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General light-cone model for spin-dependent leptoproduction

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A general expression for the spin terms in the light-cone expansion of current commutators is given. The problem of current conservation is discussed. This approach is applied to deep-inelastic leptoproduction.

INTRODUCTION

The study of the expansions of products of local operators has been developed both in a heuristic[1] and rigorous[2] way beginning with the fundamental work of Wilson.[3] This problem has attracted the attention of physicists in connection with the scaling phenomenon in leptoproduction.

In this case one is interested in the light-cone expansion of the commutator of two physical currents. For nonpolarized leptoproduction, general expansions have been given in the literature.[4] The experimental results favor the quark light-cone models.[5,6] They, and their relations[7] with the quark-parton models, have been widely discussed.

In this paper we shall study the structure and properties of the spin terms in the expansion of the commutator of two currents in the neighborhood of the light cone. Our most important results are the following.

(i) We have found the most general light-cone expansion of the commutator of two currents that contributes to the spin-dependent terms in leptoproduction. The local operators that contribute to this expansion turn out to be of two kinds: $O^{(\lambda_1 \cdots \lambda_n)}$, fully symmetric, and $O^{[\rho\sigma](\lambda_1 \cdots \lambda_n)}$, antisymmetric in two indices and symmetric in the remaining.

(ii) We have proved that in the standard situation in which only the most singular terms in the expansion are considered one cannot impose current conservation. The adequate procedure is to use a weaker restriction that we call current conservation on the light cone (CCLC). This restriction has to be imposed even for nonconserved currents.

(iii) The general expansion has been explicitly worked out in order to get the scaling functions in leptoproduction. The order of scale for the nonconserved structure functions is depressed. Our treatment is not equivalent, nor can it be formulated in general in a manifestly conserved form. This question will be discussed at length elsewhere.[8]

Previously published works on this topic consider a particular case of the general situation we discuss with $\Delta - 1 = \Delta' = 2$, as suggested by the quark model. Wray[9] and, independently, Ward[10] in their study of ν production use a quark model and predict dominant scaling for nonconserved terms, in disagreement with our results. Hey and Mandula[11] have studied e production starting from an explicitly conserved light-cone expansion in terms of two bilocals. This work has been extended by Heimann.[12] Our approach can be used to give a simpler and more general proof of Heimann's results asserting that the expansion can be formulated in terms of a single bilocal in an explicitly conserved form.

The paper has been divided into two sections and one appendix. In the first section we discuss the general structure of the expansion and the constraints dictated by current conservation on the light cone (CCLC). The actual meaning of CCLC will be considered. In Sec. II the scale functions for polarized leptoproduction are computed. The causal functions used in this paper and some formulas of interest are given in the Appendix.

I. THE GENERAL CURRENT-CURRENT AND DIVERGENCE-CURRENT COMMUTATOR EXPANSIONS

As a result of the works on the light-cone expansion[1,2] of the operator products in terms of local operators, we know

$$A(x)B(0) = \sum_n C_n(x) O^n(0) ,$$

where the $C_n(x)$ are c-number functions, singular at $x^2 = 0$. The order of the singularity is given by the scale dimension and the spin of the local operators in the expansion.

We choose for the expansion a local-operator basis $\{O^n(0)\}$ transforming with the Lorentz irreducible representations $[D(j, j')]$, i.e. traceless and with symmetries associated with Young patterns with at most two rows.[13] These operators have a well-defined scale dimension for all their components,[14] and the traceless property will keep the expansion singularities when the Lorentz contractions are performed. Note that only two kinds of such irreducible local operators have non-vanishing expectation values (e.v.)

$$\langle \vec{p}, s | O^n(0) | \vec{p}, s \rangle$$

and therefore contribute to inclusive leptoproduction.

(i) the full symmetric ones, transforming with the $D(\frac{1}{2}n, \frac{1}{2}n)$ representations, and

(ii) the operator antisymmetric in two indices and symmetric in the remaining indices, transforming with the $D(\frac{1}{2}n, \frac{1}{2}n+1) \oplus D(\frac{1}{2}n+1, \frac{1}{2}n)$ representations.

Using similar arguments, we can see easily that only full symmetric operators contribute to the spin-independent terms in leptoproduction. Taking into account these considerations, the most singular terms in the current-commutator light-cone expansion (CCE) can be written as follows:

