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# Broken SU(3) in hyperon magnetic moments

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It is pointed out that certain confusing features of SU(3) in hyperon magnetic moments can be explained by the inclusion of SU(3) $\Lambda$-$\Lambda'$ mixing, where $\Lambda'$ is the SU(3)-singlet ninth $\frac{1}{2}^+$ baryon.

The precise test of SU(3), especially in the fundamental properties of hyperons, is important for a deeper understanding of broken SU(3) symmetry. The Coleman-Glashow (CG) formulas[1] for the hyperon moments were derived using exact-SU(3)-symmetry arguments [thus rendering their interpretation in the real world of broken SU(3) symmetry ambiguous]. In addition, the experimental values of the hyperon moments reveal a rather confusing situation; while the $\Sigma^+$ magnetic moment is consistent with the CG formula, the $\Lambda$ moment which has been measured to good precision deviates significantly from the CG value (measured in *nuclear magnetons*) $\mu_\Lambda = \frac{1}{2}\mu_N = -0.957$. (Although this disagreement between the CG result and experiment is somewhat mitigated by the use of natural magnetons, there is at present no theoretical justification for the use of this system of units.) The present world average[2] gives $\mu_\Lambda = -0.67 \pm 0.06$, whereas the recent Fermilab experiment[3] reports $\mu_\Lambda = -0.57 \pm 0.05$. The measurement of the transition moment $\mu_{\Sigma\Lambda}$ from the $\Sigma \to \Lambda\gamma$ decay has been undertaken and there is now a preliminary experimental result.[4] Unfortunately the errors in the experimental values of $\Sigma^-$ and $\Xi^-$ moments are still too large[5] to draw any definite conclusion.

In this note we present a nonperturbative derivation of broken-SU(3) hyperon-anomalous-magnetic-moment sum rules based on asymptotic SU(3) and current algebra. For other approaches to the sum rules, refer to the papers by Caldi and Pagels and Cheng and Pagels[6] and the references cited therein. We point out that in our particular sum rules (based on the so-called good-good commutators) the violation of the CG sum rules (in the unit of *nuclear magneton*) arises solely from the SU(3) mixings between the hyperons and the higher-lying $\frac{1}{2}^+$ baryons. In particular, the deviations of $\mu_\Lambda$ and $\mu_{\Sigma\Lambda}$ from the CG predictions can be explained in terms of the small but important $\Lambda$-$\Lambda'$ mixing, where $\Lambda'$ is a (still to be found) higher-lying $\frac{1}{2}^+$ baryon belonging to the $L=0$ SU(3) *singlet* which should appear in the popular $qqq$ decomposition $3\otimes 3\otimes 3 = 1\oplus 8\oplus 8\oplus 10$. In the SU(6)$\otimes$O(3) classification the $\Lambda'$ may belong to the 70 $L^P=0^+$ $(1,\frac{1}{2}^+)$.

Our present result acquires greater significance when it is combined with our following recent observation[7] of the possible significant role which may also be played by the $\Lambda'$ in other fundamental SU(3) sum rules involving hyperons: (i) the small but definite discrepancy between the mass-squared Gell-Mann–Okubo (GMO) mass formula of the hyperons and experiment; (ii) the trouble of $\Sigma$-$\Lambda$ degeneracy; and (iii) the deviation of the $D/F$ ratio of the hyperon axial-vector semileptonic couplings from the SU(6) value as exhibited by the recent high-statistics experiment[8] on the $\Sigma^- \to ne^-\bar{\nu}$ decay can be explained simultaneously if a ninth $I=0$ $\frac{1}{2}^+$ $\Lambda'$ exists[7] with a mass around say, 1700 MeV, with a broad width.

