

## Inclusive hadron spectrum in $e^+e^-$ annihilation as a test for the production and decay of heavy leptons\*

K. J. F. Gaemers and Risto Raitio<sup>†</sup>

*Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305*

(Received 25 March 1976)

The production and subsequent decay into a neutrino and hadrons of a heavy lepton in  $e^+e^-$  colliding beams gives a contribution to the inclusive hadron spectrum  $s d\sigma/dx$ . We calculate this contribution assuming the usual  $V-A$  structure for the weak hadronic current and assuming that the "hadronic" decays of the heavy lepton are dominated by the  $\pi$ ,  $\rho$ ,  $A_1$ , and  $\rho'$  mesons. Owing to the decay into a single pion and a neutrino the inclusive energy spectrum of hadrons has a distinct maximum. The position of this maximum is directly related to the heavy-lepton velocity. Experimental determination of the position of this maximum will therefore determine the heavy-lepton mass. Our calculation also shows that the inclusive hadron spectrum decreases with increasing  $x$  less rapidly than the total measured inclusive hadron spectrum. Measuring the spectrum of a hadron in  $e^+e^- \rightarrow \text{hadron} + e(\mu) + \text{anything}$  and determining the position of the maximum offers a test for the existence of heavy leptons and a way of determining their mass.

### I. INTRODUCTION

A quantity which has been measured extensively in  $e^+e^-$  colliding-beam experiments is the inclusive momentum spectrum of single hadrons.<sup>1</sup> If the anomalous  $e\mu$  events, detected at SPEAR,<sup>2,3</sup> indicate the production of a pair of new fermions, which we call heavy leptons henceforth, the decays of these heavy leptons into a neutrino and hadrons will give a contribution to the inclusive hadron spectrum. Since the measured spectra show a "bulge" at c.m. energies between 4 and 4.4 GeV (see Ref. 1), it is worthwhile to calculate how much of this effect could be due to hadronic decay products of heavy leptons. More important, it is possible to look at this distribution by itself if one considers events where only one muon or electron is present, together with one or more hadrons. The detected lepton then serves as a possible indicator for the production of a heavy-lepton pair, one decaying leptonically the other decaying into a neutrino and hadrons. (We will call this latter mode hadronic decay from now on.)

However, it is to be noted that if at energies above 4 GeV, heavy leptons as well as a new type of hadrons (charm?) are produced, and if these hadrons have an appreciable leptonic and semileptonic branching ratio, these will contaminate the inclusive spectrum from the heavy leptons even if a lepton is detected along with hadrons. However, the calculations in this paper indicate that the hadron spectrum from heavy leptons has several distinct features, which should help to disentangle possible background effects.

Measuring the inclusive hadron spectrum from the decays of heavy leptons will establish the existence of hadronic decay modes. It is necessary to

show the existence of hadronic decay modes if one wants to explain the observation of  $e\mu$  events by heavy leptons.

The general differential cross section for the reaction

$$e^+e^- \rightarrow h + X \quad (h = \pi, K, N, \dots) \quad (1)$$

can be written as<sup>4</sup>

$$\frac{d\sigma}{dx d\Omega} = \sigma_T + \sigma_L + (\sigma_T - \sigma_L)(\cos^2\theta + P^2 \sin^2\theta \cos 2\phi). \quad (2)$$

In this expression  $\sigma_{T,L}$  are positive quantities which depend only on  $x = E_h/E$  and the beam energy  $E$ . The energy of the detected hadron is  $E_h$ . The transverse polarization of the beams is given by  $P$ . The variables  $\theta$  and  $\phi$  are the polar and azimuthal angles of the hadron.

Instead of presenting results as a function of  $E$ ,  $x$ ,  $\theta$ , and  $\phi$ , we will present the behavior of

$$s \frac{d\sigma}{dx}(x, E) = s \int \frac{d\sigma}{dx d\Omega} d\Omega \quad (s = 4E^2) \quad (3a)$$

and

$$\alpha(x, E) = \frac{\sigma_T - \sigma_L}{\sigma_T + \sigma_L}. \quad (3b)$$

We obtain the following results. The spectrum  $s d\sigma/dx$  shows a clear maximum as a function of  $x$ . The position of this maximum is related to the heavy-lepton velocity in a simple way. The mass of the heavy lepton can thus be determined from the position of this maximum.

