Infrared behavior in non-Abelian theories for forward processes*

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We show, by means of a simple example, that the Bloch-Nordsieck program cannot be applied to non-Abelian theories in an identical manner to quantum electrodynamics. The simple example is single-gluon bremsstrahlung in quark-quark scattering to lowest order in perturbation theory, the cross section for which is found to be *quadratically* divergent, i.e., $\sim 1/\lambda^2$, where λ is the fictitious gluon mass. Radiation from *internal* gluon lines is found to contribute to this divergence. Application to the scattering of color-singlet states is discussed.

I. INTRODUCTION

The most popular class of models for the strong interaction today is that of the non-Abelian (Yang-Mills[1]) gauge theories.[2] Their renormalizability and ability to explain the short-distance phenomenon of Bjorken scaling in terms of "asymptotic freedom"[3] makes them particularly attractive. However, no one has yet demonstrated that their long-distance behavior can account for the feature of "quark confinement," essential for any realistic description of hadron physics. The hope is that somehow the infrared (IR) divergences of these theories are so severe that they prevent asymptotic states which are not singlets of the gauge group. To this end we are studying the most severe infrared divergences in these theories. In this paper we present the results of a simple calculation, which demonstrates that indeed *these divergences are worse in non-Abelian (NA) theories than in QED*, so that they cannot be cured by the standard techniques used in QED.[4,5,6] The simple calculation is the evaluation of the cross section for the process

quark + quark (antiquark)
→ quark + quark (antiquark) + gluon ,

which is found to be quadratically divergent (i.e., $\sim 1/\lambda^2$, where λ is the fictitious gluon mass).[7] This divergence comes entirely from the forward region. The cross section for the analogous process in QED is only logarithmically divergent, and this divergence is cancelled by virtual-gluon contributions, in the way first discovered by Bloch and Nordsieck.[4]

Among the diagrams which contribute to this quadratic divergence in NA theories is one in which the soft gluon is radiated from an internal gluon line, whereas in QED only radiation from external lines contributes to the worst IR-divergent terms. This is due to the fact that photons couple only to massive particles, whereas the gluons which carry the group charge can couple to themselves through 3 (or 4) gluon couplings.

In order to put our work in perspective, we start by briefly reviewing other related results. A detailed study of infrared divergences in NA theories away from the forward direction has been undertaken by Cornwall and Tiktopoulos.[8] By explicitly evaluating lowest-order perturbation-theory diagrams and studying all orders by means of a certain differential equation, these authors are able to isolate the leading infrared divergences to each order in perturbation theory. The validity of their differential equation has not been established rigorously for NA theories; nevertheless, it seems exceedingly plausible. The leading infrared divergences from each order of perturbation theory are then summed, and one finds that these nonforward processes have associated with them a factor of the form

$$\exp\left(-Ag^2\sum_j c_j f(\lambda)\right) , \tag{1.1}$$

where λ is the fictitious gluon mass introduced as an IR cutoff, and $f(\lambda)\sim\ln\lambda$ or $\ln^2\lambda$, depending upon the process and region of phase space being studied. A is a kinematic factor independent of λ, and c_j is the eigenvalue of the quadratic Casimir operator for the group representation to which the jth external particle belongs. Thus if at least one of the external particles is not a singlet, this exponential factor goes to zero as $\lambda\to 0$, a feature which the authors interpret as confinement. It will be extremely interesting to see if a similar result can be established when forward processes are included, since that is where the worst IR divergences occur. It is, of course, divergences in the forward direction which are responsible for the infinite Coulomb cross section in electrodynamics.

In spite of the appealing results mentioned above, there remain the usual doubts about the validity of summing a series of leading logarithms (especially since the series is alternating in sign and its sum is much smaller than the neglected terms individually), together perhaps with the doubts about the differential equation. Several authors[9,10,11] have tried to understand the infrared structure of NA theories order by order in perturbation theory, analogously to QED. In particular, Yao[9] has studied nonforward quark-quark and off-shell gluon-quark scattering to lowest nontrivial order in the coupling constant (sixth order in the cross section), and found that the logarithmic IR divergences which appear cancel between the real and virtual gluons, just as in QED, as long as the observer cannot detect the group charge. For quark-quark scattering the calculation is then essentially the same as in QED; none of the "new" diagrams (i.e., those with 3-gluon vertices, etc.) are divergent. If the result of Cornwall and Tiktopoulos[8] that "... cross sections of nonforward processes involving non-neutral (i.e., nongroup singlet) particles, whether or not an indefinite number of soft gauge mesons are included, *vanish* in the limit $\lambda \to 0$" is correct, then the analogy with the situation in QED will break down in higher orders. However, worse divergences appear in the forward direction, and it is these which are the subject of study in this paper.