The derivation of the sum rules is based on the hypothesis of asymptotic SU(3) and the validity, in broken SU(3), of the chiral SU(3)$\otimes$SU(3) current algebra, especially the SU(3) charge-current commutators

$$[V_i, V_j^\mu(0)] = if_{ijk}V_k^\mu(0). \tag{1}$$

A similar approach was taken by De Alfaro and others[9] without using the notion of asymptotic SU(3). Asymptotic SU(3) makes the dynamical assumption[10] that the linearity of SU(3) transformations is still preserved in broken SU(3) but only in the infinite-momentum limit. The SU(3) multiplet classification carried out in the infinite-momentum limit may still be recognizable in the real world. Namely, if the asymptotic behavior [under SU(3) transformation] of the creation or annihilation operator $a_\alpha(\vec{k}, s)$ of the physical hadron $B_\alpha$ with SU(3) index $\alpha$, mass $m_\alpha$, the helicity $s$ were adequate, the mass-squared Gell-Mann–Okubo (GMO) mass formula including the possible SU(3) mixing would emerge[10] as an exact constraint (rather than a first-order perturbative theoretic formula) for the multiplet $B_\alpha$. The SU(3) sum rules obtained from the chiral SU(3)$\otimes$SU(3) algebra by using asymptotic SU(3) are then valid in the presence of GMO mass splitting.

Thus, according to asymptotic SU(3), we assume for the $a_\alpha(\vec{k}, s)$ of hyperons that

$$[V_i, a_\alpha(\vec{k}, s)] = i\sum_\beta u_{i\alpha\beta}(\vec{k}, s)a_\beta(\vec{k}, s) + \delta u_{i\alpha}(\vec{k}), \tag{2}$$

where

$$\delta u_{i\alpha}(\vec{k}) \to |\vec{k}|^{-(1+\epsilon)}\ (\epsilon > 0)\ \text{ for }\ \vec{k}\to\infty. \tag{3}$$

$\sum_\beta$ should be extended, in principle, over all the higher-lying $\frac{1}{2}^+$ baryons. However, as mentioned, we assume that the most important SU(3)-mixing contribution comes from the $\Lambda'$. From Eq. (3) $a_\Lambda(\vec{k}, s)$ and $a_{\Lambda'}(\vec{k}, s)$ can then be related linearly (only at $\vec{k}\to\infty$) to the SU(3) representation operator $a_8(\vec{k}, s)$ and $a_0(\vec{k}, s)$ by

$$\begin{pmatrix} a_\Lambda(\vec{k}, s) \\ a_{\Lambda'}(\vec{k}, s)\end{pmatrix} = \begin{pmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega\end{pmatrix}\begin{pmatrix} a_8(\vec{k}, s) \\ a_0(\vec{k}, s)\end{pmatrix},\quad \vec{k}\to\infty. \tag{4}$$

By inserting Eq. (1) between the states $\langle B_\alpha(\vec{k}, s)|$ and $|B_\beta(\vec{l}, s')\rangle$ with $\vec{k}\to\infty$ and $\vec{l}\to\infty$, we obtain broken-SU(3) sum rules for the matrix elements of the current $V_i^\mu(0)$, i.e.,

$$\sum_\sigma \langle B_\alpha(\vec{k}, s)|V_i|B_\sigma(\vec{k}, s)\rangle\langle B_\sigma(\vec{k}, s)|V_j^\mu(0)|B_\beta(\vec{l}, s')\rangle - \sum_\rho \langle B_\alpha(\vec{k}, s)|V_j^\mu(0)|B_\rho(\vec{l}, s')\rangle\langle B_\rho(\vec{l}, s')|V_i|B_\beta(\vec{l}, s')\rangle = if_{ijk}\langle B(\vec{k}, s)|V_k^\mu(0)|B_\beta(\vec{l}, s')\rangle.$$

The matrix elements of the vector charge $V_i$ at $\vec{k}\to\infty$ and $\vec{l}\to\infty$ can be parametrized in terms of SU(3) Clebsch-Gordan coefficients and mixing angles [Eq. (4)] by virtue of asymptotic SU(3). However, to derive magnetic-moment sum rules it is more convenient to use, instead of Eq. (1), the commutator involving directly the electromagnetic current $j^\mu(x) = j_3^\mu(x) + (1/\sqrt{3})J_8^\mu(x)$:

$$[[j^\mu(x), V_{K^+}], V_{K^-}] + [[j^\mu(x), V_{\pi^+}], V_{\pi^-}] = 3j^\mu(x). \tag{5}$$

