PHYSICAL REVIEW D VOLUME 14, NUMBER 5 1 SEPTEMBER 1976

Hierarchy of lepton masses in a vectorlike theory with Majorana particles*

T. P. Cheng
Physics Department, University of Missouri—St. Louis, St. Louis, Missouri 63121
(Received 22 April 1976)

We demonstrate that a slightly modified version of the vectorlike weak-interaction theory of Fritzsch, Gell-Mann, and Minkowski can provide a framework for understanding the lepton mass spectrum. We conjecture that electrons and neutrinos are massless in the absence of weak interactions, and acquire masses through radiative corrections. But because there are heavy Majorana leptons in such a theory, electron mass is of the second order and neutrino masses are of the fourth order. One of the more immediate predictions of our theory is that among the dominant three-body decay products of the 1.8-GeV lepton U of Perl *et al.* there should be a Majorana lepton of intermediate mass (~ 100 MeV?) which decays into $\bar{\nu}_e\gamma$ and $\nu_e\gamma$ with a lifetime $\sim 10^{-5}$ sec. Heavy Majorana particles and the attendant lepton-number violations are in fact desired for a number of reasons in an $SU_2 \times U_1$ vectorlike theory where all fundamental fermions transform as doublets. We discuss lepton-number-violation processes such as $\alpha \to \beta\mu^\pm\mu^\pm$. We calculate the fourth-order induced neutrino masses. The possibility of their being finite and calculable quantities in a theory with an enlarged gauge group is also examined. We remark that our estimates indicate that the observation of "neutrino beam oscillation" will be hopelessly difficult in a laboratory setting.

I. INTRODUCTION

Recently Fritzsch, Gell-Mann, and Minkowski[1,2] proposed a gauge theory of weak and electromagnetic interactions. The gauge group is $SU_2^{\text{weak}} \times U_1$ and there are equal numbers of weak isodoublets of the left-handed and right-handed leptons and quarks. Their "minimal theory" has six tricolored quarks,[3] u, c, and t (of charge $+\frac{2}{3}$) and d, s, and b (of charge $-\frac{1}{3}$), which transform under SU_2^{weak} as[4]

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L, \quad \begin{pmatrix} t \\ d \end{pmatrix}_R, \begin{pmatrix} c \\ s \end{pmatrix}_R, \begin{pmatrix} u \\ b \end{pmatrix}_R. \tag{1.1}$$

There are also "six" leptons, e^-, μ^-, M^-, ν_e, (ν_μ, N_μ), and N_M; they are arranged into doublets as follows:

$$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} N_\mu \\ \mu \end{pmatrix}_R \tag{1.2}$$

and

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} N_M \\ M \end{pmatrix}_L, \quad \begin{pmatrix} N_M \\ e \end{pmatrix}_R, \begin{pmatrix} \nu_e \\ M \end{pmatrix}_R, \tag{1.3a}$$

where N_μ is a heavy Majorana lepton. We shall accept the plausible identification of the heavy lepton M^- as the U particle[5] (~ 1.8 GeV), which is supposed to be the intermediary of the $e\mu$ events discovered in the high-energy e^+e^- annihilations.[6] (In fact we shall also assume, for definiteness, that the neutral leptons N_M, N_μ have masses in the 1–10-GeV range.)

In this paper we shall concentrate our attention on the leptons of this "vectorlike doublet theory." Some of the outstanding leptonic features, namely induced neutrino masses and lepton-number-nonconservation effects, were described a while back in an unpublished report.[7] Here these results will be presented in some detail and also we shall take up the general question of the lepton mass spectrum and mixing angles.

One of the more conspicuous mysteries about the leptons is the hierarchy of their masses: There are the extremely light (possibly massless) neutrinos[8] and the very light electron; on the other hand, we have the muon and other possible heavy leptons in the general GeV range. Does this doublet vectorlike theory provide a framework for understanding this? We shall demonstrate that a modification of the theory in (1.3a) can yield a plausible interpretation of these vast mass differences.

The modification of (1.3a) is as follows:

$$\begin{pmatrix} \nu_e + \epsilon N_M \\ e \end{pmatrix}_L, \begin{pmatrix} N_M - \epsilon\nu_e \\ M \end{pmatrix}_L, \begin{pmatrix} N_M \\ e \end{pmatrix}_R, \begin{pmatrix} N_e \\ M \end{pmatrix}_R. \tag{1.3b}$$

That is, the right-handed component of M^- is now coupled to another Majorana lepton N_e (of moderate mass, comparable to that of the muon); there is a slight mixing between the left-handed neutral leptons with $\epsilon \simeq \frac{1}{20}$ (see Ref. 9) (viz., the mixing angle ϵ is of a size that makes it plausible that this effect is related to the mass discrepancies between the muon and the heavy leptons). Given this, we show that the following scenario is possible: e, ν_e, and ν_μ are massless in the absence of weak interactions, while all other leptons have masses of $O(1)$ or $O(\epsilon)$ from spontaneous symmetry breaking (e.g., vacuum expectation values of Higgs) and subsequent diagonalization. When the weak interactions in the form of (1.2) and (1.3b) are turned on, the electron becomes massive, of $O(\epsilon\alpha/\pi)$, through second-order radiative processes, and the neutrinos acquire masses of $O(\epsilon^2\alpha^2/\pi^2)$ in the neighborhood of 10 eV or less through the fourth-order corrections. Thus the hierarchy of lepton masses is pictured to have its origin in accidental symmetries at the tree level and successively higher-order radiative corrections.

Our picture of the lepton mass spectrum resembles that of quarks constructed by Zee,[10,11] who was the first to propose a vectorlike theory with six quarks as in (1.1).[10] His basic working premise is that the light quarks u, d, and s are all massless at the tree level and there are small mixings $\theta \simeq O(10^{-1})$ among the left-handed u and t quarks; thus, we make the replacements $u_L \to (u+\theta t)_L$ and $t_L \to (t-\theta u)_L$ in (1.1). Then u, d, and s all acquire masses through the second-order radiative corrections: $m_u = O(m_b\theta\alpha/\pi)$, $m_d = O(m_t\theta\alpha/\pi)$, and $m_s = O(m_c\alpha/\pi)$. If the heavy quarks c, b, and t are of the same order then we obtain $m_u \simeq m_d \simeq \theta m_s$. Zee also introduces mixings between c_L and t_L; thus there are also induced $s \rightleftarrows d$ transitions and the subsequent mass matrix diagonalization leads to Cabibbo angles.