Appelquist *et al.*[10] have studied IR divergences to lowest order in perturbation theory in the famous process

$$e^+e^- \to \gamma \to \text{hadrons}\,,$$

and found that the cross section for this process is IR finite, diagram by diagram for the vacuum polarization. Contributions to particular final states may, however, be IR divergent in a given order. This is similar to the situation in QED.[12,13] Thus it seems that for this process, confinement cannot be seen in any finite order of perturbation theory. Since this calculation was done in one order of perturbation theory, there is, of course, no conflict with the results of Cornwall and Tiktopoulos.[8]

In Sec. II we review very briefly the process $e^-e^-(e^+) \to e^-e^-(e^+)\gamma$ in QED. In Sec. III we present our results for the analogous process in non-Abelian theories and show that it is quadratically divergent. Finally, in Sec. IV we present our conclusions, in particular, those relating the results of Sec. III to the scattering of group singlets.

II. THE PROCESS $e^-e^-(e^+) \to e^-e^-(e^+) + \gamma$

Infrared divergences in QED are now well understood[4,5,6] both classically and quantum mechanically. The only infinite cross section is the elastic one, which makes manifest the infinite range of the Coulomb potential. Here we will not present any classical examples, but will study the process

$$e^-e^- \to e^-e^- + \gamma \tag{2.1}$$

in terms of Feynman diagrams. The amplitude for this process in the infrared limit can be written as the sum of four diagrams [Figs. 1(i)–1(iv)], and we start by considering the first two. Working in the Feynman gauge, we write

$$\begin{aligned} T_1 &= -\,\bar{u}(p_3)\epsilon\cdot\gamma(\not{p}_3+\not{k}+m)\gamma^\mu u(p_1) \\ &\quad\times \bar{v}(p_2)\gamma_\mu v(p_4)\frac{1}{2p_3\cdot k}\frac{1}{(p_4-p_2)^2} \\ &\simeq -\frac{\epsilon\cdot p_3}{p_3\cdot k}\bar{u}(p_3)\gamma^\mu u(p_1)\,\bar{v}(p_2)\gamma_\mu v(p_4) \end{aligned} \tag{2.2a}$$

in the infrared limit, where the $\not{k}$ in the numerator has been neglected. Similarly,

$$T_2 \simeq \frac{\epsilon\cdot p_1}{p_1\cdot k}\bar{u}(p_3)\gamma^\mu u(p_1)\,\bar{v}(p_2)\gamma_\mu v(p_4)\,. \tag{2.2b}$$

Calculating the contribution of T_1 to the cross section (it itself is, of course, not gauge invariant), one finds that it is quadratically divergent. This divergence is not a physical one, however,

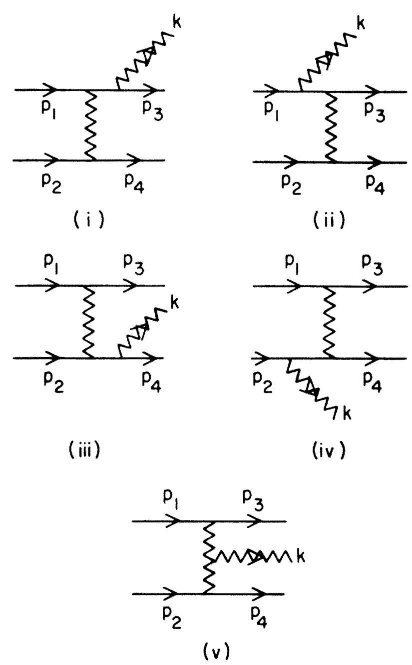


FIG. 1. (i)–(iv) are the diagrams which contribute to to the bremsstrahlung of a single soft photon in electron-electron scattering. (i)–(v) are the diagrams which contribute to the bremsstrahlung of a single soft gluon in quark-quark scattering.