The calculated spectrum due to a heavy lepton falls slower with increasing  $x$  than the measured total inclusive spectrum. The average number of charged hadrons in the decay of a heavy lepton is

$\approx 1.1$ ; this is less than the expected average charged multiplicity in the decay of charmed hadrons.<sup>5</sup>

The outline of this paper is as follows. In Sec. II we give a general expression for the inclusive hadron spectrum in terms of structure functions for the weak hadronic current. In Sec. III we derive expressions for the decay distributions from specific channels. We assume that these channels are dominated by a single  $\pi$  or a single resonance. For the vector current we assume dominance by the  $\rho$  and the  $\rho'$ . For the axial-vector current we take contributions from a single  $\pi$  and the  $A_1$ . All results are derived retaining the full width of the resonances involved. In Sec. IV we combine the production cross section of a heavy-lepton pair with the obtained decay distributions. We present numerical results on the inclusive  $\pi$  spectrum and discuss briefly the presence of kaons and nucleons.

## II. GENERAL FORMALISM

In order to calculate the inclusive spectrum we will first study the decay of an unpolarized heavy

lepton at rest. The amplitude for the decay

$$U^- \rightarrow \nu_U + h + X \quad (4)$$

is given by

$$T = \frac{G}{\sqrt{2}} \bar{u}(k) \gamma^\alpha (1 - \lambda \gamma^5) u(Q) \langle h(q), X | J_\alpha^{\text{weak}}(0) | 0 \rangle. \quad (5)$$

Here  $k$  and  $Q$  are the four-momenta of the neutrino and heavy lepton, respectively, and  $q$  is the momentum of the observed hadron ( $h$ ). We assume a massless neutrino  $\nu_U$ . The weak hadronic current is assumed to have the well-known  $V-A$  structure. As a consequence the *heavy-lepton* weak current can only be a combination of  $V$  and  $A$ , and we neglect the possibility of  $S$ ,  $T$ , and  $P$  interactions. We keep the relative strength of  $V$  and  $A$  as a free parameter; however,  $V, A$  interference terms cannot appear in the spectrum.

From the amplitude we get the differential decay rate

$$d\omega = \frac{G^2}{8M(2\pi)^2} \frac{d^3q}{q_0} \int \frac{d^3k}{k_0} W_{\alpha\beta}(q, Q-k) \{ (1+\lambda^2) [k^\alpha Q^\beta + Q^\alpha k^\beta - (k \cdot Q) g^{\alpha\beta}] - 2i\lambda \epsilon^{\alpha\beta\gamma\delta} k_\gamma Q_\delta \}, \quad (6)$$

where  $M$  is the heavy-lepton mass and the tensor  $W_{\alpha\beta}$  is defined by

$$W_{\alpha\beta}(q, Q-k) = \sum_X \langle 0 | J_\beta^{\text{weak}}(0) | h(q), X \rangle \langle h(q), X | J_\alpha^{\text{weak}}(0) | 0 \rangle \delta^4(q + P_X + k - Q). \quad (7)$$

In this expression  $P_X$  is the total four-momentum of the hadrons recoiling against the neutrino and the observed hadron  $h$ .

For the tensor  $W_{\alpha\beta}$  we write down a general decomposition into six real structure functions using the variable  $\bar{Q} = Q - k$ :

$$W_{\alpha\beta}(q, \bar{Q}) = - \left( g_{\alpha\beta} - \frac{\bar{Q}_\alpha \bar{Q}_\beta}{\bar{Q}^2} \right) W_1 + \frac{1}{m_h^2} \left( q_\alpha - \frac{q \cdot \bar{Q}}{\bar{Q}^2} \bar{Q}_\alpha \right) \left( q_\beta - \frac{q \cdot \bar{Q}}{\bar{Q}^2} \bar{Q}_\beta \right) W_2 \\ + \frac{i}{M^2} \epsilon_{\alpha\beta\gamma\delta} q^\gamma \bar{Q}^\delta W_3 + \frac{\bar{Q}_\alpha \bar{Q}_\beta}{M^2} W_4 + \frac{1}{M^2} (\bar{Q}_\alpha q_\beta + q_\alpha \bar{Q}_\beta) W_5 + \frac{i}{M^2} (\bar{Q}_\alpha q_\beta - q_\alpha \bar{Q}_\beta) W_6. \quad (8)$$

