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Dynamical symmetry breaking via a bound state and a comment on the O(N) σ model

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We analyze the sheet structure of the effective potential for relativistic field theories that have a bound state (mass m_B) and show that if $m_B{}^2$ goes negative the vacuum changes sheets avoiding tachyon problems and giving dynamical symmetry breaking if the bound state has quantum numbers. We illustrate a transition of this type in the O(N)-symmetric σ model for large N. We resolve a tachyon problem in this model noted by Coleman, Jackiw, and Politzer by finding the vacuum state of lowest energy. We show this model has no symmetry breaking of any kind in the leading $1/N$ approximation. Any symmetry breaking in the tree approximation is restored dynamically.

I. INTRODUCTION

Spontaneous symmetry breaking plays a central role in many contemporary ideas in particle physics. This is the phenomenon in which the vacuum state breaks an exact symmetry of the Hamiltonian.[1] There are two ways in which the noninvariance of the vacuum can manifest itself: One is through the nonvanishing of the vacuum expectation value of a field carrying quantum numbers, $\langle \phi_i \rangle \neq 0$; or alternatively, it is possible to have $\langle T\phi_i\phi_j \cdots \rangle$ not invariant even though the $\langle \phi_i \rangle = 0$.[2] The former can occur at the classical (tree) level or as a consequence of dynamics. The latter necessarily has dynamical origins. It is fortunate that we have a classical picture of the former type since it has significant pedagogical value for developing one's intuition about this phenomenon. One starts with a symmetric Hamiltonian, and concocts the nonderivative part, $V(\phi)$, to have a minimum for $\phi_i \neq 0$. The classical ground-state value of ϕ_i becomes the vacuum expectation value of the quantum field. One can "see" the transition from normal to spontaneously broken vacuums take place by varying μ^2 in the mass term $-\frac{1}{2}\mu^2\phi_i{}^2$ from positive to negative values. The theory "saves itself" from a tachyon disaster by choosing a new vacuum.

The central purpose of this paper is to show that an equally simple picture exists for the latter type of symmetry breaking, i.e., noninvariant, vacuum with $\langle \phi_i \rangle = 0$. We show the following: given a theory with a spin-zero bound state, if the bound-state mass m_B can be driven to $m_B{}^2 < 0$, the theory always saves itself from the tachyon disaster, and spontaneous symmetry breaking will occur if the bound state carries quantum numbers. This is the analogous situation to the above discussion but with the bound state playing the role of the scalar field and hence this is not a surprising result. However, the simple picture becomes sufficiently disguised in the bound-state case to warrant discussion. Also, a new approach to dynamical symmetry breaking is suggested. The majority of this paper deals with the transition between the two vacuums in the absence of internal-symmetry considerations. The generalization to spontaneous symmetry breaking is immediate.

In order to check these general ideas we looked for a model that has this transition between vacuums. The O(N) σ model is soluble as an expansion in $1/N$ and the leading term displays the desired effects.[3-5] The transition does not break a symmetry because the bound state in question is an O(N) singlet. In this illustration the Lagrangian parameters were chosen to be those which have a normal (symmetry-preserving) vacuum.

Finally, we were led to a very interesting conclusion concerning the O(N) σ model when parameters are chosen to give symmetry breakdown of the tree level. It is this. *The O(N)-symmetric σ model in 4-space has no symmetry breaking of any kind in the large-N limit.* This result is not an illustration of the above-stated objective of this paper, but fell out as a consequence of our analysis. In studying this case, Coleman, Jackiw, and Politzer[5] noted that the model (in 4-space) has an asymmetric vacuum and that the theory defined from that vacuum has a tachyon. They concluded correctly that there must be another vacuum of lower energy and speculated that it occurs in higher order in $1/N$. They failed to note that their own solution has another vacuum which is always lower in energy. The correct vacuum has a bound state, not a tachyon. The interesting thing is that the true vacuum is O(N) *symmetric*. Dynamics restores the symmetry that is broken in the tree approximation. If this restoration of symmetry were also true for small N, it would cast doubt on the validity of the classical argument for spontaneous symmetry breaking in quantum theories. This would bring us full circle to the awkward position of questioning the validity of the very picture that we wished to generalize in the first place. Unfortunately, we do not know if the restoration of symmetry does or does not occur for small N.

