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Applications of a fixed-sphere bag model*

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We extend previous applications of the fixed-sphere MIT bag model to analyses of the nucleon axial radius, spectroscopy of the new heavy hadrons, and weak nonleptonic decays. The nucleon axial-charge radius is calculated to be nearly equal to the electromagnetic-charge radius. Using the ground state of charmonium as input, we predict the masses and excitation energies of new particles. Comparison of the model with observed states reveals that the model has insufficient vector-pseudoscalar mass splitting and inadequate excitation energies. Finally, both $\Delta S = 1$ and $\Delta C = 1$ nonleptonic decays of mesons and baryons are studied using a weak Hamiltonian enhanced by short-distance strong-interaction effects. Variation of matrix elements with the hadronic radius is investigated. The model is found to be unable to reproduce hyperon and kaon decay amplitudes.

I. INTRODUCTION

The MIT bag model[1] is a description of hadrons which manifestly allows for the confinement of conjectured hadronic constituents, such as quarks and gluons. Appropriate boundary conditions serve to ensure their confinement, and the internal dynamics of the hadron is governed by local interactions of the constituent quantum fields. At the present level of sophistication, a phenomenological parameter, B, is present in the energy-momentum tensor in order to provide stability for the hadron bag. We are not aware of any realistic attempt at this time to remove the need for this parameter. The bag model is relativistically invariant. However, applications have thus far tended not to utilize the relativistic invariance in any essential manner.

Subsequent to the introduction of the MIT bag model, there has been a good deal of effort to explore its phenomenological content. Various studies[2-4] have been carried out of the static properties of ground-state baryons in a fixed-sphere model of noninteracting quarks. Using the same theoretical framework, Donoghue *et al.*[5] have performed a more ambitious calculation, considering radiative decays of various excited states as well as hyperon nonleptonic decay amplitudes. The latter calculation is the first to consider matrix elements of four-quark operators in the bag model. More recently, a sophisticated approach in which quark-quark interactions and zero-point fluctuations are taken into account was used to provide a detailed fit to the masses of the ground-state baryons and mesons.[6]

These fixed-sphere phenomenological calculations have for the most part yielded satisfactory results. Matrix elements of two-quark operators, such as the magnetic moment, charge radius, etc., typically differ from the corresponding experimental values by no more than about 30%. Moreover, as shown in Ref. 6, the observed mass spectra of ground-state baryons and mesons can be fitted to an impressive level of accuracy, especially in comparison with alternative models of hadron dynamics. These successes not only lend credence to the bag-model approach to hadronic phenomenology, but also reinforce the hypothesis that ground-state baryons and mesons behave as three-quark and quark-antiquark composites. That is, at least at this level, there is little motivation for a more complicated wave function.

However, there are problems. The likelihood of finally being able to generate a sufficiently light pion, and indeed to meaningfully study chiral symmetry, remains an open question. Of more concern here are the difficulties, first detected in Ref. 5, associated with hyperon nonleptonic decay amplitudes and, separately, with excited states of the hadrons. The latter problem is most severe for the nucleon excited state $N^*(\frac{3}{2}^-; 1520)$, where the bag model predicts an insufficient excitation energy and a very poor $E1$ transition amplitude in radiative decay.[5] It is clearly important to test the performance of the bag model with other hadronic configurations for which sufficient information regarding excited states exists. This provides a key motivation for our bag-model study of charmonium (see Sec. II). In Ref. 5, bag-model calculations of S-wave hyperon nonleptonic decay amplitudes gave results too small by factors of 3 to 5. It was evident that any one of several possibilities could be responsible for this, e.g., our bag-model approach, the renormalization-group-enhanced nonleptonic weak Hamiltonian, etc. We have made some progress in further understanding the nature of this result, especially with respect to the role played by the fixed-sphere bag wave functions. This matter is discussed in Sec. III.

In order to introduce some notation and also to present a rather typical example of determining the static properties of hadrons in the bag model, we shall calculate the squared radius, $\langle r^2\rangle_{ax}$, associated with the distribution of axial charge within a nucleon. There exists some experimental data regarding this quantity, and it does not appear to have been calculated yet using the bag approach. We use the definition

$$\langle r^2\rangle_{ax}=\left\langle p(s_z)\right|\int d^3x\,\vec{x}^2\psi^+(\vec{x})\tau_3\sigma_z\psi(\vec{x})\left|p(s_z)\right\rangle, \quad (1)$$

where the integration takes place over the bag volume, p represents a proton, and $\psi(x)$ is a quark Heisenberg field operator summed over color, flavor, helicity, and all frequency modes accessible to the quarks within the proton bag. Only nonstrange quarks contribute to Eq. (1). Evaluation is straightforward, and we find

$$\langle r^2\rangle_{ax}=\frac{10}{27}\frac{R^2}{\omega^2-m^2R^2}\frac{C(\omega,mR)}{2\omega^2-2\omega+mR}, \quad (2)$$

where R is the radius of the spherical bag, ω is the lowest frequency mode in which a quark can occur, m is the mass of the nonstrange quarks, and $C(\omega,mR)$ is the quartic polynomial

$$\begin{aligned}C(\omega,mR)&=\omega^4+2\omega^3(1+mR)-\omega^2(4+\tfrac{1}{2}mR+m^2R^2)\\&\quad+\omega(3-mR-2m^2R^2-2m^3R^3)\\&\quad+\tfrac{1}{2}mR(-\tfrac{3}{2}+mR+m^2R^2). \quad (3)\end{aligned}$$

This expression simplifies in the limit of massless nonstrange quarks to

$$\langle r^2\rangle_{ax}=\frac{5R^2}{27}\frac{\omega^3+2\omega^2-4\omega+3}{\omega^2(\omega-1)}. \quad (4)$$

We have evaluated these formulas numerically, taking parameters first from the calculation of Ref. 5 ($\omega=2.206$, $mR=0.3234$, $R^{-1}=136.4$ MeV). Eq. (1) gives $\langle r^2\rangle_{ax}=1.01$ fm^2, which is practically equal to the charge-radius squared, $\langle r^2\rangle_{em}$, found in Ref. 5. One might anticipate a rather different result upon employing the parameters associated with the $m=0$ calculation of Ref. 6 ($\omega=2.043$, $R^{-1}=200$ MeV), since there one finds a rather different squared charge radius, $\langle r^2\rangle_{em}=0.517$ fm^2. We obtain $\langle r^2\rangle_{ax}=0.512$ fm^2. Evidently, for any reasonable fixed-sphere bag model, we have

$$\langle r^2\rangle_{ax}^{bag}\cong\langle r^2\rangle_{em}^{bag}. \quad (5)$$

That this result is not an obvious or trivial consequence of the quark model can be understood by passing to the nonrelativistic limit, $m\to\infty$. This kinematic situation was first studied in Ref. 3 in order to show that the usual nonrelativistic-quark-model results could be regained by the bag model in the appropriate limit. One of the results was

$$\langle r^2\rangle_{em}\to\frac{(2\pi^2-3)R^2}{6\pi^2}\text{ as }m\to\infty. \quad (6a)$$

The nonrelativistic limit of Eqs. (2) and (3) is easily seen to be

$$\langle r^2\rangle_{ax}\to\frac{5(2\pi^2-3)R^2}{18\pi^2}\text{ as }m\to\infty, \quad (6b)$$

or more precisely,

$$\langle r^2\rangle_{ax}\to\tfrac{5}{3}\langle r^2\rangle_{em}\text{ as }m\to\infty. \quad (7)$$

Thus, not only are the zero-momentum transfer values of the proton's axial-vector and charge form factors related by a factor of $\frac{5}{3}$ in the nonrelativistic limit, so are their first derivatives. This is to be expected if indeed the entire nonrelativistic axial-vector and charge form factors have a ratio of $\frac{5}{3}$.