In view of this work, it is satisfying that in the complete vectorlike theory (1.1), (1.2), and (1.3b) we can have a unified picture of mass spectra for all elementary fermions. We can postulate that the same symmetry-breaking mechanism generates masses for the heavy quarks and heavy leptons, giving masses in the 1–10-GeV range; the light fermions then pick up mass through radiative corrections. The principal difference between the spectra of quarks and leptons has its origin in the presence of Majorana particles among leptons—they push the neutrino masses to the fourth order. In fact the presence of heavy Majorana leptons in such a theory is desirable from a number of viewpoints. Some of these will be discussed in the next section. Another notable difference between quark and lepton states is that while there is some mixing between the d and s systems (the Cabibbo angle) the corresponding mixing between ν_e and ν_μ is probably small (see additional remarks below).

Some introductory remarks and motivations for vectorlike theories are discussed in Sec. II. Here we emphasize the connection of a doublet theory to lepton-number nonconservation. Specific reactions such as $\alpha \to \beta\mu^\pm\mu^\pm$ are examined and it is concluded that just about all the lepton-number-violation signals predicted by the theory in Eq. (1.2) will be extremely difficult to detect. In Sec. III we present the fourth-order calculations of the induced neutrino mass. The result, as expected, is formally divergent. In an appendix the possibility that it is a finite and calculable quantity in a theory with an enlarged gauge group is then examined. We found that this can be achieved with minimal distortion of the original theory. The lesson is that to obtain a numerical estimate it is probably reasonable to cut off the infinities encountered in the $SU_2\times U_1$ theory with a mass comparable to that of the intermediate vector boson. This leads to a muon-neutrino mass in the range of a few electron volts.

In Sec. IV, the full lepton theory (1.2) and (1.3) is studied. The question of general lepton mass spectrum and mixing angles mentioned above is taken up. In our new vectorlike theory we expect that one of the two neutrals in the three-body decay products of the 1.8-GeV U particle of Perl *et al* is a Majorana particle having an intermediate mass (100 MeV?) and decaying through a weak-electromagnetic process: $N_e \to \nu_e\gamma$ and $\bar{\nu}_e\gamma$ with a lifetime $\sim 10^{-5}$ sec. Finally we consider neutrino beam oscillation[12,13] and conclude that it is unlikely that the ν_e, ν_μ oscillation length will be short enough for such an effect to be observable in a laboratory setting. Some remarks about induced neutrino mass in a general vectorlike theory form the conclusion of the paper.

II. VECTORLIKE DOUBLET THEORY AND LEPTON-NUMBER NONCONSERVATION

Theories such as (1.1)–(1.3), where for each representation of the gauge group there are equal numbers of left-handed $(V-A)$ and right-handed $(V+A)$ couplings of elementary fermions, are often called "vectorlike."[14] Anomaly divergences are automatically absent in these theories. They also offer the framework for spontaneous parity violation.[15] Six-quark vectorlike theories have also been discussed by other authors.[3] However, only in Refs. 1 and 2 has the lepton-quark symmetry in a vectorlike theory as shown in (1.1)–(1.3) been so emphasized.[16] This symmetry leads to the important relation

$$Q = T_3^{\text{weak}} + \tfrac{1}{2}(B-L), \tag{2.1}$$

where Q is the electric charge operator, T_3^{weak} is the third component of the weak isospin, B is the baryon number, and L is the lepton number. In other words, in this particular $SU_2\times U_1$ theory the U_1 generator is just $(B-L)$. Since T_3^{weak} must be broken, the conservation of charge and baryon number implies that lepton-number conservation must be violated. Thus, in this restricted version of the vectorlike theory there is an intimate connection between the requirements of left-right and quark-lepton symmetries and lepton-number nonconservation.

Before spontaneous symmetry breaking all fermions are presumed to be massless and the Lagrangian has the very attractive feature of being maximally symmetric. Besides being chirally symmetric, it is invariant under space reflection (and it is also natural to assume *CP* invariance at this stage as well). Spontaneous symmetry breaking is supposed to take place (for example, through the Higgs mechanism[17]). The broken symmetries observed in particle physics are then all consequences of "dynamics." (For example, they are due to specific features of the Higgs potential.) The pattern of symmetry breaking is determined by the resultant fermion mass matrix.

Although no explicit Higgs scheme will be constructed here, we presume that whatever is the mechanism of spontaneous symmetry breaking, the end result must be that in the fermion mass matrix there will be scalars and pseudoscalars. The diagonalization of such a matrix brings about apparent axial-vector currents. (The "universal" $V-A$ character of weak interactions is then a low-energy phenomenon. It just so happens that light particles e, ν_e, μ, ν_μ, and u, d, and s are coupled together through their left-handed components, while all $V+A$ currents involve at least one heavy fermion.)

It is further assumed that the zeroth-order symmetry breaking gives rise to a mass term for the heavy "μ-like" lepton of the form

$$(\overline{N}_\mu)_R(N_\mu)^c_R+\text{H.c.}=\overline{N}_\mu(1-\gamma_5)N^c_\mu+\text{H.c.},\qquad(2.2)$$

where N^c_μ is the charge conjugate field of N_μ. Such a term would simultaneously fulfill two desired functions in such a theory: (i) This term manifestly violates lepton-number conservation; it creates or destroys two $(N_\mu)_R$ at the same time. (ii) In this six-lepton vectorlike theory, since right-handed e^-, μ^- must be coupled to heavy leptons and only one [namely, $(N_M)_R$] is available, at least one of them must be coupled to a two-component massive fermion. Expression (2.2) is just a mass term for such a two-component Majorana field.[18] Furthermore, as we shall demonstrate in Sec. IV, only with the presence of Majorana leptons can we obtain a reasonable picture for the lepton mass spectrum: $m_e \gg m_\nu$.