The structure functions  $W_i$ ,  $i=1 \dots 6$  are functions of  $q \cdot \bar{Q}$  and  $\bar{Q}^2$ .  $W_3$  and  $W_6$  are nonzero only if there is  $V, A$  interference;  $W_4$  and  $W_5$  are nonzero if the currents contributing are not conserved. If we combine (6) and (7) the expression for the decay rate becomes

$$d\omega = \frac{G^2}{8M(2\pi)^2} \frac{d^3q}{q_0} \int \frac{d^3k}{k_0} \left\{ (1+\lambda^2) \left[ k \cdot Q \left( 2 + \frac{M^2}{\bar{Q}^2} \right) W_1 + \frac{1}{m_h^2} [2(q \cdot Q)(k \cdot q) - 2(k \cdot q)M^2\delta - (k \cdot Q)m_h^2 + (k \cdot Q)M^2\delta^2] W_2 \right. \right. \\ \left. \left. + (k \cdot Q)W_4 + 2(k \cdot q)W_5 \right\} \right. \\ \left. + 4\lambda \frac{1}{M^2} [(q \cdot Q)(k \cdot Q) + (k \cdot q)(k \cdot Q) - (k \cdot q)M^2] W_3 \right\}, \quad (9)$$

where

$$\delta \equiv q \cdot \bar{Q} / \bar{Q}^2.$$

Without further information the number of structure functions entering the problem is such that

this expression is not very useful. However, we will show that for the channels we will consider, only one structure function dominates. It is to be noted that the structure function  $W_6$  does not appear in Eq. (9).

### III. INCLUSIVE DECAY DISTRIBUTIONS

The  $e\mu$  data suggest that the mass of the heavy lepton is smaller than 2 GeV.<sup>1</sup> From earlier estimates of the various branching ratios,<sup>6</sup> we see that this implies that the hadronic decay modes are dominated by resonances, and that the continuum is not very important.

We consider three types of hadronic final states in the decay of a heavy lepton. (We will throughout have a negative  $U$  in mind.) In decreasing order of importance they are (a) only pions, (b) kaons and pions, (c) baryons and mesons. In general we can say that final states (c) will have a very small branching ratio due to the limited phase space if they are at all possible. The ratio of final states of type (a) and type (b) is set by the Cabibbo angle. From these qualitative arguments we see that final states of type (a) will be the most important. These final states have the following property: If we assume that the hadronic weak currents are first class an even number of pions can be produced only through the vector current, whereas an odd number of pions will be produced only through the axial-vector current. As a result, in final states of type (a) there is no  $V, A$  interference. For an even number of pions, this means that only  $W_1$  and  $W_2$  contribute; for an odd number there will also be contributions from  $W_4$  and  $W_5$ .

The decay rate into a single pion,  $U^- \rightarrow \nu_U \pi^-$ , can be calculated without reference to the general expression (9).<sup>6</sup> In the heavy-lepton rest frame it is

$$d\omega = \frac{G^2 f_\pi^2 (1 + \lambda^2)}{8(2\pi)^2} M^3 \left(1 - \frac{m_\pi^2}{M^2}\right) \times \delta(M^2 + m_\pi^2 - 2q \cdot Q) \frac{d^3q}{q_0} \quad (10)$$

Here  $m_\pi$  and  $f_\pi$  are the pion mass and pion decay constant, respectively;  $f_\pi$  includes the Cabibbo angle.

From this we get the distribution normalized to unity,

$$\frac{d\omega}{\omega} = \frac{1}{\pi} \left(1 - \frac{m_\pi^2}{M^2}\right)^{-1} \delta(M^2 + m_\pi^2 - 2q \cdot Q) \frac{d^3q}{q_0} \quad (11)$$

The normalized distribution has the advantage that it is Lorentz invariant, which is useful when these distributions have to be combined with the production cross section of a heavy-lepton pair.

