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Possibility of a ninth $J^P = \frac{1}{2}^+$ baryon. II

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We demonstrate that the hypothesis of asymptotic level realization of SU(3) in the algebra $[A_i, A_j] = if_{ijk}V_k$ added to the assumption that a ninth $I = 0$, $J^P = \frac{1}{2}^+$ baryon Λ' exists with a mass around 1700 MeV implies that deviations from the good results of SU(6) or SU(6)$\otimes$O(3) schemes can be derived theoretically, without invoking the notion of SU(6) symmetry. The Cabibbo angles are determined to be $\sin\theta_V = 0.227 \pm 0.008$ and $\sin\theta_A = 0.224 \pm 0.037$. We find that $D/(D+F) \simeq 0.678$. Hyperon semileptonic decays in the presence of the Λ' are analyzed in detail. Partial widths for several $\frac{3}{2}^+ \to \frac{1}{2}^+ + \pi$ decays and branching ratios for the strong decay modes of the Λ' are discussed.

I. INTRODUCTION AND SUMMARY

In our previous paper[1] (cited as I), (a) chiral SU(3)$\otimes$SU(3) charge algebra,[2] (b) asymptotic SU(3) symmetry,[3-5] (c) "exotic" commutation relations[3] (CR's), and the existence of a ninth $I=0$, $J^P=\frac{1}{2}^+$ baryon were used to show that several types of SU(6) problems[1] had a common solution. Here we demonstrate that the addition of a new theoretical constraint (d) —the hypothesis[3,6] of asymptotic level realization of SU(3) in the algebra $[A_i,A_j] = if_{ijk}V_k$ $(i,j,k=1,2,\ldots,8)$—allows us to derive theoretically the deviations from the good results of SU(6) or SU(6)$\otimes$O(3) symmetry schemes *without* invoking the notion of exact SU(6) symmetry. We emphasize that the use of asymptotic level realization permits a theoretical derivation (with a minimum of experimental input) of the successful but phenomenological fit to the semileptonic hyperon decays discussed in paper I. We briefly explain the hypothesis. Upon insertion of this CR between the baryon states $\langle B_{\alpha,s}(\vec{q},\lambda)|$ and $|B_{\beta,s}(\vec{q},\lambda)\rangle$ belonging to the same SU(3)-irreducible representation [α, β, and λ denote the physical SU(3) indices, i.e., Σ^+, Λ, Λ', etc. and helicity, and s denotes the J^P and other quantum numbers], we obtain taking the limit $\vec{q}\to\infty$

$$\lim_{\vec{q}\to\infty}\langle B_{\alpha,s}(\vec{q},\lambda)|[A_i,A_j]|B_{\beta,s}(\vec{q}',\lambda)\rangle = if_{ijk}\lim_{\vec{q}\to\infty}\langle B_{\alpha,s}(\vec{q},\lambda)|V_k|B_{\beta,s}(\vec{q}',\lambda)\rangle \equiv g_{\alpha\beta}, \quad (1.1)$$

where the $g_{\alpha\beta}$ are pure numbers according to asymptotic SU(3). As one varies in Eq. (1.1) the SU(3) indices α and β (for a given s), the ratios of the $g_{\alpha\beta}$'s may be regarded as representing the asymptotic SU(3) contents of the CR $[A_i,A_j]=if_{ijk}V_k$. We write the left-hand side of Eq. (1.1) as a sum over the sets of the intermediate single-particle states sandwiched between the factors A_i and A_j. The hypothesis[3,6] is that these sets of intermediate states can be grouped into the levels $R_0, R_1, \ldots$ such that *each* R_i *separately* realizes the asymptotic SU(3) ratios of the $g_{\alpha\beta}$'s. The success of this hypothesis will depend in large measure on whether there exists an approximate asymptotic higher symmetry for hadrons or not. With the quark model (in which the baryons have a qqq structure) as a guide, the R_i may be distinguished by a simple angular-momentum classification. Thus, R_0 is characterized by $L=0$, R_1 by $L=1$, etc. In this paper we take the ground state R_0 as consisting of the $J^P=\frac{1}{2}^+$ singlet Λ', the $J^P=\frac{1}{2}^+$ octet (N,Λ,Σ,Ξ), and the $J^P=\frac{3}{2}^+$ decuplet ($\Delta,\Sigma^*,\Xi^*,\Omega$). We neglect the possible existence of higher-lying $L^P=0^+$ states. The motivation to include the Λ' comes, of course, from the fact that qqq, i.e., $3\otimes3\otimes3=1+8+8+10$, involves a singlet. Without considering the existence of the Λ', Matsuda and Oneda[5] have indeed derived under this hypothesis the SU(6) result $D/F=\frac{3}{2}$, etc., but *without* encountering the clearly bad result of SU(6), i.e., $(G_A/G_V)_\beta=\frac{-5}{3}$.