How does our bag-model result, Eq. (5), compare with experiment? There have been several attempts to measure $\langle r^2\rangle_{ax}$ over the past decade. In principle, the most straightforward approach is to obtain the momentum-transfer dependence of the axial-vector form factor $F_A(q^2)$ in quasi-elastic neutrino scattering, where

$$\langle p(k)\,|A_+^\mu\,|n(q)\rangle=\bar{u}(k)\gamma^5[F_A\gamma^\mu+F_T\sigma^{\mu\nu}q_\nu+F_pk^\mu]u(q), \quad (8)$$

and to use $\langle r^2\rangle_{ax}=-6F_A'(0)$. A typical parametrization for such measurements is in terms of a pole fit, $F_A(t)\propto(M_A^2-t)^{-n}$ with $n=1,2$. The numerical relation between M_A and $\langle r^2\rangle_{ax}$ is

$$M_A=m_\pi\left(\frac{12nF_A(0)}{\langle r^2\rangle_{ax}}\right)^{1/2}, \quad (9)$$

where $\langle r^2\rangle_{ax}$ is expressed in Fermis. Upon taking $F_A(0)=1.25$ and using the experimental value for $\langle r^2\rangle_{em}=0.66$ fm^2 along with the bag-model result $\langle r^2\rangle_{ax}\cong\langle r^2\rangle_{em}$, we find

$$\begin{aligned}M_A&=667\text{ MeV if }n=1,\\M_A&=944\text{ MeV if }n=2.\end{aligned} \quad (10)$$

Derrick[7] quotes a best fit of $M_A=0.57\pm0.1$ GeV($n=1$) and $M_A=0.95\pm0.12$ GeV($n=2$) to differential and total cross sections for the reaction $\nu_\mu n\to\mu^-p$. The values are consistent, particularly for the dipole form factor ($n=2$), with the bag-model predictions of Eq. (10).

We conclude this section by summarizing the remaining contents of the paper. In Sec. II, we shall apply the bag formalism to a study of the vector mesons $\psi(3095)$, $\psi'(3684)$ recently observed at several laboratories. This is followed in Sec.

III by a discussion of weak decays of baryons and mesons, especially in relation to what the bag model can tell us. Our concluding remarks are presented in Sec. IV.

II. CHARMED-QUARK SPECTROSCOPY

We shall assume for the calculations performed in this section that the 2.8,..., 4.2 GeV resonances discovered in the past year at Brookhaven,[8] SLAC,[9] and DESY[10] consist of a quark-antiquark pair and that each member of the pair carries charm.[11,12] From this, we can determine the mass, m_c, of a charmed quark phenomenologically, and then calculate the spectrum of hadrons associated with such pairs. Of special interest are the bag-model predictions for masses of the excited states corresponding to a quark or antiquark occupying an excited frequency mode, and for mass splittings produced by a spin-spin interaction between the quarks. The latter effect implies that the mass of a hadron with aligned quark spins (an "ortho" state, $J=1$) will differ from the configuration in which the quarks occupy the same frequency modes, but are anti-aligned (a "para" state, $J=0$).

In order to obtain an estimate of the charmed-quark mass we shall use as input the following experimental values of the charmonium ground state: $M=3095$ for $J=1$, and $M=2800$ for $J=0$. Although the former value is known precisely,[9] the latter has been extracted from a limited number of events and should be regarded as tentative.[10] It is convenient to remove the effect of the "ortho-para" mass splitting from our fit to the charmed-quark mass. Since the quark spin-spin interaction must have the form $\vec{\sigma}(1)\cdot\vec{\sigma}(2)$, it follows that the appropriate combination of charmonium masses to consider is $\bar{M}=\frac{1}{4}[3M(J=1)+M(J=0)]=3025$ MeV.

There is no need to reproduce here the fundamental bag-model equations, which by now have appeared in several publications. However, an outline of the steps necessary to determine m_c is given to assist the reader. For each quark mode, one has a linear boundary condition which relates frequency ω to the dimensionless quantity m_cR. A single nonlinear boundary condition relates BR^4 to a known function of ω and mR. We assume that B can be determined *a priori* by means of a phenomenological fit to the ground-state hadrons. Finally, there is an energy equation which, if we are in the position of knowing all quark masses in the problem, fixes the mass of the hadron under consideration. Otherwise, we can use the energy equation to fit the quark mass. We follow the latter procedure in fitting m_c to the mean charmonium mass of 3025 MeV.

The only ambiguity left in our calculation amounts to choosing which contributions to the energy to take into account. In the calculations to be described below, we have investigated a variety of situations in order to test the sensitivity of our results to any particular choice. The dominant contribution for a given quark is always $E_Q=\omega/R$, which is the energy of a noninteracting particle confined to a sphere of radius R. It turns out that from this term alone we can determine whether the excitation energies in the bag model are sufficient to agree with spectroscopic data compiled at SLAC and DESY. The volume energy, $E_v=\frac{4}{3}\pi BR^3$, is associated with the pressure parameter, B, which ensures the stability of the bag. Clearly, it must be included in any bag-model calculation. The remaining sources of energy that we shall consider here were first taken into account in Ref. 6. That one should include an energy, E_0, associated with zero-point fluctuations is admittedly unconventional. Although a good deal of work clearly remains to be done on this subject, we shall take this effect into account in several of our calculations. The form $E_0=-Z_0/R$ was used in Ref. 6 with the constant Z_0 being determined in the overall fit to hadron masses. It was found that $Z_0\cong 2$. The final contribution to the energy which we shall upon occasion consider in this paper[13] arises from gluon coupling. Unfortunately, it is difficult to study this interaction in full generality. Perhaps its most interesting aspect is that it alone contributes to mass splittings within a given mode, as discussed previously. Use of lowest-order perturbation theory[6] implies for mesons that "para" states lie below the corresponding "ortho" states in mass. This automatically precludes a recent suggestion[14] that charmed vector-meson states are lighter than their pseudoscalar counterparts and thus decay weakly, giving rise to the $\mu^\pm, e^\mp$ pairs observed in e^+e^- reactions at SLAC. In principle, it is possible for gluon exchange to generate the type of tensor forces needed to drive vector-meson masses beneath pseudoscalar masses, although such graphs would be of higher order. If, as found in Ref. 6, the dimensionless quark-gluon coupling α_c is less than unity, it becomes difficult to see how tensor forces could dominate the lower-order processes. Estimation of the coupling α_c is at present one of the most important areas of bag phenomenology. As we shall see, existing experimental knowledge of $J=0,1$ mass splitting only further clouds the issue.

The results to our fit of the charmed-quark mass, m_c, are presented in Table I. The energy operator used to obtain these results has the form

$$E=E_{\text{volume}}+E_{\text{quark}}+E_{\text{zero-pt.}}, \tag{11}$$

and thus depends upon the constants $B^{1/4}$ and Z_0.

We have chosen to display the dependence of the output variables m_c, R^{-1}, and ω upon variations in the input parameter. The values chosen for $B^{1/4}$ and Z_0 cover the range covered by these variables in Refs. 5 and 6. The variation in m_c is seen to be no more than 10% above or below the average of the values appearing in Table I. The same is also true of R^{-1} and ω. The charmed-quark mass is seen to rise as Z_0 increases for fixed $B^{1/4}$ and to fall as $B^{1/4}$ increases for fixed Z_0. Our analysis implies a charmed-quark mass in the vicinity of 1400–1500 MeV with the kind of "error bars" mentioned above.

Historically, the discovery of charmonium was soon followed by the announcement[9] of an excited state with an accurately measured mass of 3684 MeV. The spin-parity (J^P) of this excited state is known to be 1^-. Accordingly, we interpret it as a state with quark configuration $(1S)^1(2S)^1$. Given this assignment, we can calculate the mass of $\psi'(3684)$ in the bag model. The prescription for handling quarks in excited modes is discussed in Refs. 2 and 5. Upon performing the numerical analysis for a variety of bag parameters, we find a mass value for the excited state which is consistently too low. Results for two representative values of input parameters are presented in the second row of Table II. Whereas an excitation energy of almost 600 MeV is needed, the fixed-sphere bag model can sustain excitation energies of no more than 350 MeV.

Of course there are excited states in the bag model, the $(1S)^1(1P)^1$ configurations, whose energies lie between the $(1s)^2$ ground state and its $(1S)^1(2S)^1$ excitation. Experimental evidence for such states exists.[15] However, in view of the poor results for the $(1S)^1(2S)^1$ excitation energy, we shall not present here the numerical values of the $(1S)^1(1P)^1$ masses.

