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Relativistic quantum transport theory approach to multiparticle production

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The field-theoretic description of multiparticle production processes is cast in a form analogous to ordinary transport theory. Inclusive differential cross sections are shown to be given by integrals of covariant phase-space distributions. The single-particle distribution function $F(p, R)$ is defined as the Fourier transform of a suitable correlation function in analogy with the nonrelativistic (Wigner) phase-space distribution function. Its transform $F(p, q)$ is observed to be essentially the discontinuity of a multiparticle scattering amplitude. External-field problems are studied to exhibit the physical content of the formalism. When $q = 0$ one recovers the single-particle distribution exactly. The equation of motion for $F(p, R)$ generates an infinite hierarchy of coupled equations for various distribution functions. In the Hartree approximation one obtains nonlinear integral equations analogous to the Vlasov equation in plasma physics. Such equations are convenient for exhibiting collective motions; in particular it appears that a collective mode exists in a ϕ^4 theory for a uniform infinite medium. It is speculated that such collective modes could provide a theoretical basis for clustering effects in multiparticle production.

I. INTRODUCTION

The covariant transport equation approach[1] to multiparticle production allows the unification of many features of contemporary field-theoretic and statistical-hydrodynamic models. Moreover, the basic objects of the theory (the covariant Boltzmann distribution functions) are defined for both configuration-space and momentum coordinates, allowing a fuller use of classical intuition than is usual.

The present work is the outgrowth of efforts to study under what circumstances the Landau hydrodynamical model might be true in a field-theoretic context.[2] As in classical fluid mechanics one looks for a microscopic transport theory from which hydrodynamics can be derived for averaged macroscopic observables such as density, fluid velocity, etc. The hydrodynamic equations of motion are quite insensitive to the detailed structure of matter and to the interparticle forces, provided these are of short range. (For example, the collision term of the Boltzmann equation drops out completely in the deduction of the hydrodynamic equations by virtue of energy-momentum conservation.) Therefore, to the extent that such a description is true one expects to learn little about the microscopic laws of nature from the study of such bulk properties as the one- and two-particle inclusive differential cross sections. At the same time one might expect to achieve partial success even in the absence of a detailed microscopic theory.

It is immediately clear that the transport theory is free of many of the limitations of the statistical-hydrodynamical approach. In particular, local thermodynamic equilibrium is not assumed, but rather can be derived when appropriate to the physics of the problem. In addition, the scattering boundary conditions are exactly incorporated. Not so obvious is the result that the distributions $F(p,R)$, $F(p_1R_1, p_2R_2)$, etc., are directly related to observable single, double, etc., inclusive differential cross sections. The inclusive distributions then result from solving a set of coupled transport equations. As in the usual transport theory of gases there is an infinite hierarchy of coupled equations for certain distribution functions. This infinite hierarchy gives an exact formulation of the multiparticle production process. Unfortunately, to close the hierarchy it is necessary to introduce auxiliary distribution functions not directly related to observable inclusive cross sections. Although we obtain a formally exact theory, the practical necessity of truncating the hierarchy introduces approximations whose validity is presently very difficult to assess.

The transport equations lead automatically to a natural description of collective motions induced by interactions among the produced particles. This is a decisive conceptual improvement over the multiperipheral model, which completely neglects the final-state interactions among the produced particles despite the fact that these particles surely overlap geometrically during the production process. The cost of all these good features is having to deal with a highly intractable nonlinear set of integro-differential equations, even after compromising approximations (e.g., Hartree) have been made.