We note that the formalism presented here has one degree of freedom *less* than the methods of I [i.e., the theoretical model presented here has the same number of degrees of freedom as in an exact SU(3) two-angle Cabibbo analysis], resulting from the addition of the theoretical constraint—the asymptotic level realization of SU(3) in the CR $[A_i,A_j]=if_{ijk}V_k$. In Sec. II, we use the broken SU(3) sum rules developed in I in combination with the "realization" constrainst derived with the aid of our new theoretical constraint (d) plus some of the latest hyperon semileptonic data[7,8] to fix the mass of the Λ' and to determine the axial-vector and vector Cabibbo angles θ_A and θ_V, respectively. We find that $\Lambda'=1.736$ GeV (we use $m_{\Lambda'}\equiv\Lambda'$, etc.), $\sin\theta_V=0.227\pm0.008$, and $\sin\theta_A=0.224\pm0.037$. Using the calculated values of Λ', θ_A, and θ_V, we obtain and present in Sec. III axial-vector matrix elements and branching ratios for various hyperon semileptonic decays. In Sec. IV we calculate the values of the newly defined D and F couplings, the D/F ratio, and also the coupling G^* between the $\frac{3}{2}^+$ decuplet and $\frac{1}{2}^+$ octet, defined by

$$G^{*2} \equiv \lim_{\vec{q}\to\infty} \langle p(\vec{q})|A_{\pi^+}|\Delta^0\rangle\langle\Delta^0|A_{\pi^-}|p(\vec{q}')\rangle.$$

We find that $D = 0.678$, $F = 0.322$, $D/F = 2.101$, and $G^{*2} = 0.455$. We also find that the fractional contribution of the ground state R_0 (denoted by f_0) in Eq. (1.1) is $f_0 \simeq 65\%$. In Sec. V, using our value for G^{*2}, normalizing $\Gamma(\Sigma^{*+}\to\Lambda\pi^+)$ to 28 MeV, and using p^3 (p is the decay momentum in the c.m. frame) averaged over the mass distribution for the particular decay mode under consideration, we calculate partial widths, which are in good agreement with experiment,[9] for several $\frac{3}{2}^+ \to \frac{1}{2}^+ + \pi$ decays.

II. ASYMPTOTIC LEVEL REALIZATION OF SU(3), THE MASS OF Λ′, THE AXIAL-VECTOR CABIBBO ANGLE, AXIAL-VECTOR MATRIX ELEMENTS, AND HYPERON SEMILEPTONIC PARTIAL WIDTHS

We single out for illustration the CR ($A_{\pi^+} = A_1 + iA_2$, etc.)

