

obviously fits the data better than an expression containing only a $\ln s$ term would.

Although we have only investigated clusters that decay into fewer than five particles, this difficulty is quite general with a fixed-cluster-size (FCS) LCEX model. There does not appear to be any mechanism in this scheme to generate terms quadratic in λ_V for large values of λ_V. To see why this occurs, we refer to Table III. It shows that while the expression for f_2^{--} contains a term qua-

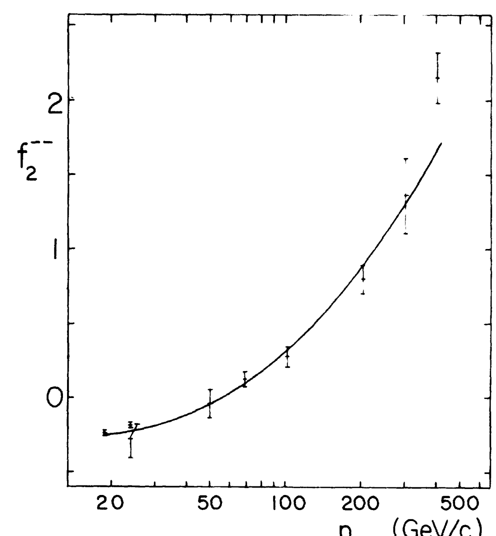

FIG. 4. The experimental momentum dependence of f_2^{--}. The line is Whitmore's (Ref. 9) fit to the experimental data [see Eq. (3.3)].

dratic in λ_V, there is the factor $[1-\tanh(\frac{2}{3}\lambda_V)]^2$ which goes to zero at large values of λ_V. The reason for this factor is that in the calculation of $f_2^{--}=\langle n^2\rangle-\langle n\rangle^2-\langle n\rangle$, $\langle n^2\rangle$ contributes the $\lambda_V{}^2$ term while $\langle n\rangle$ has a $\lambda_V\tanh(\frac{2}{3}\lambda_V)$ factor so that at high values of λ_V, the cancellation occurs between the $\langle n^2\rangle$ and $\langle n\rangle^2$ terms. It is easy to show that this is a general feature and will occur for all larger sized clusters.

In addition to the above difficulty, we have also found that this FCS LCEX model contradicts the apparent Koba-Nielsen-Olesen (KNO) scaling[12] of topological cross-section data[13] that has been observed for p-p interactions over the energy range of 50–405 GeV/c incident momenta. This behavior of the experimental data requires a phenomenological model to at least mimic KNO scaling for this energy range even though the theoretical basis of KNO scaling is controversial. We have checked on this requirement for the FCS LCEX model by calculating the topological cross-section predictions for one-, two-, three-, and four-particle isovector clusters at different energies. Each energy was determined by fixing the average number of clusters produced so that the resulting value of $\langle n\rangle$ would correspond to the experimental values for the various incident momenta. A typical prediction is the case of three-particle clusters at 102 and 405 GeV/c which is plotted in Fig. 5 using the KNO variables $\psi=\langle n\rangle\sigma_n/\sigma_{\text{inel}}$ and $x=n/\langle n\rangle$. Since KNO scaling predicts that topological cross-section data displayed in this fashion are energy independent, the predictions at the two energies should lie on top of one another. The strong shift between them clearly demonstrates the lack of KNO scaling over this energy range.

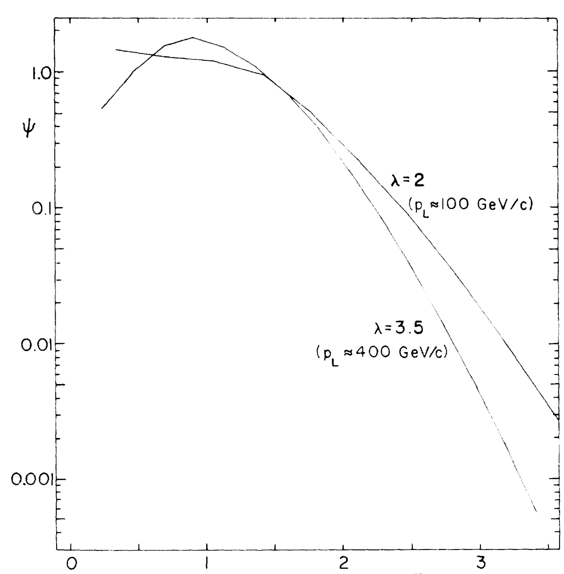

FIG. 5. A KNO plot of topological cross-section predictions of the fixed-cluster-size LCEX model for three-particle isovector clusters. The curves are straight lines connecting the points for the various n which were calculated with $p_3=0.67$.