We now turn to the decay into two pions,  $U^- \rightarrow \nu_U + \pi^- + \pi^0$ . We describe the final state by a Breit-Wigner resonance with the  $\rho$  mass ( $M_\rho$ ) and width ( $\Gamma_\rho$ ). The tensor  $W_{\alpha\beta}$  now has the form

$$W_{\alpha\beta} = \int \frac{d^3p}{(2\pi)^3 2p_0} \delta^4(q + p + k - Q)(q - p)_\alpha (q - p)_\beta \times \frac{f_{\nu\rho}^2 f_{\rho\pi\pi}^2}{|(Q - k)^2 - M_\rho^2 + iM_\rho\Gamma_\rho|^2}, \quad (12)$$

from which we can see that here only the structure function  $W_2$  is nonzero. The integration variable  $\vec{p}$  is the  $\pi^0$  momentum. The coupling constants  $f_{\nu\rho}$  and  $f_{\rho\pi\pi}$  describe the couplings of a  $\rho$  to the vector current and two-pion system, respectively. The resulting decay rate is

$$d\omega = \frac{G^2 (1 + \lambda^2) f_{\nu\rho}^2 f_{\rho\pi\pi}^2}{8M(2\pi)^5} \frac{d^3q}{q_0} \times [2(2q \cdot Q - M^2)^2 A(q \cdot Q) + (8q \cdot Q - 3M^2 - 4m_\pi^2) B(q \cdot Q)]. \quad (13)$$

The functions  $A$  and  $B$  are defined by

$$\left. \begin{matrix} A \\ B \end{matrix} \right\} = \int \frac{d\vec{k}}{k_0} \frac{\delta((Q - k - q)^2 - m_\pi^2)}{|(Q - k)^2 - M_\rho^2 + iM_\rho\Gamma_\rho|^2} \times \left\{ \begin{matrix} 1 \\ k \cdot Q \end{matrix} \right. \quad (14)$$

These functions depend only on the invariant  $q \cdot Q$ , which in the heavy-lepton rest frame reduces to  $q_0 M$ . In this frame,  $q_0$  can vary between  $m_\pi$  and  $M/2$ . The bounds on the  $k_0$  integration in (14) are given as a function of  $q_0$  by

$$k_\pm = \frac{M}{2} \frac{(M - 2q_0)}{M - q_0 \mp |\vec{q}|}. \quad (15)$$

If we introduce the quantity

$$\mathfrak{M} = \frac{M^2 - M_\rho^2 + iM_\rho\Gamma_\rho}{2M}, \quad (16)$$

we find for  $A$  and  $B$

$$A = \frac{\pi}{4M^2 |\vec{q}|} \frac{1}{\text{Im}\mathfrak{M}} \arctan \frac{k_0 - \text{Re}\mathfrak{M}}{\text{Im}\mathfrak{M}} \Big|_{k_-}^{k_+}, \quad (17)$$

$$B = \frac{\pi}{4M |\vec{q}|} \left( \ln |k_0 - \mathfrak{M}| + \frac{\text{Re}\mathfrak{M}}{\text{Im}\mathfrak{M}} \arctan \frac{k_0 - \text{Re}\mathfrak{M}}{\text{Im}\mathfrak{M}} \right) \Big|_{k_-}^{k_+}.$$

Combining (13), (15), and (17), we find the single-pion spectrum for this channel.

If we now turn to three pions in the final state, we see that because here the axial-vector current contributes we have in general contributions from  $W_1$ ,  $W_2$ ,  $W_4$ , and  $W_5$ . We will now assume that the three pions are always coming from the cascade decay

$$U^- \rightarrow \nu_U + A_1, \quad A_1 \rightarrow 3\pi. \quad (18)$$

As can be seen,<sup>6</sup> the matrix element of the axial-vector current between the vacuum and the  $A_1$  state has the form

$$\langle A_1 | A^\mu(0) | 0 \rangle \propto \epsilon^\mu. \quad (19)$$

Here  $\epsilon^\mu$  is the  $A_1$  polarization vector. This implies that, although the axial-vector current is not conserved, the particular matrix element (19) may be considered to be conserved. We therefore restrict ourselves to the structure functions  $W_1$  and  $W_2$ .

$$W_{\alpha\beta} = \int \frac{d^4p}{(2\pi)^3} \delta^{(+)}(p^2 - M_X^2) \delta^4(p + q + k - Q) \left[ -q_{\alpha\beta} + \frac{(Q-k)_\alpha(Q-k)_\beta}{(Q-k)^2} \right] \frac{F_{A_1}^2}{|(Q-k)^2 - M_{A_1}^2 + iM_{A_1}\Gamma_{A_1}|^2}. \quad (20)$$

