Resonance model for $e^+e^- \to$ hadrons*

A. C. D. Wright
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
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A model is presented in which e^+e^- annihilation proceeds through the process $e^+e^- \to \pi R$, where R is a resonance. The contribution to the single-pion inclusive distribution from resonance decay products is calculated. Scale breaking in $s d\sigma/dx$ and $\sigma_{tot}(s)$ is attributed to a threshold for production of a new class of resonances.

Recent results for e^+e^- annihilation from SPEAR[1] have confirmed that R, the ratio of hadronic to μ-pair cross sections, rises (except for the high-energy side of resonances) in the range 3 GeV $\lesssim \sqrt{s} \lesssim$ 5 GeV, and may be continuing to rise slightly up to 7.8 GeV. In the parton model,[2] this behavior is commonly interpreted as due to a threshold for production of one or more pairs of heavy pointlike objects, in addition to the "old" light hadronic constituents. Unfortunately, this picture reveals little about the single-particle inclusive spectrum, which depends on the probability distribution function for finding the hadron in a quark.[3] In contrast, thermodynamic or statistical[4] models naturally provide the inclusive distribution but cannot predict the behavior of the total cross section.

In the present work, we explore a model which shares many features of the statistical,[4] resonance,[5] and vector-dominance[6] models of e^+e^- annihilation, and which affords a satisfactory description of both the total hadronic cross section and the details of single-pion production. We have in mind a picture in which the inclusive distribution results from the buildup of resonances in the missing-mass channel.[7] This may be motivated in the following way. We assume, in the manner of Bloom and Gilman,[8] that resonances dominate inelastic electroproduction in the Bjorken limit. Therefore, the Compton amplitude is of the form[9]

$$T_2(s,0,q_1^2,q_2^2)=\sum_n \frac{G_n(q_1^2)G_n(q_2^2)}{s-M_n^2+iM_n\Gamma_n}\ , \tag{1}$$

where $G_n(q^2)$ is the excitation form factor of the nth resonance. The structure function for the process $e^+e^- \to \pi(p)+$hadrons is given by[10]

$$\begin{aligned}\overline{W}_2(M^2,s)&=\frac{1}{\pi}\Delta_{M^2}T_2(M^2,0,s+i\epsilon,s-i\epsilon')\\ &=\frac{1}{\pi}\sum_n \Delta_{M^2}\frac{G_n(s+i\epsilon)G_n(s-i\epsilon')}{M^2-M_n^2+iM_n\Gamma_n}\ ,\end{aligned} \tag{2}$$

where now $s=q^2$ and $M^2 \equiv (q-p)^2$. Therefore, $e^+e^- \to \pi X$ is dominated by resonances in the missing-mass channel (poles in M^2).[7] This is depicted in Fig. 1(a).

As it stands, the above picture is somewhat inconsistent in that it takes no account of the possibility that the *observed* pion can be a decay product of the resonance, as is shown in Fig. 1(b). In fact, diagrams such as Fig. 1(b) should contribute due to the presence of anomalous singularities in the virtual Compton amplitude.[11] There remains the question of whether their inclusion constitutes double counting. Such is the case in a pure dual-resonance picture, where the inclusive cross section is entirely accounted for by Fig. 1(a). More generally, some fraction of the contribution from Fig. 1(b) is already accounted for by Fig. 1(a), so that there is partial double counting. However, for want of information on the amount of double counting involved, we shall assume that pion production is given by the sum of the diagrams in Fig. 1.

According to this picture, upon neglecting interference between the diagrams of Fig. 1, we have

$$E\frac{d\sigma_i}{d^3p}=E\frac{d\sigma_i^0}{d^3p}+\sum_j\int\frac{d^3p'}{E'}E'\frac{d\sigma_j^0(p')}{d^3p'}E\frac{dN_{ij}(p,p')}{d^3p}\ , \tag{3}$$

where $Ed\sigma_i^0/d^3p$ is the "bare" cross section for production of a pion of charge i corresponding to Fig. 1(a). Also, $E\,dN_{ij}(p,p')/d^3p$ is the probability distribution for observing a pion of momentum p