In this and in two subsequent sections we shall mostly be concerned with the muon sector of the theory (1.2), so a simplified notation will be followed ($\nu\equiv\nu_\mu$, $N\equiv N_\mu$):

$$\begin{pmatrix}\nu\\ \mu\end{pmatrix}_L,\quad\begin{pmatrix}N\\ \mu\end{pmatrix}_R.\qquad(2.3)$$

This will be considered as a self-contained theory; namely, we shall at first ignore all leptonic mixing angles, which, on phenomenological ground, must be small. Some comments in this respect will be given in Sec. IV.

Thus, to zeroth order, we assume that the fermion mass term is of the form

$$m_\mu\overline{\mu}_L\mu_R+m_N\overline{N}_RN^c_R+\text{H.c.}\qquad(2.4)$$

To this order ν is a massless two-component fermion. We shall presently show that higher-order radiative corrections will always induce a transition of ν_L to ν^c_L. Hence ν cannot remain massless. In fact, it is a massive Majorana fermion itself. In this sense the induced muon-neutrino mass reflects the lepton-nonconservation feature of the theory. Detailed discussion of this subject will be given later in Sec. III and in the Appendix. Clearly, the most prominent violations of lepton-number conservation involve the massive neutral lepton N itself. We observe the occurrence of

$$N\to\mu^++x^-$$

and

$$N\to\mu^-+x^+$$

when $x^\pm$ is some collection of particles carrying total lepton number zero (e.g., $x^+=e^++\nu_e$). But the trouble is that N will be difficult to produce. It can be produced in muon inclusive scattering experiments, but its cross sections have to compete with the dominant μ electroproductions (i.e., the one-photon exchange processes).

One can also consider other nonconservation effects: The bilinear term $\overline{N}N^c$ in the Lagrangian will induce, as higher-order weak-coupling effects, nonconservation of muon number as well.[7] The physical processes of the type

$$\alpha\to\beta\mu^\pm\mu^\pm\qquad(2.5)$$

can take place, where α and β are some appropriate particle states differing by two units of charge. This comes about through the basic second-order diagram for $W^\pm W^\pm\to\mu^\pm\mu^\pm$ as shown in Fig. 1. From this one can easily work out the decay and scattering processes involving only known particles by attaching various quark or lepton lines to the W bosons (Fig. 2).

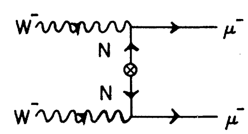

FIG. 1. The diagram which transmits N-lepton nonconservation into nonconservation of muons. The symbol $\otimes$ denotes the $\overline{N}N^c$ mass insertion; it acts as a "sink" or "source" of fermion lines.

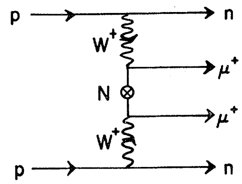

FIG. 2. Lepton-nonconservation effect in the proton-proton (or quark-quark) scattering process.

Among the elementary particle decay processes, the one with the most clear signals[19] is $K^{\pm} \to \pi^{\mp}\mu^{\pm}\mu^{\pm}$. A straightforward calculation yields a branching ratio $\Gamma(K \to \pi^{\mp}\mu^{\pm}\mu^{\pm})/\Gamma(K \to \mu\nu)$ in the neighborhood of 10^{-16}, if we assume that N_μ has a mass of about 2 GeV.[20] The branching ratios for similar processes such as $\Xi^- \to p\mu^-\mu^-$ and $\Sigma^- \to p\mu^-\mu^-$ are all expected to be so small as to make a search for them hopeless.[21] One can also consider scattering processes such as $pp \to \mu^+\mu^+h^0h^0$ (Fig. 2) and $\mu^-p \to \mu^+h^-$ (h^{0-} denote hadrons, or a collection of hadrons with total baryon number 1). But again all the cross sections are so small, making experimental detection impractical.

In the above we have considered reactions corresponding basically to the attachment of quark lines to the W's in Fig. 1. If we attach lepton lines instead we obtain reactions such as

$$\nu\nu \to 2\mu^+2\mu^-. \tag{2.6}$$

This may have some astrophysical implications. They are only two orders down as compared with the basic lepton-number-conserving process of $\nu\nu \to \nu\nu$ which proceeds by exchanging a neutral intermediate vector boson Z in the t channel.

In short, the heavy (~2 GeV) Majorana fermion coupled as in Eq. (2.3) will not lead to any prominent signals of lepton-number violations which car be detected in laboratory experiments. The only hope will be indirect evidence such as neutrino mass and neutrino beam oscillations to be discussed below—but even here we are not optimistic.

III. FOURTH-ORDER CALCULATION OF NEUTRINO MASS

In the vectorlike theory Eq. (2.3) the neutrino cannot stay massless.[7] The presence of N in the intermediate state will induce a $\nu \rightleftharpoons \nu^c$ transition. This gives rise, in the Lagrangian, to a term which is bilinear in the neutrino field, i.e., an effective mass term. This also is an example of a lepton-number-nonconservation effect as discussed in Sec. II. In fact, the fourth-order diagram in Fig. 3 may be obtained by simply joining the muon lines of $\nu\nu \to 2\mu^+2\mu^-$ in (2.6).