The only issue left to be discussed regarding charmonium is the ortho-para mass splitting. This effect is produced by the magnetic coupling of gluons to quarks, contributing a spin-spin interaction to the energy operator of the form

$$E=\overline{E}+\vec{\sigma}(1)\cdot\vec{\sigma}(2)\Delta \ . \tag{12}$$

The object is to reproduce, in the bag model, the experimental value of

$$\Delta\,(\text{charmonium})=\tfrac{1}{4}(3095-2800)=75\ \text{MeV} \ . \tag{13}$$

Working to lowest-order perturbation theory in the quark-gluon coupling α_c, one can derive the following expression for $\Delta'=\Delta/\alpha_c$ in the specific case of charmonium,[16]

$$\Delta'=\frac{16\mu^2(\omega,m_cR)}{R^3}(1+J)\ , \tag{14}$$

where

$$\mu(\omega,m_cR)=\frac{R}{6}\,\frac{4\omega+2m_cR-3}{2\omega^2-2\omega+m_cR}\ , \tag{15}$$

$$J=[p\sin^2p-\tfrac{3}{2}(p-\sin p\cos p)]^{-2}\{-2p^2\sin^4p-\tfrac{3}{2}(p-\sin p\cos p)^2+\tfrac{1}{2}p^2[4p\,\mathrm{Si}(2p)-2p\,\mathrm{Si}(4p)]\}\ , \tag{16}$$

and

$$p=(\omega^2-m_c{}^2R^2)^{1/2}\ .$$

As with the other calculations described in this section, we have studied Eqs. (14)–(16) for a variety of input parameter values. In all cases, the calculated values are far too small to reproduce the experimental value for the range of couplings $\alpha_c<1$. For example, if $B^{1/4}=114$ MeV and $Z_0=0.0$, we find $\Delta'=20$ MeV, whereas for $B^{1/4}=135$ MeV and $Z_0=1.9$, we obtain $\Delta'=26$ MeV. Even for $\alpha_c=1$, these values are about a factor of 3 too small. Fitting the orthopara mass splitting with a value of α_c exceeding unity is prohibited because the perturbation expansion would be invalid. Moreover, such a value would be inconsistent with fits to the hadron ground state. We shall return to this subject in the Conclusion.

The charmed-quark mass introduces a new energy scale into hadron physics. This is precisely the interpretation we have given to the narrow resonances with a mass near 3 GeV and their excited states. However, internal consistency of this scheme implies the existence of still more new energy scales—those associated with meson states containing one charmed and one uncharmed quark (to wit, $\bar{u}c$, $\bar{d}c$, $\bar{s}c$, and antiparticles), and baryon states with one, two, and finally three charmed quarks. It is fair to question just how fruitful a fixed-sphere bag-model calculation of these new energy scales can be in view of our admitted failure to reproduce the charmonium excitation energies. Our feeling is that, while the bag

TABLE I. Determination of the charmed-quark mass. The entries are solutions of the bag-model equations for the ortho-para average mass of ground-state charmonium, $\overline{M}=3025$ MeV. Dependence on input parameters, $B^{1/4}$ and Z_0, to the energy operator is displayed.

$B^{1/4}$ (MeV)	Z_0	M_c (MeV)	R^{-1} (MeV)	ω
115	0.0	1351	181.5	8.00
	1.0	1436	183.3	8.42
	2.0	1544	185.2	8.85
125	0.0	1327	194.4	7.43
	1.0	1430	196.5	7.85
	2.0	1537	198.6	8.28
135	0.0	1303	207.1	6.93
	1.0	1414	209.5	7.35
	2.0	1529	211.9	7.79
145	0.0	1277	219.8	6.49
	1.0	1396	222.4	6.92
	2.0	1519	225.1	7.36

TABLE II. Spectrum of meson states. Masses of mesons associated with various quark configurations are exhibited. Quarks written in lower case are in the $1S$ mode, whereas capitalized quarks pertain to the $2S$ mode. In columns A and B, we take $Z_0=0.0$, 1.9; $B^{1/4}=114$ MeV, 135 MeV; $m_c=1353$ MeV, 1517 MeV, respectively. Common to both columns are the quark masses $m_u=44$ MeV, $m_s=300$ MeV.

State	Meson mass (MeV) A	B
$c\bar{c}$	3025	3025
$c\bar{C}$	3257	3369
$u\bar{c}$	1954	1865
$U\bar{c}$	2433	2495
$u\bar{C}$	2156	2160
$s\bar{c}$	2113	2016
$S\bar{c}$	2513	2567
$s\bar{C}$	2319	2316

model, at its present level of sophistication, appears to have problems with excited states, it reproduces many observed properties of the hadron ground state. Therefore, we have proceeded to calculate the masses of charm-carrying hadrons in the bag model. Selected results are given in Tables II and III, and, as above, we exhibit values corresponding to two different sets of input variables. These masses pertain to averages over the spin value; the quark spin-spin interaction has not been taken into account yet.

We first take notice of the overall scale of the spectrum. The lightest mesons ($\bar{u}c$) are lighter than 2 GeV in mass; next come the strange mesons with a mass slightly above 2 GeV, then the corresponding baryon states with masses beginning near 2300 MeV, doubly charmed baryons near 3500 MeV, and finally triply charmed baryons at around 4500 MeV. These predictions are compared in Sec. IV with the meagre amount of data available at this time. Tables II and III also contain predictions for excited $(1S)^1(2S)^1$ meson and $(1S)^2(2S)^1$ baryon configurations. It seems to us that the most sensible way to interpret these numbers is to consider their ratio to the charmonium excitation energy, $(1S)^2 \to (1S)^1(2S)^1$. In this way we can at least hope to arrive at a qualitative insight as to the relative excitation energies to be expected from various charmed states. It is also worthwhile to notice that if the ground state consists of quarks with markedly different masses (such as the u and c quarks), the excitation energy will vary quite a bit as we lift one quark and then the other to the $2S$ level. Denoting quarks in the $2S$ configuration with capital letters, we find for mesons[17]

$$c\bar{c} \to C\bar{c} : u\bar{c} \to U\bar{c} : u\bar{c} \to u\bar{C} :: 1. : 1.8 : 0.85 \,, \qquad (17)$$

whereas for baryons, relative to the charmonium excitation energy,

$$c\bar{c} \to C\bar{c} : uuc \to uuC : uuc \to Uuc :: 1 : 0.7 : 1.4 \,. \qquad (18)$$

One can now estimate various $1S \to 2S$ excitation energies by normalizing with respect to the measured value, 580 MeV, of the charmonium excitation energy and using the ratios of Eqs. (15) and (16). Since there are no pertinent data at this time, it is impossible to judge the merit of these results. The physical reason for the difference in energy in raising, say, a u quark to an excited mode relative to raising a c quark to the same mode is that in the ground states of these charmed particles the motion of noncharmed quarks is generally relativistic, whereas the charmed quarks move nonrelativistically. The momentum of a quark of mass m in frequency mode ω can be ex-

TABLE III. Spectrum of charmed baryon states. Masses of baryons associated with various quark configurations are exhibited. The notation is identical to that of Table II.