$$[A_{\pi^+}, A_{\pi^-}] = 2V_{\pi^0} \qquad (2.1)$$

for the study of Eq. (1.1), choose $B_{\alpha,s}(\vec{q},\lambda)$ and $B_{\beta,s}(\vec{q},\lambda)$ to be the $\frac{1}{2}^+$ octet, and vary the indices α and β. The independent "realization" equations are obtained, for example, by taking $\alpha=\beta=p$, Σ^+, and Ξ^0. Then with $R_0 = \{\frac{1}{2}^+ \text{ octet}, \frac{3}{2}^+ \text{ decuplet}, \Lambda'\}$ we obtain in the limit $\vec{q}\to\infty$

$$|\langle p|A_{\pi^+}|\Lambda\rangle|^2 + |\langle p|A_{\pi^+}|\Delta^0\rangle|^2 - |\langle p|A_{\pi^-}|\Delta^{++}\rangle|^2 = f_0, \qquad (2.2)$$

$$\tfrac{1}{2}\{|\langle\Sigma^+|A_{\pi^+}|\Lambda\rangle|^2 + |\langle\Sigma^+|A_{\pi^+}|\Lambda'\rangle|^2 + |\langle\Sigma^+|A_{\pi^+}|\Sigma^0\rangle|^2 + |\langle\Sigma^+|A_{\pi^+}|\Sigma^{*0}\rangle|^2\} = f_0, \qquad (2.3)$$

$$|\langle\Xi^0|A_{\pi^+}|\Xi^-\rangle|^2 + |\langle\Xi^0|A_{\pi^+}|\Xi^{*-}\rangle|^2 = f_0, \qquad (2.4)$$

where f_0 is the fractional contribution of R_0 to the sum over the intermediate states in Eq. (2.1). From Sec. II of I, we have the following asymptotic SU(3) parametrization ($\vec{q}\to\infty$) of the axial-vector matrix elements. We note that we have used for the parametrization not only the CR $[V_i, A_j] = if_{ijk}A_k$ but also the constraints obtained from the exotic CR's $[\mathring{V}_{K^0}, A_{\pi^-}] = 0$, etc.:

$$\langle p(\vec{q})|A_{\pi^+}|n\rangle = G\sqrt{2}\,(D+F), \qquad (\text{I}.2.19)$$

$$\langle\Sigma^+(\vec{q})|A_{\pi^+}|\Sigma^0\rangle = \frac{2G}{\epsilon'}F, \qquad (\text{I}.2.20)$$

$$\langle\Sigma^+(\vec{q})|A_{\pi^+}|\Lambda\rangle = \frac{2}{\sqrt{3}}\,\frac{G}{\epsilon'(\beta-1)\cos\omega}D, \qquad (\text{I}.2.20)$$

$$\langle\Sigma^+|A_{\pi^+}|\Lambda'\rangle = \beta\cot\omega\langle\Sigma^+(\vec{q})|A_{\pi^+}|\Lambda\rangle + \frac{\delta'\beta'}{\sin\omega}\langle p(\vec{q})|A_{\pi^+}|n\rangle, \qquad (\text{I}.2.18)$$

where

$$\beta \equiv \frac{(\Sigma^0)^2-(\Lambda)^2}{(\Sigma^0)^2-(\Lambda')^2}, \quad \beta' \equiv \frac{(\Sigma^0)^2-(n)^2}{(\Sigma^0)^2-(\Lambda')^2},$$

$$\delta' \equiv \frac{2}{3}\left[1 - \frac{(p)^2-(\Sigma^+)^2}{(n)^2-(\Sigma^0)^2}\right] \simeq 0,$$

$$\epsilon' \equiv [1-(\tfrac{3}{2})^{1/2}\delta'\beta']^{-1} \simeq 1,$$

$D+F\equiv 1$. In exact SU(2) $\delta' = 0$ and $\epsilon' = 1$. If we consider SU(2) breaking then Λ'-Λ-Σ^0 mixing takes place. In this paper we only consider SU(3) Λ-Λ' mixing and neglect the SU(2) mixing. However, we keep SU(2) breaking in the masses (i.e., $\delta'\neq 0$) in order to study a *partial* effect of SU(2) mixing. G is simply a scale factor. In the absence of the Λ' our D and F couplings can be shown to coincide with the familiar D and F couplings of exact SU(3). ω is the SU(3) Λ-Λ' mixing angle which appears in the quadratic Gell-Mann–Okubo (GMO) mass formula as follows:

$$\sin^2\omega[(\Lambda')^2-(\Lambda)^2] = \tfrac{1}{3}\{[(n)^2+(\Xi^0)^2] - [(\Sigma^0)^2+3(\Lambda)^2]\}\equiv\alpha. \qquad (\text{I}.2.15)$$