Finally, we consider a more general model in which only the average cluster size is constant. To achieve this, we choose energy-independent values for the d_i; thus, more than one size of cluster is produced but the relative amounts of each size are constant. This generalization fails, however, because the failure of the FCS LCEX model detailed above does not depend on cluster size. Therefore any energy-independent combination of different-sized clusters will also fail.

C. Fixed average number of clusters

The second way suggested by Levy[7] to account for the energy dependence of particle production is to have a varying cluster size. To investigate this situation, we have set λ_V equal to a constant and $\lambda_S=0$ so that we are looking at an isovector cluster model having a fixed average number of clusters (FANC) with limited charge exchange.

The increase of $\langle n\rangle$ with energy must now be achieved by increasing the cluster size. To illustrate this procedure, we refer to Fig. 3 and note that if $\lambda_V=3$ then $\langle n\rangle$ is approximately equal to the size of the cluster. This particularly simple case occurs since λ_V is large enough so that $\tanh(\frac{2}{3}\lambda_V)\simeq 1$. Under this condition

$$\langle n\rangle=(d_1+2d_2+3d_3+4d_4)\tfrac{1}{3}\lambda_V, \qquad (3.4)$$

and we see that small values of $\langle n\rangle$ occur when d_1 dominates over the other d_i. Then as the incident energy increases, first d_2 and then d_3 and d_4 dominate over the other d_i, yielding an increase in $\langle n\rangle$.

Since $\langle n\rangle$ has a logarithmic energy increase, the dominance of the d_i must occur at approximately equal ratios of the energy. As a simple example, if $d_1=1$ at $s=s_1$ (all other $d_i=0$) and $d_2=1$ at $s=s_2=\alpha s_1$, then the energy at which $d_3=1$ is given by $s_3=\alpha s_2$. Noting that experimentally[14,15]

$$\langle n\rangle=1.03\ln s-3.23 \qquad (3.5)$$

for s in GeV^2, we have

$$\alpha=e^{\lambda_V/3.09}\,. \qquad (3.6)$$

In order to learn something about the average number of clusters produced, we compare the predictions of the model with Slattery's KNO curve[13] representing the experimental data. By referring to Fig. 3, we find that the experimental value of $\langle n\rangle\simeq 3.5$ at 405 GeV/c requires $\lambda\simeq 10.5$, 5.25, 3.5, or 2.6 for the case where only one-, two-, three-, or four-particle clusters are produced. Each of these cases is plotted in Fig. 6. Although the conclusion is not decisive, we take the three-particle clusters as forming the best fit to the data and thus arrive at a value of $\lambda_V \simeq 3.5$. It follows from (3.6) that $\alpha \simeq 3.10$ and the values of the s_i where the respective d_i equal unity[16] are given in Table IV. The table also contains calculated values of the two-particle correlation coefficient at these energies, assuming that the respective d_i equals unity. For this crude model, we are encouraged by the comparison of this correlation coefficient to the experimental values listed for nearby energies. We show in Fig. 7 the model's predictions for the average number of neutral pions produced for a given number of negative pions $\langle m \rangle_n$ and experimental values[17] with approximately the same energy. We note that the general trends of each are comparable.

To gain further insight into this model, we have investigated an oversimplified choice for the energy dependence of the d_i in which no more than two different sizes of clusters are produced at one time. The specific energy dependence of the d_i which is shown in Fig. 8 gives complete dominance of the production of each size cluster at the energies listed in Table IV and causes each type to disappear at the energy at which the next one dominates. This particular choice of the d_i gives a logarithmic energy dependence to $\langle n \rangle$ under the conditions leading to Eq. (3.4) and allows the calculation of the energy dependence of the two-particle correlation coefficients. These are compared to experimental values[18] in Fig. 9. Again, we note that the trends of the data and predictions are comparable.

