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Phase-shift and spin-rotation phenomena in neutron interferometry*

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The perfect-crystal neutron interferometer was used to study characteristic phenomena arising from simultaneous phase shift and spin rotation of neutron waves. In accordance with theoretical predictions, the beams leaving the interferometer became partially polarized even with unpolarized incident neutrons. The intensity and the polarization as a function of phase shift and spin rotation have been found to oscillate with the same period, displaying a mutual beat pattern.

I. INTRODUCTION

The experimental possibilities of neutron physics were generally extended by the successful development of neutron interferometers utilizing the neutron coherence properties. The first attempt to construct such a neutron interferometer with coherent beams produced by a Fresnel biprism arrangement[1,2,3] proved to be only partially successful. The separation of the two coherent partial beams within this interferometer was extremely small (about 60 μ) and only first-order interferences could be observed. Even the zero-order interference pattern was vanishing rapidly with increasing thickness of inserted phase-shift materials.

The possibility of developing an effective neutron interferometer, based on x-ray interferometer principles as developed by Bonse *et al.*,[4,5] allowing wide spatial separation of the two coherent partial beams and permitting observation of several hundred orders of interference, was suggested by Bonse and Hart[6] and analyzed in detail by Rauch,[7] Rauch and Suda,[8] and Bauspiess, Bonse, and Graeff.[9] The first experimental tests of this device, dealing with the measurements of nuclear phase shifts and of the influence of magnetic fields on the interference effects have been previously described.[10,11] Subsequently, Colella *et al.*[12] have observed gravitationally induced phase shifts of neutron waves, while Kikuta *et al.*[13] reported general test measurements; both groups used a similar type of interferometer.

The possibility of studying the transformation properties of spinors under general rotations in neutron interferometry requires special attention. In this connection some discussion exists in the literature concerning the observability of 2π rotations of physical systems.[14-18] With the perfect-crystal interferometer, now placed at one guide tube of the high-flux-beam reactor at Grenoble, we could clearly verify the change of sign of a spinor wave function for rotations of 2π and odd multiples of it,[19] an effect which has also recently been demonstrated by other researchers.[20,21]

The purpose of the research reported here is to investigate the general case of simultaneous nuclear phase shifting and magnetic spin rotation.[18,22,23] Some preliminary results of the measurements have recently been reported.[24] After developing the necessary formalism in Sec. II, we discuss its consequences in Sec. III. In Sec. IV, we describe the experimental facilities used to measure the intensity and polarization of the emerging beams. The experimental results are then presented in Sec. V.

II. THEORY

The incoming neutrons are split in the first crystal plate of the interferometer into two coherent parts I and II, Fig. 1. The second plate acts as a mirror by Laue diffraction. The neutron waves leaving the interferometer at this stage are of no importance for the interference effect and will not be further considered. After interference in the analyzer crystal, a forward (0) and deviated (H) diffracted beam emerge from the interferometer. Their wave functions are described by the relations

$$\psi_0 = \psi_0^{\mathrm{I}} + \psi_0^{\mathrm{II}} \tag{1}$$

and

$$\psi_H = \psi_H^{\mathrm{I}} + \psi_H^{\mathrm{II}}, \tag{2}$$

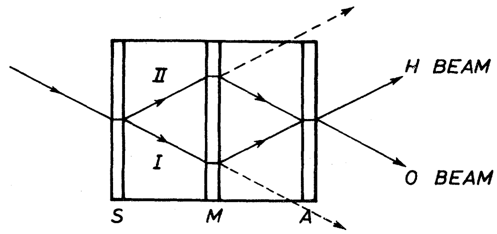

FIG. 1. Schematic representation of the neutron interferometer showing the individual beams.

where I and II denote those parts of the wave which passed the interferometer along beam paths I and II, respectively. For the case of total absence of phase shift and spin rotation we have for the ideal case

$$\psi_0^{\mathrm{I}} = \psi_0^{\mathrm{II}}. \tag{3}$$

For the deviated beam, a more complicated relation can be shown to exist[8,25]; the intensity and polarization of this beam are complementary to the corresponding values of the 0 beam as governed by conservation laws. For that reason, the forward diffracted beam will be considered only.