since in the sum $T_1 + T_2$ there appears a factor

$$\frac{\epsilon\cdot p_1}{p_1\cdot k} - \frac{\epsilon\cdot p_3}{p_3\cdot k}, \tag{2.3}$$

which vanishes in the forward direction, which is where T_1 and T_2 individually are most singular. In potential scattering, for example, the factor (2.3) corresponds to

$$\frac{\epsilon\cdot v_i}{\omega - \vec{k}\cdot\vec{v}_i} - \frac{\epsilon\cdot v_f}{\omega - \vec{k}\cdot\vec{v}_f},$$

where v_i (v_f) is the initial (final) velocity of the electron, ω is the conjugate variable to time, and $\vec{k} = \omega\hat{x}$, where $\vec{x}$ is position in space. This factor appears in the expression for the vector potential, and hence in many physical quantities. The contribution of $T_1 + T_2$ to the cross section is only logarithmically divergent.

Altarelli and Bucella[14] have calculated the cross section for this process to this order in perturbation theory at high energy and obtain

$$\frac{d\sigma}{d\omega} = \frac{8\alpha r_e^2}{\omega}\frac{E-\omega}{E}\left(\frac{E}{E-\omega} + \frac{E-\omega}{E} - \frac{2}{3}\right) \times \left(\ln\frac{4E^2(E-\omega)}{m^2\omega} - \frac{1}{2}\right), \tag{2.4}$$

where ω is the photon energy in the c.m. frame, $r_e = e^2/m$, and $E = \frac{1}{2}\sqrt{s}$. The logarithmic divergence as $\omega \to 0$ is explicitly demonstrated. For $e^+e^- \to e^+e^-\gamma$ there are also the annihilation diagrams. These diagrams individually do not lead to any quadratic divergences.

In non-Abelian theories T_1 and T_2 now have different group-theory matrices associated with them so that the cancellation (2.3) as $p_1 \to p_3$ does not occur. However, there is a new diagram [Fig. 1(v)], and the question which will be studied in the next section is whether the five diagrams of Fig. 1 can conspire to remove the quadratic divergence.

III. THE PROCESS $qq(\bar{q}) \to qq(\bar{q}) + \gamma$

We consider in this section the on-shell process

quark + quark (antiquark)

→ quark + quark (antiquark) + gluon

in lowest-order perturbation theory and isolate the most infrared-divergent part. For this process we do not have any obvious classical limit to guide us. The theory is defined by the Lagrangian density

$$-\tfrac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} + \bar{q}(i\partial\!\!\!/ - M - ig A\!\!\!/^i T^i)q, \tag{3.1}$$

which couples a set of gluon fields A^i_μ to a multiplet of quark fields q. The matrices T^i are the generators of the gauge group normalized by

$$[T^i, T^j] = ic_{ijk}T^k, \tag{3.2}$$

where c_{ijk} are the structure constants of the group. $F^i_{\mu\nu}$ is defined by

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + gc_{ijk}A^j_\mu A^k_\nu. \tag{3.3}$$

For definiteness the calculation is carried out with SU(n) as the gauge group, but it will become clear that our main results are independent of the group. The charge associated with this group will be called color, even for $n \neq 3$.

In the infrared limit there are five diagrams for this process [Figs. 1(i)–1(v)]. Each of the diagrams of Fig. 1 can be written as a product of a group-theoretic weight and a quantity which depends on the kinematic variables. Evaluation of the group-theoretic factor is particularly simple using the graphical techniques of Cvitanović,[15] and we would like briefly to illustrate this here. For simplicity we assume that the initial state is a quark-antiquark system in a singlet state, although the reader can readily generalize this (in particular, one can average over the colors of the initial quarks), and as will be shown below it does not alter our conclusions. The group-theoretic weight is a product of factors[15]:

(i) δ_{ab} for each internal quark leg,

(ii) δ_{ij} for each internal gluon,

(iii) $(T_i)_{ab}$ for each quark-quark-gluon vertex, and

(iv) $-ic_{ijk}$ for each 3-gluon vertex.[16]