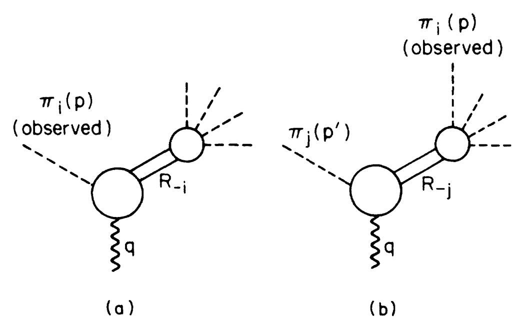

FIG. 1. Contributions to the process $e^+e^- \to \pi_i(p)X$. (a) π_i produced in conjunction with a resonance; (b) π_i resulting from resonance decay.

and charge i from the decay of a resonance of charge $-j$, as in Fig. 1(b). Henceforth, we assume that the bare cross sections for positive, negative, and neutral pions are identical, and that $E\,dN_{ij}/d^3p$ is independent of j. Furthermore, we shall specialize to negative pions and therefore suppress the index i, giving

$$E\frac{d\sigma}{d^3p}=E\frac{d\sigma^0}{d^3p}+3\int\frac{d^3p'}{E'}E'\frac{d\sigma^0(p')}{d^3p'}E\frac{dN(p,p')}{d^3p}\ . \tag{4}$$

The probability distribution satisfies the sum rules[12]

$$\int E\frac{dN}{d^3p}\frac{d^3p}{E}=\tfrac{1}{2}\bar{n}_c \tag{5}$$

and, in the resonance rest frame,

$$\int E\frac{dN}{d^3p}d^3p=\beta m_R\ . \tag{6}$$

Here $\bar{n}_c$ is the charged multiplicity, β is the fraction of resonance energy carried away by negative pions, and m_R is given by

$$m_R^{\,2}=s+m_\pi^{\,2}-2E'\sqrt{s}\ . \tag{7}$$

To compare (4) with experiment, we need to know the form of $E\,dN/d^3p$ and $E\,d\sigma^0/d^3p$. For the latter, we assume the Callan-Gross relation[13]

$$\bar{F}_1^0(x,s)=-x\bar{F}_2^0(x,s)\ , \tag{8}$$

which gives

$$E\frac{d\sigma^0}{d^3p}=\frac{2\alpha^2}{s^2}\left[1-\frac{1}{2}\left(1-\frac{4m_\pi^{\,2}}{x^2s}\right)\sin^2\theta\right]\bar{F}_1^0(x,s)\ , \tag{9}$$

where $x\equiv 2p\cdot q/s=1-(M^2-m_\pi^{\,2})/s$. Furthermore, to connect electroproduction to annihilation, we assume that in the scaling region the Gribov-Lipatov reciprocity relation[14] holds:

$$\bar{F}_2^0(x)=-x^{-3}F_2(\omega=1/x)\ , \tag{10}$$

where $F_2(\omega)$ is the structure function for electroproduction off pions. Since R is not constant at SPEAR energies, $\bar{F}_1^0$ cannot scale there. However, the existence of the narrow resonances[15] $\psi(3.1)$ and $\psi'(3.7)$ suggests, in our resonance-dominance picture, that a threshold for production of a new class of resonances occurs at $\sqrt{s}\approx 3.5$ GeV. Assuming that the contribution from each class of resonances scales separately, and taking a square-root-type effective threshold behavior for the new resonances, we get

$$\bar{F}_1^0(x,s)=\bar{F}_{1O}^0(x)+\theta(M-3.5\text{ GeV})\times[1-M^2/(3.5\text{ GeV})^2]^{1/2}\bar{F}_{1N}^0(x)\ , \tag{11}$$

where $\bar{F}_{1O}^0(x)$ and $\bar{F}_{1N}^0(x)$ are the scaling functions for the "old" and "new" resonances, respectively. By (10), there exist electroproduction structure functions $F_{2O}(\omega)$ and $F_{2N}(\omega)$ corresponding to $\bar{F}_{1O}^0(x)$ and $\bar{F}_{1N}^0(x)$, and for these we take

$$F_{2O}(\omega)=CF_{2N}(\omega)=C_1\frac{(\omega-1)}{\omega^2}\ , \tag{12}$$

where C and C_1 are constants. The threshold behavior of (12) corresponds to a pion form factor which behaves as $F_\pi(t)\propto t^{-1}$ at large t. The ω^{-1}-type asymptotic behavior of (12) is that given by an effective Regge-pole exchange with $\alpha_{\text{eff}}(0)=0$. The highest-lying Regge trajectory (aside from the Pomeron) contributing to $\gamma\pi$ scattering is the P',[7] which probably has $0\leq\alpha_{P'}(0)\leq 0.5$, so we consider (12) to represent the contribution of the resonances dual to *all* nondiffractive exchanges. Assuming (8) and (10) to apply to F_{2O} and F_{2N} separately, we have from (12)