A phase shift and a spin rotation in one of the beam paths can be described by the unitary operator

$$U = e^{i\chi} e^{-\vec{\sigma}\cdot\vec{\alpha}/2}\,, \tag{4}$$

where χ is the phase shift, $\vec{\sigma}$ is the Pauli spin vector and $\vec{\alpha}$ is the rotation vector. Without loss of generality we can assume that the phase shift and the spin rotation are acting on the beam in path I only. Thus, the emerging two partial waves are

$$\psi_0^{\mathrm{I}\prime} = e^{i\chi} e^{-i\vec{\sigma}\cdot\vec{\alpha}/2}\psi_0^{\mathrm{I}} \tag{5a}$$

and

$$\psi_0^{\mathrm{II}\prime} = \psi_0^{\mathrm{II}}. \tag{5b}$$

Here the primed symbols are used to characterize the case for which the phase shift and/or spin rotation is present while the unprimed symbols apply where neither of these influences is present. Thus, using Eq. (3) and Eq. (5) the wave function of the forward beam is written as

$$\begin{aligned}\psi_0' &= \psi_0^{\mathrm{I}\prime} + \psi_0^{\mathrm{II}\prime}\\ &= (e^{i\chi}e^{-i\vec{\sigma}\cdot\vec{\alpha}/2} + 1)\psi_0^{\mathrm{I}}\\ &= \{e^{i\chi}[\cos(\tfrac{1}{2}\alpha) - i\vec{\sigma}\cdot\hat{\alpha}\sin(\tfrac{1}{2}\alpha)] + 1\}\psi_0^{\mathrm{I}}.\end{aligned} \tag{6}$$

Here, α is the angle of rotation around the direction of the unit vector $\hat{\alpha} = \vec{\alpha}/\alpha$.

Thus, the intensity of the forward beam is given by

$$\begin{aligned}I' &= \psi_0'^{\dagger}\psi_0'\\ &= \psi_0^{\mathrm{I}\dagger}(1+U^{\dagger})(1+U)\psi_0^{\mathrm{I}}.\end{aligned} \tag{7}$$

Using Eqs. (4) and (6), it can be shown that I' can be expressed as

$$I' = \tfrac{1}{2}I\,[+\cos\chi\cos(\tfrac{1}{2}\alpha) + \hat{\alpha}\cdot\vec{\mathrm{P}}\sin\chi\sin(\tfrac{1}{2}\alpha)], \tag{8}$$

where we have introduced the intensity

$$I = \psi_0^{\dagger}\psi_0 = 4\psi_0^{\mathrm{I}\dagger}\psi_0^{\mathrm{I}} \tag{9}$$

and the polarization

$$\begin{aligned}\vec{\mathrm{P}} &= \psi_0^{\dagger}\vec{\sigma}\psi_0/\psi_0^{\dagger}\psi_0\\ &= \psi_0^{\mathrm{I}\dagger}\vec{\sigma}\psi_0^{\mathrm{I}}/\psi_0^{\mathrm{I}\dagger}\psi_0^{\mathrm{I}}.\end{aligned} \tag{10}$$

To find the polarization $\vec{\mathrm{P}}'$, given by

$$\begin{aligned}\vec{\mathrm{P}}' &= \psi_0'^{\dagger}\vec{\sigma}\psi_0'/\psi_0'^{\dagger}\psi_0'\\ &= \psi_0^{\mathrm{I}\dagger}(1+U^{\dagger})\vec{\sigma}(1+U)\psi_0^{\mathrm{I}}/I',\end{aligned} \tag{11}$$

the term

$$\begin{aligned}(1+U^{\dagger})\vec{\sigma}(1+U) = \{1 + e^{-i\chi}[\cos(\tfrac{1}{2}\alpha) + i\vec{\sigma}\cdot\hat{\alpha}\sin(\tfrac{1}{2}\alpha)]\}\vec{\sigma}\\ \times\{1 + e^{i\chi}[\cos(\tfrac{1}{2}\alpha) - i\vec{\sigma}\cdot\hat{\alpha}\sin(\tfrac{1}{2}\alpha)]\}\end{aligned} \tag{12}$$

has to be evaluated using the relations

$$(\vec{\sigma}\cdot\vec{\alpha})\vec{\sigma} = \vec{\alpha} + i(\vec{\sigma}\times\vec{\alpha}), \tag{13a}$$

$$\vec{\sigma}(\vec{\sigma}\cdot\vec{\alpha}) = \vec{\alpha} - i(\vec{\sigma}\times\vec{\alpha}). \tag{13b}$$