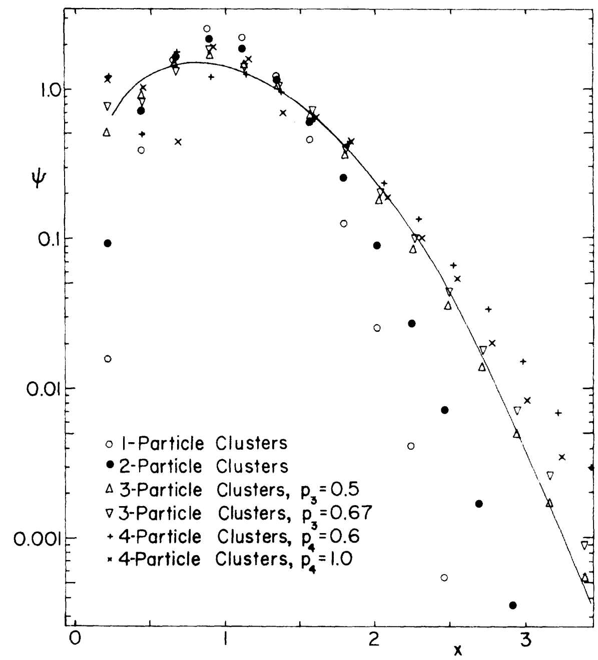

FIG. 6. Comparison of predictions for different sizes of clusters with experimental data. The line is Slattery's empirical fit (Ref. 12) to experimental data.

D. Variation of both number and size of clusters

We have shown that the FCS LCEX model fails on comparison with experiment in two ways, whereas these data can be explained by the FANC LCEX model. However, there are suggestions within the model that the more general case of a variable number of clusters with a variable size is required. Although this case may prove to be the one closest to describing nature, the increase in the number of parameters causes difficulty in determining them uniquely. Consequently, in this section, we look at two reasons for considering this case, but postpone a full study of it to a future report.

We look first at the probability for producing no negative or neutral pions, which is $P(0,0)$ in Eq. (2.21). Using Eqs. (2.17) and (2.16) with $\lambda_S = 0$, we find

$$P(0,0) = \frac{e^{-\lambda_V/3}}{\cosh(\frac{2}{3}\lambda_V)} \simeq 0.06 \qquad (3.7)$$

TABLE IV. Effects of the dominance of different sizes of isovector clusters at different energies.

No. of pions in cluster	Value of s where cluster dominates (GeV^2)	Predicted value of f_2^{--}	Exp. value of f_2^{--}	Value of s at which exp. was performed (GeV^2)
1	79.2	−0.52	−0.04 ±0.09	95.8
2	245.9	−0.52	0.28 ±0.07	193.5
3	763.2	0.77−1.23	2.15∓0.17	763.2
4	2368.9	2.16−2.91	⋯	⋯