Within the context of this $SU_2 \times U_1$ theory of weak and electromagnetic interactions, we obtain the following result:

Figure 3(a) yields

$$\begin{aligned} m_\nu^{(a)} &= -\frac{1}{m_N}\left[\frac{\alpha}{\pi} m_\mu (M_W^2 - m_\mu^2)^{-1}\left(M_W^2 \ln\frac{\Lambda^2}{M_W^2} - m_\mu^2 \ln\frac{\Lambda^2}{m_\mu^2}\right)\right]^2 \\ &\simeq -\frac{1}{m_N}\left(\frac{\alpha}{\pi} m_\mu \ln\frac{\Lambda^2}{M_W^2}\right)^2; \end{aligned} \tag{3.1}$$

Fig. 3(b) yields

$$m_\nu^{(b)} = 2m_N\left(\frac{\alpha}{\pi} m_\mu M_W^{-1}\right)^2 \zeta(2). \tag{3.2}$$

α is the fine-structure constant, m_μ, m_N, and M_W are the masses for μ^-, the heavy lepton, and the vector intermediate boson, and $\zeta(2)$ is the Riemann zeta function and equals 1.644.... The following comments about the computation are in order:

(i) The calculation is done in the Feynman-'t Hooft gauge. Thus, we have not bothered to compute explicitly the corresponding diagrams where the W lines are replaced by those of the Goldstone bosons; they are suppressed by powers of (m_μ/M_W).[22]

(ii) The diagram in Fig. 3(a) is the familiar one one, and is divergent.

(iii) The diagram in Fig. 3(b) is the nontrivial one to calculate. In doing the computation we have used the "hypersphere method."[23] Namely, instead of combining denominators by Feynman parameters, we have proceeded to the Euclidean space directly and performed the four-dimensional angular integrations after expanding the denominators in Gegenbauer polynomials. In presenting the result of Eq. (3.2), we have assumed that all fermion masses are small when compared to M_W. The result is easy to understand: It is finite because there must be three mass insertions on the fermion lines (due to the chiral structure of the vertices); this also explains why we have the mass combination $m_N m_\mu^2$ in the numerator. M_W^{-2} ap-

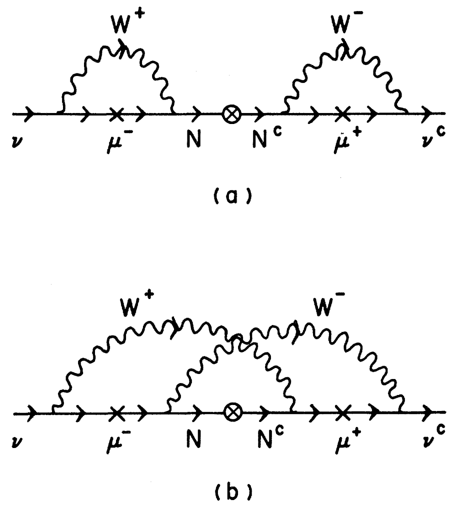

FIG. 3. Feynman diagrams representing the fourth-order contributions to the neutrino mass. The symbol $\otimes$ denotes the $\bar{\mu}_L \mu_R$ mass insertion.

pears then basically because this is the only remaining mass scale.

(iv) $m_\nu^{(a)}$ and $m_\nu^{(b)}$ have opposite signs.

Since the neutrino mass $m_\nu = m_\nu^{(a)} + m_\nu^{(b)}$ is infinite, the usual renormalization procedure says that it is an arbitrary parameter of this theory. Alternatively, we may hope that a natural cutoff will arise in the theory and that the expressions given in Eqs. (3.1) and (3.2) will tell us something about the neutrino mass. One way to implement this idea is to construct a theory such that the neutrino's masslessness in the zeroth order follows from a *natural* symmetry of the theory, and consequently the higher-order corrections must necessarily be finite.[24] In the Appendix, we present a model in which precisely such a phenomenon occurs. What does this exercise of computing radiative correction to "natural" zeroth-order symmetry teach us? For one thing, it demonstrates that a minimal modification of the original theory can lead us to a finite result for m_ν, and furthermore that the result is to a good approximation equivalent to cutting off the divergence in the $SU_2 \times U_1$ theory with a mass comparable to that of the intermediate vector boson. In other words, when it comes to estimating the numerical value of m_ν, it is probably reasonable to expect that the $\ln \Lambda^2 / M_W^2$ factor in Eq. (3.1) is less than 1. For the rest of this paper we shall simply take this to be 1 and regard it as a reasonable upper bound. If we take $m_N \sim 2$ GeV then we expect $m_\nu \lesssim 25$ eV.

IV. LEPTON MASS SPECTRUM AND MIXING ANGLES

In Secs. II and III we have explicitly restricted ourselves to the μ-type particles; that is, we have considered the two μ doublets in Eq. (2.3) a self-contained theory. In this last section we shall extend our discussions to the other leptons.

A. Electron cannot couple significantly to a heavy Majorana lepton

It has been observed that the e-type leptons cannot form a self-contained theory such as Eq. (2.3), namely

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} N_e \\ e \end{pmatrix}_R , \tag{4.1}$$

Where N_e is a massive Majorana fermion. This will lead to phenomenological disaster: $O(G_F^2)$ nuclei no-neutrino double-beta decay can take place.[25,26] (Empirical evidence has been accumulating for years now that such reactions must be severely suppressed.[27]) The basic mechanism for such a reaction is, of course, the same as the one shown in Fig. 1 (or, more directly Fig. 2, with muons replaced by electrons, and where p's and n's are bound protons and neutrons of the nuclei). We note that this is different from the traditional considerations for neutrinoless double-beta decay, where the decay proceeds through the emission and reabsorption of a Majorana neutrino. Although the theory in (4.1) will predict ν_e to be a Majorana neutrino, the contribution from the ν-induced double-beta decay will be completely negligible because the decay amplitude will be proportional to the tiny induced ν mass, i.e., this particular amplitude is in fact of $O(G_F^4)$. On the other hand, for a heavy intermediate state (Figs. 1 and 2) the amplitude is *inversely* proportional to the mass of the Majorana fermion. Indeed, to get a suppression factor compatible with phenomenology we need the neutral lepton to have a mass $\simeq 10^5$ GeV (see Ref. 26)—clearly an unacceptable feature in any reasonable theory.