Following some analysis we finally obtain

$$\begin{aligned}\vec{\mathrm{P}}' = \frac{I}{2I'}\{&[\cos^2(\tfrac{1}{2}\alpha) + \cos(\tfrac{1}{2}\alpha)\cos\chi]\vec{\mathrm{P}}\\ &+ [\sin^2(\tfrac{1}{2}\alpha)\hat{\alpha}\cdot\vec{\mathrm{P}} + \sin\chi\sin(\tfrac{1}{2}\alpha)]\hat{\alpha}\\ &+ [\cos\chi\sin(\tfrac{1}{2}\alpha) + \cos(\tfrac{1}{2}\alpha)\sin(\tfrac{1}{2}\alpha)](\hat{\alpha}\times\vec{\mathrm{P}})\}.\end{aligned} \tag{14}$$

For unpolarized incident neutrons, which were used in our experiment, the intensity then can be found to be

$$I'(\vec{\mathrm{P}} = \vec{0}) = \frac{I}{2}[1 + \cos\chi\cos(\tfrac{1}{2}\alpha)] \tag{15}$$

and the polarization is

$$\vec{\mathrm{P}}'(\vec{\mathrm{P}} = \vec{0}) = \frac{\sin\chi\sin(\tfrac{1}{2}\alpha)}{1 + \cos\chi\cos(\tfrac{1}{2}\alpha)}\hat{\alpha}\,. \tag{16}$$

III. PHYSICAL CONSEQUENCES

We now consider the possibility of measuring phase changes of particle wave functions with the neutron interferometer to clarify the question of whether 2π rotations of physical systems are observable. It is well known that a 2π rotation of a half-integral-spin system introduces the phase factor -1 to its wave function. This phase factor normally is canceled since observables in quantum mechanics are quadratic in the wave function.

In view of Eqs. (8) and (14) one can note that the rotation angles α of odd multiples of 2π give values of intensity and polarization which differ from the initial state. The experimentally most direct approach designed to demonstrate this effect is to measure the intensity I' with unpolarized incident neutrons and no phase shift χ while performing a controlled rotation α by means of a variable magnetic field. It is worth noting that even with unpolarized incident neutrons, the emerging neutron beams are at least partially polarized. According to our preceding analysis, in particular the results represented by Eq. (15) and Eq. (16), we conclude that the intensity oscillations as well as the polarization oscillations are modulated. Both effects follow from the property that spin rotations do not combine in a scalar additive manner in their contribution to the phase shift. The following experimental analysis is specifically concerned with the measurement of these phenomena.

IV. EXPERIMENTAL FACILITY AND PROCEDURE

The measurements were performed using the perfect-crystal neutron interferometer[10,11] located at one guide tube of the high-flux-beam reactor at Grenoble. Unpolarized incident neutrons reflected from the (220) plane of a perfect Si crystal were used with a wavelength of 1.83 Å and a beam cross section of 5×1.5 mm^2. Figure 2 shows the schematic layout of the arrangement and was used to determine simultaneously the polarization of the forward diffracted beam and the intensity of the deviated diffracted beam. Rotation of a 10-mm-thick Al plate produced a relative phase shift $\Delta\chi$ between the two partial beams by nuclear interaction given by

$$\Delta\chi = -N\lambda b_c \Delta D, \tag{17}$$

where N is the number of nuclei per cm^3, λ is the neutron wavelength, b_c is the coherent scattering length of Al, and ΔD is the path difference defined by

$$\Delta D = D\left(\frac{1}{\cos(\vartheta_B+\epsilon)} - \frac{1}{\cos(\vartheta_B-\epsilon)}\right) \tag{18}$$

with D being the thickness of the phase shifter, ϵ its rotation angle (Fig. 2), and ϑ_B the Bragg angle.

Beam I was passed through a 1-cm gap of an electromagnet allowing the rotation of the neutron spin by the angle

$$\alpha = -\gamma \int B dt$$
$$= (-\gamma/v) \int B ds \tag{19}$$

around the direction $\vec{\alpha} = \vec{B}/B$ which was chosen as the z axis. In this equation γ is the neutron gyromagnetic ratio, B is the magnetic induction and v is the velocity of the neutrons. The magnetic field distribution within the interferometer was measured by means of a small Hall probe mounted on an x-y drive. By this procedure we could determine the angle α_{eff} as the difference of the rotation angles along the two beam paths as a function of the current in the magnet coil. The polarization of the forward beam was analyzed using