The central object we study in this paper is the effective potential $V(\phi_i)$.[6] $V(\phi_i)$ is a c-number function of a c-number field ϕ_i, whose global minimum determines the vacuum expectation value of the quantum field ϕ_i. For all cases of interest in this paper, $V(\phi)$ has a $\frac{3}{2}$-power branch point in ϕ at a finite real positive value of ϕ. It was noted in an earlier paper that the branch point was a consequence of a deep bound state.[7] It was also noted to occur in the O(N) σ model.[3,5] The significance of the branch point was not fully appreciated in either case; all the new results in this paper follow from a careful look at the second sheet of this branch point.

The transition of vacuums described above is due to the following circumstance: $V(\phi)$ has two local minima at $\phi=0$ on the two sheets of the aforementioned branch point. As m_B^2 passes through zero to negative values, the global minimum shifts from one sheet to the other. In the new vacuum the bound-state mass is positive. The wrong vacuum is not a local maximum of $V(\phi)$, it is just not the global minimum. The interesting vacuums we found for the O(N) σ model lie on the second sheet of the branch point in $V(\phi)$.

In Sec. II we present the argument for the switching of vacuums as a bound state passes through zero. Also, we give the generalization to dynamical symmetry breaking. In Sec. III we illustrate these ideas with the O(N) σ model. The desired features are found in published solutions which made life easy. In Sec. IV, we show our contention that the O(N)-symmetric σ model has no symmetry breaking in the large-N limit.

II TRANSITION OF VACUUM-DYNAMICAL SYMMETRY BREAKING

Let us illustrate this mechanism first in the absence of internal symmetry with a pseudoscalar field ϕ interacting in an unspecified way producing a scalar bound state at mass m_B. m_B^2 is a function of coupling constants, but let us adopt m_B^2 as a parameter-measuring attraction which is the bound-state mass when it takes on positive values.[8] To see how the transition at $m_B^2=0$ arises, let us look at the effective potential $V(\phi)$:

$$V(\phi)=-\sum_{n=1}^{\infty}\frac{\phi^{2n}}{(2n)!}\Gamma^{(2n)}(0)\ . \tag{2.1}$$

V is in fact the generating function of the n-point one-particle-irreducible vertices (1PIV) $\Gamma^{(2n)}(P_1\cdots P_{2n})$ with all momenta zero. In a recent paper we derived an expression for $V(\phi)$ under the assumption that a theory has a deep bound state and that the bound-state poles dominate $\Gamma^{(2n)}$ at zero momentum[7] (although the 1PIV's do not have poles corresponding to elementary fields, they do have bound-state poles.) We found[9] the following expression for V:

$$V(\phi)=\tfrac{1}{2}m_\phi^2\phi^2-\frac{m_B^6}{6\gamma^2}\left[3\xi-2+2(1-\xi)^{3/2}\right], \tag{2.2}$$

where $\xi=\beta\gamma\phi^2/m_B^4$, β is the (ϕ–ϕ–bound-state) coupling, and γ the (three bound-state) coupling. Equation (2.2) has the features stated above but before discussing them let us write Eq. (2.2) in an equivalent but much more transparent way. Introduce a scalar classical field χ and define $V(\phi,\chi)$:

$$V(\phi,\chi)\equiv\tfrac{1}{2}m_\phi^2\phi^2+\tfrac{1}{2}m_B^2\chi^2+\tfrac{1}{2}\beta\phi^2\chi+\tfrac{1}{6}\gamma\chi^3\ . \tag{2.3}$$