State	Baryon mass (MeV) A	B
uuc	2288	2299
uuC	2458	2536
uUc	2726	2788
usc	2513	2530
ssc	2613	2610
ucc	3386	3502
ccc	4472	4688

pressed as

$$\frac{p}{m}=\left[\left(\frac{\omega}{mR}\right)^2-1\right]^{1/2} . \tag{19}$$

For example, the charmed quarks in charmonium (using the mean mass of 3025 MeV) move with $p/m \cong 0.4$, and those in the lightest charmed meson $\bar{u}c$ move with $p/m \cong 0.4$ for the charmed quark and $p/m \cong 8.5$ for the uncharmed quark. These values are characteristic for the spectrum of charmed hadrons as described in the fixed-sphere bag model. Finally, let us consider the mass splittings between the $J=0, 1$ states for mesons and $J=\frac{1}{2}, \frac{3}{2}$ states for baryons. Again, we shall rely upon ratios of these splittings relative to that of ortho and paracharmonium. Upon using the extension of Eqs. (14)–(16) to the case where the constituent quarks do not all have the same mass, and performing some elementary spin calculations, we find[17] for the $E(J=1)$-$E(J=0)$ mass splitting

$$\bar{c}c : \bar{u}c : \bar{s}c :: 1.0 : 2.2 : 1.9 . \tag{20}$$

Before giving baryon mass splittings, we observe that there exist not just two, but three states containing a single charmed quark together with two noncharmed, nonstrange quarks. These three baryons have a spin-isospin content ($J=\frac{3}{2}$, $T=1$), ($J=\frac{1}{2}$, $T=1$), and ($J=\frac{1}{2}$, $T=0$). Thus, there are two mass splittings to consider here–one occurs between the $J=\frac{3}{2}$ state and each of the $J=\frac{1}{2}$ states. Relative to the charmonium splitting we find[17]

$$c\bar{c} : (cuu)_{T=1} : (cdu)_{T=0} : ccu :: 1.0 : 1.3 : 3.3 : 1.5 . \tag{21}$$

If the tentative ortho-para charmonium mass splitting of 300 MeV is indeed correct, the fixed-sphere bag model is seen to predict, for the singly charmed baryon states, spin-dependent mass splittings of rather substantial magnitude. We remind the reader that the $\rho-\pi$ mass difference is 620 MeV, whereas for $\Delta - N$, the value is 300 MeV.

We shall return to the subject of charmed-quark spectroscopy in Sec. IV. Our next topic concerns weak decays of charmed and uncharmed hadrons.

III. WEAK DECAYS

It has long been hoped that models of hadron structure will add to our understanding of the weak interaction. Already, quark models such as the present one have helped elucidate the form of baryon-to-baryon matrix elements of the weak currents.[5] However, no theory has been successful in explaining the nonleptonic weak processes. In a previous paper, we examined the hyperon hadronic decays with a fixed-sphere bag model and found that the amplitudes were too small by a factor of at least 3. In the first part of this section, we re-examine these amplitudes in greater detail and extend our approach to include kaon decays. Then, we apply what we have learned to the weak semileptonic and nonleptonic decays of charmed particles.

In both $\Delta S=1$ and $\Delta C=1$ transitions only the charged currents contribute,

$$J_\mu(x)=\bar{u}(x)\gamma_\mu(1+\gamma_5)[d(x)\cos\theta_C+s(x)\sin\theta_C] + \bar{c}(x)\gamma_\mu(1+\gamma_5)[s(x)\cos\theta_C-d(x)\sin\theta_C] . \tag{22}$$

For nonleptonic processes, the weak Hamiltonian density, appearing in the integrand of

$$H_w(0)=g^2\int d^4x\, D_F(x,M_w)T[J^+_\mu(x)J^\mu(0)], \tag{23}$$

can be written by means of a Wilson expansion[18] in terms of multiplicatively renormalizable local operators. For $\Delta S=1$ processes, renormalization group techniques have been used to show that[19]

$$H_w=c_-H_-+c_+H_+ , \tag{24}$$

where (we sum over color indices i,j)

$$H_\pm=\frac{G}{2\sqrt{2}}\cos\theta_C\sin\theta_C[\bar{d}_i\gamma_\mu(1+\gamma_5)u_i\bar{u}_j\gamma^\mu(1+\gamma_5)s_j \pm\bar{d}_i\gamma_\mu(1+\gamma_5)s_i\bar{u}_j\gamma^\mu(1+\gamma_5)u_j] , \tag{25}$$

and

$$c_\pm=\left[1+\frac{25g^2}{24\pi^2}\ln\left(\frac{M_w}{\mu}\right)\right]^{d_\pm},$$
$$d_-=0.48,\quad d_+=-0.24 . \tag{26}$$

Various estimates of the quantity in brackets have placed it in the range from 2 to 10, the smaller values being probably more reasonable.[20] It is within this framework that we explore the weak interactions.

In studying parity-violating hyperon nonleptonic decays, we use soft-pion techniques to change $B'B\pi$ vertices into $B'B$ amplitudes,

$$\lim_{q_\mu\to 0}\langle B'\pi^0(q)|H_w^{\text{p.v.}}|B\rangle=-\frac{i}{F_\pi}\langle B'|[F_3{}^5,H_w^{\text{p.v.}}]|B\rangle =\frac{i}{2F_\pi}\langle B'|H_w^{\text{p.c.}}|B\rangle , \tag{27}$$

with $F_\pi=94$ MeV. The last matrix element is readily calculated in the bag model,[5] leading to the SU(3) parametrization[21]

$$\langle n\pi^0|H_w^{\text{p.v.}}|\Lambda\rangle = d+3f\,,$$
$$\langle p\pi^0|H_w^{\text{p.v.}}|\Sigma^+\rangle = \sqrt{6}(d-f)\,, \tag{28}$$
$$\langle \Lambda\pi^0|H_w^{\text{p.v.}}|\Xi^0\rangle = d-3f\,,$$

with $d=-f$ given by

$$d=-f=-\frac{\sqrt{3}G}{4F_\pi}\sin\theta_C\cos\theta_C c_- I\,, \tag{29}$$

where I is the bag-overlap integral

$$I=\prod_{i=1}^{4}N_i\int_0^R dr\,r^2\left[j_0^2(pr)+\frac{\omega-mR}{\omega+mR}j_1^2(pr)\right]\left[j_0(pr)j_0(p'r)+\left(\frac{(\omega-mR)(\omega'-m'R)}{(\omega+mR)(\omega'+m'R)}\right)^{1/2}j_1(pr)j_1(p'r)\right]. \tag{30}$$

Unprimed and primed quantities refer to nonstrange and strange quarks, respectively. Evaluation of Eq. (30) using the bag-model parameters of Ref. 5 gives

$$d=-f=8.5\times10^{-9}c_-\,. \tag{31}$$

This is too small by a factor of 3 to 5 even for the largest reasonable enhancement. *However, it is important to realize that our result depends on the parameters of the model*. In Ref. 6, the quarks are described in terms of rather different parameters, most notably a smaller bag radius. Using these Ref. 6 values, we obtain

$$d=-f=-2.5\times10^{-8}c_-\,, \tag{32}$$

which is a factor 3 larger than the result given originally in Ref. 5. This large increase is readily understandable. If one has a function ψ normalized as

$$\int_V d^3x\,\psi^+(x)\psi(x)=1\,,$$

then it is easy to see that the quantity

$$\int_V d^3x\,(\psi^+(x)\psi(x))^2\,,$$

which has dimensions of $(\text{vol.})^{-1}$, increases as the radius decreases. In the simplest case, $\psi=$ constant, the latter integral transforms like R^{-3}. Our previous model had $R=7.3\ \text{GeV}^{-1}$ $(\langle r^2\rangle^{1/2}=1.02\ \text{fm})$ while the model of Ref. 6 employed $R=5\ \text{GeV}^{-2}$ $(\langle r^2\rangle^{1/2}=0.72\ \text{fm})$, roughly accounting for the factor of 3 increase. Clearly, the radius sets the scale for the four-quark overlap integrals.

The bag model results of Refs. 5 and 6 are compared with the experimental S-wave hyperon amplitudes in Table IV. The contents of this table imply that the bag-model results are still generally too small, even for the parameters of Ref. 6, to fit the decay amplitudes. We feel that it is possible to extract a more precise conclusion from this analysis by examining the SU(3) content of the experimental amplitudes. Such a fit has been performed previously[22] to both the S-wave and P-wave amplitudes simultaneously, with the result that $f/d\approx-1.2$ to -1.3. We might even take the more extreme position of performing a weighted least-squares fit to the S-wave data, where we find[5]

$$f=9.5\times10^{-8},\quad d=-4.45\times10^{-8}\,,$$

or $f/d\cong-2$. Taken together, these findings imply that $|f/d|>1$, which is a result unattainable in our model or in any other that uses SU(6) quark wave functions. Moreover, if we take Eq. (32) seriously, then comparison with our results indicates that the bag model can generate a d parameter of the necessary magnitude without appreciable difficulty. It is the f amplitude whose magnitude theory is evidently unable to reproduce.