$$\bar{F}_{1O}^0(x)=C\bar{F}_{1N}^0(x)=C_1(1-x)/x\ . \tag{13}$$

Note that, in this case, (10) agrees with the simple crossing relation[16]

$$\bar{F}_2^0(x)=F_2(x)\ . \tag{14}$$

For resonance decay, we assume an isotropic distribution inspired by the thermodynamic model[4]

$$E\frac{dN}{d^3p}=Ae^{-bE}(\text{rest frame})\ . \tag{15}$$

The (m_R-dependent) constants A and b are determined by (5) and (6). In (5), we take

$$\bar{n}_c(m_R)=1.9+\ln m_R^{\,2}\ , \tag{16}$$

which is consistent with the charged multiplicity measured in $\bar{p}p$ collisions.[17] For β, we take $\beta_O=0.3$ for the old resonances as is found in $\bar{p}p$ scattering,[17] while β for the new resonances is left as a parameter β_N. Furthermore, for simplicity we average β for both old and new resonances above the new resonance threshold so that

$$\beta(m_R^{\,2})=\frac{C\beta_O+\theta(m_R-3.5\text{ GeV})[1-m_R^{\,2}/(3.5\text{ GeV})^2]^{1/2}\beta_N}{C+\theta(m_R-3.5\text{ GeV})[1-m_R^{\,2}/(3.5\text{ GeV})^2]^{1/2}} \tag{17}$$

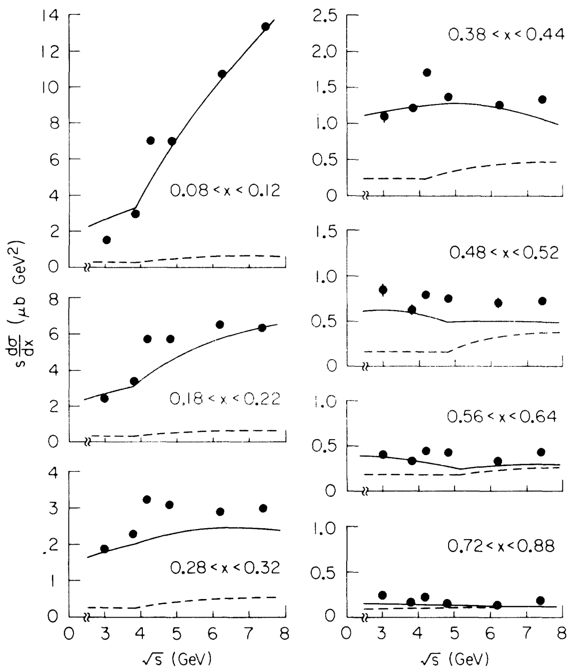

FIG. 2. Data from Ref. 1 for $s\,d\sigma/dx$ for charged pions plotted versus $\sqrt{s}$, where $x \equiv 2p/\sqrt{s}$. The solid lines are our fit, and the dashed lines are the "bare" contribution $s\,d\sigma^0/dx$.

for all values of m_R^2.

By using (15), (13), (11), and (9), we may apply (4) to the data[1] for $e^+e^- \to \pi^- X$ and determine the free parameters. Upon performing the integration in (4) numerically, we find

$$\beta_N = 0.15\ ,\quad C = 1\ ,\quad C_1 = 2.3\ . \tag{18}$$

Note that $\beta_N = 0.15$ corresponds to 70% of the new resonances' energy being carried away by neutral pions, compared with 40% for the old resonances. Assuming that isospin is conserved in the resonance decay, and that the resonances have $I = 1$, then the upper bound for the energy carried by neutral pions is[18]

$$\langle E_{\pi^0} \rangle / E_{\text{tot}} < (9 + \sqrt{41})/20 \approx 0.77 \tag{19}$$

if the resonances are neutral. For charged resonances there is no upper bound.[18] Therefore, the decay of the new resonances satisfies the bound (19), although why so much of the resonance energy goes into neutrals remains somewhat mysterious.