These can be represented graphically as in Fig. 2. The lie algebra for the group SU(n), for example, can then be translated into diagrammatic identities, which in turn can be used to evaluate the group-theoretic weights of the diagrams. We do not reproduce these identities here; the relevant ones for this calculation can be found in Figs. 2, 3, 14, and 16 of Ref. 15. Derivation of the group-theoretic weights for the five diagrams of Fig. 1 can be found in Fig. 3. Thus T_1 and T_3 have a factor

$$\frac{1}{\sqrt{n}}\left(n - \frac{1}{n}\right)(T_i)_{ab}$$

(the $1/\sqrt{n}$ comes from the normalization of the initial state), T_2 and T_4 have

$$-\frac{1}{n\sqrt{n}}(T_i)_{ab},$$

and T_5 has

$$\frac{1}{\sqrt{n}}n(T_i)_{ab}.$$

But it is clear from the example of Sec. II that the sum of the momentum-space weights of $T_1 + T_2$ and $T_3 + T_4$ does not produce a quadratic divergence;

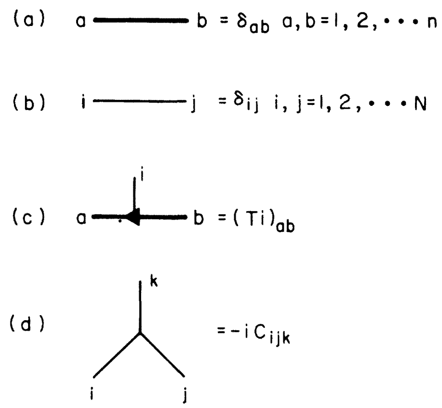


FIG. 2. Diagrammatic rules for the evaluation of the group-theoretic weights for Feynman diagrams in non-Abelian theories. The thick lines represent the quarks, and the thin lines represent the gluons.

thus in order to study any possible quadratic divergence we must evaluate T_1, T_3, and T_5 with the same group-theoretic weight $\sqrt{n}(T_i)_{ab}$. Moreover, it can be easily shown that for any choice of initial colors the result is essentially the same; in order to evaluate the quadratically divergent terms, one must evaluate T_1, T_3, and T_5 each with the same group-theoretic factor, which, in general, however, will not be $\sqrt{n}(T_i)_{ab}$. The sum of these three terms does not produce a factor in the numerator which vanishes in the forward direction [cf. (2.3) in QED], which would indicate that there is no quadratic divergence.[17] Thus to check whether this amplitude yields a quadratically divergent cross section we evaluated this cross section explicitly.

Of course, one must make a choice of an IR cutoff. Although dimensional regularization, or giving the gluon a small mass, may be more generally applicable, for this process it proves convenient to calculate the differential cross section $d\sigma/d\mu^2$, where μ is the invariant mass of the quark-gluon pair. Provided μ is not equal to m, the mass of the quark, this cross section is finite and gauge invariant, and is a measurable quantity. This is equivalent to calculating $d\sigma/d\omega$, where ω is the gluon's energy in some defined frame. Any quadratic divergence in the total cross section then manifests itself as a cubic divergence in $d\sigma/d\mu^2$ of the form $1/(\mu^2-m^2)^3$. We calculate the spin-averaged cross section, summed over the final color states.

Let

$$\Sigma_{ij}=\lim_{\mu^2\to m^2}\frac{d\sigma_{ij}}{d\mu^2}\,,\quad i,j=1,3,5\,, \tag{3.4}$$

where $d\sigma_{ij}/d\mu^2$ is $T_iT_j^*$ integrated over the three-body phase space with the constraint that $(p_3+k)^2=\mu^2$, and each Σ_{ij} has the same group-theoretic factor $n(n^2-1)$. We have evaluated the Σ_{ij} and find the following:

$$\text{(a)}\quad K\Sigma_{11}=-16m^2\sigma^2\int\frac{d\phi_3\delta^{(+)}((p_3+k)^2-\mu^2)}{(2p_3\cdot k)^2[(p_2-p_4)^2]^2}\,, \tag{3.5}$$

where

$$K=(2\pi)^5\times\text{flux}\times\left(\frac{1}{n(n^2-1)}\right)$$

and appears for all the Σ_{ij}, $\sigma\equiv s-2m^2$, and $d\phi_3$ is the infinitesimal three-body phase space (except for factors of 2π which are included in K)

$$d\phi_3=\frac{d^3p_3d^3p_4d^3k}{2E_32E_42\omega}\delta^{(4)}(p_1+p_2-(p_3+p_4+k))\,.$$