Consequently, the right-handed electron must couple predominantly to a four-component Dirac particle: The minimal vectorlike theories satisfying this requirement can be either

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} N_M \\ M \end{pmatrix}_L , \begin{pmatrix} N_M \\ e \end{pmatrix}_R , \begin{pmatrix} \nu_e \\ M \end{pmatrix}_R \tag{4.2a}$$

(this is the choice made in Ref. 2) or

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} N_M \\ M \end{pmatrix}_L , \begin{pmatrix} N_M \\ e \end{pmatrix}_R , \begin{pmatrix} N_e \\ M \end{pmatrix}_R \tag{4.2b}$$

(this is basically the theory we will advocate).

We have ignored the possible presence of mixing angles. At this level there is little reason to choose one over the other. ν_e is massless at the tree level, and both theories will induce a higher-order mass (albeit at different orders). Choice (4.2a) yields (ignoring intrinsic fourth-order diagrams which, as we saw in Sec. III, were small)

$$m(\nu_e) \simeq \frac{m(e)m(M)}{m(N_M)}\left(\frac{\alpha}{\pi}\ln\frac{\Lambda^2}{M_W{}^2}\right)^2$$
$$\simeq 3 \text{ eV}. \tag{4.3a}$$

Choice (4.2b) yields (in the same approximation)

$$m(\nu_e) \simeq \frac{1}{m(N_e)}\left[\frac{m(e)m(M)}{m(N_M)}\left(\frac{\alpha}{\pi}\ln\frac{\Lambda^2}{M_W{}^2}\right)^2\right]^2$$
$$\simeq 10^{-13} \text{ eV}. \tag{4.3b}$$

B. Understanding the lepton mass spectrum

We cannot have N_M lighter than the mass of K, so *a priori* all we can say is that it is probably of the same order as M^- mass (it can easily be heavier). Since we have identified $M^\pm$ with the $U^\pm$ particles of Perl *et al.*,[6] i.e.,

$$e^+e^- \to U^+U^-, \quad U^- \to \mu^-\bar{\nu}_\mu\nu_U, \quad U^+ \to e^+\nu_e\bar{\nu}_U,$$

theory (4.2a) would have $\nu_U = \nu_e$ and theory (4.2b) would have $\nu_U = N_e$. Even though the experimental data are still very sketchy, there is no particular evidence for ν_U being very heavy. Since in the following discussion of the lepton mass spectrum an extremely light N_e is not favored either, a reasonable guess will be that the mass of N_e (if such a particle does exist) should be comparable to that of the muon. Thus the lepton masses have the following hierarchical structure:

the heavy leptons, M, $(N_\mu, N_M?) \simeq 2$ GeV,
the medium-heavy leptons, μ, $(N_e?) \simeq 10^{-1}$ GeV,
the electron, $e \simeq 5\times 10^{-4}$ GeV,
the neutrinos, $\nu_\mu, \nu_e \lesssim 10^{-8}$ GeV (?).

The vast mass differences suggest that e and the ν's are massless in the absence of weak interactions, but radiative processes endow them with masses;[28] the electron at second order,[29] and the ν's at fourth order or higher.[30] It is not difficult to see that theories (4.2a) and (4.2b), the way they stand, cannot have such a feature since the massless e_L is coupled to another massless ν_L. We must introduce a small mixing:

$$\begin{pmatrix} \nu_e + \epsilon N_M \\ e \end{pmatrix}_L, \quad \begin{pmatrix} N_M - \epsilon\nu_e \\ e \end{pmatrix}_L. \tag{4.4}$$

However, this scheme cannot work in conjunction with (4.2a) since it would imply that e and ν_e have comparable masses:

$$m(e) \simeq m(N_M)\frac{\epsilon\alpha}{\pi}\ln\frac{\Lambda^2}{M_W{}^2} \tag{4.5}$$

and

$$m(\nu_e) \simeq m(M)\frac{\epsilon\alpha}{\pi}\ln\frac{\Lambda^2}{M_W{}^2}. \tag{4.6a}$$

On the other hand, there is no such problem for scheme (4.2b). Here the result for the electron mass remains as (4.5), but the dominant term for the ν mass is now pushed to the fourth order:

$$m(\nu_e) \simeq \frac{1}{m(N_e)}\left[m(M)\frac{\epsilon\alpha}{\pi}\ln\frac{\Lambda^2}{M_W{}^2}\right]^2. \tag{4.6b}$$

If we take $\epsilon = m(\mu)/m(M) \simeq \frac{1}{20}$, which is small enough to be reasonably safe from phenomenological troubles and reflects our suspicion that the tree-level mixing may have something to do with the tree-level mass ratios, we obtain $m(e) \simeq 0.5$ MeV, $m(\nu_e) \simeq 500$ eV, and of course our previous result, $m(\nu_\mu) \simeq 25$ eV. These are the results if we take all the above mass listings at their face value. The electron-neutrino mass is a bit too large. Just to illustrate the range of variations: If we make another reasonable choice of input parameters such as $m(N_\mu) \simeq m(N_M) \simeq 10$ GeV, $m(N_e) \simeq 0.5$ GeV, and $\epsilon \simeq \frac{1}{50}$ then we have $m(\nu_e) \simeq 20$ eV and $m(\nu_\mu) \simeq 5$ eV. These numbers are not in any direct conflict with experiments.

C. Some characteristics of N_e

It should be noted that in Eq. (4.6b) $m(\nu_e) \propto m^{-1}(N_e)$ a very small value for $m(N_e)$ is definitely not favored. It probably lies in the range 50–500 MeV. When sufficient statistics exist for (μe) events in e^+e^- annihilations, it will be extremely interesting to see if we can determine the mass of ν_U. This should provide us with the first opportunity to distinguish between the two theories (1.3a) and (1.3b).

We note that mixing for the right-handed leptons must also be much smaller, i.e.,

$$\begin{pmatrix} N_\mu + \epsilon' N_e \\ e \end{pmatrix}_R, \quad \begin{pmatrix} N_e - \epsilon' N_\mu \\ M \end{pmatrix}_R, \tag{4.7}$$

where the mixing angle ϵ' must not be larger than 10^{-3} in order to avoid too high a rate for the nuclei no-neutrino double-beta decays discussed in Sec. IV A.