Here  $M_X$  is the average mass recoiling against the  $\nu\pi$  system. We integrate over the recoil momentum  $p$ . This leads to a decay rate of the form

$$d\omega = \frac{G^2(1+\lambda^2)F_{A_1}^2}{8M(2\pi)^5} \frac{d^3q}{q_0} \int \frac{d^3k}{k_0} \frac{\delta((Q-q-k)^2 - M_X^2) k \cdot Q}{|(Q-k)^2 - M_{A_1}^2 + iM_{A_1}\Gamma_{A_1}|^2} \left( 2 + \frac{M^2}{(Q-k)^2} \right). \quad (21)$$

In these expressions  $F_{A_1}$  is a combination of the couplings of the  $A_1$  to the axial-vector current and the  $A_1$  decay constant. If we approximate  $2 + M^2/(Q-k^2) \approx 2 + M^2/M_{A_1}^2$ , the integral can be evaluated analytically and the resulting expression is

$$d\omega = \frac{G^2(1+\lambda^2)F_{A_1}^2}{8M(2\pi)^5} \frac{d^3q}{q_0} \left( 2 + \frac{M^2}{M_{A_1}^2} \right) B'. \quad (22)$$

Here  $B'$  is the same expression as  $B$  in (17) except for a change in integration limits. Instead of (15) we now have

$$k_\pm = \frac{M}{2} \frac{M - 2q_0 + \Delta}{M - q_0 \mp |\mathbf{q}|}, \quad \Delta = \frac{m_\pi^2 - M_X^2}{2M}. \quad (23)$$

The case of four pions in the final state can be treated in the same way. We now assume that the four pions come from a  $\rho'$ . Because the  $\rho'$  decay goes via an  $s$ -wave  $\rho\epsilon$  state, we use the same argument as for the  $A_1$  to restrict ourselves to a pure  $W_1$  contribution. This means that (22) will also describe the four-pion case after the appropriate modifications of masses, width, and coupling constant. As can be seen from the estimates in Ref. 6, the contribution from more than four pions will be negligible. We will therefore not include effects of more than four pions in the inclusive distribution.

As far as decays of type (b) are concerned we will consider only the decays

$$U \rightarrow \nu_U K, \quad (24)$$

$$U \rightarrow \nu_U K^*, \quad K^* \rightarrow K\pi. \quad (25)$$

The expression for inclusive single- $K$  production is the same as for single pions, (10) and (11), provided we change the masses and coupling constants. In the same way we use expression (13) for the decay (25).

Since the decay of the  $A_1$  proceeds via a  $\pi\rho$   $s$ -wave state, the pions from the  $A_1$  will be distributed isotropically, independent of the  $A_1$  polarization. We therefore take for the  $A_1$  only a  $W_1$  contribution to account for the isotropic pion distribution. In analogy with Eq. (12) for the  $\rho$ , for the  $A_1$  contribution we get

#### IV. NUMERICAL RESULTS AND DISCUSSION

In order to calculate the inclusive hadron spectrum from heavy leptons in an  $e^+e^-$  colliding-beam experiment, we have to combine the distributions from Sec. III with the production cross section of heavy-lepton pairs. The production cross section of a heavy lepton of mass  $M$  is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E^2} \left( 1 - \frac{M^2}{E^2} \right)^{1/2} \left( 1 + \cos^2\theta + \frac{M^2}{E^2} \sin^2\theta \right). \quad (26)$$

In order to get  $d\sigma/dxd\Omega$  for a particular hadron, we proceed as follows. From the expressions derived in Sec. III we can form the distributions normalized to one for each channel ( $\pi, \rho, A_1, \rho', K, K^*$ ) and each particle type. We multiply this normalized distribution in each channel with the corresponding branching ratio and the average number of observable hadrons in that channel. For instance, the  $A_1^-$  decays with equal probability in  $\pi^- \pi^0 \pi^0$  and  $\pi^- \pi^- \pi^+$ ; in this case we therefore have  $\langle N_{\pi^-} \rangle = 1.5$ . For each channel we multiply the obtained expres-

TABLE I. The branching ratios (b.r.), average multiplicities  $\langle N \rangle$ , and recoil masses ( $M_X$ ) as used in the calculations.

	b.r.	$\langle N_{\pi^-} \rangle$	$\langle N_{K^-} \rangle$	$M_X$ in MeV
$\pi^-(137)$	0.13	1	0	0
$\rho^-(770)$	0.24	1	0	137
$A_1^-(1100)$	0.10	$\frac{3}{2}$	0	700
$\rho'^-(1500)$	0.05	$\frac{3}{2}$	0	900
$K^-(494)$	0.01	0	1	0
$K^{*-}(892)$	0.02	$\frac{1}{3}$	$\frac{2}{3}$	137

sion by the production cross section and integrate over all angles of the heavy lepton, keeping in mind that due to kinematical restrictions this integration is not always over  $4\pi$ .