In addition we parametrize (using the CR $[V_i, A_j] = if_{ijk}A_k$) the matrix elements $\langle\frac{1}{2}^+|A_i|\frac{3}{2}^+\rangle$, in our

TABLE I. Axial-vector matrix elements for $\frac{1}{2}^+ \to \frac{1}{2}^+ + e\nu$ transitions calculated with the mass of $\Lambda' = 1.736$ GeV and the SU(3) Λ-Λ' mixing angle $= 6.4300°$.

Axial-vector matrix element	
$g_{\Sigma^+\Sigma^0}$	+0.5713
$g_{\Sigma^- n}$	−0.4411
$g_{p\Lambda}$	+0.8925
$g_{p\Sigma^0}$	−0.3119
$g_{p\Lambda'}$	+0.4211
$g_{\Lambda'\Sigma^+}$	+0.5924
$g_{\Xi^0\Xi^-}$	−0.4411
$g_{\Sigma^0\Xi^-}$	+0.8832
$g_{\Lambda\Xi^-}$	+0.0908
$g_{\Lambda'\Xi^-}$	−0.5319
g_{np}	+1.2490
$g_{\Sigma^+\Lambda}$	−0.6276

asymptotic limit, by writing

$$\langle p(\vec{q})|A_{\pi^+}|\Delta^0\rangle\langle\Delta^0|A_{\pi^-}|p\rangle \equiv G^{*2}, \tag{2.5}$$

where $\vec{q}\to\infty$. Then Eqs. (2.2), (2.3), and (2.4) become

$$2G^2(D+F)^2 - 2G^{*2} = f_0, \tag{2.6}$$

$$\frac{2G^2F^2}{\epsilon'^2} + \frac{2G^2D^2\alpha_1^{\,2}}{\epsilon'^2} + \frac{\alpha^2}{\sin^2\omega}\alpha_2 + \frac{2GD\beta}{\sqrt{3}\,\epsilon'(\beta-1)} + \frac{G^{*2}}{4} = f_0, \tag{2.7}$$

$$\frac{2G^2(F-D)^2}{\epsilon'^2} - \frac{2\sqrt{3}\alpha_1 G(F-D)}{\epsilon'} + \frac{3\alpha_1^{\,2}}{2} + G^{*2} = f_0, \tag{2.8}$$

where

$$\alpha_1^{\,2} \equiv \frac{1+\beta^2\cot^2\omega}{3(\beta-1)^2\cos^2\omega}, \quad \alpha_2 \equiv \delta'\beta' G\sqrt{2}\,(D+F).$$

With the broken-SU(3) sum rules of I and our parametrizations given by Eqs. (I.2.18), (I.2.19), (I.2.20), and (2.5), Eqs. (2.6) and (2.8) yield, after tedious calculation and assuming that $G^{*2}\neq 0$,

$$\gamma F^2\{R(8-4\sqrt{6}\delta'\beta') + (\delta'\beta')[4\sqrt{6} - 12(\delta'\beta')]\} = 3, \tag{2.9}$$