$$
\begin{aligned}
[J_\mu^\dagger(x), J_\nu(0)] \simeq \sum_{\min\tau} & [E(x^2,\Delta) a_n x_\mu x_\nu x_{\lambda_1} \cdots x_{\lambda_n} + E(x^2, \Delta-1) c_n g_{\mu\nu} x_{\lambda_1} \cdots x_{\lambda_n} + E(x^2,\Delta-2) d_n g_{\mu\lambda_1} g_{\nu\lambda_2} x_{\lambda_3} \cdots x_{\lambda_n} \\
& + E(x^2,\Delta-1) b_n^{\pm}(x_\mu g_{\nu\lambda_1} \pm x_\nu g_{\mu\lambda_1}) x_{\lambda_2} \cdots x_{\lambda_n} + E(x^2,\Delta-1) l_n \epsilon_{\mu\nu\lambda_1 k} x^k x_{\lambda_2} \cdots x_{\lambda_n}] O^{(\lambda_1 \cdots \lambda_n)}(0) \\
+ \sum_{\min\tau'} & \{E(x^2,\Delta'-1) 2 g_n^- g_{\mu[\rho} g_{\sigma]\nu} x_{\lambda_1} \cdots x_{\lambda_n} + E(x^2,\Delta'-1) 2 g_n^+ [g_{\mu[\rho} x_{\sigma]} g_{\nu\lambda_1} + (\mu \leftrightarrow \nu)] x_{\lambda_2} \cdots x_{\lambda_n} \\
& + E(x^2,\Delta') 2 h_n x_{[\rho} \epsilon_{\sigma]\mu\nu k} x^k x_{\lambda_1} \cdots x_{\lambda_n} + E(x^2,\Delta') 2 k_n^{\pm} [x_\mu g_{\nu[\rho} x_{\sigma]} \pm (\mu \leftrightarrow \nu)] x_{\lambda_1} \cdots x_{\lambda_n} \\
& + E(x^2,\Delta') l_n^{\mp} [x_\mu \epsilon_{\nu\rho\sigma k} x^k \mp (\mu \leftrightarrow \nu)] x_{\lambda_1} \cdots x_{\lambda_n}\} O^{[\rho\sigma](\lambda_1 \cdots \lambda_n)}(0),
\end{aligned} \tag{1}
$$

where

$$\Delta = d_j + 1 - \tfrac{1}{2}\tau, \quad \tau = d_n - n, \quad \Delta' = d_j - \tfrac{1}{2}\tau', \quad \tau' = d_n' - (n+2).$$

The parameter τ associated to an operator with scale dimension d and transforming with the $D(j, j')$ representation is defined as

$$\tau = d - (j + j') - |j - j'|. \tag{2}$$

For a symmetric operator, this number coincides with the "twist" defined by Gross and Treiman,[15] and it is a generalization in the general case ($j \neq j'$). As we shall see, operators with the same τ give contributions of the same scaling order to the Bjorken limit of the leptoproduction structure functions. Henceforth we shall refer to the number defined in (2) as "twist".

The $E(x^2, \Delta)$ casual functions are defined in the Appendix.

The coefficients b_n^-, g_n^-, k_n^-, and l_n^+ vanish according to the usual hypothesis of normal currents under time reversal. Therefore we shall omit the upper indices of b_n^+, g_n^+, k_n^+, and l_n^- without ambiguity.

Each set of values for τ, τ' and $a_n, b_n, \ldots$ leads to a particular light-cone model (for instance, the Fritzsch-Gell-Mann[5] model corresponds to $\tau = \tau' = 2$, $a_n = c_n + b_n = d_n = g_n = l_n = 0$).

Similarly to (2), we can construct the expansion for the divergence-current commutator:

$$
\begin{aligned}
[D^\dagger(x), J_\nu(0)] \simeq \sum_{\min\tau} & [E(x^2,\bar\Delta) \bar a_n x_\nu x_{\lambda_1} \cdots x_{\lambda_n} + E(x^2,\bar\Delta-1) \bar c_n g_{\nu\lambda_1} x_{\lambda_2} \cdots x_{\lambda_n}] O^{(\lambda_1 \cdots \lambda_n)}(0) \\
+ \sum_{\min\tau'} & [E(x^2,\bar\Delta') 2 \bar g_n g_{\nu[\rho} x_{\sigma]} x_{\lambda_1} \cdots x_{\lambda_n} + E(x^2,\bar\Delta') \bar h_n \epsilon_{\nu\rho\sigma k} x^k x_{\lambda_1} \cdots x_{\lambda_n}] O^{[\rho\sigma](\lambda_1 \cdots \lambda_n)}(0),
\end{aligned} \tag{3}
$$

where

$$\bar\Delta = \tfrac{1}{2}(\delta + d_j + 1 - \tau), \quad \bar\Delta' = \tfrac{1}{2}(\delta + d_j - \tau'),$$

with δ the scale dimension of the current divergence

$$D(x) \equiv \partial^\mu J_\mu(x).$$

Comparing (3) with the derivative of (1),

$$
\begin{aligned}
[\partial^\mu J_\mu^\dagger(x), J_\nu(0)] \simeq \sum_{\min\tau} & \{E(x^2,\Delta) x_\nu x_{\lambda_1} \cdots x_{\lambda_n} [(5+n-2\Delta) a_n + 2(c_n + b_n)] \\
& + E(x^2,\Delta-1) g_{\nu\lambda_1} x_{\lambda_2} \cdots x_{\lambda_n} [n c_n + 2 d_n + (6+n-2\Delta) b_n]\} O^{(\lambda_1 \cdots \lambda_n)}(0) \\
+ \sum_{\min\tau'} & \{E(x^2,\Delta') 2 g_{\nu[\rho} x_{\sigma]} x_{\lambda_1} \cdots x_{\lambda_n} [(6+n-2\Delta') k_n + 2 g_n] \\
& + E(x^2,\Delta') \epsilon_{\nu\rho\sigma k} x^k x_{\lambda_1} \cdots x_{\lambda_n} [(n+4-2\Delta') l_n - (n+2) h_n]\} O^{[\rho\sigma](\lambda_1 \cdots \lambda_n)}(0).
\end{aligned} \tag{4}
$$

We can see that if $\delta < d_j + 1$ (and, therefore, $\bar\Delta < \Delta$, and $\bar\Delta' < \Delta'$) the coefficients in expansion (4) must vanish. Then, the following relations arise:

$$(5+n-2\Delta)a_n+2(c_n+b_n)=0,\quad nc_n+(n+6-2\Delta)b_n+2d_n=0,$$
$$(n+6-2\Delta')k_n+2g_n=0,\quad (n+4-2\Delta')l_n-(n+2)h_n=0. \tag{5}$$

They are the formulation in coordinate space of the concept of current conservation on the light cone (CCLC).