Sandwich Eq. (5) between the $\frac{1}{2}^+$ baryon states $\langle B_\alpha(\vec{k}, s)|$ and $|B_\beta(\vec{l}, s')\rangle$ with $\vec{k}\to\infty$ and $\vec{l}\to\infty$. We take such combinations as $\alpha=\beta=(p, n, \Sigma^+, \Sigma^0, \Sigma^-, \Lambda, \Lambda', \Xi^0, \Xi^-)$, $\alpha=\Sigma^0$ and $\beta=\Lambda$, $\alpha=\Sigma^0$ and $\beta=\Lambda'$, and $\alpha=\Lambda$ and $\beta=\Lambda'$. We then obtain twelve sum rules. Let us define

$$\langle B_\alpha\rangle \equiv \langle B_\alpha(\vec{k}, s)|j^\mu(0)|B_\alpha(\vec{l}, s')\rangle,\quad \langle t\rangle \equiv \langle \Sigma^0(\vec{k}, s)|j^\mu(0)|\Lambda(\vec{l}, s')\rangle,$$

$$\langle t'\rangle \equiv \langle \Sigma^0(\vec{k}, s)|j^\mu(0)|\Lambda'(\vec{l}, s')\rangle,\quad \langle t''\rangle \equiv \langle \Lambda(\vec{k}, s)|j^\mu(0)|\Lambda'(\vec{l}, s')\rangle.$$

For illustration, we write one of the twelve equations obtained for the case $\alpha=\beta=p$:

$$3\langle p\rangle - \langle n\rangle - \tfrac{1}{2}\langle\Sigma^0\rangle - \tfrac{3}{2}\cos^2\omega\langle\Lambda\rangle - \tfrac{3}{2}\sin^2\omega\langle\Lambda'\rangle - \sqrt{3}\cos\omega\langle t\rangle + \sqrt{3}\sin\omega\langle t'\rangle + 3\cos\omega\sin\omega\langle t''\rangle = 3\langle p\rangle,\ \text{etc.} \tag{6}$$

The anomalous magnetic moment $k_\alpha$ of $B_\alpha$ measured, for example, in nuclear magnetons is defined through

$$\langle B_\alpha\rangle = \bar{u}_\alpha(\vec{k}, s)[\gamma^\mu F_1^\alpha(q^2) + k_\alpha(e/2m_p)F_2^\alpha(q^2)i\sigma^{\mu\nu}q^\nu]u_\alpha(\vec{l}, s'),$$

where $q^2 = (k-l)^2$, $F_1^\alpha(0)$ is the charge of the $B_\alpha$, and $F_2^\alpha(0)=1$. [In general, the *total* magnetic moment $\mu_\alpha$ is given by $\mu_\alpha = (m_p/m_\alpha)F_1^\alpha(0) + k_\alpha$.] For the transition moment we also define (in the *same* unit), for example,

$$\langle t\rangle = \bar{u}_{\Sigma^0}(\vec{k}, s)[\gamma^\mu F_1^{\Lambda\Sigma}(q^2) + k_{\Sigma\Lambda}(e/2m_p)F_2^{\Sigma\Lambda}(q^2)i\sigma^{\mu\nu}q^\nu + \cdots]u_\Lambda(\vec{l}, s')$$

with $F_2^{\Sigma\Lambda}(0)=1$ and $F_1^{\Lambda\Sigma}(0)=0$. We now take the limit $\vec{l}\to\infty$, $k_z = \lambda|\vec{l}|$, and $\lambda>0$ ($\vec{l}$ is taken along the $z$ axis and $\vec{k}$ lies in the $zx$ plane, i.e., $k_y=0$). Since $\mu=0$ or $z$ usually give[9] the most reliable results (good-good commutator), we will restrict our study to the $\mu=0$ sum rules.[11] $\mu=0$ and $\mu=z$ give identical results. We then note that ($|\vec{l}|\to\infty$) for $s=\frac{1}{2}$ and $s'=-\frac{1}{2}$,

$$\langle B_\alpha(\vec{k}, \tfrac{1}{2})|j^0(0)|B(\vec{l}, -\tfrac{1}{2})\rangle \simeq (\lambda|\vec{l}|^2)^{1/2}[-k_\alpha(e/2m_p)(1+\lambda)\lambda^{-1}k_x F_2^\alpha(q_\alpha^{\,2})],$$

where $q_\alpha^{\,2} = -(1-\lambda)^2\lambda^{-1}m_\alpha^{\,2} - k_x^{\,2}\lambda^{-1}$ and, for example,

$$\langle \Sigma^0(k, \tfrac{1}{2})|j^0(0)|\Lambda(\vec{l}, -\tfrac{1}{2})\rangle \simeq (\lambda|\vec{l}|^2)^{1/2}[-k_{\Sigma\Lambda}(e/2m_p)(1+\lambda)\lambda^{-1}k_x F_2^{\Sigma\Lambda}(q_{\Sigma\Lambda}^{\,2})]$$

where $q_{\Sigma\Lambda}^{\,2} = m_\Sigma^{\,2}(1-1/\lambda) + m_\Lambda^{\,2}(1-\lambda) - k_x^{\,2}\lambda^{-1}$.