where

$$\gamma \equiv G^2/G^{*2} \text{ and } R \equiv D/F.$$

Similarly, from Eqs. (2.6) and (2.7), we obtain

$$\gamma F^2\Bigg[R^2\Bigg(2(1-\alpha_1^{\,2}) + (\delta'\beta')\left\{2\sqrt{6}\alpha_1^{\,2} - (\tfrac{8}{3})^{1/2}\left[\frac{\beta}{(\beta-1)\sin^2\omega}\right]\right\} + (\delta'\beta')^2\left\{2\left[\frac{\beta}{(\beta-1)\sin^2\omega}\right] - 3\alpha_1^{\,2} - \frac{2}{\sin^2\omega}\right\}\Bigg)$$
$$+R\Bigg(4 - (\delta'\beta')\left\{(\tfrac{8}{3})^{1/2}\left[\frac{\beta}{(\beta-1)\sin^2\omega}\right]\right\} + (\delta'\beta')^2\left\{2\left[\frac{\beta}{(\beta-1)\sin^2\omega}\right] - \frac{4}{\sin^2\omega}\right\}\Bigg) + \left[2\sqrt{6}(\delta'\beta') - (\delta'\beta')^2\left(3 + \frac{2}{\sin^2\omega}\right)\right]\Bigg] = \tfrac{9}{4}. \tag{2.10}$$

Equations (2.9) and (2.10) imply that

$$R^2[8(1-\alpha_1^{\,2}) + (\delta'\beta')(\alpha_1^{\,2} - d/3)8\sqrt{6} + (\delta'\beta')^2(-12\alpha_1^{\,2} + 8/d\beta)]$$
$$+R[-8 + (\delta'\beta')(-2/3d+3)4\sqrt{6} + (\delta'\beta')^2(8\{2-\beta\}/d\beta)] + [-4\sqrt{6}(\delta'\beta') + (\delta'\beta')^2(24 - 8\{\beta-1\}/d\beta)] = 0, \tag{2.11}$$

TABLE II. Hyperon semileptonic branching ratios: (i) from experiment, (ii) from a one-angle Cabibbo fit, and (iii) from an asymptotic algebraic realization of SU(3) and broken-SU(3) sum rules, where $\Lambda' = 1.736$ GeV.

Decay process	Branching ratio		
	(i) [a]	(ii) [a]	(iii) [b]
$\Sigma^-\to ne^-\bar{\nu}$	$(1.082\pm0.038)\times10^{-3}$	1.07×10^{-3}	1.08×10^{-3}
$\Sigma^-\to n\mu^-\bar{\nu}$	$(4.47\ \pm0.43)\times10^{-4}$	4.95×10^{-4}	4.80×10^{-4}
$\Lambda\to pe^-\bar{\nu}$	$(8.13\ \pm0.29)\times10^{-4}$	8.13×10^{-4}	7.74×10^{-4}
$\Lambda\to p\mu^-\bar{\nu}$	$(1.57\ \pm0.35)\times10^{-4}$	1.34×10^{-4}	1.25×10^{-4}
$\Sigma^-\to\Lambda e^-\bar{\nu}$	$(6.04\ \pm0.60)\times10^{-5}$	6.98×10^{-5}	6.11×10^{-5}
$\Sigma^+\to\Lambda e^+\bar{\nu}$	$(2.02\ \pm0.47)\times10^{-5}$	2.28×10^{-5}	2.00×10^{-5}
$\Xi^-\to\Lambda e^-\bar{\nu}$	$(1.15^{+0.90}_{-0.55})\times10^{-3}$	0.46×10^{-3}	0.41×10^{-3}
$\Xi^-\to\Sigma^0 e^-\bar{\nu}$	$<0.5\times10^{-3}$	...	0.08×10^{-3}
$\Xi^-\to\Lambda e^-\bar{\nu}$ } $\Xi^-\to\Sigma^0 e^-\bar{\nu}$ }	$(0.68\ \pm0.22)\times10^{-3}$	0.55×10^{-3}	0.49×10^{-3}
$\Xi^-\to\Lambda\mu\bar{\nu}$	$<1.3\times10^{-3}$	...	1.11×10^{-4}
$\Xi^-\to\Sigma^0\mu^-\bar{\nu}$	<0.005	...	1.01×10^{-6}
$\Xi^0\to\Sigma^+ e^-\bar{\nu}$	$<1.5\times10^{-3}$	...	0.25×10^{-3}

$\tau_\Lambda = 2.624\times10^{-10}s$ $\tau_{\Sigma^+} = 0.80\times10^{10}s$ $\tau_{\Sigma^-} = 1.482\times10^{-10}s$
$\tau_{\Xi^-} = 1.652\times10^{-10}s$ $\tau_{\Xi^0} = 2.96\times10^{-10}s$

[a] See Ref. 7.
[b] Values presented in this paper.