These integrations can be performed, and one finds

$$K\Sigma_{11}=-\frac{4\sigma^2\lambda\pi^2}{m^2}\frac{1}{(\mu^2-m^2)^3}\,, \tag{3.6}$$

where $\lambda=[s(s-4m^2)]^{1/2}$, the usual triangle function.

$$\text{(b)}\quad K\Sigma_{13}=K\Sigma_{31}=\frac{2\sigma^3\pi^2}{m^2}\ln\left(\frac{\sigma+\lambda}{\sigma-\lambda}\right)\frac{1}{(\mu^2-m^2)^3} \tag{3.7}$$

Because the real gluon is so soft this contribution is not negligible compared to Σ_{11}, as happens for example if the gluon energy is fixed and $s\to\infty$.

$$\text{(c)}\quad K\Sigma_{33}=-\frac{4\sigma^2\lambda\pi^2}{m^2}\frac{1}{(\mu^2-m^2)^3}-\frac{4\sigma^2\lambda^3\pi^2}{3m^6}\frac{1}{(\mu^2-m^2)^3} \tag{3.8}$$

Since the constraint $(p_3+k)^2=\mu^2$ is obviously not symmetric in p_3 and p_4, there is no reason why Σ_{33} should equal Σ_{11}, and in fact it does not.

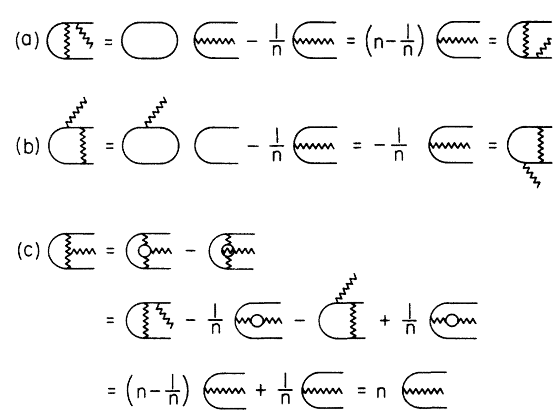


FIG. 3. Evaluation of the group-theoretic weights for the five diagrams of Fig. 1 in a theory with SU(n) as the gauge group.

$$\text{(d)}\quad K\Sigma_{15}=K\Sigma_{51}=+\frac{4\sigma^2\lambda\pi^2}{m^2}\frac{1}{(\mu^2-m^2)^3}-\frac{16\sigma m^2\pi^2}{(\mu^2-m^2)^3}\ln\left(\frac{\sigma+\lambda}{\sigma-\lambda}\right)+\frac{4\pi^2\sigma^2m^2}{\lambda}\frac{1}{(\mu^2-m^2)^3}\ln^2\left(\frac{\sigma+\lambda}{\sigma-\lambda}\right)$$
$$+4\sigma^2\int\frac{d\phi_3\delta^{(+)}((p_3+k)^2-\mu^2)}{(p_1-p_3)^2[(p_2-p_4)^2]^2} \tag{3.9}$$

Although the last term on the right-hand side of (3.9) is straightforward to evaluate, it will be cancelled in Σ_{55}, so we leave it in the unintegrated form. Here we have the first example of radiation from an internal line contributing to the most infrared-divergent term of the cross section.

$$\text{(e)}\quad K\Sigma_{35}=K\Sigma_{53}=+\frac{4\sigma^2\lambda^3\pi^2}{3m^6}\frac{1}{(\mu^2-m^2)^3}-\frac{2\lambda^2\sigma\pi^2}{3m^2}\frac{1}{(\mu^2-m^2)^3}\ln\left(\frac{\sigma+\lambda}{\sigma-\lambda}\right)+4\sigma^2\int\frac{d\phi_3\delta^{(+)}((p_3+k)^2-\mu^2)}{[(p_1-p_3)^2]^2(p_2-p_4)^2}. \tag{3.10}$$