The transport approach, in addition to readily exhibiting the collective motion of the entire medium (hydrodynamics), gives internal collective motions in a natural way. The most familiar prototype of this sort of behavior is the Vlasov equation for a plasma and the plasmon collective excitation. In fact, our whole approach is closely patterned after the Vlasov theory in which a given particle moves in the time-dependent self-consistent potential due to all the other particles.[3] Application of this method to an infinite uniform system of mesons interacting via a ϕ^4 interaction leads to a massive collective excitation for nonzero temperature (Sec. V). We point out that this sort of effect could provide a theoretical basis for the existence of clusters in many-hadron final states.[4,5]

The basic object of the theory is the covariant field-theoretic analog of the Wigner phase-space distribution function.[6] The latter is in turn the quantum-mechanical substitute for the single-particle Boltzmann distribution function. In a nonrelativistic second-quantized theory we may write

$$f(\vec{p}, \vec{R}, t) = \int \frac{d^3r}{(2\pi)^3} e^{-i\vec{p}\cdot\vec{r}}\,\mathrm{Tr}\rho \times \psi^*(\vec{R}-\tfrac{1}{2}\vec{r}, t)\psi(\vec{R}+\tfrac{1}{2}\vec{r}, t), \tag{1.1}$$

where we work in the Heisenberg picture; ρ is a (fixed) density matrix. For a pure normalized scattering state $\rho = |\psi_{\rm in}\rangle\langle\psi_{\rm in}|$ and the trace reduces to the usual expectation value. Throughout the present work we shall use this special density matrix. The construction (1.1) clearly obeys

$$\int d^3p\, f(\vec{p}, \vec{R}, t) = \langle n(\vec{R}, t)\rangle , \tag{1.2}$$

where $n(\vec{R}, t) = \psi^*\psi(\vec{R}, t)$ and $\langle A\rangle = \mathrm{Tr}\rho A$. Expanding ψ in terms of the destruction operator $a(\vec{p}, t)$ according to

$$\psi(\vec{R}, t) = \int \frac{d^3p}{(2\pi)^{3/2}} e^{i\vec{p}\cdot\vec{R}} a(\vec{p}, t) \tag{1.3}$$

allows one to rewrite (1.1) in the form

$$f(\vec{p}, \vec{R}, t) = \int \frac{d^3q}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{R}} \times\langle a^*(\vec{p}+\tfrac{1}{2}\vec{q}, t) a(\vec{p}-\tfrac{1}{2}\vec{q}, t)\rangle . \tag{1.4}$$

Integration over space now gives

$$\int d^3R\, f(\vec{p}, \vec{R}, t) = \langle n(\vec{p}, t)\rangle . \tag{1.5}$$

Hence as $t\to\infty$ (1.5) is the differential cross section $d\sigma/d^3p$ times a flux factor, when the initial wave packets are taken to be sharply peaked in momentum space.

To calculate the cross section one solves the equation of motion for f (which for potential scattering is almost the same as the usual collisionless Boltzmann equation) and extracts the flux factor from (1.5). Details of this procedure are given in the second reference of footnote 1.

The utility of the phase-space distribution technique in scattering problems does not seem to be widely appreciated, although the possibility was noted about twenty years ago.[7,8] The method has been revived independently (for nuclear reaction theory) by Remler and Sathe.[9] The use of $f(\vec{p}, \vec{R}, t)$ permits one to use classical intuition as to the joint behavior in $\vec{p}$ and $\vec{R}$ while maintaining the quantum integrity of the conjugacy of the variables $\vec{p}$ and $\vec{R}$. The advantages of this technique are clearly exhibited in applications to quantum transport theory.[10]

The extension of the foregoing ideas to the relativistic domain is straightforward, although the physical meaning of the position coordinate is less direct than in the case of nonrelativistic quantum mechanics. In addition, covariance requires the use of two time coordinates in the basic correlation function. One can invent relativistic versions of the phase-space distribution function based on single-particle wave functions,[11] but since we are especially interested in inclusive reactions we make our definitions in a manner suggested by the reduction formula expression for cross sections (Sec. II). In particular, the obscurity of the notion of the space localization of bosons is no impediment to the construction of a consistent theory.