From these considerations we also expect that N_e decays mainly through the weak-electromagnetic process into $\nu_e + \gamma$ (see Ref. 31) (i.e., a photon is attached to the virtual intermediate states that causes the mixing between N_e and ν_e). For $N_e \simeq 100$ MeV, a simple calculation yields[32]

$$\tau(N_e \to \nu_e + \gamma) \simeq 10^{-5} \text{ sec.}$$

For such a mass the decay $N_e \to e^+e^-\nu_e$ is suppressed by the extremely small mixing angle ϵ' to a width of about 10^{-6} of the above dominant radiative decay process.[33]

In view of our previous discussion on the difficulties of producing the Majorana lepton N_μ, it should be noted that if N_e is indeed produced in the e^+e^- annihilations through the intermediaries of $M^\pm$, this will be our only practical access to a heavy Majorana particle. Unfortunately there is no practical method of neutrino detection[34] to observe the Majorana character of N_e by its decay modes

$$N_e \to \nu_e\gamma$$

and

$$N_e \to \bar{\nu}_e\gamma.$$

D. Mixings between μ-type and e-type particles

Finally we examine possible mixings between μ and (e, M) particles, viz., between doublets in (1.2) and (1.3b). Here the principal constraint comes from the extremely tight experimental upper limit for $\Gamma_{\mu\to e\gamma}/\Gamma_\mu^{\text{tot}}(<10^{-8})$. From this we can immediately conclude that there cannot be any significant mixing between $(\nu_\mu)_L$ and $(N_M)_L$. But let us ask about mixing between N_e and N_μ in the right-handed doublets

$$\begin{pmatrix} N_\mu(\theta) \\ \mu \end{pmatrix}_R, \quad \begin{pmatrix} N_e(\theta) \\ M \end{pmatrix}_R, \qquad (4.8)$$

where

$$N_\mu(\theta) = \cos\theta\, N_\mu + \sin\theta\, N_e\,,$$

$$N_e(\theta) = -\sin\theta\, N_\mu + \cos\theta\, N_e\,.$$

This may be possible, since the $\mu \to e\gamma$ reaction is now a fifth-order process; namely a photon is attached to one of the charged lines in the chain:

$$\mu^- \underset{\theta}{\to} (N_e W^-) \to M^- \to (N_M W^-) \to e^-\,.$$

Still a crude estimate informs us that even if θ is not small the resultant rate will be too high unless $\ln(\Lambda^2/M_W^2)$ is significantly less than unity $(<10^{-2})$. In this scheme the radiatively induced $\nu_\mu\nu_e$ mixing comes about by

$$\nu_\mu \to (\mu^- W^+) \underset{\theta}{\to} N_e \to (M^- W^+) \underset{\epsilon}{\to} \nu_e\,.$$

If θ is not small, then indeed the off-diagonal term $\nu_\mu\nu_e$ may not be much smaller than the diagonal terms $\nu_\mu\nu_\mu$, $\nu_e\nu_e$. In summary, our consideration of the mixing problem between μ-e systems leads us to conclude that $\mu \to e\gamma$ can be suppressed either

(i) By having substantial mixing (θ is not small) but $\ln\Lambda^2/M_W^2 \ll 1$; this implies that $m_{\nu_e\nu_\mu} \approx m_{\nu_\mu\nu_\mu} \ll 1$ eV, or

(ii) by having an extremely small θ, so basically there is no mixing between the e-μ system; this implies that $m_{\nu_e\nu_\mu} \ll m_{\nu_\mu\nu_\mu}$.

(Of course, it is possible that both θ and $\ln\Lambda^2/M_W^2$ are small.) In any event, we find that because of the constraint which comes from $\mu \to e\gamma$, the transitional mass between ν_e and ν_μ must be several orders of magnitude less than 1 eV. This has the following implication for "neutrino beam oscillation": Pontecorvo[12] and others first suggested that mixing between the e-μ system will lead to ν_e, ν_μ beam oscillation (much like the K^0-$\bar{K}^0$ oscillation). Fritzsch and Minkowski[13] have recently discussed this phenomenon more specifically within the framework of the vectorlike theory. They suggested that the oscillation length for the reactor ν beam may be as small as 10 m. However, their conclusion was based on the assumptions that there will be significant mixing[35] *and* that the mass difference $m^2(\nu_\mu) - m^2(\nu_e) \simeq O(1 \text{ eV}^2)$. From our discussion we feel that this is much too optimistic—the oscillation length is more likely to be many orders of magnitude longer than 10 m, making its observation in a laboratory setting hopeless. On the other hand, oscillation itself should be a natural feature in such a theory. An expectation of a factor of 2 reduction of the solar neutrino flux[12,13] seems reasonable to us.

E. Neutrino mass in theories of spontaneous parity violation

We conclude this paper with the observation that a neutrino with nonzero mass is the most natural feature in *most* vectorlike theories. So long as there is ν_R or Majorana N_R coupled nontrivially, it is difficult to avoid an induced mass term. The only way known to us to avoid (to all orders) a massive neutrino is to put ν_R in a singlet—so effectively it is decoupled from the weak interaction. The simplest example of such a theory is

$$\begin{pmatrix} \nu(\theta) \\ l \end{pmatrix}_L, \quad N(\theta)_L, \quad \begin{pmatrix} N \\ l \end{pmatrix}_R, \nu_R\,. \qquad (4.9)$$

So we can conclude that the requirement of spontaneously broken parity invariance does not automatically lead to nonzero neutrino mass. But in doublet vectorlike theories such as those discussed in this paper this is a feature almost impossible to avoid.