Inserting these expressions into the 12 equations [Eq. (6), etc.], we obtain an infinite set of sum rules (for given $\lambda$ and $k_x$) among the form factors $F_2^\alpha(q_\alpha^{\,2})$, $F_2^{\Sigma\Lambda}(q_{\Sigma\Lambda}^{\,2})$, $F_2^{\Sigma\Lambda'}(q_{\Sigma\Lambda'}^{\,2})$, and $F_2^{\Lambda\Lambda'}(q_{\Lambda\Lambda'}^{\,2})$, which are valid for the present approximation in which only the $\Lambda$-$\Lambda'$ mixing is taken into account.

The broken-SU(3) anomalous-magnetic-moment sum rules are the special cases of these sum rules for which we take the limit $\lambda\to 1$ and $k_x\to 0$ to produce $q_\alpha^{\,2} = q_{\Sigma\Lambda}^{\,2} = q_{\Sigma\Lambda'}^{\,2} = q_{\Lambda\Lambda'}^{\,2} = 0$. We then obtain

$$k_{\Lambda'} = (-1 + \tfrac{3}{2}\sin^2\omega)k_N + \frac{2}{\sqrt{3}}\cos\omega k_{\Sigma\Lambda}, \tag{7}$$

$$k_\Lambda = \tfrac{1}{2}k_n - k_{\Lambda'}, \tag{8}$$

$$k_{\Sigma^+} = k_p, \quad k_{\Sigma^0} = -\tfrac{1}{2}k_n, \quad k_{\Sigma^-} = -k_p - k_n, \tag{9}$$

$$k_{\Xi^0} = k_n, \quad k_{\Xi^-} = -k_p - k_n,$$

and

$$k_{\Sigma\Lambda'} = \left(\frac{\sqrt{3}}{4\sin\omega}\right)[(3\sin^2\omega - 1)k_n + 2k_\Lambda], \tag{10}$$

$$k_{\Lambda\Lambda'} = \frac{-1}{4\cos\omega\sin\omega}[(1-\sin^2\omega)k_n - 2(1 - 2\sin^2\omega)k_\Lambda]. \tag{11}$$

Equations (7) and (8) represent the modification of the CG sum rules due to the $\Lambda$-$\Lambda'$ mixing considered. Except for these modifications the CG sum rules should be valid (under the present approximation) for the *anomalous* magnetic moment and transition moments measured in the *same* unit of nuclear magneton. However, as will be mentioned later, there will be a further effect of mixing which is less important than the $\Lambda$-$\Lambda'$ mixing considered.

For deriving only the magnetic-moment sum rules we can also use, instead of Eq. (5), the charge-charge commutator $[[D_+, V_K^+], V_{K^-}] + [[D_+, V_{\pi^+}], V_{\pi^-}] = 3D_+$ where $D_+ \equiv (-i/\sqrt{2})\int (x+iy) j^0(\vec{x})d^3x$ is the electric-dipole-moment operator. Note that $\langle B_\alpha(\vec{k}, \frac{1}{2})|D_+|B_\alpha(\vec{k}, -\frac{1}{2})\rangle$ is proportional to $k_\alpha$ for $\vec{k} \to \infty$. By using essentially this argument Matsuda also obtained[12] Eqs. (7)–(9) in the absence of the $\Lambda'$.