We shall assume henceforth that the inequality $\delta < d_j+1$ is satisfied, as it would be, for instance, if the current conservation in the Lagrangian were broken by a mass term.[16]

II. COMPUTATION OF THE SCALE FUNCTIONS

In this section we shall find expressions for the scaling limit of the spin-dependent structure functions in leptoproduction:

$$W^s_{\mu\nu}(p,q)=\frac{1}{4\pi}\int dx\, e^{iq\cdot x}\langle\vec{p},s\,|[J^+_\mu(x),J_\nu(0)]\,|\vec{p},s\rangle_{\rm spin}$$
$$=-i\epsilon_{\mu\nu\rho\sigma}q^\rho s^\sigma\frac{Y_1}{2M}-i(q\cdot s)\epsilon_{\mu\nu\rho\sigma}q^\rho p^\sigma\frac{Y_2}{2M^2}-i\epsilon_{\mu\nu\rho\sigma}p^\rho s^\sigma\frac{Y_3}{2M}-g^\perp_{\mu\nu}(q\cdot s)\frac{Y_4}{M}+(q\cdot s)p^\perp_\mu p^\perp_\nu\frac{Y_5}{M^3}$$
$$+(p^\perp_\mu s^\perp_\nu+p^\perp_\nu s^\perp_\mu)\frac{Y_6}{M}+q_\mu q_\nu(q\cdot s)\frac{Y_7}{M^3}+(q_\mu p_\nu+q_\nu p_\mu)\frac{Y_8}{M^3}(q\cdot s)+(q_\mu s_\nu+q_\nu s_\mu)\frac{Y_9}{M}, \tag{6}$$

where Y_i, $i=1-9$, are dimensionless functions of q^2 and $\nu=p\cdot q$, and $g^\perp_{\mu\nu}=g_{\mu\nu}-q_\mu q_\nu/q^2$, $a^\perp_\mu=a_\mu-q_\mu(a\cdot q)/q^2$.

The scale functions $G_i(\omega)$, $i=1-9$, are the limit of the functions Y_i, $i=1-9$, in the Bjorken region:

$$\lim_{\rm Bj}\nu^{\alpha_i}Y_i(\nu,q^2)\to G_i(\omega),\quad i=1-9,$$

where ω is the "scaling variable", $\omega=2\nu/(-q^2)$, and the numbers α_i, $i=1-9$, are the scale order.

To compute the e.v. of the CCE (2), we shall use

$$\langle\vec{p},s\,|O^{(\lambda_1\cdots\lambda_n)}(0)\,|\vec{p},s\rangle=s^{(\lambda_1}p^{\lambda_2}\cdots p^{\lambda_n)}+\text{terms with one or more }g^{\lambda_i\lambda_j}$$
$$=\frac{1}{n}\sum_i p^{\lambda_1}\cdots s^{\lambda_i}\cdots p^{\lambda_n}+\sum_{i<j}g^{\lambda_i\lambda_j}\cdots,$$

$$\langle\vec{p},s\,|O^{[\rho\sigma](\lambda_1\cdots\lambda_n)}(0)\,|\vec{p},s\rangle=\tfrac{1}{2}(p^\rho s^\sigma-p^\sigma s^\rho)p^{\lambda_1}\cdots p^{\lambda_n}+\frac{M^2}{n+2}\sum_i g^{\lambda_i[\rho}s^{\sigma]}p^{\lambda_1}\cdots p^{\lambda_{i-1}}p^{\lambda_{i+1}}\cdots p^{\lambda_n}$$
$$-\frac{2M^2}{3(n+2)}\sum_{i<j}g^{\lambda_i\lambda_j}p^{[\rho}s^{\sigma]}p^{\lambda_1}\cdots p^{\lambda_{i-1}}p^{\lambda_{i+1}}\cdots p^{\lambda_{j-1}}p^{\lambda_{j+1}}\cdots p^{\lambda_n}$$
$$+\frac{M^2}{6(n+2)}\sum_{i,j}[g^{\rho\lambda_i}p^{[\lambda_j}s^{\sigma]}p^{\lambda_1}\cdots-(\rho\leftrightarrow\sigma)]+\text{terms with two or more }g^{\lambda_i'\lambda_j}$$
$$(\lambda_i'=\rho,\sigma,\lambda_i)\,. \tag{7}$$

The structure of the right-hand side of (7) is a consequence of the irreducibility of $O^{(\cdots)}$ and $O^{[\;](\cdots)}$.