This paper is organized as follows. In Sec. II the method is outlined and applied to some examples involving c-number external potentials. A generalization (and interpretation of) Mueller's result[12] on the relation of the inclusive cross sections to certain "absorptive parts" of multiparticle amplitudes is discovered. More precisely, it is found that the Fourier transform $F(p, q)$ of the Boltzmann function coincides, when the external legs are amputated and q is set equal to zero, with the single-particle inclusive differential cross section. Generalizations to more particles are straightforward. In this manner the evaluation of the absorptive parts entering into the inclusive cross section is seen to be accomplished by the solution of various transport equations.

In Sec. III we work out appropriate definitions for relativistic multiparticle phase-space distributions, define correlation functions in $\{p, R\}$ coordinates, and exhibit the connection with the conventional integrated correlation functions. Section IV is concerned with the "pairing approximation" by means of which the hierarchy of equations is truncated. At present the only justification for this *ad hoc* procedure is the precedent of success in ordinary many-body problems. In Sec. V we illustrate how the method may be used to find collective modes in the case of the ϕ^4 theory with an external-source coupling of the form $V\phi^2$. We find a collective massive excitation for finite temperature and illustrate how it enhances the mass distribution in the double-differential cross section, thereby suggesting a dynamical mechanism for clustering effects.

II. TRANSPORT THEORY ASPECTS OF MULTIPARTICLE PRODUCTION

We define the covariant one-particle distribution function $F(p,R)$ by

$$F(p,R)=\int d^4r\, e^{ip\cdot r}\langle\psi|\,\phi(R-\tfrac{1}{2}r)\phi(R+\tfrac{1}{2}r)\,|\psi\rangle\,. \tag{2.1}$$

Here $|\psi\rangle$ is a normalized Heisenberg "in" state, and ϕ is the Heisenberg field operator for the particle under discussion. No mass-shell restriction on the four-vector p is implied.

The choice of $|\psi\rangle$ depends on the problem to be solved. For example, if the subject is particle production by some space-dependent source, then $|\psi\rangle$ is the vacuum state. If the subject is particle production in collisions, then $|\psi\rangle$ will be an "in" state describing the two incident particles in the collision. Note, however, that we will, in general, have to describe these particles by wave packets, because if $|\psi\rangle$ is a momentum eigenstate, then translation invariance leads $F(p,R)$ to become independent of R. Spatial localization of the incident particles is essential to a transport description of the collision process. Only after the collision is over, and when one wishes to extract from the distribution function a physical observable, such as an inclusive cross section, it is possible to take a sharp (in momentum space) packet limit.

In classical physics, integrating the Boltzmann distribution function over space produces the number density in momentum, and vice versa. Its covariant analog $F(p,R)$ has a similar property. The number density in momentum is

$$\frac{dN}{d^3p}=\langle\psi|\,a^\dagger_{\text{out}}(\vec{p})a_{\text{out}}(\vec{p})|\,\psi\rangle\,. \tag{2.2}$$

Use of the reduction formula permits one to rewrite this as

$$2\omega\frac{dN}{d^3p}=\frac{1}{(2\pi)^3}\int d^4x_1\int d^4x_2\exp[ip\cdot(x_2-x_1)]\times\langle\psi|\,j(x_1)j(x_2)|\,\psi\rangle\,, \tag{2.3}$$

where $j(x)\equiv(\Box+\mu^2)\phi(x)$. The right-hand side may be expressed in terms of F:

$$2\omega\frac{dN}{d^3p}=\frac{(p^2-\mu^2)^2}{(2\pi)^3}\int d^4R\,F(p,R)\Big|_{p^2=\mu^2}. \tag{2.4}$$

In particular, if $|\psi\rangle$ describes the two incident particles in a collision process, $2\omega\, dN/d^3p$ is simply the one-particle inclusive cross section, times a flux factor.