Our soft-pion method for calculating parity-violating hyperon decay amplitudes is expected to yield the dominant contribution to these amplitudes. However, it is not the only contribution. In the following, a method is presented which allows for explicit calculation of several correction terms. Unfortunately, we are unable to determine the complete amplitude so our treatment is necessarily limited. However, we feel that there is pedagogical value in presenting it and in observing the size of the terms which we can, in fact, calculate. The easiest way to present our ideas is in terms of quark diagrams. A set of possible contributions to nonleptonic decays in a three-quark model of baryons is exhibited in Fig. 1. Even at this level, approximations have been made. For example, in diagram 1(e), there could exist gluon lines connecting the constituent quarks of the pion to those of the baryons. Analogous gluon lines can contribute to diagrams 1(a)–1(d).

TABLE IV. Experimental hyperon nonleptonic decay amplitudes, and the bag model predictions using the input parameters of Ref. 5 and Ref. 6. c_- is the enhancement factor, defined in the text, and is expected to be in the range $1\lesssim c_-\lesssim3$.

	$10^7A(\Lambda_0^0)$	$10^7A(\Xi_0^0)$	$10^7A(\Sigma_0^+)$
Experimental	2.39 ± 0.05	-3.39 ± 0.07	-3.28 ± 0.11
Ref. 5	$0.18\,c_-$	$-0.36\,c_-$	$-0.44\,c_-$
Ref. 6	$0.54\,c_-$	$-1.1\,c_-$	$-1.3\,c_-$

However, the difficulty in calculating such terms forces us to consider only the processes depicted in Fig. 1. Incidentally, if we employ the assumptions which justify the appearance of short-distance expansions in the derivation of the effective nonleptonic Hamiltonian (24), we can cast Fig. 1(d) in the form implied by Fig. 1(e). The W boson is taken to be so massive that its propagator is shrunk to a point. The ensuing four-fermion vertex can be reordered by a Fierz transformation to yield a factorized structure analogous to that in Fig. 1(e). In the limit of zero pion momentum $q^\mu \to 0$, only Figs. 1(a)–1(c) are nonzero. It is these, given our approximate model, which must correspond to the current-algebra contribution. Upon passing back to the physical limit $q^2 = m_\pi^2$, Figs. 1(a)–1(c) are modified in ways which we cannot determine. However, it is possible to take account of Figs. 1(d) and 1(e). By using

$$\langle \pi^0(q)\,|A_3^\mu|\,0\rangle\langle n\,|V_\mu|\,\Lambda\rangle = F_\pi g_V(\Lambda \to n) q^\mu \bar{u}(n)\gamma_\mu u(\Lambda)\,, \tag{33}$$

we can evaluate these diagrams. Since they depend on the mass difference between baryons, they are explicitly SU(3)-breaking. However, if the $\frac{1}{2}^+$ baryon masses satisfy the Gell-Mann–Okubo mass formula, and if we use the SU(3) values of g_V, then the $\Delta I = \frac{1}{2}$ components can be parametrized by

$$d = -\frac{GF_\pi}{8\sqrt{3}}\cos\theta_C \sin\theta_C (M_\Xi - M_\Lambda)(c_- + \tfrac{2}{3}c_+)\,,$$
$$f = \frac{GF_\pi}{6\sqrt{3}}\cos\theta_C \sin\theta_C [\tfrac{1}{4}(3M_\Lambda + M_\Xi) - M_N](c_- + \tfrac{2}{3}c_+)\,, \tag{34}$$

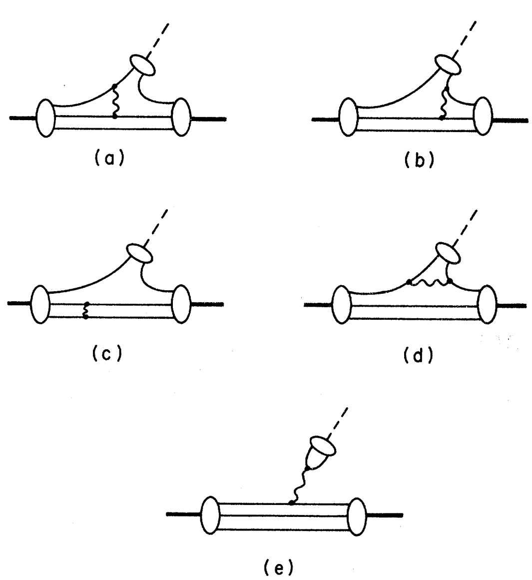

FIG. 1. The quark diagrams that contribute to baryon nonleptonic decay.

so that

$$d = -1.3\times 10^{-9}(c_- + \tfrac{2}{3}c_+), \quad f = 4.5\times 10^{-9}(c_- + \tfrac{2}{3}c_+)\,. \tag{35}$$

There is an overall minus-sign uncertainty associated with these terms, so that we cannot know if they help or hinder the d/f problem. In any case, their magnitude is too small to be of much effect for reasonable values of the enhancement.

It does not appear to have been previously noticed that the presence of $\Delta I = \frac{3}{2}$ terms in H_w ensures violation of the $\Delta I = \frac{1}{2}$ sum rules. For example, if we evaluate the two modes for Λ decay using the factorization diagrams, we obtain[23]

$$A(\Lambda_-^0) + \sqrt{2}A(\Lambda_0^0) = 7c_+ \times 10^{-8}\,. \tag{36}$$

This is considerably larger than the experimental value,[24] $(1.2\pm 0.8)\times 10^{-8}$, of the left-hand side of Eq. (36) for the usual values of c_+. This indicates that $\Delta I = \frac{3}{2}$ effects are not suppressed enough in this version of the nonleptonic weak Hamiltonian. It is conceivable that the necessary cancellations can be generated by yet other correction terms to the current-algebra result, but this is not *a priori* evident.

To discuss kaon decays in the bag framework, we must again use soft-pion techniques. These have previously been studied for the kaon system by Holstein,[25] and we will use his notation. The relevant amplitudes are ($i = \frac{1}{2}, \frac{3}{2}$)

$$\langle 0\,|H_w^{1/2}|K^n\rangle = A_{1/2} m_K{}^3 \bar{s}_{1/2} K^n\,,$$
$$\langle \pi^a |H_w^i|K^n\rangle = {}^iA_n^a + {}^iB_n^a k\cdot q_a + {}^iC_n^a q_a{}^2\,,$$
$$\langle \pi^a\pi^b|H_w^i|K^n\rangle = {}^iA_n^{ab} + {}^iB_n^{ab}k\cdot q_a + {}^iB_n^{ba}k\cdot q_b + {}^iC_n^{ab}q_a{}^2 + {}^iC_n^{ba}q_b{}^2 + {}^iD_n^{ab}q_a\cdot q_b + (K\text{ pole})^i\,, \tag{37}$$

where $\bar{s}_{1/2}$ is the $I = \frac{1}{2}$ spurion $\bar{s}_{1/2} = (0,1)$, and $K^n = (1,0)$ for K^+, $K^n = (0,1)$ for K^0. The momenta k and q are those of the physical $K \to 2\pi$ process. Using current algebra in the soft-pion limit, we obtain

$${}^{1/2}A_n^{ab} = \frac{A_{1/2}m_K{}^3}{4F_\pi{}^2}\delta^{ab}\bar{s}_{1/2}K^n\,,$$
$${}^{1/2}B_n^{ab} = \frac{B_{1/2}m_K}{4F_\pi{}^2}\bar{s}_{1/2}\tau^b\tau^a K^n\,, \tag{38}$$
$${}^{1/2}C_n^{ab} = \frac{C_{1/2}}{B_{1/2}}{}^{1/2}B_n^{ab}\,,$$

where

$${}^{1/2}B_n^a = \frac{B_{1/2}m_K}{2F_\pi}\bar{s}_{1/2}\tau^a K^n\,, \tag{39}$$

and $C_{1/2}$ is defined likewise. Similarly, for the

$\Delta I = \frac{3}{2}$ components,

$$^{3/2}A_n^{ab} = 0\ ,$$

$$^{3/2}B_n^{ab} = \frac{B_{3/2} m_K}{4F_\pi^{\ 2}} (\bar{s}^a_{3/2} \tau^b K^n - 2i\epsilon^{abc} \bar{s}^c_{3/2} K^n)\ , \qquad (40)$$