Inserting the e.v. of these local operators in the CCE (2) and making the Lorentz contractions, we obtain the following expression for the e.v. of that current commutator:

$$\langle\vec{p},s\,|[J^\dagger_\mu(x),J_\nu(0)]\,|\vec{p},s\rangle\simeq\sum_n\bigg\{E(x^2,\Delta)a_n x_\mu x_\nu(s\cdot x)(p\cdot x)^{n-1}+E(x^2,\Delta-1)c_n g_{\mu\nu}(s\cdot x)(p\cdot x)^{n-1}$$
$$+E(x^2,\Delta-2)d_n\left[\frac{1}{n}(s_\mu p_\nu+s_\nu p_\mu)(p\cdot x)+\frac{n-2}{n}p_\mu p_\nu(s\cdot x)\right](p\cdot x)^{n-3}$$
$$+E(x^2,\Delta-1)b_n\left[\frac{1}{n}(x_\mu s_\nu+x_\nu s_\mu)(p\cdot x)+\frac{n-1}{n}(x_\mu p_\nu+x_\nu p_\mu)(s\cdot x)\right](p\cdot x)^{n-2}$$
$$+E(x^2,\Delta-1)e_n\left[\frac{1}{n}(p\cdot x)\epsilon_{\mu\nu\lambda k}s^\lambda x^k+\frac{n-1}{n}(s\cdot x)\epsilon_{\mu\nu\lambda k}p^\lambda x^k\right](p\cdot x)^{n-2}$$
$$+E(x^2,\Delta'-1)g_n[2p_\mu p_\nu(s\cdot x)-(p_\mu s_\nu+p_\nu s_\mu)(p\cdot x)](p\cdot x)^{n-1}$$

$$+E(x^2\Delta')h_n[(p\cdot x)\epsilon_{\sigma\mu\nu k}s^\sigma x^k-(s\cdot x)\epsilon_{\sigma\mu\nu k}p^\sigma x^k](p\cdot x)^n$$
$$+E(x^2,\Delta')k_n[(x_\mu p_\nu+x_\nu p_\mu)(s\cdot x)-(x_\mu s_\nu+x_\nu s_\mu)(p\cdot x)](p\cdot x)^n$$
$$+E(x^2,\Delta')l_n[x_\mu\epsilon_{\nu\rho\sigma k}p^\rho s^\sigma x^k-(\mu\leftrightarrow\nu)](p\cdot x)^n$$
$$+\frac{M^2}{n+2}\{E(x^2,\Delta'-1)g_n[x_\mu s_\nu+s_\nu x_\mu-2(x\cdot s)g_{\mu\nu}](p\cdot x)^{n-1}$$
$$+E(x^2,\Delta')nh_n[-x^2\epsilon_{\sigma\mu\nu k}s^\sigma x^k](p\cdot x)^{n-1}$$
$$+E(x^2,\Delta')nk_n[(x_\mu s_\nu+x_\nu s_\mu)x^2-2(s\cdot x)x_\mu x_\nu](p\cdot x)^{n-1}\}$$
$$\left.+\frac{(n-1)M^2}{n+2}E(x^2,\Delta'-1)g_n[(s_\mu p_\nu+s_\nu p_\mu)x^2-(x_\mu p_\nu+x_\nu p_\mu)(s\cdot x)](p\cdot x)^{n-2}\right\}$$
$$+\text{nonleading terms}. \tag{8}$$

From this e.v. we select the leading coefficients of all independent tensorial terms. They have been collected in Table I. Therefore, after expanding the current commutator and computing its e.v. we get

$$\begin{aligned}\langle\vec{p},s\,|[J_\mu^\dagger(x),J_\nu(0)]\,|\vec{p},s\rangle=&(s_\mu p_\nu+s_\nu p_\mu)\tilde{f}_1(p\cdot x)E(x^2,\lambda_1)+x_\mu x_\nu(s\cdot x)\tilde{f}_2(p\cdot x)E(x^2,\lambda_2)+g_{\mu\nu}(s\cdot x)\tilde{f}_3(p\cdot x)E(x^2,\lambda_3)\\&+p_\mu p_\nu(s\cdot x)\tilde{f}_4(p\cdot x)E(x^2,\lambda_4)+(x_\mu s_\nu+x_\nu s_\mu)\tilde{f}_5(p\cdot x)E(x^2,\lambda_5)\\&+(x_\mu p_\nu+x_\nu p_\mu)(s\cdot x)\tilde{f}_6(p\cdot x)E(x^2,\lambda_6)+\epsilon_{\mu\nu\lambda k}s^\lambda x^k\tilde{f}_7(p\cdot x)E(x^2,\lambda_7)\\&+\epsilon_{\mu\nu\lambda k}p^\lambda x^k(s\cdot x)\tilde{f}_8(p\cdot x)E(x^2,\lambda_8)+\epsilon_{\mu\nu\lambda k}s^\lambda p^k\tilde{f}_9(p\cdot x)E(x^2,\lambda_9)\,.\end{aligned} \tag{9}$$

Each $\tilde{f}_i(x\cdot p)$, $i=1-9$, corresponds to the addition of the most singular contributions to each term in Table I. Therefore, one or two kinds of coefficients contribute to each function depending on the values taken by Δ and Δ'. The parameters λ_i, $i=1-9$, denote the order of the leading singularity in each term. For instance, $\lambda_1=\max(\Delta-2,\Delta'-1)$, $\lambda_2=\max(\Delta,\Delta')$, etc.

In order to find the Bjorken limit of the structure functions we have to Fourier-transform (9).

TABLE I. Light-cone dominant contributions to the current-commutator expectation value.