APPENDIX: FINITE AND CALCULABLE NEUTRINO MASS IN A THEORY WITH AN ENLARGED GAUGE GROUP

Throughout this section we shall use the Higgs mechanism[17] explicitly for spontaneous symmetry breaking. In obtaining a finite and "calculable" neutrino mass we shall follow closely Zee's work on calculating the Cabibbo angle.[10] The basic strategy is due to Georgi and Glashow.[36]

So as not to disturb the symmetric features of the original vectorlike $SU_2 \times U_1$ theory, we shall keep the same pattern of fermions but enlarge the gauge group to $SU_2^L \times SU_2^R \times SU_2^C \times U_1$. (The reason this group gives finite results is given in Ref. 36.) The lepton in Eq. (2.3) now transforms as $(\frac{1}{2}, 0, 0)$ and $(0, \frac{1}{2}, 0)$ representations of the new group. A reflection symmetry between SU_2^L and SU_2^R will be imposed so that our Lagrangian is still invariant under space inversion just as in the original vectorlike theory. In particular, we have the equality between the gauge couplings $g_L = g_R \equiv g$ with $g^2/4\pi = \alpha$. There will also be a host of Higgs mesons giv giving masses to W's the heavy fermions. In particular, there are three sets of mesons,[37] $\chi_1 \sim (\frac{1}{2}, 0, \frac{1}{2})$, $\chi_2 \sim (0, \frac{1}{2}, \frac{1}{2})$, and $\chi_3 \sim (\frac{1}{2}, \frac{1}{2}, 0)$ causing mixing between the three sets of gauge bosons,

$$W_a^\pm = \sum_i T_{ai} W_i^\pm , \tag{A1}$$

where T_{ai} is a rotation matrix with $a = L, R, C$ and $i = \mathrm{I}, \mathrm{II}, \mathrm{III}$, and where $W_i^\pm$ are diagonal with respect to the mass matrix (with masses M_i). In this theory Eqs. (3.1) and (3.2) are modified trivially to read (we have also made a sign change of the $\bar{\nu}\nu^C$ term by a γ_5 transformation)

$$m = \frac{m_\mu^2}{m_N}\left(\frac{\alpha}{\pi}\right)^2 \left\{ \left(\sum_i T_{iL} T_{iR} \ln \frac{\Lambda^2}{M_i^2} \right)^2 - m_N^2 \sum_{ij} T_{iL} T_{jR} T_{iR} T_{jL} \left[(M_i^{-2} + M_j^{-2}) \zeta(2) - \frac{\Delta_{ij}^2}{2M_i^2} + O(\Delta^3) \right] \right\} , \tag{A2}$$

where $\Delta_{ij}^2 \equiv (M_i^2 - M_j^2)^2 / M_i^2 M_j^2$.

The orthogonality condition on the rotation matrix T immediately leads to the expected cancellation of $\ln\Lambda^2$ terms. (This just expresses the fact that the divergent part reflects the underlying symmetric theory where there is no mixing of W_L and W_R.) To display the result in a more transparent manner, we assume that only W_L and W_R mix to form W_{I} and W_{II}, so that Eq. (4.2) now takes on the simple form

$$m_\nu = \frac{m_\mu^2}{m_N}\left(\frac{\alpha}{\pi}\right)^2 c^2 \left[\frac{M_{\mathrm{I}}^2 - M_{\mathrm{II}}^2}{M_{\mathrm{I}}^2 M_{\mathrm{II}}^2} + O\left(\frac{m_N^2}{M^2} \Delta^3 \right) \right] , \tag{A3}$$

where $c \equiv T_{L\mathrm{I}} T_{R\mathrm{I}}$ is some product of the angular factors. The magnitude of m_ν in Eq. (4.3) will of course depend on the details of mixing.

---

*Work supported in part by the National Science Foundation.

[1]H. Fritzsch and P. Minkowski, Caltech Report No. CALT-68-503, 1975 (unpublished).

[2]H. Fritzsch, M. Gell-Mann, and P. Minkowski, Phys. Lett. 59B, 256 (1975).

[3]F. Wilczek, A. Zee, R. L. Kingsley, and S. Treiman, Phys. Rev. D 12, 2768 (1975); A. De Rújula, H. Georgi, and S. L. Glashow, *ibid.* 12, 3589 (1975); S. Pakvasa, L. Pilachowski, W. A. Simmons, and S. F. Tuan, *ibid.* 13, 156 (1976).

[4]For simplicity of notation we have ignored the mixing angles.

[5]If this 1.8-GeV U particle of Perl *et al.* turns out not to be a heavy lepton, all our predictions can be trivially modified when the mass of the M^- lepton is correctly identified.

[6]M. L. Perl *et al.*, Phys. Rev. Lett. 35, 1489 (1975); M. L. Perl, in Proceedings of the 1975 Summer Institute on Particle Physics at SLAC, Report No. SLAC PUB-191, 1975 (unpublished).

[7]T. P. Cheng, Univ. of Missouri—St. Louis report, 1975 (unpublished).

[8]The experimental upper limit of the electron neutrino mass is about 60 eV [Karl-Erik Bergkvist, Nucl. Phys. B39, 317 (1972)]. The muon-neutrino mass is set to be less than 0.23 MeV from π decays. [M. Daum *et al.*, Phys. Lett. 60B, 380 (1976)]. The bounds are much more stringent if we accept cosmological considerations. They are set in the neighborhood of 10 eV. For a recent report see K. Cowsik and J. McClelland, Phys. Rev. Lett. 29, 669 (1972).

[9]Small mixings of this order in no way contradict the known weak-interaction phenomenology. This also holds for possible mixings in the quark sector.

[10]A. Zee, Phys. Rev. D 9, 1772 (1974).

[11]A. Zee, Phys. Rev. D 13, 713 (1976).

[12]B. Pontecorvo, Zh. Eksp. Teor. Fiz. 53, 1717 (1967) [Sov. Phys.—JETP 26, 984 (1968)]; V. Gribov and B. Pontecorvo, Phys. Lett. B28, 493 (1969); see also S. Pakvasa and K. Tennakone, Phys. Rev. Lett. 28, 1415 (1972); Lett. Nuovo Cimento 6, 675 (1973). Neutrino beam oscillation was considered earlier by M. Nakagawa, H. Okonagi, S. Sakata, and A. Toyoda,

Prog. Theor. Phys. 30, 727 (1963).

[13]H. Fritzsch and P. Minkowski, Phys. Lett. 62B, 72 (1976); S. M. Bilenky and B. Pontecorvo [*ibid.* 61B, 248 (1976)] also considered maximal mixing and $|m_{\nu_e}{}^2 - m_{\nu_\mu}{}^2| \simeq 1\ \mathrm{eV}^2$, thus reaching a conclusion similar to that of Fritzsch and Minkowski.