It will be convenient to define an auxiliary function $\tilde{F}(p,R)$ by

$$\tilde{F}(p,R)=\int d^4r\, e^{ip\cdot r}\langle\psi|\,j(R-\tfrac{1}{2}r)j(R+\tfrac{1}{2}r)|\psi\rangle; \tag{2.5}$$

This is the same as $F(p,R)$ but with the "external legs" taken off. It will also frequently be convenient to work with the Fourier transform of the distribution function and of $\tilde{F}$:

$$F(p,q)=\int d^4R\, e^{-iq\cdot R}F(p,R), \tag{2.6}$$

and similarly for $\tilde{F}$. We evidently have

$$\tilde{F}(p,q)=[(p+\tfrac{1}{2}q)^2-\mu^2][(p-\tfrac{1}{2}q)^2-\mu^2]F(p,q), \tag{2.7}$$

so that (2.4) can be rewritten as

$$2\omega\frac{dN}{d^3p}=\frac{1}{(2\pi)^3}\tilde{F}(p,q)|_{q=0} \tag{2.8}$$

An equation of motion which parallels the classical Boltzmann equation can be readily obtained for the function $F(p,R)$. Subtracting the equations of motion

$$(\Box_1+\mu^2)\langle\phi(x_1)\phi(x_2)\rangle=\langle j(x_1)\phi(x_2)\rangle$$

and

$$(\Box_2+\mu^2)\langle\phi(x_1)\phi(x_2)\rangle=\langle\phi(x_1)j(x_2)\rangle$$

and recognizing the identity

$$\Box_2^{\,2}-\Box_1^{\,2}=2\frac{\partial}{\partial R}\cdot\frac{\partial}{\partial r},$$

where

$$R=\frac{x_1+x_2}{2},\quad r=x_2-x_1$$

yields the result

$$2ip\cdot\frac{\partial}{\partial R}F(p,R)=\int d^4r\, e^{ip\cdot r}\langle j(R-\tfrac{1}{2}r)\phi(R+\tfrac{1}{2}r)-\phi(R-\tfrac{1}{2}r)j(R+\tfrac{1}{2}r)\rangle . \tag{2.9}$$

The operator $p\cdot\partial/\partial R=p_0(\partial/\partial t+\vec{p}/p_0\cdot\vec{\nabla})$ is evidently the covariant analog of the conventional $\partial/\partial t+\vec{v}\cdot\vec{\nabla}$ of the nonrelativistic transport equation. The right-hand side depends on the interaction, and is the analog of the usual collision term.

The form of the operator $j(x)$ is determined by the interaction Lagrangian, and in general is such that the right-hand side of (2.9) cannot be re-expressed solely in terms of the one-particle distribution function $F(p,R)$, but will involve higher distribution functions (as well as other types of objects). Thus (2.9) becomes the first of an infinite set of coupled transport equations, connecting multiparticle distribution functions to each other. Obviously one does not avoid the infinite number of degrees of freedom in field theory simply by adopting the language of transport theory.

We shall launch into a full discussion of the hierarchy of transport equations presently, but for the remainder of this section let us illustrate the use of the transport language with two examples, in which Eq. (2.9) does close.

The simplest possible case involves a Lorentz scalar source $V(x)$ coupled linearly to the meson field, i.e., $\mathcal{L}_{\rm int}(x)=V(x)\phi(x)$. In this case the current $j(x)=V(x)$ and the formal solution to the equation of motion

$$\phi(x)=\phi_{\rm in}(x)-\int d^4x'\Delta(x-x')j(x') \tag{2.10}$$

[where $\Delta(x)$ is the retarded Green's function with $\Delta^{-1}(p)=p^2-\mu^2$] is an explicit solution. We compute particle production from the vacuum by the action of $V(x)$ by substituting (2.10) into (2.9) and solving by Fourier transformation on R. The result is expressible as