Term index	Tensorial terms	Contributions from symmetric O: Singularity	Contributions from symmetric O: Coefficient	Contributions from mixed-symmetry O: Singularity	Contributions from mixed-symmetry O: Coefficient
1	$s_\mu p_\nu+s_\nu p_\mu$	$E(x^2,\ \Delta-1)$	$\left[\frac{d_n}{n}(p\cdot x)^{n-2}\right]$	$E(x^2,\ \Delta'-1)$	$[-g_n\ (p\cdot x)^n]$
2	$x_\mu x_\nu\ (s\cdot x)$	$E(x^2,\ \Delta)$	$[a_n\ (p\cdot x)^{n-1}]$	$E(x^2,\ \Delta')$	$\left[\frac{-2nM^2}{n+2}k_n(p\cdot x)^{n-1}\right]$
3	$g_{\mu\nu}\ (s\cdot x)$	$E(x^2,\ \Delta-1)$	$[c_n\ (p\cdot x)^{n-1}]$	$E(x^2,\ \Delta'-1)$	$\left[\frac{-2M^2}{n+2}g_n\ (p\cdot x)^{n-1}\right]$
4	$p_\mu p_\nu\ (s\cdot x)$	$E(x^2,\ \Delta-2)$	$\left[\frac{n-2}{n}d_n\ (p\cdot x)^{n-3}\right]$	$E(x^2,\ \Delta'-1)$	$[2g_n\ (p\cdot x)^{n-1}]$
5	$x_\mu s_\nu+x_\nu s_\mu$	$E(x^2,\ \Delta-1)$	$\left[\frac{b_n}{n}(p\cdot x)^{n-1}\right]$	$E(x^2,\ \Delta')$	$[-k_n\ (p\cdot x)^{n+1}]$
6	$(x_\mu p_\nu+x_\nu p_\mu)\ (s\cdot x)$	$E(x^2,\ \Delta-1)$	$\left[\frac{n-1}{n}b_n\ (p\cdot x)^{n-2}\right]$	$E(x^2,\ \Delta')$	$[k_n\ (p\cdot x)^n]$
7	$\epsilon_{\mu\nu\lambda k}\ s^\lambda x^k$	$E(x^2,\ \Delta-1)$	$\left[\frac{e_n}{n}(p\cdot x)^{n-1}\right]$	$E(x^2,\ \Delta')$	$[(h_n-l_n)\ (p\cdot x)^{n+1}]$
8	$\epsilon_{\mu\nu\lambda k}\ p^\lambda x^k\ (s\cdot x)$	$E(x^2,\ \Delta-1)$	$\left[\frac{n-1}{n}e_n\ (p\cdot x)^{n-2}\right]$	$E(x^2,\ \Delta')$	$[(l_n-h_n)\ (p\cdot x)^n]$
9	$\epsilon_{\mu\nu\lambda k}\ s^\lambda p^k$	$\cdots$		$E(x^2,\ \Delta'-1)$	$[(1-\Delta')\,l_n\ (p\cdot x)^n]$

From the first term we obtain

$$(s_\mu p_\nu + s_\nu p_\mu)\int e^{iq\cdot x}dx\, E(x^2,\lambda_1)\tilde{f}_1(p\cdot x) = (s_\mu p_\nu + s_\nu p_\mu)\int f_1(\xi)d\xi \int dx\, e^{i(q+\xi p)\cdot x}E(x^2,\lambda_1)$$

$$= (2\pi)^2 2^{2(1-\lambda_1)} i\,(s_\mu p_\nu + s_\nu p_\mu)\int f_1(\xi)d\xi\, E(k^2, 2-\lambda_1)\,,$$

where

$$k = q+\xi p, \quad \tilde{f}_i(p\cdot x) = \int d\xi\, e^{i\xi p\cdot x} f_i(\xi)\,.$$

Using

$$(-k^2 + i\epsilon k^0)^\lambda = [-(q^2 + 2\xi p\cdot q + q^2M^2) + i\epsilon(q\cdot p + q^2M^2)]^\lambda = (2\nu)^\lambda\left[-\left(\xi - \omega^{-1} + \frac{\xi^2M^2}{2\nu}\right) + i\epsilon\left(1 + \frac{\xi^2+M^2}{\nu}\right)\right]^\lambda,$$

we have

$$\lim_{\mathrm{Bj}} E(k^2, 2-\lambda_i) = (2\nu)^{\lambda_i - 2}\hat{E}(\xi - \omega^{-1}, 2-\lambda_i)\,,$$

where $\hat{E}$ is defined in the Appendix.

If we define

$$\psi_i(\omega^{-1}) = 2^{2-\lambda_i}\pi^2 i\int f_i(\xi)d\xi\, \hat{E}(\xi - \omega^{-1}, 2-\lambda_i)\,, \tag{10}$$

the contribution from the first term in (9) to the scale function associated to $s_\mu p_\nu + s_\nu p_\mu$ is

$$(s_\mu p_\nu + s_\nu p_\mu)\nu^{\lambda_1 - 2}\psi_1\,.$$

Proceeding in a similar way with the remaining terms and collecting the results, we obtain

$$\lim_{\mathrm{Bj}}\int dx\, e^{iq\cdot x}\langle \vec{p}, s\,|[J_\mu^\dagger(x), J_\nu(0)]\,|\vec{p}, s\rangle = A_1 + A_2 + \cdots + A_9\,, \tag{11}$$