TABLE III. Axial-vector coupling constant to vector coupling constant, $(G_A)/(G_V)$, ratios: (i) from experiment, (ii) from a one-angle Cabibbo fit, and (iii) from an asymptotic algebraic realization of SU(3) and broken SU(3), where $\Lambda' = 1.736$ GeV.

Decay process	(i) [a]	(ii) [a]	(iii) [b]
$\Sigma^- \to n e^- \bar{\nu}$	$\pm(0.435 \pm 0.035)$	-0.394	-0.435
$\Lambda \to p e^- \bar{\nu}$	0.658 ± 0.054	0.702	0.733
$n \to p e^- \bar{\nu}$	1.250 ± 0.009	1.250	1.250
$\Xi^0 \to \Sigma^+ e^- \bar{\nu}$	...	...	1.232
$\Xi^- \to \Lambda^0 e^- \bar{\nu}$	...	...	0.074
$\Xi^- \to \Sigma^0 e^- \bar{\nu}$	...	...	1.232
$\Xi^- \to \Xi^0 e^- \bar{\nu}$	...	...	-0.441

[a] See Ref. 7.

[b] Values obtained in this paper.

where $d \equiv \alpha/[(\Sigma^0)^2 - (\Lambda)^2]$. However, from Eqs. (I.2.20) we obtain immediately that

$$D/F \equiv R \equiv \sqrt{3}(\beta - 1)\cos\omega \frac{g_{\Sigma^+\Lambda}}{g_{\Sigma^0\Sigma^+}}. \qquad (2.12)$$

With the help of Eqs. (I.2.7) and (I.3.1), Eq. (2.12) becomes

$$R = \frac{\sqrt{6}(\beta - 1)\cos\omega (G_A)_{\Sigma^+\Lambda}}{(G_A)_{np} + (G_A)_{\Sigma^- n}}. \qquad (2.13)$$

Using a computer, it is a simple matter to solve Eqs. (2.11) and (2.13) simultaneously with ω and Λ' [which are related by the GMO formula, Eq. (I.2,15)] and with the values of $(G_A)_{\Sigma^+\Lambda}$, $(G_A)_{np}$, and $(G_A)_{\Sigma^- n}$ given in paper I. We find that

$$\Lambda' = 1.736 \text{ GeV}, \quad \omega = 6.4300^\circ.$$

From the broken-SU(3) rule Eq. (I.4.2) we obtain

$$\sin\theta_A = 0.224 \pm 0.037.$$

In paper I, we obtained previously that

$$\sin\theta_V = 0.227 \pm 0.008.$$

TABLE IV. Branching ratios for the strong decay modes of $\Lambda'(1736)$ calculated using broken-SU(3) decay rate formulas.

Decay mode	Branching ratio
$\bar{K}^0 n$ [a]	0.11
$K^- p$ [a]	0.12
$\Sigma^+\pi^-$ [a]	0.26
$\Sigma^0\pi^0$ [a]	0.26
$\Sigma^-\pi^+$ [a]	0.25
$(\Sigma^*\pi)$ [b]	0.01
$\Gamma(\Lambda' \to \text{all}) \simeq 1.09$ GeV	

[a] See Eq. (I.7.1) for the decay-rate formula. The relevant axial-vector matrix elements are given in Table II.

[b] The decay rate formula is given by Eq. (4.1). The axial-vector matrix elements are presented in Table V.