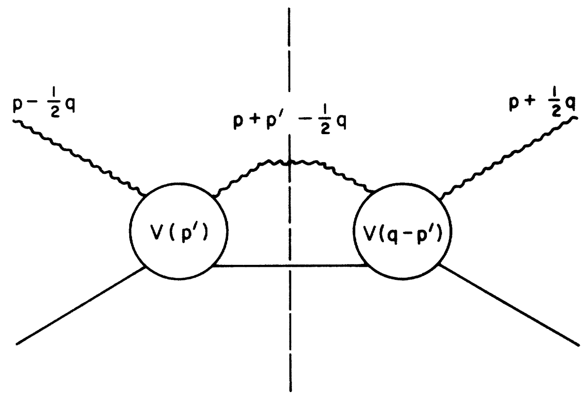

FIG. 1. The discontinuity (represented by the vertical dashed line) of the scattering amplitude for meson $p-\frac{1}{2}q$ going to meson $p+\frac{1}{2}q$ in the presence of the potential V is indicated to second order. When the external legs are amputated this quantity corresponds to $\tilde{F}_2(p,q)$.

$$\tilde{F}(p,q)=V(p+\tfrac{1}{2}q)V^*(p-\tfrac{1}{2}q), \tag{2.11}$$

so that the number distribution is, according to (2.8), the usual result

$$\frac{dN}{d^3p}=\frac{|V(p)|^2}{2\omega(2\pi)^3}. \tag{2.12}$$

The problem posed by $\mathcal{L}_{\rm int}=\frac{1}{2}V\phi^2$ is just a relativistic Schrödinger problem (of course, for the field ϕ, not the state of the system). This example is easily analyzed in perturbation series and gives a detailed illustration of the principles involved in the transport approach to particle production. The transport equation is

$$\begin{aligned}2ip\cdot\frac{\partial}{\partial R}F(p,R)&=\int d^4r\, e^{ip\cdot r}[V(R-\tfrac{1}{2}r)-V(R+\tfrac{1}{2}r)]\langle\phi(R-\tfrac{1}{2}r)\phi(R+\tfrac{1}{2}r)\rangle\\&=\int d^4r\int\frac{d^4p'}{(2\pi)^4}e^{i(p-p')\cdot r}F(p',R)[V(R-\tfrac{1}{2}r)-V(R+\tfrac{1}{2}r)].\end{aligned} \tag{2.13}$$

The resemblance to the conventional nonrelativistic equation should be noticed. A slowly varying V permits one to obtain the transport equation for a particle moving in an external field,

$$2p\cdot\frac{\partial}{\partial R}F(p,R)-\frac{\partial V}{\partial R}\cdot\frac{\partial F(p,R)}{\partial p}=0. \tag{2.14}$$

The exact equation following from (2.13) is more conveniently written when Fourier transformed on R:

$$2p\cdot qF(p,q)=\int\frac{d^4q'}{(2\pi)^4}V(q')[F(p+\tfrac{1}{2}q',q-p')-F(p-\tfrac{1}{2}q',q-q')]. \tag{2.15}$$

Again consider particle production by V from an initial vacuum. The unperturbed distribution F_0 is

$$F_0(p,q)=(2\pi)^5\delta_-(p^2-\mu^2)\delta(q), \tag{2.16}$$

where the minus subscript signifies that only the $p_0<0$ root is taken and $\delta(q)$ reflects the spatial homogeneity of the initial condition. Equation (2.15) suggests an iterative scheme; to second order we find

$$F(p,q)=(2\pi)^5\delta_-(p^2-\mu^2)\delta(q)$$

$$-2\pi\left(\frac{\delta_-((p+\frac{1}{2}q)^2-\mu^2)}{(p-\frac{1}{2}q)^2-\mu^2}+\frac{\delta_-((p-\frac{1}{2}q)^2-\mu^2)}{(p+\frac{1}{2}q)^2-\mu^2}\right)\left(V(q)-\int\frac{d^4q'}{(2\pi)^4}\frac{V(q')V(q-q')}{(p+p'+\frac{1}{2}q)^2-\mu^2}\right)$$

$$+\frac{2\pi}{[(p+\frac{1}{2}q)^2-\mu^2][(p-\frac{1}{2}q)^2-\mu^2]}\int\frac{d^4p'}{(2\pi)^4}V(q')V(q-q')\delta_-((p+q'-\frac{1}{2}q)^2-\mu^2). \tag{2.17}$$