$$^{3/2}C_n^{ab} = \frac{C_{3/2} B_n^{ab}}{B_{3/2}}\ ,$$

where $\bar{s}^1_{3/2} = (-\frac{1}{2}, 0)$, $\bar{s}^2_{3/2} = (-\frac{1}{2}i, 0)$, $\bar{s}^3_{3/2} = (0, -1)$, and $B_{3/2}$ is given by

$$^{3/2}B_n^a = \frac{B_{3/2} m_K}{2F_\pi} \bar{s}^a_{3/2} K_n\ , \qquad (41)$$

with $C_{3/2}$ defined similarly. If we also form

$$^{1/2}D_n^{ab} \equiv \frac{D_{1/2} m_K}{4F_\pi^{\ 2}} \delta^{ab} \bar{s}_{1/2} K^n\ ,$$

$$^{3/2}D_n^{ab} \equiv \frac{D_{3/2} m_K}{4F_\pi^{\ 2}} (\bar{s}^a_{3/2} \tau^b K^n + \bar{s}^b_{3/2} \tau^a K^n) \qquad (42)$$

we can write the physical $K \to 2\pi$ amplitudes as

$$\langle \pi^a \pi^b | H_w^{1/2} | K^n \rangle = \delta^{ab} \bar{s}_{1/2} K^n \frac{m_K}{4F_\pi^{\ 2}} [(A_{1/2} + B_{1/2}) m_K^{\ 2} + 2C_{1/2} m_\pi^{\ 2} + \tfrac{1}{2} D_{1/2} (m_K^{\ 2} - m_\pi^{\ 2})]$$

$$\langle \pi^a \pi^b | H_w^{3/2} | K^n \rangle = [\bar{s}^a_{3/2} \tau^b K^n + \bar{s}^b_{3/2} \tau^a K^n] \frac{m_K}{8F_\pi^{\ 2}} [B_{3/2} m_K^{\ 2} + 2C_{3/2} m_\pi^{\ 2} + D_{3/2} (m_K^{\ 2} - m_\pi^{\ 2})]\ . \qquad (43)$$

The original motivation for this analysis was to relate the $K \to 2\pi$ processes to $\langle 0 | H_w^{1/2} | K^n \rangle$ by taking both pions soft, thereby obtaining a $\Delta I = \frac{1}{2}$ rule. This requires that $A_{1/2}$ dominate the other amplitudes. *However, in any model where mesons are constructed of only a quark-antiquark pair,* $A_{1/2}$ *is zero.* The reason is that H_w contains the normal-ordered product of four quark fields. Holstein's explanation[25] of the $\Delta I = \frac{1}{2}$ rule is therefore not plausible in a $q\bar{q}$ model of the mesons such as ours.

Use of soft-pion techniques does not allow one to obtain any information about $D_{1/2, 3/2}$. There are two plausible ways in which to treat these terms: (a) in Ref. 25 it is argued that these terms vanish, (b) the full $K \to 2\pi$ decay amplitudes are required to vanish if one assumes both CP and SU(3) symmetry.[26] This latter situation implies that

$$D_{1/2} = 2A_{1/2} + 2B_{1/2} + 4C_{1/2}\ ,$$

$$D_{3/2} = B_{3/2} + 2C_{3/2}\ , \qquad (44)$$

in the SU(3) limit. By assuming that the SU(3) breaking inherent in a nonvanishing $K \to 2\pi$ rate is due only to kinematic effects ($m_K^{\ 2} \neq m_\pi^{\ 2}$), we can use (44) as a prediction for the D terms in the physical processes. Our conclusions are sufficiently general that they do not depend greatly on the choice of method (a) or (b). Finally, we follow Ref. 25 and set all C terms to zero. One motivation for doing so is that in the soft-kaon limit, the $K \to \pi$ matrix element turns into one of $\pi \to$ vacuum, which is zero in our model. This precludes any C terms in $K \to \pi$. Collecting the relevant formulas together gives

$$\langle \pi^a | H_w^{1/2} | K^n \rangle = B_{1/2} \frac{m_K}{2F_\pi} k \cdot q_a \bar{s}_{1/2} \tau^a K^n\ ,$$

$$\langle \pi^a | H_w^{3/2} | K^n \rangle = B_{3/2} \frac{m_K}{2F_\pi} k \cdot q_a \bar{s}^a_{3/2} K^n\ . \qquad (45)$$

Thus, we find for method (a),

$$\langle \pi^a \pi^b | H_w^{1/2} | K^n \rangle = \delta^{ab} \bar{s}_{1/2} K^n \frac{m_K^{\ 3}}{4F_\pi^{\ 2}} B_{1/2}\ ,$$

$$\langle \pi^a \pi^b | H_w^{3/2} | K^n \rangle = [\bar{s}^a_{3/2} \tau^b K^n + \bar{s}^b_{3/2} \tau^a K^n] B_{3/2} \frac{m_K^{\ 3}}{8F_\pi^{\ 2}}\ , \qquad (46)$$

whereas for method (b)

$$\langle \pi^a \pi^b | H_w^{1/2} | K^n \rangle = \delta^{ab} \bar{s}_{1/2} K^n \frac{m_K}{2F_\pi^{\ 2}} B_{1/2} (m_K^{\ 2} - m_\pi^{\ 2})\ ,$$

$$\langle \pi^a \pi^b | H_w^{3/2} | K^n \rangle = [\bar{s}^a_{3/2} \tau^b K^n + s^b_{3/2} \tau^a K^n] \frac{B_{3/2}}{4F_\pi^{\ 2}} (m_K^{\ 2} - m_\pi^{\ 2})\ . \qquad (47)$$

Thus, in Eqs. (46) and (47) we have expressed the $K \to 2\pi$ amplitudes in terms of the K-to-π matrix elements of H_w.

To obtain the bag-model prediction for the $K\pi$ transitions we proceed as with the $B'B$ matrix elements. The quark diagrams that enter are shown in Fig. 2. We obtain

$$B_{1/2} = (c_- + \tfrac{2}{3} c_+) B\ ,$$

$$B_{3/2} = -\tfrac{8}{3} c_+ B\ , \qquad (48)$$

where

$$B = \frac{-4F_\pi}{m_K k \cdot q} G \cos\theta_C \sin\theta_C (m_K m_\pi)^{1/2} \prod_{i=1}^{4} N_i I\ , \qquad (49)$$

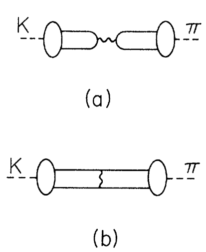

FIG. 2. The quark diagrams for the K-π transition.

$$I=\int_0^R r^2dr\,(u_1u_2-v_1v_2)(u_3u_4-v_3v_4)$$
$$-(u_1v_2+u_2v_1)(u_3v_4+u_4v_3)\,, \qquad (50)$$

and

$$u_i=j_0(p_ir)$$
$$v_i=\left(\frac{E_i-m_i}{E_i+m_i}\right)^{1/2}j_1(p_ir)\,. \qquad (51)$$

Numerically,[27] the bag model gives

$$B=-1.6\times10^{-10}\,, \qquad (52)$$

for parameters fitted as in Ref. 5, and

$$B=4.4\times10^{-9}\,, \qquad (53)$$

for the parameters of Ref. 6.

As can be readily seen, the most secure and most fatal prediction is the ratio $B_{3/2}/B_{1/2}$. To reproduce the experimental K decay amplitudes we need

$$\left|\frac{B_{3/2}}{B_{1/2}}\right|=0.064\pm0.002\,, \qquad (54)$$

while in our model this ratio is appreciably larger for reasonable values of the enhancement. For example, we find $B_{1/2}/B_{3/2}=1.6$, 0.8, and 0.45, respectively, for the cases of $c_-=1$ (no enhancement), 2, and 3. We feel that this is not as much the fault of the quark model as it is the failure of the octet-enhancement mechanism. In models such as ours the $\Delta I=\frac{3}{2}$ reduced matrix element will be of the same order as that with $\Delta I=\frac{1}{2}$ unless the $\Delta I=\frac{3}{2}$ operator is suppressed. The amplitudes needed to give the observed $K\to2\pi$ decay rates are given in Table V, along with the bag-model results. The significant model dependence of the results is unfortunate, but it is obvious in all cases that the $\Delta I=\frac{1}{2}$ component is not large enough for reasonable values of the enhancement.