where

$$A_1 \equiv (s_\mu p_\nu + s_\nu p_\mu)\nu^{\lambda_1-2}\psi_1\,,$$

$$\begin{aligned} A_2 \equiv & -i(s_\mu p_\nu + s_\nu p_\mu)[(\lambda_2 - 3)\psi_2' - \chi\psi_2'']\nu^{\lambda_2-4} + i(s_\mu q_\nu + s_\nu q_\mu)\psi_2''\nu^{\lambda_2-4} \\ & + i(q_\mu p_\nu + q_\nu p_\mu)(q\cdot s)[(\lambda_2-4)\psi_2'' - \chi\psi_2''']\nu^{\lambda_2-5} + ig_{\mu\nu}(q\cdot s)\psi_2''\nu^{\lambda_2-4} \\ & - ip_\mu p_\nu(q\cdot s)\{(\lambda_2-4)[(\lambda_2-3)\psi_2' - \chi\psi_2''] - \chi[(\lambda_2-3)\psi_2' - \chi\psi_2'']\}\nu^{\lambda_2-5} - iq_\mu q_\nu(q\cdot s)\psi_2'''\nu^{\lambda_2-5}\,, \end{aligned}$$

$$A_3 \equiv ig_{\mu\nu}(q\cdot s)\nu^{\lambda_3-3}\psi_3'\,,$$

$$A_4 \equiv ip_\mu p_\nu(q\cdot s)\nu^{\lambda_4-3}\psi_4'\,,$$

$$A_5 \equiv (p_\mu s_\nu + p_\nu s_\mu)[-i(\lambda_5-2)\psi_5 + i\chi\psi_5']\nu^{\lambda_5-3} + (q_\mu s_\nu + q_\nu s_\mu)[i\psi_5']\nu^{\lambda_5-3}\,,$$

$$A_6 \equiv 2p_\mu p_\nu(q\cdot s)[(\lambda_6-3)\psi_6' - \chi\psi_6'']\nu^{\lambda_6-4} - (q_\mu p_\nu + q_\nu p_\mu)(q\cdot s)\psi_6''\nu^{\lambda_6-4} + (p_\mu s_\nu + p_\nu s_\mu)\nu^{\lambda_6-3}\psi_6'\,,$$

$$A_7 \equiv i\epsilon_{\mu\nu\lambda k}s^\lambda p^k[(\lambda_7-2)\psi_7 - \chi\psi_7']\nu^{\lambda_7-3} + i\epsilon_{\mu\nu\lambda k}s^\lambda q^k\psi_7'\nu^{\lambda_7-3}\,,$$

$$A_8 \equiv \epsilon_{\mu\nu\lambda k}p^\lambda s^k\nu^{\lambda_8-3}\psi_8' - \epsilon_{\mu\nu\lambda k}p^\lambda q^k(q\cdot s)\nu^{\lambda_8-4}\psi_8''\,,$$

$$A_9 \equiv \epsilon_{\mu\nu\lambda k}s^\lambda p^k\nu^{\lambda_9-2}\psi_9\,,$$

with

$$\chi = \omega^{-1} \text{ and } \psi' = (d/d\chi)\psi(\chi).$$

Looking to the powers of ν in each term of (11) and selecting the leading ones, we get the contributions to the scale functions $G_i(\omega)$, $i = 1$–9, shown in Table II. In that table we distinguish between the functions coming from the symmetric operators and the ones generated by the mixed-symmetry operators.

As long as the values for Δ and Δ' are not specified, both kinds of functions have to be separately considered.

The functions $\hat{G}_7, \hat{G}_8, \hat{G}_9$ in Table II are not the true scale functions of Y_7, Y_8, Y_9. In order to get these we have to subtract from $\hat{G}_7, \hat{G}_8, \hat{G}_9$ the contributions to the explicitly conserved terms in $W_{\mu\nu}$. This mechanism is indicated in Table III.

TABLE II. Dominant contributions to the scale functions G_i, $i=1$–6 and $\hat{G}_7$, $\hat{G}_8$, $\hat{G}_9$.