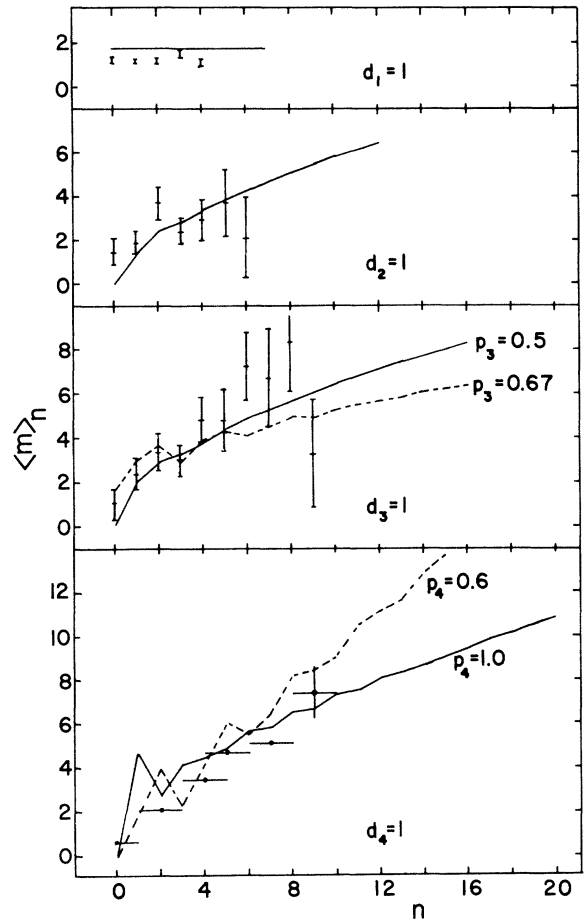

FIG. 7. The average number of neutral particles produced for a given number of negative pions as a function of the number of negative pions for various sizes of isovector clusters. The experimental data are taken from Ref. 17.

for $\lambda_V = 3.5$. In our model, this is the probability for an inelastic process such as the charge-exchange reaction $p + p \rightarrow \Delta^{++} + n$. Dao *et al.*[19] have measured the inclusive Δ^{++} production cross section at 303 GeV/c and have found $\sigma(p+p \rightarrow \Delta^{++} + \text{anything})/\sigma_{\text{inel}} \simeq 0.14$. Since $P(0,0)$ is the probability for exclusive Δ^{++} production, there is no serious disagreement with this experimental value. However, Regge theory says that this reaction requires Reggeon exchange; thus, asymtotically, the cross section should fall as a power of s. In the FANC LCEX model, $P(0,0)$ = const. Therefore, λ_V must grow logarithmically with s in order to obtain agreement with this prediction of Regge theory. This model already contained a variation in the cluster size, so we now have both the size and number of clusters growing with energy.

A second indication of the need for this more general case comes from a closer look at the apparent KNO scaling of topological cross sections in the 50–405 GeV/c p-p data. We have used three-particle clusters in Fig. 5 to illustrate the general tendency for the KNO plots of cluster-produced pions to become narrower as the average number of clusters (λ) increases. Figure 10, on the other hand, shows the effects of constant λ_V and varying cluster size on the KNO plot. We point out that an increasing cluster size causes a slight broadening of the plot. Thus, increasing both the size and number of clusters produces

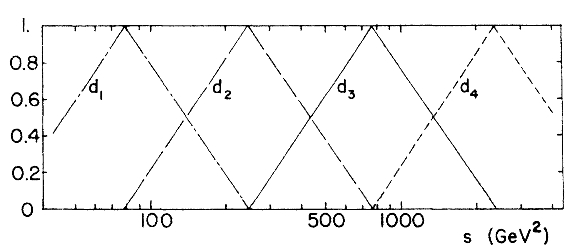

FIG. 8. The functional dependence on s of the probability for producing each size of isovector cluster which is used in Sec. III C.

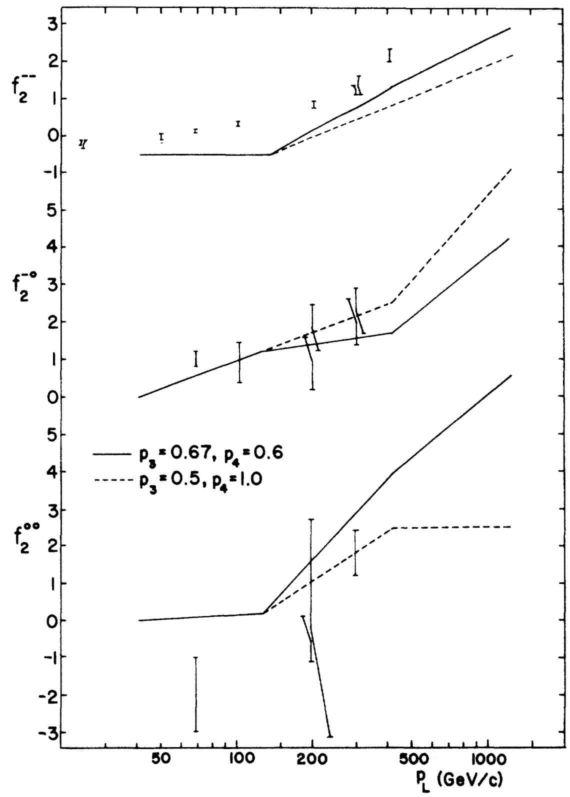

FIG. 9. The dependence of the integrated two-particle correlation coefficients on energy for the model of Sec. III C. The experimental data are from Ref. 18.