Using θ_A and θ_V, we can easily compute axial-vector matrix elements and hence $(G_A)/(G_V)$ ratios and partial widths for a variety of hyperon semileptonic decays. The results presented in Tables I, II, III, and IV are in good agreement with the experimental data available. The broad width of the $\Lambda' \simeq 1.1$ GeV [we include the decays $\Gamma(\Lambda' \to \Sigma^*\pi)$] will obviously make its detection difficult.

III. THE D AND F COUPLINGS AND THE D/F RATIO

Using the value of Λ', we obtain from Eq. (2.11) or (2.12) that

$$R \equiv D/F = 2.101,$$

and from $D + F = 1$ and Eq. (2.9) we find that

$$D = 0.678, \quad F = 0.322,$$

and

$$\gamma \equiv G^2/G^{*2} = 1.714.$$

Thus, with $\langle p | A_{\pi^+} | n \rangle = 1.249$ from Table I and Eq. (I.2.19) we find that

$$G^2 = 0.780, \quad G^{*2} = 0.455.$$

Thus, Eq. (2.6) implies that

$$f_0 \simeq 65\%.$$

Thus, the ground state R_0 contributes 65% of the sum over the intermediate states in the CR $[A_{\pi^+}, A_{\pi^-}] = 2V_{\pi^0}$. The general realization of the CR's $[A_i, A_j] = if_{ijk}V_k$ ($[A_{\pi^+}, A_{K^0}] = v_{K^+}$, etc.) does not yield a further constraint.

Previously, Oneda and Matsuda[6]—with the hypothesis that R_0 consisted of the $\frac{1}{2}^+$ octet and the $\frac{3}{2}^+$ decuplet only (i.e., no Λ') (and also without using the constraint $[\dot{V}_{K^0}, A_{\pi^-}] = 0$ which leads to the Σ-Λ degeneracy)—found that γ and D/F attained their exact-SU(6) values $\gamma = \frac{25}{16}$ and $D/F = \frac{3}{2}$ (which are now not in good agreement with experiment) but that $(G_A/G_V)_\beta = \frac{5}{3}\sqrt{f_0}$.

Thus, with $\theta_A = \theta_V$ and $f_0 \simeq 56\%$, they are able to reproduce the experimental result $(G_A/G_V)_\beta = 1.25$. Now our inclusion of Λ' to the ground state R_0 explains the deviation of the D/F ratio from its SU(6) value $\frac{3}{2}$. The latest one-angle fit to the experimental data[7,8] gives (when $D+F$ is normalized to 1)

$$D = 0.651 \pm 0.009, \quad F = 0.349 \pm 0.008.$$

In comparing the experimental data to our results, it should be noted that our D and F couplings are not defined in the usual manner because of the existence of the Λ' [see Eqs. (I.2.20)]. In the absence of the Λ', our D and F couplings coincide with the familiar D and F couplings of exact SU(3). For a complete comparison, the data would have

TABLE V. Axial-vector matrix elements for $\frac{3}{2}^+ \to \frac{1}{2}^+ + P_\gamma$ and $\Lambda' \to \frac{3}{2}^+ + P_\gamma$ transitions, with the mass of Λ' =1.736 GeV and the SU(3) Λ-Λ' mixing angle ω =6.4300°. See Eq. (4.1).

Axial-vector matrix element squared	
${G_{\Lambda'\Sigma^*\pi}}^2$	0.0086
${G_{\Sigma^{*+}\Lambda\pi^+}}^2$	0.6742
${G_{\Sigma^{*+}\Sigma\pi}}^2$	0.2276
${G_{\Delta^{++}p\pi^+}}^2$	1.3656
${G_{\Xi^{*0}\Xi\pi}}^2$	0.4552

TABLE VI. Width prediction for some $\frac{3}{2}^+$ decuplet[a] strong decays with $\Gamma(\Sigma^{*+} \to \Lambda\pi^+)$ normalized to 28 MeV. Experimental values of p^3 averaged over the mass distribution of the decay mode[b] are used in our broken-SU(3) decay-rate formula.[c]