Inspection of these contributions (and higher-order terms) immediately shows that (2.17) represents the sum of the discontinuities of a scattering amplitude of mesons of initial and final momenta $p\mp\frac{1}{2}q$ on a potential V (Fig. 1). $\bar{F}(p,q)$ corresponds to including discontinuities of internal lines only and amputating external legs. In the present problem the lowest-order contribution to $\bar{F}$ is

$$\bar{F}(p,q)=2\pi\int\frac{d^4p'}{(2\pi)^4}V(p')V(q-p')\delta_-((p+p'-\frac{1}{2}q)^2-\mu^2), \tag{2.18}$$

and the number distribution of produced particles is

$$dN/d^3p=\frac{1}{2\omega(2\pi)^6}\int d^4q'|V(p+q')|^2\delta_+((q')^2-\mu^2). \tag{2.19}$$

$\bar{F}(p,q)$ is the absorptive part of a scattering amplitude whose perturbation expansion leads directly to the graphical structure exhibited in Fig. 1.

III. N-PARTICLE DISTRIBUTION FUNCTIONS AND INCLUSIVE REACTIONS

As we remarked earlier, in any real field theory, the transport equation for the one-particle distribution function cannot be written in terms of the one-particle distribution function only, but of necessity involves expectation values of products of more than two fields. For example, if we take as an interaction Lagrangian $\mathcal{L}_I=\frac{1}{4}\lambda\phi^4$, the transport equation (2.9) reads

$$2ip\cdot\frac{\partial}{\partial R}F(p,R)=\lambda\int d^4r\,e^{ip\cdot r}\langle\phi^3(R-\tfrac{1}{2}r)\phi(R+\tfrac{1}{2}r)-\phi(R-\tfrac{1}{2}r)\phi^3(R+\tfrac{1}{2}r)\rangle\!\rangle,$$

so that we are forced to deal with expectations of four fields. We must therefore turn next to a discussion of multiparticle distribution functions.

Consider the reaction

$$\Psi\to p_1+p_2+\cdots+p_n+X, \tag{3.1}$$

where Ψ is a normalized incoming state (assumed to contain no particles of the field ϕ) and X represents other possible products in addition to the n particles having momentum $p_1,p_2,\ldots,p_n$. The S matrix for reaction (3.1) is

$$S_{fi}=\langle X_{\text{out}}|a_{\text{out}}(p_1)\cdots a_{\text{out}}(p_n)|\Psi_{\text{in}}\rangle$$

$$=\int\prod_i d^4x_i f^*_{p_i}(x_i)\prod_i K_{x_i}\langle X_{\text{out}}|T\phi(x_1)\cdots\phi(x_n)|\Psi_{\text{in}}\rangle, \tag{3.2}$$

where $f_p(x)=\exp(-ip\cdot x)/[2\omega(2\pi)^3]^{1/2}$ and $K_x=\Box_x+\mu^2$.

The inclusive probability is

$$P(\Psi\to p_1+\cdots+p_n+\text{anything})=\sum_X|S_{fi}|^2$$

$$=\int\prod_i d^4x_i\,d^4y_i f^*_{p_i}(x_i)f_{p_i}(y_i)\prod_i K_{x_i}K_{y_i}\langle\Psi_{\text{in}}|T\phi(y_1)\cdots\phi(y_n)T\phi(x_1)\cdots\phi(x_n)|\Psi\rangle. \tag{3.3}$$

The time-ordered product in (3.3) could be replaced by a multiple retarded commutator. From Eq.