Tensorial term	Scale function	Power of ν	Symmetric-$\mathcal{O}$ contributions (above) Mixed-symmetry-$\mathcal{O}$ contributions (below)
$-i\epsilon_{\mu\nu\rho\sigma}q^\rho s^\sigma$	$G_1(\omega)$	$\Delta-4$	ψ_7'
		$\Delta'-3$	ψ_7'
$-i\epsilon_{\mu\nu\rho\sigma}q^\rho p^\sigma(q\cdot s)$	$G_2(\omega)$	$\Delta-5$	$i\psi_8''$
		$\Delta'-4$	$i\psi_8''$
$-i\epsilon_{\mu\nu\rho\sigma}p^\rho s^\sigma$	$G_3(\omega)$	$\Delta-4$	$-[(\Delta-3)\psi_7-\chi\psi_7']+i\psi_8'\equiv 0$
		$\Delta'-3$	$-[(\Delta-3)\psi_7-\chi\psi_7']-i\psi_9+i\psi_8'$
$-g^{\perp}_{\mu\nu}(q\cdot s)$	$G_4(\omega)$	$\Delta-4$	$-i(\psi_2''+\psi_3')$
		$\Delta'-4$	$-i(\psi_2''+\psi_3')$
$p^{\perp}_\mu p^{\perp}_\nu(q\cdot s)$	$G_5(\omega)$	$\Delta-5$	$-i\{(\Delta-4)[(\Delta-3)\psi_2'-\chi\psi_2'']-\chi[(\Delta-3)\psi_2'-\chi\psi_2'']'\}$ $+i\psi_4'+2[(\Delta-4)\psi_6'-\chi\psi_6'']$
		$\Delta'-4$	$i\psi_4'+2[(\Delta'-3)\psi_6'-\chi\psi_6'']$
$p^{\perp}_\mu s^{\perp}_\nu+p^{\perp}_\nu s^{\perp}_\mu$	$G_6(\omega)$	$\Delta-4$	$\psi_1-i[(\Delta-3)\psi_2'-\chi\psi_2'']-i[(\Delta-3)\psi_5-\chi\psi_5']+\psi_6'$
		$\Delta'-3$	$\psi_1-i[(\Delta'-2)\psi_5-\chi\psi_5']+\psi_6'$
$q_\mu q_\nu(q\cdot s)$	$\hat{G}_7(\omega)$	$\Delta-5$	$-i\psi_2'''$
		$\Delta'-5$	$-i\psi_2'''$
$(q_\mu p_\nu+q_\nu p_\mu)(q\cdot s)$	$\hat{G}_8(\omega)$	$\Delta-5$	$i[(\Delta-4)\psi_2''-\chi\psi_2''']-\psi_6''$
		$\Delta'-4$	$-\psi_6''$
$q_\mu s_\nu+q_\nu s_\mu$	$\hat{G}_9(\omega)$	$\Delta-4$	$i\psi_2''+i\psi_5'$
		$\Delta'-3$	$i\psi_5'$

As we can see in Tables II and III, the connection between the leading singularities in the CCE and the scaling orders of each Y_i is obvious.

In general, the values of Δ and Δ' are independent; hence the contributions of both kinds of operators will depend on the actual relation between Δ and Δ'. The most reasonable assumption $\tau=\tau'$ means that $\Delta=\Delta'+1$, and both kinds of operators contribute simultaneously to all scale functions except G_4, where the contributions from mixed-symmetry operators are trace-like and therefore nonleading.

Looking at Table I, the following relations can be obtained:

$$
\begin{array}{cc}
\Delta & \Delta' \\
\hline
\tilde{f}_4\equiv\tilde{f}_1' & \tilde{f}_7+(x\cdot p)\tilde{f}_8\equiv 0 \\
\tilde{f}_8\equiv\tilde{f}_7' & 2\tilde{f}_1+(x\cdot p)\tilde{f}_4\equiv 0 \\
\tilde{f}_6\equiv\tilde{f}_5' & \tilde{f}_5+(x\cdot p)\tilde{f}_6\equiv 0
\end{array}
\tag{12}
$$

and the formulas (5) become

$$
\begin{array}{cc}
\Delta & \Delta' \\
\hline
(x\cdot p)\tilde{f}_2'+(6-2\Delta)\tilde{f}_2+2[\tilde{f}_2+\tilde{f}_5+(x\cdot p)\tilde{f}_6]=0 & (5-2\Delta')\tilde{f}_6-\tilde{f}_5'-2\tilde{f}_1=0 \\
\tilde{f}_3+(7-2\Delta)\tilde{f}_5+(x\cdot p)(\tilde{f}_6+2\tilde{f}_1)=0 & \tilde{f}_8-\tilde{f}_7'+2\tilde{f}_9=0\,.
\end{array}
\tag{13}
$$

Fourier-transforming (12) and (13) we obtain

$$
\begin{array}{cc}
\Delta & \Delta' \\
\hline
f_4-i\xi f_1\equiv 0 & f_7+if_8'\equiv 0 \\
f_8-i\xi f_7\equiv 0 & 2f_1+if_4'\equiv 0 \\
f_6-i\xi f_5\equiv 0 & f_5+if_6'\equiv 0
\end{array}
\tag{14}
$$

and

$$
\begin{array}{cc}
\Delta & \Delta' \\
\hline
(6-2\Delta)f_2-(\xi f_2)'+2(f_3+f_5+if_6')=0 & (5-2\Delta')f_6-i\xi f_5-2f_1=0 \\
f_2+(7-2\Delta)f_5+i(f_6'+2f_1)=0 & f_8-i\xi f_7+2f_9=0\,.
\end{array}
\tag{15}
$$

(14) and (15) can be translated to relations between the $\psi_i(\omega)$, $i=1-9$, and therefore to relations between the scale functions $G_i(\omega)$, $i=1-9$. The use of (15) that has been obtained from the CCLC relations (5) will cancel out the contributions in Tables II and III to the function G_3, G_7, G_8, G_9.

In this situation some unknown contributions to those functions arise. These come from less-singular terms, not specified in (1). The scaling orders of the nonconserved structure functions will be connected with $\overline{\Delta}$ and $\overline{\Delta}'$ in (3) and therefore ruled by the scale dimension δ of the conservation-breaking term in the Lagrangian.

For example, the symmetrical leading contribution to G_3

$$i[(\Delta-3)_{\psi_7}-\chi\psi_7']+\psi_8'=2^{3-\Delta}\pi^2 i\int(i\xi f_7-f_8)\hat{E}(\xi-\omega^{-1},4-\Delta)d\xi$$

vanishes by (14). This conclusion is not a surprise because a full symmetric operator cannot give any contribution to $\epsilon_{\mu\nu\rho\sigma}p^\rho s^\sigma$.