If we take the present value of the world average of $\mu_\Lambda$ (including Ref. 3) $\mu_\Lambda = -0.611 \pm 0.039$, Eq. (8) implies that $\mu_{\Lambda'} \simeq -0.346 \pm 0.039$. The ratio of $\tau_{\Sigma^0}$ of the $\Sigma \to \Lambda\gamma$ decay to its SU(3) value[13] $\tau_{\Sigma^0}[\mathrm{SU}(3)] = 0.8 \times 10^{-19}$ sec (based on $k_{\Sigma\Lambda} = [-(3/2)^{1/2}]k_n$) is given by

$$\tau_{\Sigma^0} = \left(\frac{1 - \frac{3}{2}\sin^2\omega}{\cos\omega} + \frac{\mu_{\Lambda'}}{\mu_n\cos\omega}\right)^{-2} \tau_{\Sigma^0}[\mathrm{SU}(3)]. \tag{12}$$

Since $\omega$ is small, to a good approximation $\tau_{\Sigma^0} \simeq (1 + \mu_{\Lambda'}/\mu_N)^{-2}\tau_{\Sigma^0}[\mathrm{SU}(3)]$. In Ref. 7, we have found that $\omega \simeq 6.2°$ when no $q^2$ dependence was introduced for the couplings of semileptonic hyperon decays.[14] The GMO mass formula is

$$\sin^2\omega[(\Lambda')^2 - (\Lambda)^2] = \tfrac{1}{3}\{2[(n)^2 + (\Xi)^2] - [(\Sigma^0)^2 + 3(\Lambda^0)^2]\}.$$

For $\omega \simeq 6.2$ we find[14] $\tau_{\Sigma^0} = 0.584 \times 10^{-19}$ sec. Present preliminary experiment[4] gives $\tau_{\Sigma^0} = (0.63 \pm 0.30) \times 10^{-19}$ sec. Conversely, in terms of $\tau_{\Sigma^0}$, $k_\Lambda$ is given by

$$k_\Lambda = \tfrac{1}{2}k_n\left\{1 - 2\cos\omega\left[\left(\frac{\tau_{\Sigma^0}[\mathrm{SU}(3)]}{\tau_{\Sigma^0}}\right) - \left(\frac{1 - \frac{3}{2}\sin^2\omega}{\cos\omega}\right)\right]\right\} \tag{13}$$

$$\simeq \tfrac{1}{2}k_n\left\{1 - 2\left[\left(\frac{\tau_{\Sigma^0}[\mathrm{SU}(3)]}{\tau_{\Sigma^0}}\right) - 1\right]\right\}.$$

With $\tau_{\Sigma^0} = 0.63 \times 10^{-19}$ sec we obtain $\mu_\Lambda = -0.693$ and $\mu_{\Lambda'} = -0.263$.

We, therefore, see that values of $\mu_{\Lambda'}$ around $-0.2$ to $-0.3$, which explain the deviation of $\mu_\Lambda$ from the CG result, are consistent with the nominal value of the latest (though preliminary) experiment on $\tau_{\Sigma^0}$. Therefore, a reduction in the error of $\tau_{\Sigma^0}$ is certainly desired. The sign and magnitude of $\mu_{\Lambda'}$ seem reasonable. Our result indicates that the $\langle\text{singlet}|j^\mu|\text{octet}\rangle$ coupling is comparable with the usual $\langle\text{octet}|j^\mu|\text{octet}\rangle$ coupling. In fact, using the value $\mu_\Lambda \simeq -0.611$ we find that for $\omega \simeq 6.2°$ $k_{\Sigma\Lambda'} \simeq 2.52$ and $k_{\Lambda\Lambda'} \simeq 1.63$ from Eqs. (10) and (11), respectively. This is certainly in line with our observation in Ref. 7 that among the hyperon semileptonic axial-vector couplings, the $\langle\text{singlet}|A_i|\text{octet}\rangle$ couplings are comparable with the $\langle\text{octet}|A_i|\text{octet}\rangle$ couplings which also imply that singlet-octet baryon–ps-meson couplings $\simeq$ octet-octet baryon–ps-meson couplings. Then it is expected that a value of $\mu_{\Lambda'}$ of the order of $-0.2$ to $-0.3$ is induced through the mixing characterized by the strength $\sin\omega \simeq 0.1$. Other, possibly significant, SU(3) mixing (though certainly smaller than the $\Lambda$-$\Lambda'$ mixing) is the one with the decuplet,[7] i.e., with the 70 $L^P = 0^+$ $(10, \frac{1}{2}^+)$. The octet-octet mixing will be less important.[7] For the octet-decuplet mixing, the mixing takes place *only* for the $\Sigma$ and $\Xi$. At present the experimental value of $\mu_{\Sigma^+}$, $2.62 \pm 0.41$, is *consistent* with our prediction of 2.58 from Eq. (9), but the experimental error is still large $\simeq 16\%$. If $\mu_{\Sigma^+}$ is indeed reasonably close to our value (i.e., the effect of the above-mentioned octet-decuplet mixing on the $\Sigma^+$ moment is *not* large) we expect that $\mu_{\Sigma^-}$, $\mu_{\Xi^-}$, and $\mu_{\Xi^0}$ will also be close to our predicted values, $\mu_{\Sigma^-} = -0.66$, $\mu_{\Xi^-} = -0.59$, and $\mu_{\Xi^0} = -1.91$. [$k_{\Sigma\Lambda}$ will also be slightly modified from the prediction Eq. (7) by the octet-decuplet mixing.] More precise measurements on $\Sigma$ and $\Xi$ moments will be enlightening. The study of the effect of the $\Lambda'$ in the other processes such as the hyperon nonleptonic decays and hyperon production reactions should also be interesting.