(3.2) we see that this expression is simply the (normal-ordered) outgoing differential number distribution for the initial state $\Psi_{\rm in}$:

$$\sum_{X} |S_{fi}|^2 = \langle \Psi_{\rm in}| : n_{\rm out}(p_1)\cdots n_{\rm out}(p_n) : |\Psi_{\rm in}\rangle, \tag{3.4}$$

and as such is equal to the inclusive differential cross section $d\sigma/d^3p_1\cdots d^3p_n$ times a flux factor in the limit that $\Psi_{\rm in}$ represents a two-particle state sharply peaked in momentum space.

Alternatively, one can begin directly from (3.4), the latter considered as the fundamental physical quantity. (The normal ordering removes inconvenient disconnected parts.) Explicitly we have (leaving $\Psi_{\rm in}$ tacit)

$$\begin{aligned} 2\omega_1\langle n_{\rm out}(p_1)\rangle &= \frac{1}{(2\pi)^3}\int d^4x_1 d^4y_1 \exp[ip_1\cdot(y_1-x_1)]K_{x_1}K_{y_1}\langle\phi(x_1)\phi(y_1)\rangle\,, \\ 2\omega_1 2\omega_2\langle : n_{\rm out}(p_1)n_{\rm out}(p_2):\rangle &= \frac{1}{(2\pi)^6}\int d^4x_1 d^4x_2 d^4y_1 d^4y_2 \exp[ip_1\cdot(y_1-x_1)]\exp[ip_2\cdot(y_2-x_2)] \\ &\quad\times K_{x_1}K_{x_2}K_{y_1}K_{y_2}\langle T\phi(x_1)\phi(x_2)T\phi(y_1)\phi(y_2)\rangle, \end{aligned} \tag{3.5}$$

and so forth.

We now define a hierarchy of phase-space distribution functions analogous to the Boltzmann functions by isolating from (3.5) the Fourier transform of the correlation functions $\langle T\phi(x_1)\cdots\phi(x_n)T\phi(y_1)\cdots\phi(y_n)\rangle$ with respect to the relative coordinates $y_1-x_1\equiv r_1$, $y_2-x_2\equiv r_2$, $y_i-x_i\equiv r_i$. Writing $R_i=\frac{1}{2}(x_i+y_i)$, we define

$$\begin{aligned} F(p_1R_1) &\equiv \int d^4r_1 \exp(ip_1\cdot r_1)\langle\phi(R_1-\tfrac{1}{2}r_1)\phi(R_1+\tfrac{1}{2}r_1)\rangle, \\ F(p_1R_1,p_2R_2) &\equiv \int d^4r_1 d^4r_2 \exp(ip_1\cdot r_1+ip_2\cdot r_2)\langle T\phi(R_1-\tfrac{1}{2}r_1)\phi(R_2-\tfrac{1}{2}r_2)T\phi(R_1+\tfrac{1}{2}r_1)\phi(R_2+\tfrac{1}{2}r_2)\rangle, \end{aligned} \tag{3.6}$$

and so forth.

As we have seen in Sec. II, it is useful to further Fourier transform on the coordinate R giving

$$\begin{aligned} F_1(p_1q_1) &= \int dR_1 dr_1 \exp(iq_1\cdot R_1+ip_1\cdot r_1)\langle\phi(R_1-\tfrac{1}{2}r_1)\phi(R_1+\tfrac{1}{2}r_1)\rangle \\ &= \int dx_1 dy_1 \exp[i(p_1+\tfrac{1}{2}q_1)\cdot y_1 - i(p_1-\tfrac{1}{2}q_1)\cdot x_1]\langle\phi(x_1)\phi(y_1)\rangle, \\ F_n(p_1q_1,\ldots,p_nq_n) &= \int\prod_{j=1}^{n} d^4x_j\, d^4y_j \exp[i(p_j+\tfrac{1}{2}q_j)\cdot y_j - i(p_j-\tfrac{1}{2}q_j)\cdot x_j]\langle T\phi(x_1)\cdots\phi(x_n)T\phi(y_1)\cdots\phi(y_n)\rangle. \end{aligned} \tag{3.7}$$