Furthermore, the remaining expression for G_3 coming from the mixed-symmetry operators results in

$$i[(\Delta'-2)\psi_7-\chi\psi_7']-\psi_9+\psi_8'=2^{2-\Delta'}\pi^2 i\int(i\xi f_7-f_8)\hat{E}(\xi-\omega^{-1},3-\Delta')d\xi+2^{3-\Delta'}\pi^2 i\int f_9\hat{E}(\xi-\omega^{-1},3-\Delta')d\xi\ ,$$

which vanishes after the application of the CCLC relations (15). Therefore, using CCLC, the scaling orders for the nonconserved structure functions get depressed in the following way:

$$\frac{\nu^\alpha Y_3}{2M}\xrightarrow[\mathrm{Bj}]{}G_3(\omega)\ ,\quad \frac{\nu^{\alpha+1}Y_8}{M^3}\to G_8(\omega)\ ,$$

$$\frac{\nu^{\alpha+1}Y_7}{M^3}\to G_7(\omega)\ ,\quad \frac{\nu^\alpha Y_9}{M}\to G_9(\omega)\ ,$$

where $\alpha=4-\frac{1}{2}(\delta+d_j+1-\min[\tau,\tau'])$.

The Bjorken limit of the structure functions Y_3, Y_7, Y_8, Y_9 depends on whether current divergence is zero:

If the current is conserved $[D(x)\equiv0]$, all contributions to G_3, G_7, G_8, G_9 will cancel out and these functions vanish. On the other hand, if the current divergence does not vanish $[D(x)\neq0]$, the nonconserved scale functions G_3, G_7, G_8, G_9 are unknown even for a given light-cone model, since they do not depend directly on the most-singular terms in the CCE.

APPENDIX

The causal function $E(x^2,\lambda)$ is defined as

$$E(x^2,\lambda)=\Gamma(\lambda)[(-x^2+i\epsilon x^0)^{-\lambda}-(-x^2-i\epsilon x^0)^{-\lambda}]_{\epsilon\to0^+}\ .$$

This function is analytic on the whole λ plane,[17] and has a cut on the x^2 plane along R^-. It takes the following expressions according to the values of the λ parameter:

$$E(x^2,-n)=-\frac{(x^2)^n}{n!}\,2\pi i\epsilon(x^0)\theta(x^2)\ ,\quad n=0,1,2,\ldots$$

$$E(x^2,n)=-2\pi i\epsilon(x^0)\delta^{(n-1)}(x^2)\ ,\quad n=1,2,\ldots$$

$$E(x^2,\lambda)=-2i\,\Gamma(\lambda)\,\sin\pi\lambda\;\epsilon(x^0)\theta(x^2)(x^2)^{-\lambda},\quad \lambda\neq\pm n\ .$$

The Fourier transform of $E(x^2,\lambda)$ is

$$\tilde{E}_\lambda(k^2)\equiv\int\frac{d^4x}{(2\pi)^2}\,e^{ik\cdot x}E(x^2,\lambda)$$

$$=i2^{2(1-\lambda)}E(k^2,2-\lambda)\ .$$

The function $\hat{E}(u,\lambda)$ is defined in a similar way,

TABLE III. Dominant contributions to the scale functions G_7, G_8, G_9.

Tensorial term	Scale function	Power of ν	
$q_\mu q_\nu\,(q\cdot s)$	$G_7(\omega)$	$\Delta-5$	$\hat{G}_7+\frac{1}{2\chi}\,[G_4-(G_5+G_6)/\chi]$
		$\Delta'-4$	$-(G_5+G_6)/2\chi^2$
$(q_\mu p_\nu+q_\nu p_\mu)(q\cdot s)$	$G_8(\omega)$	$\Delta-5$	$\hat{G}_8-(G_5+G_6)/2\chi$
		$\Delta'-4$	$\hat{G}_8-(G_5+G_6)/2\chi$
$q_\mu s_\nu+q_\nu s_\mu$	$G_9(\omega)$	$\Delta-4$	$\hat{G}_9-G_6/2\chi$
		$\Delta'-3$	$\hat{G}_9-G_6/2\chi$

$$\hat{E}(u,\lambda)=\Gamma(\lambda)[(-u+i\epsilon)^{-\lambda}-(-u-i\epsilon)^{-\lambda}]_{\epsilon\to 0^+}\ ,$$

and its expressions according to the λ values are

$$\hat{E}(u,n)=-2\pi i\delta^{(n-1)}(u)\ ,\quad n=1,2,\ldots$$

$$\hat{E}(u,-n)=-2\pi i\frac{u^n}{n!}\,\theta(u)\ ,\quad n=0,1,2,\ldots$$

$$\hat{E}(u,\lambda)=-2i\Gamma(\lambda)\sin\pi\lambda\, u^{-\lambda}\theta(u)\ ,\quad \lambda\neq\pm n\ .$$

Both functions E and $\hat{E}$ are easy to differenciate; for example, we have

$$\frac{\partial^n}{\partial u^n}\hat{E}(u,\lambda)=\hat{E}(u,\lambda+n)\ .$$

We shall also make use of the following relation:

$$u\hat{E}(u,\lambda)=-(\lambda-1)\hat{E}(u,\lambda-1)\ .$$

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