If one eliminates χ by demanding that $V(\phi,\chi)$ be stationary in χ,

$$0=\frac{\partial V(\phi,\chi)}{\partial\chi}=m_B^2\chi+\tfrac{1}{2}\beta\phi^2+\tfrac{1}{2}\gamma\chi^2\ , \tag{2.4}$$

then $V(\phi)=V(\phi,\chi(\phi))$ subject to the constraint Eq. (2.4). In other words, $V(\phi)$ is simply a constrained polynomial in two variables, ϕ and χ. The polynomial $V(\phi,\chi)$ is the tree expression one would have written down had the χ been an elementary field. The constraint to eliminate χ is simply that $V(\phi,\chi)$ be stationary in χ. To paraphrase our claim in Ref. (7), Eq. (2.3) is a power series in ϕ and the auxiliary field χ, valid for small ϕ and χ, and containing a sufficient number of terms to exhibit the transition at $m_B^2=0$.[10]

Figure 1 shows the constraint curve in the ϕ^2, χ plane. The two local minima in $V(\phi,\chi(\phi))$ are marked. For $m_B^2>0$, the global minimum is $V_{\rm I}=V(0,0)=0$ by assumption. For $m_B^2<0$, $V_{\rm II}=V(0,-2m_B^2/\gamma)=2m_B^6/3\gamma^2$ is lower. We have assumed in this illustration that β and γ are finite and nonzero as $m_B^2\to0$. Figure (2a) shows schematically the two real branches of $V(\phi)$. The two minima pass through each other as m_B^2 goes negative.

Higher terms in the expansion of $V(\phi,\chi)$ that were neglected in Eq. (2.3) are:

$$\frac{\delta_1}{4!}\phi^4+\frac{\delta_2}{4}\phi^2\chi^2+\frac{\delta_3}{4!}\chi^4+\cdots\ . \tag{2.5}$$

The argument for a transition depends on the behavior of $V(\phi,\chi)$ in an arbitrarily small neighborhood of $\phi^2,\chi=0$ as $m_B^2\to0$. Hence, as long as the expansion converges, higher terms will not affect the argument.[11]