We now turn our attention to the weak decay of charmed particles. Because of the still speculative nature of these objects, we will not attempt an exhaustive survey, but rather limit ourselves to a few illustrative examples. We begin with a study of the singly charmed, nonstrange baryon C. This particle is expected to be one of the most abundantly produced of the charmed baryons in neutrino-induced reactions.

The C^{++} can decay semileptonicly to $\Sigma^+l^+\nu$. We can use the bag model to calculate g_V and g_A in

$$\langle\Sigma^+|V_\mu+A_\mu|C^{++}\rangle=\bar{u}(\Sigma)\gamma_\mu[g_V+g_A\gamma_5]u(c)\,. \qquad (55)$$

The techniques are the same as in Ref. 5, where good results are obtained for the noncharmed baryons. Here, we find

$$g_V(C^{++},\Sigma^+)=1.04,\quad g_A(C^{++},\Sigma^+)=0.32\,. \qquad (56)$$

Note that whereas g_A for noncharmed baryons is decreased from the nonrelativistic SU(6) prediction by about 30%, our result here is essentially the SU(8) prediction ($g_V=1$, $g_A=\frac{1}{3}$). This is because the charmed quark is almost nonrelativistic, so that the relativistic correction that lowered g_A previously is not important here. That is, the V^μ, A^μ matrix elements describe the conversion of a charmed quark into an uncharmed quark. In the bag model, the upper Dirac-spinor components of the affected quarks multiply each other, as do the lower components. However, here the charmed quark has a very small lower component, so only the upper components [which yield the SU(8) results] contribute substantially.

The predominant decay channels of charmed particles are expected to be purely hadronic if we assume analogy with the weak decays of strange particles. Despite our earlier criticisms of the octet-enhancement mechanism, we shall use a weak Hamiltonian determined from similar considerations to obtain a picture of the charm-changing nonleptonic decays. Renormalization-group techniques imply[28]

$$H_w^{\Delta C=1}=c_-H_-^{\Delta C=1}+c_+H_+^{\Delta C=1}\,, \qquad (57)$$

where the Cabibbo-favored term is

$$H_\pm^{\Delta C=1}=\frac{G}{2\sqrt{2}}\cos^2\theta_C[\bar{c}\gamma_\mu(1+\gamma_5)s\bar{d}\gamma^\mu(1+\gamma_5)u$$
$$\pm\bar{c}\gamma_\mu(1+\gamma_5)u\bar{d}\gamma^\mu(1+\gamma_5)s]\,, \qquad (58)$$

and $c_\mp$ are as before.[29] The enhanced operator

TABLE V. The first two columns give the $B_{1/2}$ and $B_{3/2}$ needed to reproduce experimental $K\to2\pi$ decay amplitudes using methods a and b, described in the text. The last two columns are the bag-model predictions for $B_{1/2}$ and $B_{3/2}$ obtained from the $K\to\pi$ matrix elements for the bag parameters of Refs. 5 and 6.

	(a)	(b)	Ref. 5	Ref. 6
$B_{1/2}$	7.8×10^{-8}	4.3×10^{-8}	$(c_-+\frac{2}{3}c_+)\times1.6\times10^{-10}$	$(c_-+\frac{2}{3}c_+)\times4.4\times10^{-9}$
$B_{3/2}$	4.9×10^{-9}	2.7×10^{-9}	$c_+\times4.4\times10^{-10}$	$c_+\times1.2\times10^{-8}$

transforms as a member of an SU(3) sextet [an SU(4) 20-plet], so that a sextet rule is predicted. Because of the large phase space available, multihadron decays are expected to be predominent. However, we are unable to handle these so we will look only at two-body decays.

There are two contributions to $C \rightarrow B\pi$ that we can calculate, the commutator term of Eq. (27) and the factorization diagram of Fig. 1(e). The C system is particularly interesting in this respect because $C^{++} \rightarrow \Sigma^+\pi^+$ decay contains no commutator term, whereas $C^+ \rightarrow \Lambda\pi^+$ contains no factorization diagram. Using techniques described previously, we find

$$A(C_+^{++}) = \tfrac{1}{3}(G\cos^2\theta_C)F_\pi g_v(C^{++},\Sigma^+)(c_- + 2c_+)(M_C - M_\Sigma) \tag{59}$$

and

$$\frac{A(C_+^+)}{A(\Lambda_-^0)} = \sqrt{2}\cot\theta_C \frac{I(c,s)}{I(s,d)}\,, \tag{60}$$

where

$$I(q,q') = \prod_{i=1}^{4} N_i \int d^3x\,(u_q u_{q'} + v_q v_{q'})(u_u u_d + v_u v_d)\,, \tag{61}$$

with u_i and v_i given in Eq. (51). The numerical value found for the ratio in Eq. (60) is $I(c,s)/I(s,d) = 1.4$ or 1.1 if $Z_0 = 0.0$, $B^{1/4} = 114.0$ MeV, and $Z_0 = 1.87$, $B^{1/4} = 135$ MeV, respectively. In particular, we find

$$A(C_+^{++}) = 1\times 10^{-6}\,,\quad A(C_+^+) = 2\times 10^{-6}\,, \tag{62}$$

upon choosing $c_- = 2$, $Z_0 = 1.9$, and $B^{1/4} = 135$ MeV. The fact that $A(C_+^{++})$ is almost as large as $A(C_+^+)$ is noteworthy because it provides an example of a term which vanishes in the soft-pion limit being comparable to the soft-pion estimate. This is due to the large baryon mass difference here, which forces the pion momentum components to be far from zero, and is in marked contrast to our earlier results involving hyperon decay.

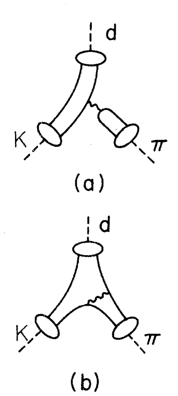

FIG. 3. Contributions to $d^0 \rightarrow K^-\pi^+$ that vanish in the soft-pion limit.

Charmed-meson two-body decays can also be examined in the soft-pion limit, keeping in mind that there may be large terms that disappear as we let $q_\pi^\mu \rightarrow 0$ (see Fig. 3). Here, we shall study the π^+K^- decay mode of the nonstrange, singly charmed meson d^0. The soft-pion techniques allow us to write as $q_\pi^\mu \rightarrow 0$,

$$\begin{aligned}\langle \pi^+K^- | H_w^{\Delta C=1} | d^0\rangle &\rightarrow -\frac{1}{F_\pi}\langle K^-|[F_-^5, H_w^{\Delta C=1}]|d^0\rangle \\ &= -\frac{1}{F_\pi}\langle K^-|[F_-, H_w^{\Delta C=1}]|d^0\rangle \\ &= -\frac{\sqrt{2}}{F_\pi}\langle \bar{K}^0|H_w^{\Delta C=1}|d^0\,.\end{aligned} \tag{63}$$

Similarly, we note that as $q_\pi^\mu \rightarrow 0$,

$$\langle \pi^0\pi^0 | H_w^{\Delta S=1}|K_S^0\rangle \rightarrow \frac{-1}{\sqrt{2}F_\pi}\langle \pi^0|H_w^{\Delta S=1}|K^0\rangle\,. \tag{64}$$

By considering a ratio, we can hopefully avoid the enhancement problem since we expect the enhancement to be the same in both $H_w^{\Delta S=1}$ and $H_w^{\Delta C=1}$. Thus, we have

$$\frac{\langle \pi^+K^-|H_w^{\Delta C=1}|d^0\rangle}{\langle \pi^0\pi^0|H_w^{\Delta S=1}|K_S^0\rangle} = 2\frac{\langle \bar{K}^0|H_w^{\Delta C=1}|d^0\rangle}{\langle \pi^0|H_w^{\Delta S=1}|K^0\rangle}\,. \tag{65}$$