Decay mode	Prediction[c] (MeV)	Experiment[b] (MeV)
$\Delta^{++}(1211) \to p\pi^+$	92.4	99.0 ± 3.6
$\Sigma^{*+}(1381) \to \Sigma^0\pi^0$	2.7	2.8 ± 0.7
$\Sigma^{*+}(1381) \to (\Sigma\pi)^+$	5.8	5.8 ± 1.5
$\Xi^{*0} \to (\Xi\pi)^0$	9.9	9.1 ± 0.5
$\Sigma^{*+} \to \Sigma^+\pi^0$	3.0	...

[a] We take the "pole" value for the mass of Δ^{++}.
[b] See Ref. 9.
[c] See Eq. (4.1).

to be parametrized within the context of our formalism. One should also note, in contrast to the methods of I, that although we have introduced a new coupling G^* into our formalism, *the realization hypothesis adds two constraints*. Thus, our theoretical frameworks have the same number of degrees of freedom as those in the exact SU(3) two-angle Cabibbo analysis. We also note that the realization among the $\frac{3}{2}^+$ decuplet [i.e., the case when $B_{\alpha,s}$ and $B_{\beta,s}$ in Eq. (1.1) belong to the $\frac{3}{2}^+$ decuplet] is automatic even in the presence of the Λ' and produces no further constraints.

IV. $\frac{3}{2}^+ \to \frac{1}{2}^+ + \pi$ DECAY PARTIAL RATES

In calculating the partial rates for the strong decays $\Gamma(\Sigma^{*+} \to \Sigma^0\pi^+)$, $\Gamma(\Sigma^{*+} \to \Sigma^+\pi^-)$, $\Gamma(\Delta^{++} \to p\pi^+)$, and $\Gamma(\Xi^{*0} \to (\Xi\pi)^0)$, we use the broken-SU(3) decay-rate formula[10] given by

$$\Gamma(B \to B' + P_\gamma) = \frac{{G_{BB'P_\gamma}}^2}{4\pi {f_{P_\gamma}}^2} p^3 , \qquad (4.1)$$

where P_γ is a $J^P = 0^-$ meson, f_{P_γ} is the appropriate meson decay constant (we take $f_\pi = 0.132$ GeV, $f_K = 0.157$ GeV), p is the center-of-mass momentum of the final-state baryon, and $G_{BB'P_\gamma}$ is the appropriate axial-vector matrix element. Since ${G_{\Delta^0 p\pi}}^2 = G^{*2} \simeq 0.455$, we can easily calculate other relevant matrix elements via relations derived in our asymptotic limit from

$$\langle \tfrac{3}{2}^+ | [V_i, A_j] | \tfrac{1}{2}^+ \rangle = i f_{ijk} \langle \tfrac{3}{2}^+ | A_k | \tfrac{1}{2}^+ \rangle .$$

For instance,

$$\langle \Delta^{++} | [V_{K^+}, A_{\pi^+}] | \Lambda \rangle = 0$$

yields

$$\langle \Sigma^{*+} | A_{\pi^+} | \Lambda \rangle = -(\tfrac{3}{2})^{1/2} \cos\omega \langle \Delta^0 | A_{\pi^-} | p \rangle ,$$

so that ${G_{\Sigma^{*+}\Lambda\pi}}^2 = \frac{3}{2}\cos^2\omega\, G^{*2}$, etc. We give in Table V values for a few $\frac{3}{2}^+ \to \frac{1}{2}^+ + P_\gamma$ and $\Lambda' \to P_\gamma$ transitions. We normalize $\Gamma(\Sigma^{*+} \to \Lambda\pi^+)$ to 28 MeV and use experimental values for p^3 averaged over the mass distribution for the particular decay mode under consideration.[9] For the mass of Δ^{++}, we take the pole value $\Delta(1211)$.[9,10]

Our results are presented in Table VI and are in good agreement with the data.[9]

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