For $m_B^2<0$, we can reexpress Eq. (2.3) in terms

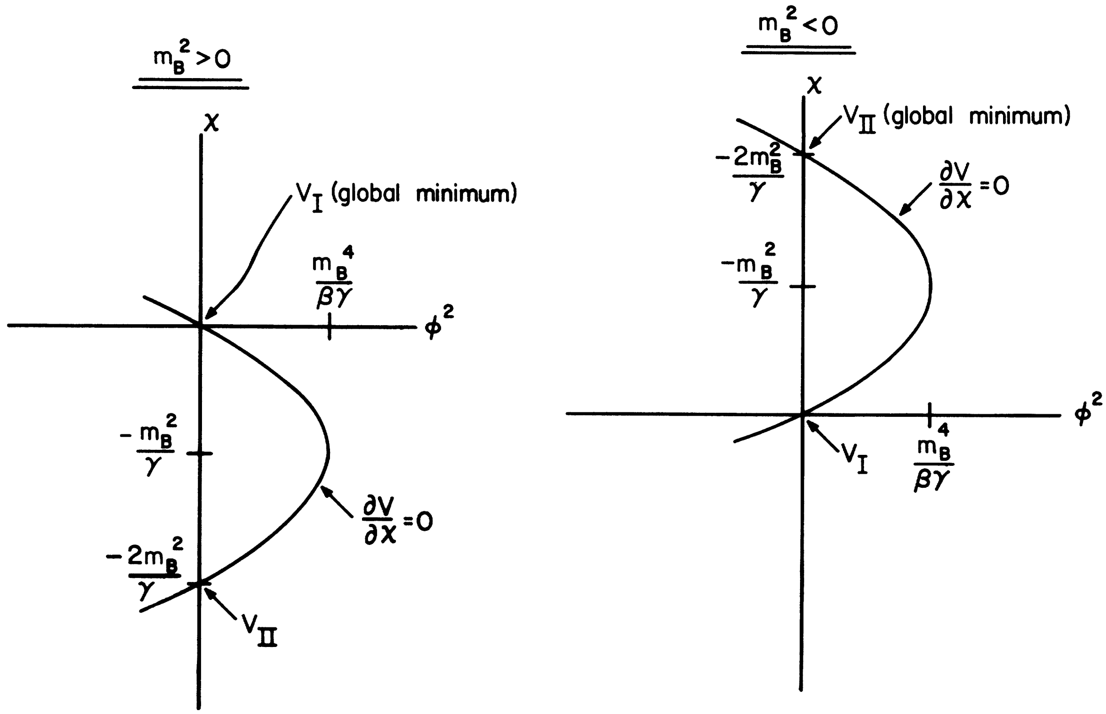

FIG. 1. Plot of the constraint curve $\partial V(\phi, \chi)/\partial\chi = 0$ in the ϕ^2, χ plane for $m_B^2 > 0$ and $m_B^2 < 0$. (We have taken β and γ as positive numbers.) $V(\phi) = V(\phi, \chi(\phi))$ subject to this constraint. $V(\phi)$ has a branch point at $\phi^2 = \phi_{br}^2 = m_B^4/\beta\gamma$ and is real for $0 \le \phi^2 \le \phi_{br}^2$. The two local minima on the two sheets of ϕ are marked. As $m_B^2 \to 0$ the two minima coalesce at $\phi^2 = \chi = 0$.

of a shifted χ to see some gross features of the new vacuum: Define $\tilde{\chi} \equiv \chi + 2m_B^2\gamma$; then

$$V(\phi, \chi) = \frac{2}{3}\frac{m_B^6}{\gamma^2} + \frac{1}{2}\left(m_\phi^2 - 2\beta\frac{m_B^2}{\gamma}\right)\phi^2 - \frac{m_B^2}{2}\tilde{\chi}^2 + \frac{\beta}{2}\phi^2\tilde{\chi} + \frac{\gamma}{6}\tilde{\chi}^3 . \tag{2.6}$$

The coefficient of $\tilde{\chi}^2$ is positive, giving a positive bound-state mass. The mass of the ϕ field picks up a correction of order m_B^2.[9] If we had included higher-order terms [Eq. (2.5)], then β, γ, and higher-order terms would pick up corrections of order m_B^2.

In models for which the bound states and fields belong to irreducible representations of a symmetry group, Eq. (2.3) would generalize to an invariant form of V:

$$V(\phi_i, \chi_i) = \tfrac{1}{2}m_\phi^2\phi_i^2 + \tfrac{1}{2}m_B^2\chi_i^2 + \tfrac{1}{2}\beta g_{ijk}\phi_i\phi_j\chi_k + \tfrac{1}{6}\gamma h_{ijk}\chi_i\chi_j\chi_k , \tag{2.7}$$

together with constraints to eliminate the χ_i's: $\partial V/\partial\chi_i = 0$. The *un*constrained problem leads exactly to the classical picture of symmetry breaking, where the parameters are free rather than arising dynamically. Since the constraints demand that V be stationary in χ_i, the stationary points of the constrained problem and unconstrained problem are identical. Hence the intuition developed from the classical picture can be carried over for this form of dynamical symmetry breaking. One must be careful in that the type of stationary point—maximum, minimum, or inflection—need not be identical in the constrained and unconstrained problems. In fact, the vacuum with the tachyon in the constrained problem is a local