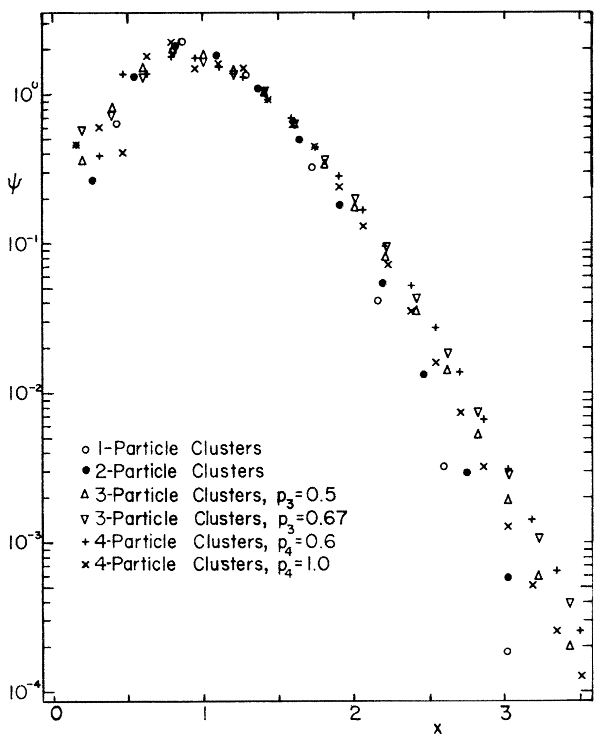

FIG. 10. A KNO plot of topological cross-section predictions showing the small deviation from scaling of the model with the average number of clusters fixed at $\lambda_V = 4$.

effects which can cancel and thereby satisfy to a greater degree the apparent scaling.

These two indications for a more general model show the need for determining the energy dependence of λ. We point out that such dependence can be resolved experimentally by investigating the energy dependence of the ratio R discussed in Sec. III A. For a constant value of λ_S/λ_V—which is not unreasonable—there is a unique relationship between R and λ. Further, the conclusion of Sec. III A that isoscalar production is suppressed serves to enhance the sensitivity of R on λ.

IV. CONCLUSIONS

From this investigation, it is easily seen that many experimental parameters are needed to describe completely a LCEX cluster model. The accurate determination of $\langle n \rangle$ tells very little about either the cluster production process or subsequent cluster decays. For example, for the sizes of clusters studied, $\langle n \rangle$ is essentially independent of the isospin characteristics of produced clusters. The two-particle correlation coefficients, however, depend much more sensitively on both the cluster production and decay parameters. Even though present experimental data do not provide enough information to evaluate uniquely all the parameters of the model, the apparent experimental $\ln^2 s$ dependence of f_2^{--} does preclude a fixed-cluster-size model such as the one considered in Sec. III B. The parameter R introduced by Pirilä *et al.*[5] is found to be extremely important since it is independent of cluster size and subsequent cluster decay and affords a direct means of determining both the average number and isoscalar-isovector admixture of clusters produced.

In a limited-charge-exchange model such as we have developed, long-range order enters in two ways. The first is via charge conservation and transfer of charge between clusters. This long-range order disappears when large numbers of clusters are produced. The second mechanism for introducing long-range order is through an energy-dependent cluster size. This mechanism can be seen as resulting from a need for information on the total energy to be passed to each cluster along the multiperipheral chain. The FCS LCEX model of Sec. III B has only the first type of long-range order while the FANC LCEX model of Sec. III C and the more general model of Sec. III D have both types. The failure of the FCS LCEX model and the agreement of the FANC LCEX model upon comparison to experiment show the need for a variation of cluster size with energy.

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