The branching ratios are determined using the expressions from Ref. 6, with the modification that we replace the continuum contribution with the  $\rho'$  contribution.

Having obtained  $d\sigma/dxd\Omega$ , the quantities in Eqs. (3a) and (3b) can now be deduced. In Table I we list the branching ratios (b.r.), average multiplicities  $\langle N \rangle$ , and recoil masses ( $M_x$ ) used in the calculations.

We now discuss the structure of  $s d\sigma/dx$ . In Fig. 1 we plot this as a function of  $x$  for beam energy  $E = 1.9$  GeV and heavy-lepton mass  $M = 1.8$  GeV. In the same figure we plot the separate contributions from the channels  $\pi$ ,  $\rho$ ,  $A_1$ , and  $\rho'$ . For comparison we also plot the inclusive spectrum due to  $\rho$  if its width is taken to be zero.<sup>6</sup> It is seen that treating the  $\rho$  in a zero-width approximation changes the spectrum considerably. It is also seen that the spectrum due to the decay  $U^- \rightarrow \nu_U \pi^-$  has a different structure from the other contributions due to the fact that this is a two-body decay as opposed to three- (and more) body decays for the other channels. As a consequence of the sharp rectangular shape of the single-pion spectrum the total spectrum shows a sharp maximum at the lowest  $x$  value where the decay  $U^- \rightarrow \nu_U + \pi^-$  is kinematically possible. If we call the  $x$  value where this maximum occurs  $x_{\max}$ , we have the following relation (up to corrections of order  $m_\pi^2/M^2$ ):

$$x_{\max} = \frac{1}{2}(1 - \beta_U). \quad (27)$$

Here  $\beta_U$  is the velocity of the heavy lepton. The

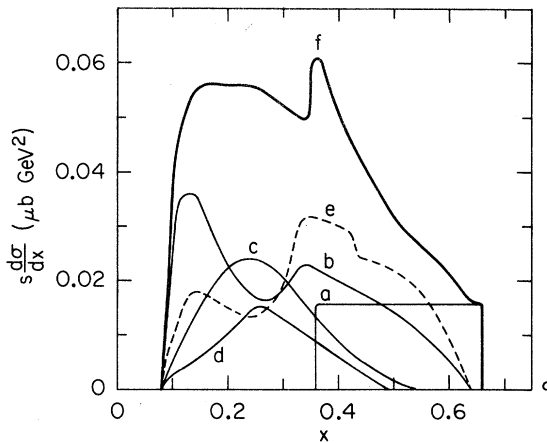


FIG. 1. The inclusive spectrum for  $M = 1.8$  GeV and  $E = 1.9$  GeV. (a)  $\pi^-$  contribution, (b)  $\rho$  contribution, (c)  $A_1$  contribution, (d)  $\rho'$  contribution, (e)  $\rho$  contribution with  $\Gamma = 0$ , (f) total.

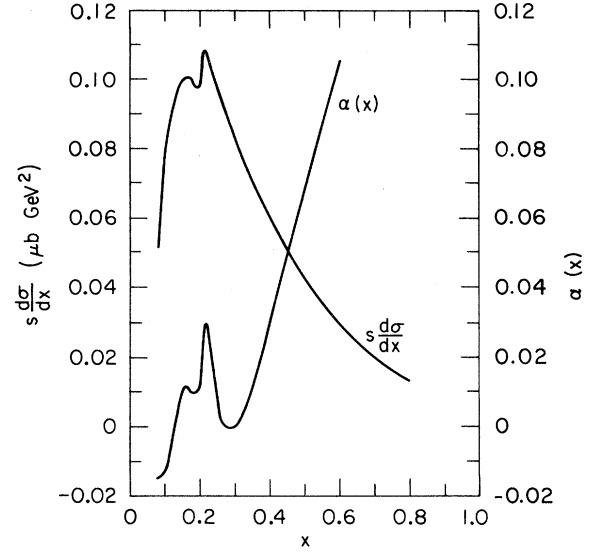


FIG. 2. The inclusive spectrum and  $\alpha(x)$  for  $M = 1.8$  GeV at  $E = 2.2$  GeV.