Finally, there is

$$\text{(f)}\quad K\Sigma_{55}=-\frac{4\lambda^5\pi^2}{3m^6}\frac{1}{(\mu^2-m^2)^3}-\frac{24\lambda\sigma^2\pi^2}{m^2}\frac{1}{(\mu^2-m^2)^3}+\frac{16\pi^2\lambda^3}{m^2}\frac{1}{(\mu^2-m^2)^3}+\frac{96m^2\sigma\pi^2}{(\mu^2-m^2)^3}\ln\left(\frac{\sigma+\lambda}{\sigma-\lambda}\right)$$
$$-\frac{8m^2\pi^2}{(\mu^2-m^2)^3}\ln^2\left(\frac{\sigma+\lambda}{\sigma-\lambda}\right)\left(\frac{3\sigma^2}{\lambda}-\lambda\right)-8\sigma^2\int\frac{d\phi_3\delta^{(+)}((p_3+k)^2-\mu^2)[(p_1-p_3)^2+(p_2-p_4)^2]}{[(p_2-p_4)^2]^2[(p_1-p_3)^2]^2} \tag{3.11}$$

Summing up the right-hand sides of (3.6)–(3.11) and writing

$$\Sigma=\lim_{\mu^2\to m^2}\frac{d\sigma}{d\mu^2}, \tag{3.12}$$

one finds

$$K\Sigma=-\frac{8\lambda\pi^2}{3m^2}(\sigma^2+32m^4)\frac{1}{(\mu^2-m^2)^3}+\frac{8\sigma\pi^2}{3m^2}\frac{(\sigma^2+26m^4)}{(\mu^2-m^2)^3}\ln\left(\frac{\sigma+\lambda}{\sigma-\lambda}\right)-\frac{8m^2\pi^2}{\lambda}\frac{(\sigma^2+4m^4)}{(\mu^2-m^2)^3}\ln^2\left(\frac{\sigma+\lambda}{\sigma-\lambda}\right). \tag{3.13}$$

Thus the coefficient of $1/(\mu^2-m^2)^3$ is *obviously not zero.* It is easy to check that the above expression is positive as required. For an arbitrary initial color configuration the result is identical except that the factor $1/n(n^2-1)$ in K is replaced by the appropriate factor for that configuration. Thus we have demonstrated that to this order in perturbation theory the single-gluon cross section is "quadratically" divergent.

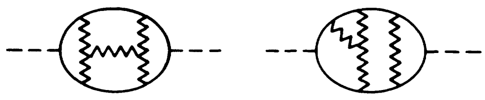
IV. CONCLUSIONS

There are numerous situations one can envisage as being possible in a theory for quarks and gluons; e.g., exact confinement of quarks, gluons and color, confinement of quarks and gluons but not of color, no confinement but inability to detect color, no confinement and ability to detect color, etc.

If quarks are eventually seen and these theories are found not to confine quarks and hence are still viable models for the strong interactions, then the results presented in Sec. III pose a serious problem. We would like to stress that this problem arises even when the colors of the initial quarks are averaged over; in this case the initial quarks are effectively neutral. Two questions immediately arise[18] (independently of whether color can be observed or not): (i) Is the total cross section finite (except for the usual Coulomb infinity) to this order in perturbation theory, i.e., are the quadratic divergences of (3.13) cancelled by interference terms in the elastic cross section? (ii) If one defines μ^2 by $\mu^2=(p_1+p_2-p_4)^2$, then does the experimentally measured $d\sigma/d\mu^2$ behave like $1/(\mu^2-m^2)^3$ as the energy resolution goes to zero for μ^2 close to m^2? This question requires the study of all orders of perturbation theory, summing over real and virtual gluons.

Perhaps when these two questions are studied, miraculously all unwanted quadratic divergences will cancel, but any cancellation mechanism will be far more complicated than the analogous one in QED. If the single-bremsstrahlung cross section was indeed quadratically infinite, then we could have the novel phenomenon that two fermions could not only "focus" each other at arbitrarily large distances, but could also radiate energy and hence decelerate.