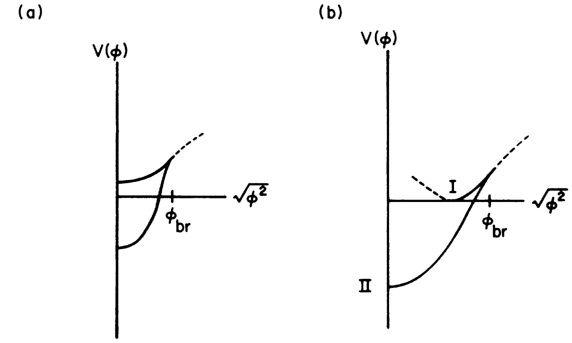

FIG. 2. (a) Behavior of $V(\phi)$ for a deep bound state—Eq. (2.2). Dashed curve indicates V is complex. (b) Behavior of $V(\phi)$ for $\mu^2 < 0$ in the O(N) model.

minimum of $V(\phi)$ but is a saddle point in $V(\phi, \chi)$ and would have been rejected in the unconstrained problem also.

There does not seem to be any problem in extending this argument to local gauge symmetries. Gauge fields would acquire a mass through couplings to χ. However, this needs to be investigated further.

III TRANSITIONS OF VACUUMS IN THE O(N) σ MODEL

The solution of this model in the large-N limit has been worked out by Schnitzer[3,4] and by Coleman, Jackiw, and Politzer.[5] In this section we are interested in the values of parameters that correspond to no classical symmetry breaking, which is treated by Schnitzer.[3] However, we follow the notation and renormalization conventions of Ref. 5, even though they did not treat this case, since this formulation is more concise.

The Lagrangian is

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi)^2 - \tfrac{1}{2}\mu_0^2\phi^2 - (\lambda_0/8N)\,(\phi^2)^2\ ,$$

where

$$\phi^2 = \sum_{i=1}^{N} \phi_i^2\ .$$

The effective potential has been calculated in Ref. 5, Eq. (2.5), and we will not repeat this calculation. We evaluate this explicitly using the definitions of renormalized μ^2 and λ [Ref. 5, Eqs. (2.9) and (2.10) in 4-space] to get the following: $V(\phi) = V(\phi, \chi(\phi))$, where

$$V(\phi, \chi) = -\frac{N}{2\lambda}\chi^2 + \tfrac{1}{2}\chi\phi^2 + \frac{N\mu^2}{\lambda}\chi + \frac{N\chi^2}{128\pi}\left[2\ln\left(\frac{\chi}{M^2}\right) - 1\right], \tag{3.1}$$

subject to the constraint to eliminate χ,

$$-2\frac{\partial V(\phi, \chi)}{\partial \chi} = -\phi^2 + \frac{2N\chi}{\lambda} - \frac{2N\mu^2}{\lambda} - \frac{N}{16\pi^2}\chi\ln\left(\frac{\chi}{M^2}\right) = 0\ . \tag{3.2}$$

The minima are found by noting

$$\frac{dV}{d\phi} = \frac{\partial V}{\partial \phi} + \frac{\partial V}{\partial \chi}\frac{d\chi}{d\phi} = \phi\chi\ ,\qquad \phi = \left(\sum_i \phi_i^2\right)^{1/2}\ . \tag{3.3}$$

Hence, all the stationary points $dV/d\phi = 0$ are on the lines $\chi = 0$ and $\phi = 0$. $V(\phi, \chi)$ is complex for $\chi < 0$, so we eliminate that region from consideration. A normal vacuum would have $\phi = 0$, $\chi > 0$; an asymmetric vacuum $\phi > 0$, $\chi = 0$. Since we are interested in the former in this section, we set $\phi = 0$ in Eq. (3.2) and solve for χ:

$$\chi - \mu^2 - \frac{\lambda}{32\pi^2}\chi\ln\frac{\chi}{M^2} = 0\ . \tag{3.4}$$

Take $\mu^2 > 0$ and choose the renormalization mass $M^2 = \mu^2$ for convenience. This has two real solutions for $0 < \lambda < \infty$,[12] $\chi_{\rm I} = \mu^2$, and $\chi_{\rm II}$. The character of the constraint curve is shown in Fig. 3 marked "2 roots." Let us define a new interaction parameter $\hat{\lambda}(\lambda)$ which is monotonic and single valued over the desired range of λ:

$$\frac{\lambda}{32\pi^2} = \frac{\hat{\lambda} - 1}{\ln\hat{\lambda}}\ . \tag{3.5}$$