fact that this maximum is very sharp and the fact that its position is in a simple way related to the heavy-lepton velocity may provide an alternative way to determine the heavy-lepton mass. If the beam energy is  $E$ , the mass is given by

$$M^2 = 4E^2 x_{\max}(1 - x_{\max}). \quad (28)$$

From Fig. 1 we also see that the parts of the spectrum due to the  $A_1$  and  $\rho'$  have a smooth behavior.

In Fig. 2 we plot  $s d\sigma/dx$  at a beam energy  $E = 2.2$  GeV. Here and in all the following figures the heavy-lepton mass is taken to be 1.8 GeV. Besides

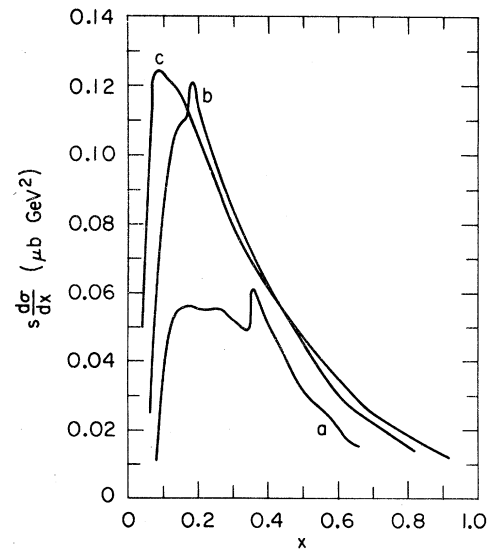


FIG. 3. The inclusive spectrum for  $M = 1.8$  GeV and energies (a)  $E = 1.9$  GeV, (b)  $E = 2.4$  GeV, (c)  $E = 3.7$  GeV.

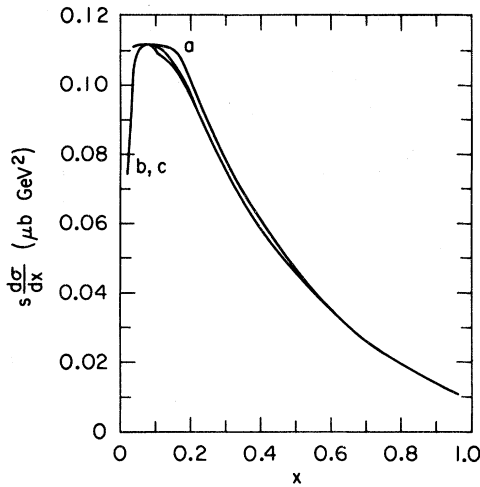


FIG. 4. The inclusive spectrum for  $M = 1.8$  GeV and energies (a)  $E = 5$  GeV, (b)  $E = 15$  GeV, (c)  $E = 100$  GeV. Curves b and c are on top of one another.

the spectrum we also plot the variable  $\alpha$  which describes the angular behavior of the inclusive hadrons. The function  $\alpha(x)$  shows some structure at low  $x$  correlated with structure in  $s d\sigma/dx$ . Experimentally at this energy a "bulge" is seen in the  $x$  region between  $x = 0.3$  and  $x = 0.5$ . From the published data we estimate an increase of  $0.25 \mu\text{b GeV}^2$  at  $x = 0.35$ . Our calculation gives  $0.19 \mu\text{b GeV}^2$ .

In Fig. 3 we present our  $s d\sigma/dx$  for three energies where experimental data are available. From the behavior with increasing  $E$  we see that the greater part of the increase takes place at low  $x$ . It is to be noted that in all curves presented we plot  $s d\sigma/dx$  for negative hadrons (pions) only, whereas the experimental data as presented in Ref. 3 refer to positive and negative charges. The behavior of  $s d\sigma/dx$  at PETRA and PEP energies and beyond is indicated in Fig. 4. We see that a scaling limit is reached, as was expected.<sup>7</sup>

In Fig. 5 we show the behavior of  $\alpha$  as a function of  $x$  at three different energies. At low energies  $\alpha$  is close to zero, but far above threshold for heavy-lepton pair production it approaches unity. This can be understood from the fact that at high energies the heavy leptons have an angular distribution proportional to  $(1 + \cos^2\theta)$ , and that at these energies the decay products move in the same direction.