Since quarks, gluons, and colored states have not been observed, the relevant question may be whether one can produce colored states starting from color-singlet states. The simplest process of this kind to study is $e^+e^-\to\gamma\to$ anything. From the results and techniques of Kinoshita[12] in QED and Appelquist *et al.*[10,11] in non-Abelian theories, one strongly suspects that the vacuum polarization diagrams of Fig. 4 (and others obtained similarly from the diagrams of Fig. 1) are infrared finite



(a) (b)

FIG. 4. Diagrams which contribute to the cross section for the process $e^+e^- \to \gamma \to$ hadrons. The dashed line represents the massive photon.

and that the singularities are suppressed by phase space. If one calculates the imaginary part of the vacuum polarization by taking discontinuities across all the physical states, then individually they may diverge, but the divergences cancel in the sum. However, even the contributions from particular intermediate states are not quadratically divergent. Thus to these orders in perturbation theory this reaction looks perfectly healthy, and there is no sign of confinement. It is reasonable that any mechanism (if one exists) which suppresses the IR divergence in the quark-inelastic-scattering example of Sec. III will also provide confinement in this process. It is an attractive speculation that the higher-order diagrams will yield factors such as (1.1) so that only color-singlet states can be produced.

The problem of hadron-hadron scattering is much more difficult, since one does not know how to treat the binding. The analogous situation in QED is the scattering of neutral particles (e.g., the scattering of positronium), the cross section for which is finite. In non-Abelian theories if the colored mesons are not degenerate in mass with the color-neutral mesons, the cross section for hadron-hadron scattering is presumably also finite, even though the exchanged gluons can couple not only to the quarks but also to the "gluon sea."

The "forward" divergences in non-Abelian theories have been shown to be worse than in QED, and in these theories radiation from internal gluon lines does contribute to these divergences. Any attempt to understand confinement and infrared behavior in NA theories must include a study of these divergences. If confinement is to be possible there must be differences from the situation in QED, and the calculation presented in this paper demonstrates that this is indeed the case.

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[1]C. N. Yang and R. Mills, Phys. Rev. 96, 191 (1954).

[2]For a recent review of non-Abelian gauge theories see E. S. Abers and B. W. Lee, Phys. Rep. 9C, 1 (1973).

[3]D. J. Gross and F. Wilczek, Phys. Rev. Lett. 26, 1343 (1973); H. D. Politzer, *ibid.* 26, 1346 (1973).

[4]F. Bloch and A. Nordsieck, Phys. Rev. 52, 54 (1937).

[5]D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. (N.Y.) 13, 379 (1961).

[6]D. R. Yennie, in *Lectures on Strong and Electromagnetic Interactions*, edited by K. Ford (Gordon and Breach, New York, 1964).

[7]We choose to impose an alternative infrared cutoff, one which is equivalent to having a minimum gluon energy.

[8]J. M. Cornwall and G. Tiktopoulos, Phys. Rev. D 13, 3370 (1976).

[9]Y. P. Yao, Phys. Rev. Lett. 36, 653 (1976).

[10]T. Appelquist, J. Carazzone, H. Kluberg-Stern, and M. Roth, Phys. Rev. Lett. 36, 768 (1976); 36, 1161(E) (1976).

[11]T. Appelquist, Yale report (unpublished).

[12]T. Kinoshita, J. Math. Phys. 3, 650 (1962).

[13]G. Sterman, Illinois Univ. Report No. ILL-TH-75-32, 1975 (unpublished).

[14]G. Altarelli and F. Bucella, Nuovo Cimento 34, 1337 (1964). See also V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Relativistic Quantum Theory* (Addison-Wesley, Reading, Mass., 1971).

[15]P. Cvitanović, Phys. Rev. D (to be published).

[16]Rules for the other vertices are also presented in Ref. 15. We do not encounter any of these vertices in this calculation.

[17]However, if we write $\sum_{i=1}^{5} T_i = \epsilon^{\mu} T_{\mu}$, then it is straightforward to demonstrate that the current-conservation relation, $k^{\mu} T_{\mu} = 0$, holds.

[18]The same questions arise for single-gluon production in the scattering of two colored mesons, under the assumption that their wave functions behave in a manner similar to that assumed for the (color-singlet) hadrons. See, e.g., S. J. Brodsky and G. Farrar, Phys. Rev. D 11, 1309 (1975).