Finally we add our simple predictions on broken SU(3) in the hyperon $F_2$ form factors. With $k_x \to 0$ and $\lambda = 1$, Eq. (6) predicts,[15] for example,

$$k_{\Sigma^+}F_2^{\Sigma^+}(q^2) = k_pF_2^p\left(\frac{m_p^2}{m_{\Sigma^+}^2}q^2\right), \tag{14}$$

$$k_{\Sigma^-}F_2^{\Sigma^-}(q^2) = -k_p F_2^p\left(\frac{m_p^2}{m_{\Sigma^-}^2}q^2\right) - k_n F_2^n\left(\frac{m_n^2}{m_{\Sigma^-}^2}q^2\right),$$

etc. (15)

We hope that these predictions can be tested in the not too distant future.

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[7]M. D. Slaughter and S. Oneda, Phys. Rev. D 14, 799 (1976); 14, 1314 (1976). The precise value of the mass of $\Lambda'$ is sensitive even to the SU(2) mixing. However, the value of the mixing angle is stable and $\sin\omega \simeq 0.1$.

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[10]S. Oneda and Seisaku Matsuda, *Fundamental Interactions in Physics*, 1973 Coral Gables conference, edited by B. Kurşunoğlu and A. Perlmutter (Plenum, New York, 1973), p. 175, S. Oneda, H. Umezawa, and Seisaku Matsuda, Phys. Rev. Lett. 25, 71 (1970). A simple model which illustrates the use of asymptotic SU(3) in conjunction with the chiral SU(3)⊗SU(3) algebras is as follows. Consider a nonet of $0^{-+}$ mesons with nondegenerate masses [SU(2) is assumed exact for simplicity] and an octet-singlet mixing interaction. The Lagrangian is given by $L = L_0 + L_{\text{int}}$, where $L_0 = \frac{1}{2}\sum_{i=0}^{8}\partial_\mu\phi_i\partial_\mu\phi_i$ and $L_{\text{int}} = \frac{1}{2}\sum_i m_i^2\phi_i^2 + g\phi_0\phi_8$. One can explicitly verify that the nonlinear terms $\delta u_{i\alpha}(\vec{k})$ in Eq. (2) of the text, which involves only the creation operators of the *physical* particles in this model, vanish as $1/|\vec{k}|^2$ for $\vec{k}\to\infty$.

[11]For the case $\mu = x, y$ (good-bad commutator) the masses of baryons will appear in the sum rules and the sum rules become more sensitive to the neglect of higher-lying $\frac{1}{2}^+$ states.

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[14]With $q^2$ dependence $\omega \simeq 4.9°$ and $\tau_{\Sigma^0} = 0.580\times 10^{-19}$ sec.

[15]The noncollinear limit $k_x \neq 0$, $\lambda\to 1$ tends to generate exact SU(3)-like relations $k_{\Sigma^+}F_2^{\Sigma^+}(-k_x^2) = k_p F_2^p(-k_x^2)$, etc. However, with the inclusion of the octet-decuplet mixing mentioned in the text, this relation will be modified and will no longer contradict Eq. (15) which will also be (slightly) modified. In general, the above noncollinear limit is highly sensitive to mixing.