A general feature of the calculated spectrum is that the falloff with  $x$  is much slower than the falloff in the observed total inclusive hadron spectrum.<sup>8</sup> If we compare at  $E = 3.7$  GeV and take the ratio of  $s d\sigma/dx$  at  $x = 0.2$  and  $x = 0.8$ , the measured hadron spectrum gives  $\approx 35$  as contrasted to 6 in

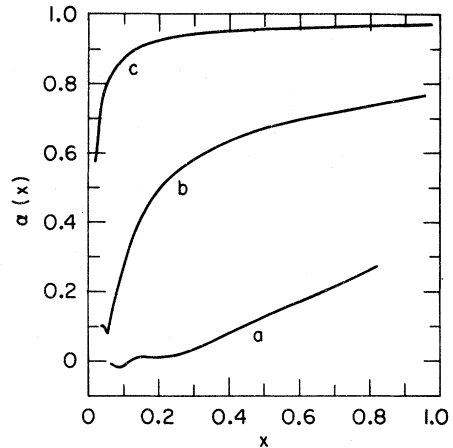


FIG. 5. The angular parameter  $\alpha(x)$  for  $M = 1.8$  GeV and energies (a)  $E = 2.4$  GeV, (b)  $E = 5$  GeV, (c)  $E = 15$  GeV.

our calculation.

About other final states we mention that although the branching ratio into a nucleon-antinucleon pair and a neutrino will be very small (or zero if  $M < 1.86$  GeV), the observation of this decay mode will give a lower bound on the heavy-lepton mass. Observation of strange-decay modes will also be difficult, owing to the small branching ratios involved.

From the various branching ratios and multiplicities in Table I it follows that the average multiplicity of charged hadrons in heavy-lepton decay is 1.1. In the leptonic decays the charged multiplicity is exactly 1. As a consequence heavy-lepton decay (leptonic and hadronic) will almost exclusively show up as events with two oppositely charged particles. Furthermore, since the neutral hadrons in a decay are mainly neutral pions, many events should be accompanied by photons from decaying  $\pi^0$ s.

In summary, we suggest that measuring the inclusive hadron spectrum from the decay of heavy leptons, for instance, by selecting events of the type  $e^+e^- \rightarrow \pi + e(\mu) + \text{anything}$ , can serve as a test of the heavy-lepton hypothesis. The position of the maximum of this spectrum will give a determination of the heavy-lepton mass.

#### ACKNOWLEDGMENTS

We wish to thank Professor S. D. Drell for warm hospitality extended to us at SLAC. We are indebted to F. Gilman and Y.-S. Tsai for discussions. (K.J.F.G.) thanks the Netherlands Organization for the Advancement of Pure Research (Z.W.O.) and (R.R.) thanks the Herman Rosenberg Foundation for financial support.

\*Work supported by the U. S. Energy Research and Development Administration.

†On leave from the University of Helsinki, Helsinki, Finland.

<sup>1</sup>M. L. Perl, SLAC Report No. SLAC-PUB-1664, 1975 (unpublished); G. Feldman, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford University*, edited by W. T. Kirk (SLAC, Stanford, California, 1975), p. 39.

<sup>2</sup>M. Cavalli-Sforza *et al.*, Phys. Rev. Lett. 36, 558

(1976).

<sup>3</sup>R. Schwitters, above proceedings (Ref. 1), p. 5.

<sup>4</sup>F. Gilman, in *Proceedings of the Summer Institute on Particle Physics, 1975*, SLAC Report No. 191, 1975 (unpublished).

<sup>5</sup>G. A. Snow, Phys. Rev. Lett. 36, 766 (1976).

<sup>6</sup>Y.-S. Tsai, Phys. Rev. D 9, 2821 (1971).

<sup>7</sup>E. A. Paschos, Phys. Rev. D 13, 745 (1976).

<sup>8</sup>S. Nussinov, R. Raitio, and M. Roos, Phys. Rev. D 14, 211 (1976).