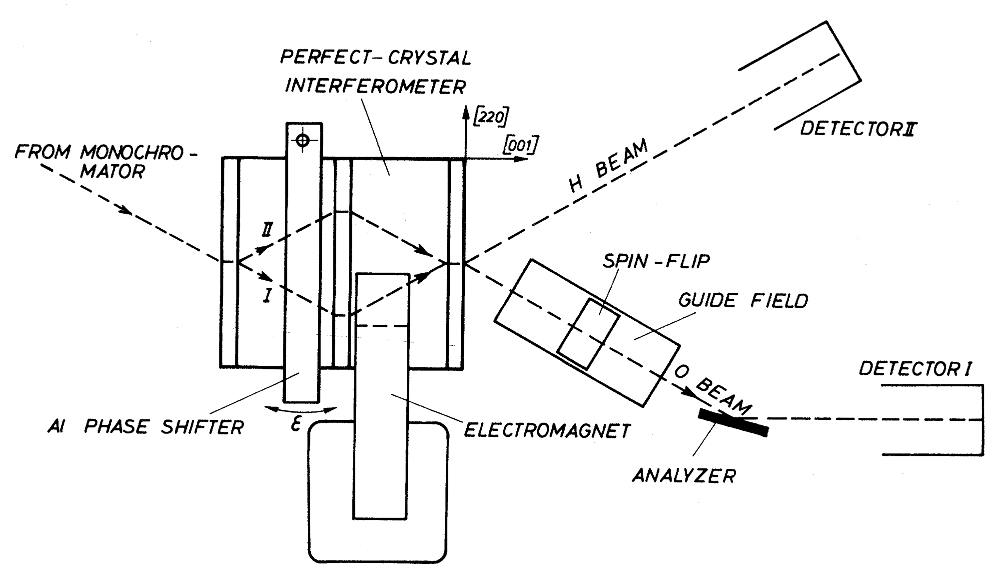

FIG. 2. Sketch of the experimental arrangement used for polarization analysis.

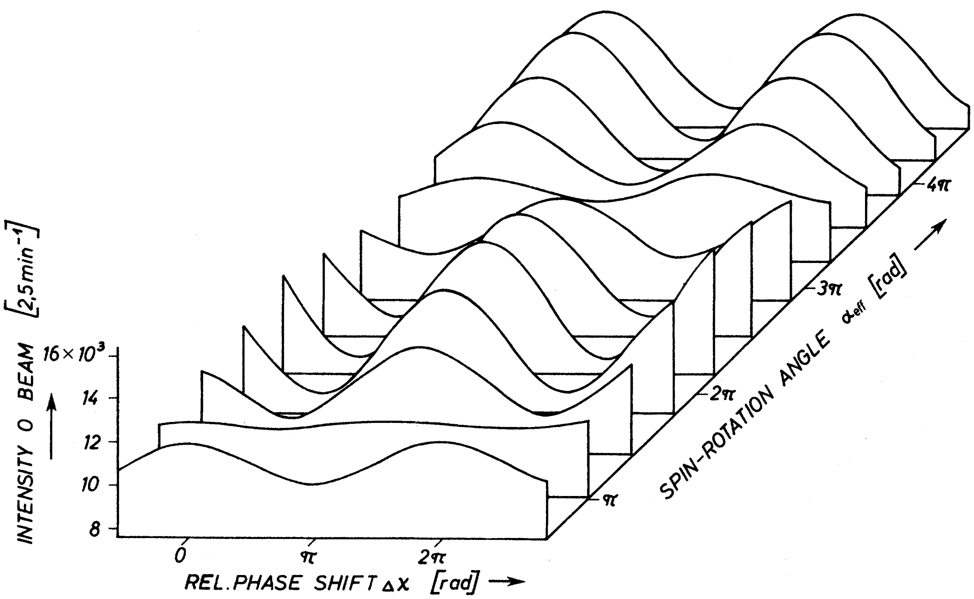

FIG. 3. Intensity of the forward beam as a function of the relative phase shift and the effective spin-rotation angle. The profiles shown were obtained by a least-square fit of the measured values; the experimental data points are not shown for clarity of presentation.

Bragg reflection at the (111) plane of a saturated Heusler alloy single crystal in combination with a dc operated spin-flip coil[26]; its intensity was measured by removing the analyzer and adjusting the detector accordingly.