Then $\chi_{\rm II} = \mu^2/\hat{\lambda}$ and all the needed formulas can be given in closed form in terms of $\hat{\lambda}$. [Solving Eq. (3.5) for $\hat{\lambda}$ gives a transcendental equation.] The results we wish to note are summarized below:

$$V_{\rm I} = V(0, \chi_{\rm I}) = \frac{N\mu^4}{128\pi^2}\left(\frac{2\ln\hat{\lambda}}{\hat{\lambda}-1} - 1\right)\ ,\qquad V_{\rm II} = V(0, \chi_{\rm II}) = \frac{N\mu^4}{128\pi^2\hat{\lambda}^2}\left(\frac{2\hat{\lambda}\ln\hat{\lambda}}{\hat{\lambda}-1} - 1\right). \tag{3.6}$$

Corresponding to each vacuum there is a ϕ mass and a bound-state mass. The bound-state mass was found by calculating the $\phi\phi$ scattering amplitude A from the generating functional, [Ref. 5, Eq. (3.1)] and expanding: $A^{-1}(s) = A^{-1}(0) + s(A^{-1}(0))'$. Then

$$m_B^2 \approx -A^{-1}(0)/(A^{-1}(0))' = 2\chi[-32\pi^2/\lambda + 1 + \ln(\chi/M^2)]\ , \tag{3.7}$$

$\chi = \chi_{\rm I}, \chi_{\rm II}$. This is a good approximation to m_B^2

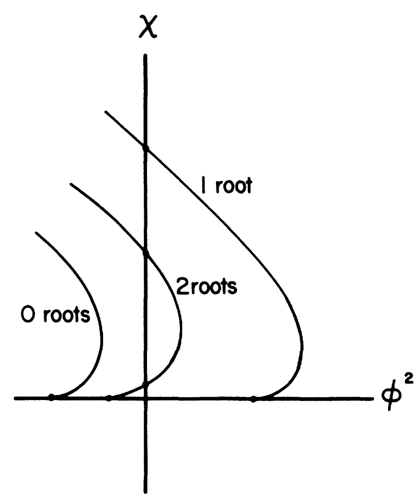

FIG. 3. Schematic behavior of the constraint curve $\partial V(\phi, \chi)/\partial\chi = 0$, Eq. (3.4), in the O(N) σ model. The number of roots of this constraint with $\phi = 0$ are used to label the curves.

when $|m_B^2|/4M_\phi^2 \ll 1$. Rather than go into the details of this calculation we can note from the analysis of Sec. II that

$$m_B^2 = \left.\frac{\partial^2 V(\phi, \hat{\chi})}{\partial \hat{\chi}^2}\right|_{\text{at minimum}}, \tag{3.8}$$

where $\hat{\chi}$ is a properly normalized field. If we disregard the normalization of χ, we get an expression for m_B^2 of the right sign, which is all we care about in this argument:

$$\frac{\partial^2 V}{\partial \chi^2} = \frac{N}{32\pi^2}\left(-\frac{32\pi^2}{\lambda} + 1 + \ln\frac{\chi}{M^2}\right). \tag{3.9}$$

This differs from Eq. (3.7) by a positive constant factor and a factor of χ. Since the minima all have $\chi > 0$, one may substitute this expression, Eq. (3.9), in place of m_B^2 for the following arguments if the reader prefers. We find

I: $m_\phi^2 = \chi_{\text{I}} = \mu^2$,

$$m_{B\text{I}}^2 = 2\mu^2\left(1 - \frac{\ln\hat{\lambda}}{\hat{\lambda}-1}\right), \tag{3.10}$$