The bag model allows us to calculate the reduced matrix elements with the result

$$\frac{\langle \pi^+K^-|H_w^{\Delta C=1}|d^0\rangle}{\langle \pi^0\pi^0|H_w^{\Delta S=1}|K_S^0\rangle} = 2\sqrt{2}\cot\theta_C\left(\frac{m_d}{m_\pi}\right)^{1/2}\frac{I'}{I}\,, \tag{66}$$

where I is the integral used in calculating the K to π matrix element Eq. (50), and I' is the same integral but with d^0 instead of kaon parameters. Numerically, $I'/I = 1.05$ if $Z_0 = 1.9$, $B^{1/4} = 135$ MeV, so that

$$\langle \pi^+K^-|H_w^{\Delta C=1}|d^0\rangle = 7.9\times 10^{-6} m_K\,. \tag{67}$$

This implies the corresponding ratio of decay widths,

$$\frac{\Gamma(d^0 \rightarrow \pi^+K^-)}{\Gamma(K_S \rightarrow \pi^0\pi^0)} \cong 65\,. \tag{68}$$

It hardly needs to be emphasized that the prediction in Eq. (68) should be viewed with caution in view of our difficulty in attempting to describe kaon decay. However, it does point out that substantial differences might occur in the decays of charmed particles relative to strange particles.

IV. CONCLUSION

In conclusion, we discuss how the fixed-sphere MIT bag model has performed in the various situations examined here.

The result

$$\langle r^2\rangle_{\rm ax} = \langle r^2\rangle_{\rm em}$$

is consistent with existing data.[7] As better data become available, the axial radius $\langle r^2\rangle_{\rm ax}$ can be useful in differentiating between various quark models, as the result is sensitive to the small components in the quark's wave function. Even the nonrelativistic quark model's prediction is consistent with the present data.

The presence of a new energy scale needed to account for the recently discovered mesons is accommodated easily by the introduction of a heavy charmed quark with mass $m_c = 1300 - 1500$ MeV. The energies of predicted charm-bearing hadrons are similar to other estimates in the literature.[30,31] Unfortunately, there is little data with which to compare our results. A Brookhaven bubble-chamber event[32] provides a candidate for a $C=1$, $S=0$ baryon of mass 2426 MeV. This is not in appreciable disagreement with the corresponding value listed in Table II. However, the fine detail of the known $c\bar{c}$ mesons does not agree with the predictions of the bag model studied here. If $\psi(3684)$ is interpreted as an excited state of $\psi(3095)$, the fixed-sphere bag model cannot reproduce the required excitation energy. This mirrors the difficulty that was encountered in constructing excited states of the uncharmed baryons. In addition, if the observed particle with mass 2800 MeV is the pseudoscalar charmonium state, our model does not predict the correct vector-pseudoscalar mass splitting for any reasonable values of the bag parameters. This is a serious failure, since these splittings are reasonably predicted in the case of ordinary hadrons, where the measure of the gluon coupling strength is found to be[6] $0.5 \lesssim \alpha_c \lesssim 0.8$. Unfortunately, the observed splitting between orthocharmonium and paracharmonium evidently requires $\alpha_c > 1$, which is inconsistent with both the ground-state results and the use of a perturbation expansion. There does exist one approach which can improve the bag prediction for the vector-pseudoscalar mass splitting. Instead of performing perturbation theory about a mean mass, it is possible to treat the $J=0,1$ states completely separately. This leads to a differing bag radius for $J=0$ and $J=1$, with the result that Δ is increased markedly,[31] although it is still too small by almost a factor of 2 to fit experiment.

It is not clear where the blame for these inadequacies lies. Perhaps the bag-model approach is wrong in its entirety, or perhaps only the spherical shape needs to be abandoned. Given the success of the bag model in describing the hadron ground state, we may even be led to question the usual interpretation of the newly observed mesons, in particular the state at 2800 MeV, about which little is known. Only future experimental and theoretical work can tell us which is correct. Construction of nonspherical bag models is underway.

In studying the nonleptonic weak decays, we have seen how the short-distance behavior and the hadronic wave functions can combine to produce an enhancement of the hyperon decay amplitudes. However, several very serious problems remain: (1) the hyperon amplitudes are still too small for reasonable values of the enhancement; (2) the SU(3) structure (d/f) is incorrect in the hyperon decays; (3) there is a $\Delta I = \frac{3}{2}$ contribution to hyperon decays which vanishes in the soft-pion limit and can be estimated via a factorization approach to be substantially larger than that found experimentally; in kaon decays, (4) the ratio of $\Delta I = \frac{3}{2}$ to $\Delta I = \frac{1}{2}$ terms is unacceptably large, and (5) the $\Delta I = \frac{1}{2}$ contribution evaluated in the soft-pion limit is roughly an order of magnitude too small.

Problems (1) and (5) are associated with specific predictions of our model, although since, as we noted, the scale of these contributions is set by the hadron radius,[33] these problems might be expected to occur in other quark models also. In particular, any claim of a successful calculation for nonleptonic decay amplitudes should be treated with caution unless accompanied by an acceptable value of, say, the charge radius. Problem (2) is common to all quark models with SU(6) wave functions. Thus, any such evaluation of the hyperon nonleptonic decay amplitude in the soft-pion limit will have $d/f = -1$. We have indicated how the existence of corrections to the soft-pion estimate can, in principle, remedy this situation, although the model correction terms considered here are too small. Problems (3) and (4) depend most heavily on the structure of the weak Hamiltonian H_w, and suggest that the $\Delta I = \frac{3}{2}$ effects are not sufficiently suppressed in the conventional short distance analysis of H_w. In view of these difficulties, perhaps it would be wise to remind the reader of the precise role played by the bag formalism here. We have consistently assumed the validity of current algebra and partial conservation of axial-vector current in relating amplitudes with varying numbers of external pions. The fixed-sphere bag model was used only to take hadron structure explicitly into account. In particular, the bag model was not used to justify the association of a pion field with a current divergence because, as mentioned in the Introduction, chiral symmetry has not yet successfully been described in the bag formalism. Just how serious this is remains an open question at this time. Although we feel that the weak Hamiltonian used in this paper is the source of trouble, the above remarks should be kept in mind.

An interesting by-product of our bag-model analysis is to reopen the question of using current algebra to dynamically generate the $\Delta I = \frac{1}{2}$ rule for kaon decay. The standard approach[25] relies upon the importance of the vacuum-to-kaon matrix element of H_w,

$$\langle 0 | H_w^{1/2} | K^n \rangle = A_{1/2} m_K{}^3 \overline{s}_{1/2} K^n . \quad (37)$$

That is, matrix elements of $H_w^{1/2}$ between kaon and one and two pion states can be expressed in terms of $A_{1/2}$. However, as we have pointed out, $A_{1/2}$ vanishes in any model where the kaon is a $\overline{q}q$ composite. Given this fact, we were led to describe kaon decays in terms of $B_{1/2}, B_{3/2}$ parameters defined in Eqs. (39), (40). Now if, in fact, the actual kaon-pion matrix elements are dominated by the energy-independent term $A_{1/2}$, we must clearly take explicit account of the $\overline{q}q$ "sea" in the kaon wave function. It will be interesting to see whether the amount of sea necessary to reproduce the phenomenological value of $A_{1/2}$ is in accord with estimates obtained, e.g., from neutrino scattering.[34] A study of this type is underway.

Finally, we have examined several possible weak-decay modes of charmed particles, giving the type of effects expected and crude estimates of each. These might be useful in testing present theoreies of the weak interaction if it proves possible to experimentally study charm-changing decays in detail.

We have encountered difficulties of several types in attempting to extend the bag model in different directions, while reinforcing its success in describing static properties of ground state baryons. Our feeling is that the era in which the simplest bag model could claim practically unqualified success has come to an end. The challenge of constructing more complex and hopefully more realistic descriptions in the bag framework confronts us. An even greater task involves welding together a satisfactory quark model with an improved weak Hamiltonian which will allow realistic calculations of nonleptonic decay rates. Perhaps progress will not be forthcoming until we increase the number of basic degrees of freedom.[35]

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