[14]H. Georgi and S. L. Glashow, Phys. Rev. D 6, 429 (1972).

[15]The primary motivation for such consideration is the ultimate unification of all interactions (except gravitation). Since the strong interaction is supposedly described by an unbroken SU_3^{color} gauge theory, it is natural to seek a simple group G which is purely vectorial that will eventually be broken down to the purely vectorial $SU_3^{\text{color}} \times U_1^{\text{proton}}$. Thus a parity-conserving vectorlike theory is very natural for weak interactions from the viewpoint of unified theory.

[16]Again, it is the ultimate unification into a simple group G that motivates this consideration. The absence of a U_1 factor requires $\Sigma Q = 0$, which is satisfied when we have the lepton-quark symmetry in this doublet theory.

[17]P. Higgs, Phys. Lett. 12, 1321 (1964).

[18]Throughout this paper fermions will be described by the two-component Weyl fields with definite helicities and distinct from their antiparticles. Thus we shall not explicitly diagonalize the mass term such as $\overline{N}_R N_R^c$ + H.c. The mass eigenstate of N is the Majorana field $N^m \equiv (1/\sqrt{2})(N + N^c)$ with mass term $\overline{N}^m N^m$. These expressions are related by the Pauli-Gürsey transformation $\psi \to (1/\sqrt{2})(\psi + \gamma_5 \psi^c)$.

[19]A stringent upper bound already exists for $K^\pm \to \pi^\pm e^\pm e^\pm$. See C. Y. Chang *et al.*, Phys. Rev. Lett. 20, 510 (1968).

[20]The calculation can be carried out by using the basic diagram in Fig. 1. In the limit of $m_\mu \approx 0$, we obtain $\Gamma \approx (768\pi^3)^{-1} \sin^2\theta_C G_F{}^4 f_\pi{}^2 f_k{}^2 m_k{}^7 m_k{}^7 m_N{}^{-2}$. This yields a branching ratio of 10^{-15}. In the original calculation (Ref. 7) there was a phase-space error of 16. I am grateful to Dr. P. B. James for pointing this out to me. The finite muon masses reduce the available phase space and push the branching ratio down to 10^{-16}.

[21]See also A. Halprin, P. Minkowski, H. Primakoff, and S. P. Rosen, Phys. Rev. D 13, 2567 (1976).

[22]See, for example, K. Fujikawa, B. Lee, and A. Sanda, Phys. Rev. D 6, 2923 (1972).

[23]See, for example, M. J. Levine and R. Roskies, Phys. Rev. D 9, 421 (1974).

[24]S. Weinberg, Phys. Rev. Lett. 29, 388 (1972); H. Georgi and S. L. Glashow, Phys. Rev. D 6, 2977 (1972).

[25]H. Fritzsch, M. Gell-Mann, and P. Minkowski (Ref. 2); also, T. P. Cheng (Ref. 7).

[26]Halprin, Minkowski, Primakoff, and Rosen (Ref. 21).

[27]For the most recent relevant result, see R. J. Cleveland, W. R. Leo, C.-S. Wu, L. R. Kasday, P. J. Gollon, and J. D. Ullman, Phys. Rev. Lett. 35, 737 (1975).

[28]We consider the alternative that all fermions receive vastly different masses through symmetry breaking as unattractive. In the framework of Higgs phenomena it seems artificial to have such different Yukawa couplings or vacuum expectation values. For dynamical symmetry breaking, where only one mass scale exists, it will be even harder to understand the smallness of the electron mass.

[29]Incidentally, the theory discussed in the Appendix which makes the induced neutrino mass finite and calculable will render the second-order induced electron mass finite also.

[30]One may regard this desire for understanding the fermion mass spectrum as another reason $V+A$ current should be introduced in a weak-interaction theory.

[31]Although $\tau(N_e)$ is longer than a typical weak lifetime, it cannot be a candidate to explain the Kolar gold mine experiments of M. R. Krishnaswany *et al.* [Phys. Lett. B57, 105 (1975)], since these events, if they are genuine signals, correspond to a neutral decay into several charged particles.

[32]The rate $\Gamma(N_e \to \nu_e \gamma) \simeq (\epsilon G_F m_M / 4\pi)^2 m_{N_e}{}^3$ up to an $O(1)$ multiplicative factor, and we have taken $\epsilon^2 \simeq 10^{-3}$ and $m_M \simeq 2$ GeV.

[33]$\Gamma(N_e \to e^+ e^- \nu) \simeq (\epsilon' G_F)^2 m_{N_e}{}^5 / 192\pi^3$, and we have taken $\epsilon' \simeq 10^{-3}$.

[34]ν_e and $\overline{\nu}_e$ can in principle be distinguished from each other by their different interactions with proton targets; only $\overline{\nu}_e$ can bring about an inverse beta process $\overline{\nu}_e p \to n e^+$.

[35]The authors in Ref. 13 stated that large mixing (i.e., $m_{\nu_e \nu_\mu} \simeq m_{\nu_\mu \nu_\mu} \simeq 1$ eV) will not cause a significant rate for $\mu \to e\gamma$. But they only considered the direct contribution of $m_{\nu_e \nu_\mu}$ in the intermediate state. This clearly will lead to an insignificant rate, since $m_{\nu_e \nu_\mu}$ is itself a higher-order process. Our point is simply that whatever mechanism causes ν_e, ν_μ to mix will also cause μ, e mixing at comparable or lower orders. The situation here is analogous to the question of no-neutrino double beta decay discussed in Sec. IV A. The contribution by $m_{\nu_e \nu_e}$ is itself negligible, but the mechanism which induces the mass term will also bring about the neutrinoless $\beta\beta$ decay to the same order.

[36]H. Georgi and S. L. Glashow, Phys. Rev. 7, 2457 (1973).

[37]Details on the Higgs potential may be found in Ref. 10.