V. EXPERIMENTAL RESULTS

Figure 3 shows the measured intensity of the forward diffracted beam as a function of the effective rotation angle α_{eff} and the relative phase

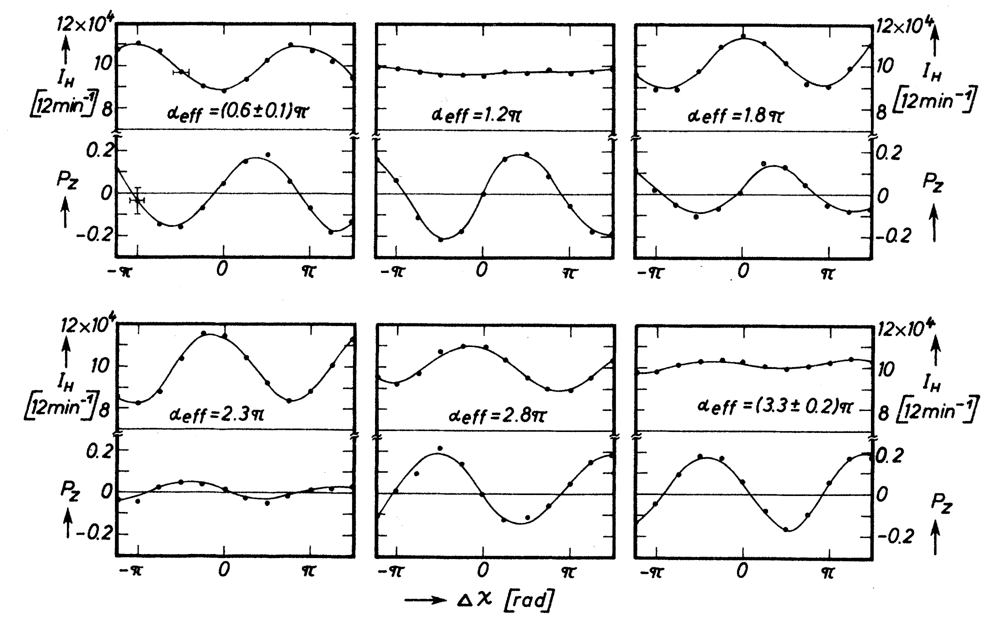

FIG. 4. Intensity of the deviated beam and polarization of the forward beam as a function of the relative phase shift for various spin-rotation angles.

shift $\Delta\chi$. Without loss of generality, the position of the origin of the $\Delta\chi$ axis was chosen to coincide with extrema of the intensity. Each of these intensity profiles was obtained by variation of the phase shift for a constant current in the magnet coil; these displayed curves have been obtained by least-square curve fitting with the discrete measured points using sinusoidal functions. The oscillation of the intensity with varying spin-rotation angle for a constant phase shift again demonstrates the 4π symmetry of spinors. In a comparison of the measured results with the theoretical prediction of the intensity Eq. (15), it is necessary to keep in mind the influence of (1) imperfections of the interferometer crystal, (2) wavelength dispersion of the neutrons, (3) inhomogeneities of the magnetic field, etc. Such effects reduce the contrast of the interference pattern, but the functional dependence is retained, showing clearly the predicted "beat" or modulation characteristic of the oscillations, Eq. (15).

The results of the polarization measurements are illustrated in Fig. 4, showing the intensity of the H beam and the polarization of the 0 beam as a function of the relative phase shift for several values of the spin-rotation angle. In accordance with the theoretical predictions, Eqs. (15) and (16), the intensity and polarization oscillations possess a lag of $\frac{1}{2}\pi$. Furthermore, the polarization oscillates with a beat pattern similar to that of the intensity, vanishing just when the oscillation amplitude of the intensity attains its maximum value; the reverse, of course, also holds. According to Eq. (16) deviations from a purely sinusoidal form should occur unless $\alpha_{\text{eff}} = (2n+1)\pi$, n integer. However, as indicated above, the experimental restrictions appreciably affect the discernible deviations from a pure sinusoidal.

VI. CONCLUDING COMMENTS

In the work reported here, we have used the neutron interferometer to verify some theoretical predictions which follow as a consequence of the effect of superposition of nuclear and magnetic interactions on neutron waves. These successful measurements suggest that a new and promising field of investigation of magnetic materials by neutron interferometry is possible. In view of our theoretical analysis, we suggest that additional fundamental information can be extracted if polarized incident neutrons were used in similar experiments.

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