The resulting cross section, integrated over angles, is shown in Fig. 2. Note that the bare cross section constitutes ~10% of the contribution from resonance decay at small x, while at large x the bare contribution dominates. Most of the resonance decay contribution comes from resonances with $m_R^2 \approx s$, that is, resonances almost at rest in the c.m. system. Consequently, the decay contribution to $E\,d\sigma/d^3p$ is almost exponential in E, with a slope close to the value of b in (15) for resonances with $m_R^2 = s$. The resulting two-component picture for e^+e^- annihilation is similar to current ideas about purely hadronic collisions.[19] The "bare" cross section, dominant at large p, corresponds to "hard" scattering in high-p_T hadronic interactions, while the resonance decay contribution resembles low-p_T hadronic physics.

Since we have taken the scaling limit in (13), no resonance peaks appear in the solid lines in Fig. 2. Therefore, we cannot reproduce the "bulge" in $s\,d\sigma/dx$ seen in the data at 4.2 GeV. Harari has suggested[20] that this bulge, and the general restriction of scale breaking to $x < \frac{1}{2}$ is due to the production of pairs of equal mass particles at rest. Above threshold, the decay products would sometimes reach the region $x > \frac{1}{2}$, but the cross section for production of pairs far above threshold might be suppressed. In our picture, the pions accompanying the new resonances do not reach $x \geq \frac{1}{2}$ until $\sqrt{s} \geq \sqrt{2 s_{\text{th}}} \approx 4.9$ GeV; however, the resulting

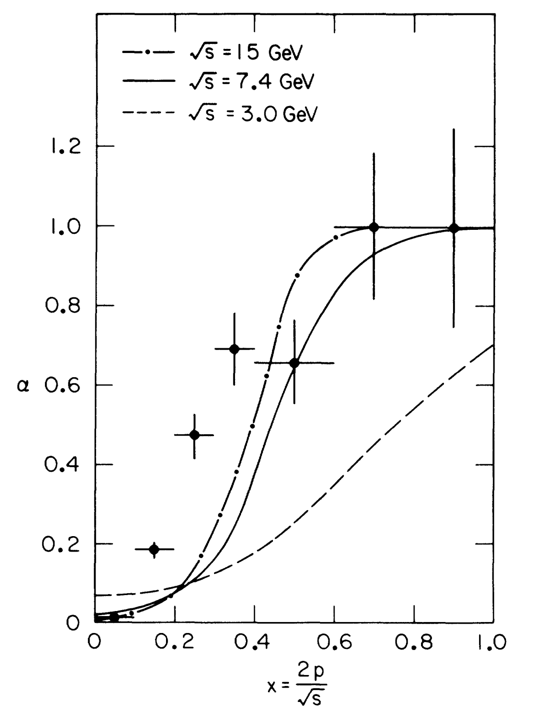

FIG. 3. Data from Ref. 21 at $\sqrt{s} = 7.4$ GeV for $\alpha \equiv (\sigma_T - \sigma_L)/(\sigma_T + \sigma_L)$ plotted versus $x \equiv 2p/\sqrt{s}$. Our predictions are for $\sqrt{s} = 7.4$ GeV (solid line), 3.0 GeV (dashed line), and 15 GeV (dashed-dotted line).

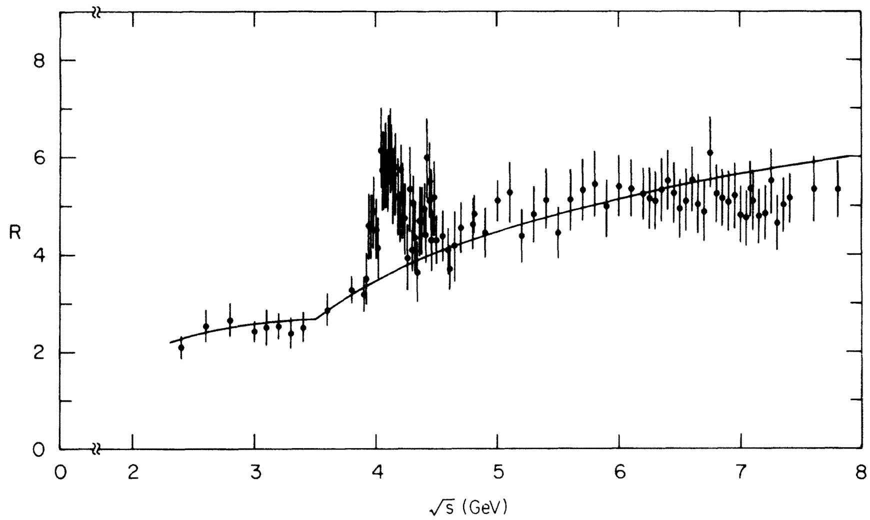

FIG. 4. Prediction for $R \equiv \sigma_{tot}/\sigma_{\mu^+\mu^-}$ plotted versus $\sqrt{s}$. The data are from Ref. 1.

scaling violation is masked somewhat by the presence of resonance decay products for $x \geq \frac{1}{2}$.