II. $m_\phi^2 = \chi_{\text{II}} = \mu^2/\hat{\lambda}$,

$$m_{B\text{II}}^2 = \frac{2\mu^2}{\hat{\lambda}}\left(1 - \frac{\hat{\lambda}\ln\hat{\lambda}}{\hat{\lambda}-1}\right). \tag{3.11}$$

For $0<\hat{\lambda}<1$, V_{II} is the global minimum[13,14] and $m_{B\text{II}}^2 > 0 > m_{B\text{I}}^2$. Similarly for $\hat{\lambda}>1$ V_{I} is the global minimum and $m_{B\text{I}}^2 > 0 > m_{B\text{II}}^2$; for $\hat{\lambda}=1$, $V_{\text{I}} = V_{\text{II}}$, $m_{B\text{I}}^2 = m_{B\text{II}}^2 = 0$,[15] which is what we wished to show. The behavior of $V(\phi)$ for $\hat{\lambda}$ near 1 is the same as for our general discussion [Fig. 2a].

It would be desirable to have a closed-form model with internal symmetry. In the O(N) model, the bound state is an O(N) singlet and hence no symmetry is broken. A useful feature of the O(N) model is that the generating functional is calculable, from which finite-momentum vertex functions can be obtained about either vacuum. Almost any two-body dynamical scheme with a bound state carrying quantum numbers will exhibit this symmetry breaking. However, only if the generating functional is known can one obtain detailed information about the asymmetric vacuum.

IV. DYNAMICAL RESTORATION OF SYMMETRY IN THE O(N) σ MODEL

Let us now look at $\lambda > 0$ as before, but $\mu^2 < 0$. The curve marked "1 root" in Fig. 3 shows the behavior of the constraint curve. There is a symmetry-breaking minimum of V at $\chi = 0$, $\phi^2 = -2N\mu^2/\lambda$:

$$V_{\text{I}}\left(\left(-\frac{2N\mu^2}{\lambda}\right)^{1/2}, 0\right) = 0. \tag{4.1}$$

This is the minimum studied in Ref. 3 and 5 and marked by "I" on Fig. 2(b). The theory has a tachyon as reported in Ref. 5. [Our Eq. (3.7) is not a valid expression for m_B^2 for this case, since there are thresholds at $s=0$ arising from zero-mass Goldstone bosons.]

Let us verify that the symmetric minimum marked II is always lower than V_{I} ($V_{\text{I}}=0$), and that the theory defined at V_{II} has a dynamical

TABLE I. Summary of the types of vacuums for all values of λ and μ^2 based on Eq. (3.4). In the "domain" column, M^2 is the renormalization mass and is left arbitrary (positive). The λ_{c_i}'s are values of λ for which the character of the roots change. They are easily calculable from Eq. (3.4). The "roots" column gives the number of symmetric minima. The "properties of roots" column are weak but useful inequalities described in the text. The last column indicates whether there exists an asymmetric minimum or not.

Domain of μ^2 and λ		No. of roots of $\partial V(\phi,\chi)/\partial\chi\|_{\phi=0}=0$	Properties of roots	Existence of asymmetric local minimum
$\mu^2 > M^2$	$\lambda_{c_1} < \lambda$	2	$\chi_1, \chi_2 < M^2$	No
	$\lambda_{c_2} < \lambda < \lambda_{c_1}$	0		No
	$0 < \lambda < \lambda_{c_2}$	2	$\chi_1, \chi_2 > \mu^2$	No
	$\lambda < 0$	1	$M^2 < \chi_1 < \mu^2$	Yes
$0 < \mu^2 < M^2$	$0 < \lambda$	2	$\chi_1 < \mu^2$; $\chi_2 > M^2$	No
	$\lambda < 0$	1	$\mu^2 < \chi_1 < M^2$	Yes
$\mu^2 < 0$	$0 < \lambda$	1	$M^2 < \chi_1$	Yes
	$\lambda_{c_3} < \lambda < 0$	0		No
	$\lambda < \lambda_{c_3}$	2	$\chi_1, \chi_2 < M^2$	No