We now turn to the angular distribution in our model. Since the bare cross section gives an angular distribution proportional to $1+\cos^2\theta$, while the resonance decay contribution is isotropic within 10%, the overall angular distribution is of the form $1+\alpha(x,s)\cos^2\theta$, where $0 \leq \alpha \leq 1$. In Fig. 3 we show our prediction for α plotted with data[21] from SPEAR at 7.4 GeV. The qualitative shape of the distribution at 7.4 GeV is reproduced by the predicted curve (solid line), although it is too low for low values of x. The prediction for 3.0 GeV (dashed line) is considerably more isotropic at high x, while the 15-GeV curve (dashed-dotted line) shows little change from the 7.4-GeV curve.

To obtain the cross section for $e^+e^- \to X$, we use the sum rule[12]

$$\sigma_{tot}(s) = \frac{1}{2\beta_1(s)} \int_{2m_\pi/\sqrt{s}}^{1} x \frac{d\sigma}{dx} dx, \tag{20}$$

where $\beta_1(s)$ is the fraction of e^+e^- energy carried away by negative pions. We take $\beta_1(s) = \beta(s)$, which should be a reasonable approximation for the overall process, to get the result for $R \equiv \sigma_{tot}/\sigma_{\mu^+\mu^-}$ shown with the data[1] in Fig. 4. The predicted R continues to rise slightly through energies which will be available at PEP, and has the value 7.16 at $\sqrt{s} = 15$ GeV. Again, because we have taken the scaling limit for the contributions from the old and new resonances, the structure in R between 3.8 and 4.5 GeV is not reproduced. This is the region where the new resonances are begining to be produced, so scaling violations might be expected here.

We conclude with several remarks:

(i) We note that in (12) we have neglected the diffractive contribution to electroproduction, which is expected to give $F_2(\omega) \to$ const as $\omega \to \infty$. Through the Gribov-Lipatov relation, this term would imply $\overline{F}_1^0(x) \to \text{const}/x^2$ as $x \to 0$, which is more singular at $x=0$ than (13). Since, in our model the resonance-decay contribution dominates the bare contribution at small x, this effect may be hidden.

(ii) We have not discussed the nature and dynamics of the new resonances beyond the creation of a threshold in R. Should further classes of resonances be produced at higher energies, associated thresholds would appear, much as in the parton model.[2]

(iii) Scale breaking in $s\,d\sigma/dx$ at small x is due to the onset of production of the new resonances and phase space available for pions from resonance decay. At large x, $s\,d\sigma/dx$ is approximately scale invariant at energies up to 7.4 GeV, although there is a slight rise at the highest energies due to the possibility of producing the new resonances accompanied by high-x pions.

(iv) Evidence[22] for jet structure in $e^+e^- \to hX$ is an important clue for the study of the dynamics of e^+e^- annihilation. Resonance models, such as ours, can lead to jets,[23] although an investigation of such structure requires a more detailed picture than we have presented here.

(v) Since the major contribution to pion production comes from resonances almost at rest in the c.m. frame, the model is similar to vector dominance,[6] in which resonances build up in the direct channel. However, some structure should remain in the missing-mass channel due to the contribution of Fig. 1(a).

(vi) Finally, the inclusive data[1] are for production of all species of hadrons, and may be significantly contaminated by K's, p's, etc. at high x. This should be kept in mind when comparing the high-x behavior of our model with the data in Fig. 2.

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[1]R. Schwitters, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford University*, edited by W. T. Kirk (Stanford Linear Accelerator Center, Stanford University, Stanford, Calif., 1976) p. 5.

[2]See for example, R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, Mass., 1972).

[3]See for example, F. J. Gilman, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford University* (see Ref. 1), p. 131. In this work a comparison between data for $e^+e^- \to hX$ and $ep \to ehX$ in the photon fragmentation region is made which suggests that the same parton fragmentation function may apply to both processes.

[4]J. D. Bjorken and S. J. Brodsky, Phys. Rev. D 1, 1416 (1970); J. Engels, H. Satz, and K. Schilling, Nuovo Cimento 17A, 535 (1973); G. Eilam *et al.*, Lett. Nuovo Cimento 14, 108 (1975).

[5]C. Ferro Fontan and H. R. Rubinstein, Nucl. Phys. B74, 378 (1974); N. S. Craigie and K. W. Rothe, *ibid.* B82, 370, 381 (1974); N. S. Craigie and T. F. Walsh, Ann. Phys. (N.Y.) 92, 340 (1975); A. DiGiacomo and K. I. Konishi, Phys. Lett. B51, 67 (1974).

[6]D. Schildknecht and F. Steiner, Phys. Lett. B56, 36 (1975); A. Bramòn, E. Etim, and M. Greco, *ibid.* B41, 609 (1972); H. Terezawa, Phys. Rev. Lett. 32, 694 (1974). A recent comprehensive review of vector-meson dominance and other models for e^+e^- annihilation is given in Springer Lecture Notes 45, *Dynamical Concepts on Scaling Violation and the New Resonances in e^+e^- Annihilation,* edited by B. Humpert (Springer, Berlin, 1976).

[7]For an analytic model in which the single-particle distribution is built up by resonances in the missing-mass channel (at SPEAR energies) see J. W. Moffat, P. Savaria, and A. C. D. Wright, Nucl. Phys. B87, 461 (1975).

[8]E. D. Bloom and F. J. Gilman, Phys. Rev. Lett. 25, 1140 (1970).

[9]See for example, M. Elitzur, Phys. Rev. D 3, 2166 (1971); H. Moreno and J. Pestieau, *ibid.* 5, 1210 (1972); Y. Avni and M. Milgrom, *ibid.* 7, 1463 (1973).

[10]P. V. Landshoff and J. C. Polkinghorne, Phys. Rev. D 6, 3708 (1972); R. Gatto and G. Preparata, Nucl. Phys. B47, 313 (1972); P. M. Fishbane and J. D. Sullivan, Phys. Rev. D 6, 3568 (1972).

[11]A. suri, Phys. Rev. D 4, 570 (1971); see also G. Schierholz and M. G. Schmidt, Nucl. Phys. B101, 429 (1975); CERN Report No. CERN-TH 2106, 1975 (unpublished). Schierholz and Schmidt consider the effect of anomalous thresholds on the threshold behavior of $\bar{W}_2$, and also study their contribution to the violation of the Callan-Gross relation.

[12]C. E. DeTar, D. Z. Freedman, and G. Veneziano, Phys. Rev. D 4, 906 (1971).

[13]This relation holds if the resonances have natural parity. See for example Y. S. Tsai, Phys. Rev. D 12, 3533 (1975).

[14]V. N. Gribov and L. N. Lipatov, Yad. Fiz. 15, 1218 (1972) [Sov. J. Nucl. Phys. 15, 675 (1972)].

[15]J.-E. Augustin *et al.*, Phys. Rev. Lett. 33, 1406 (1974); G. S. Abrams *et al.*, *ibid.* 33, 1453 (1974).

[16]S. D. Drell, D. J. Levy, and T. M. Yan, Phys. Rev. 187, 2159 (1969).

[17]See for example, G. Eilam *et al.*, Ref. 4.

[18]J. S. Bell, G. Karl, and C. H. Llewellyn Smith, Phys. Lett. B52, 363 (1974).

[19]For a model of hadronic interactions with explicit hard and soft components, see J. L. Alonso and A. C. D. Wright, Phys. Rev. D 12, 776 (1975). The two-component picture for e^+e^- annihilation proposed in the present work is similar to that of F. Elvekjaer and F. Steiner, Phys. Lett. B60, 456 (1976).

[20]H. Harari, SLAC Report No. SLAC-PUB-1589, 1975 (unpublished).

[21]R. F. Schwitters *et al.*, Phys. Rev. Lett. 35, 1320 (1975).

[22]G. Hanson *et al.*, Phys. Rev. Lett. 35, 1609 (1975).

[23]See for example, N. S. Craigie and K. W. Rothe, Ref. 5.