state of positive mass. Take $\mu^2 < 0$ ($\mu^2 \neq M^2$), $\lambda > 0$ in Eq. (3.4) and note that it has one root, $\chi = \chi_{II}$ and it satisfies

$$\chi_{II} > M^2 > 0 \; . \tag{4.2}$$

Now evaluate $V(\phi, \chi)$, Eq. (3.1), at $\phi = 0$, $\chi = \chi_{II}$, using Eq. (3.4) to eliminate λ:

$$V_{II} = V(0, \chi_{II}) = \frac{N}{128\pi^2} \chi_{II}^2 \left(\frac{2\mu^2}{\chi_{II} - \mu^2} \ln \frac{\chi_{II}}{M^2} - 1 \right) \; , \tag{4.3}$$

showing $V_{II} < 0$. Since this is a symmetric minimum with no Goldstone bosons, our formula for m_B^2, Eq. (3.7), is valid. Expressing m_B^2 in terms of χ_{II} gives

$$m_{BII}^2 = 2\chi_{II} \left(\frac{-\mu^2}{\chi_{II} - \mu^2} \ln \frac{\chi_{II}}{M^2} + 1 \right) > 0 \; . \tag{4.4}$$

Hence, choosing μ^2 negative in an attempt to generate symmetry breaking does not work in this model.

Finally, we check for other domains in λ, and μ^2 to be sure the same phenomenon takes place whenever there is an asymmetric minimum. This is all summarized in Table I and Fig. 3. The values λ_{c_1}, λ_{c_2}, and λ_{c_3} are special values of λ where the character of the χ roots of $\partial V(\phi, \chi)/\partial \chi |_{\phi=0} = 0$ changes. The above discussion corresponds to row 7 in Table I. One can easily check that for $\mu^2 > 0$, and $\lambda < 0$ (rows 4, and 6) the inequality on the root is sufficient to show $V_{II} < 0$, $m_{BII}^2 > 0$ as before.

ACKNOWLEDGMENTS

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[7]R. W. Haymaker, Phys. Rev. D 12, 1178 (1975).

[8]In Bethe-Salpeter models with simple kernels, e.g., contact or one-particle exchange, m_B^2 is analytic in coupling constants in the neighborhood of $m_B^2 = 0$, e.g., C. Schwartz, Phys. Rev. 137, B717 (1965).

[9]m_ϕ^2 is shorthand for $-\Delta_R^{-1}(0)$, the renormalized propagator at $q^2 = 0$. ϕ is the renormalized field. We call m_ϕ, loosely, the mass of the ϕ field.

[10]The simple form of V as a constrained polynomial was unfortunately not recognized in Ref. 7, and hence some aspects of that paper are unnecessarily complicated.

[11]This discussion is the analog of the tree-graph discussion in scalar field theories. This approach is also subject to quantum fluctuations [see, for example, S.-J. Chang, Phys. Rev. D 12, 1071 (1975), and references therein] which could cause this expansion to diverge.

[12]In Schnitzer's solution, Ref. 3, this range in λ corresponds to the first row in his table I; i.e., for my $0 < \lambda < \infty$, his $\bar{\lambda}$ satisfies $0 < 1 + 96\pi^2/N\bar{\lambda} < \infty$. The parameter g defined in Eq. (7.4) is what we call $\lambda/32\pi^2$.

[13]By global, we mean over the domain of ϕ^2 for which $V(\phi)$ is real.

[14]As an aside it is worth noting that for weak coupling [λ (and $\hat{\lambda}$) $\to 0$], this global minimum is not the one obtainable from perturbation theory. $V_{II} \approx -N\mu^4 \exp(64\pi^2/\lambda)/128\pi^2$, reminiscent of the superconductor gap parameter. Equation (3.2) is called the gap equation in this model by a number of authors (Refs. 3–5).

[15]A singular nature of the model at $\hat{\lambda} = 1$ was noted by Schnitzer (Ref. 3).