The functions F_n, exhibited in various forms in Eqs. (3.6) and (3.7), are covariant analogs of the usual nonrelativistic distribution functions. These functions differ from the quantities appearing in the number distributions (3.3) or (3.5) by the Klein-Gordon operators $\Box+\mu^2$. Therefore, as before, we define an auxiliary set of functions $\tilde{F}_n(p_1R_1,\ldots,p_nR_n)$ in exact analogy with (3.6) but with the inclusion of $\Pi_{i=1}^{n}K_{x_i}K_{y_i}$ in the integrand. Then we can write

$$\prod_{i=1}^{n} 2\omega_i\langle : n(p_1)\cdots n(p_n):\rangle = \frac{1}{(2\pi)^{3n}}\int\prod_{j=1}^{n} dR_j\,\tilde{F}_n(p_1R_1,\ldots,p_nR_n). \tag{3.8}$$

The Fourier transforms of the two sets of functions are related by

$$\tilde{F}_n\{p_i\,q_i\} = \prod_{i=1}^{n}[(p_i+\tfrac{1}{2}q_i)^2-\mu^2][(p_i-\tfrac{1}{2}q_i)^2-\mu^2]F_n\{p_i\,q_i\}\,. \tag{3.9}$$

As a consequence of these definitions we can write the number distributions (3.8) in the form

$$\prod_i 2\omega_i\left\langle : \prod_i n_{\rm out}(p_i):\right\rangle = \frac{1}{(2\pi)^{3n}}\tilde{F}_n\{p_i\,q_i\}|_{q_i=0}. \tag{3.10}$$

Pictorially we represent $F_n\{p_i\, q_i\}$ as a sort of scattering amplitude (but with eccentric boundary conditions) exhibited for $n=3$ in Fig. 2. To obtain the quantity $\tilde{F}_n$ we have to amputate the external legs of F_n [cf. Eq. (3.9)]; going to the limit of zero momentum transfer then gives the physical quantity (3.10).

In order to show the direct connection of our distribution functions F_n to the conventional correlations we exhibit the following equations:

$$\int d^3p_1 \langle n_{\text{out}}(p_1)\rangle = \langle N\rangle = \frac{1}{(2\pi)^3}\int d^4p\,\delta_+(p^2-\mu^2)F_1(p,q)\Big|_{q=0},$$

$$\int d^3p_1 d^3p_2 \langle :n_{\text{out}}(p_1)n_{\text{out}}(p_2):\rangle = \langle N(N-1)\rangle \tag{3.11}$$

$$= \frac{1}{(2\pi)^6}\int d^4p_1 d^4p_2\,\delta_+(p_1{}^2-\mu^2)\delta_+(p_2{}^2-\mu^2)F_2(p_1q_1,p_2q_2)\Big|_{q_1=q_2=0},$$

$$\langle N(N-1)\cdots(N-n+1)\rangle = \int\prod_{i=1}^{n}\frac{d^4p_i}{(2\pi)^4}\,2\pi\delta_+(p_i{}^2-\mu^2)F_n\{p_i\,q_i\}\Big|_{q_i=0}.$$

Here $\delta_+(p^2-\mu^2)$ is as usual $\theta(p_0)\delta(p^2-\mu^2)$.

Now it is obvious how one defines a set of correlation functions of $f_n(p,R)$ which integrate to the usual correlations f_n. Corresponding to the two-particle correlation function

$$f_2(p_1R_1,p_2R_2) = F_2(p_1R_1,p_2R_2) - F_1(p_1R_1)F_1(p_2R_2) \tag{3.12}$$

we have the corresponding equation

$$\tilde{f}_2(p_1R_1,p_2R_2) = \tilde{F}_2(p_1R_1,p_2R_2) - \tilde{F}_1(p_1R_1)\tilde{F}_1(p_2R_2), \tag{3.13}$$

and the Fourier-transformed quantities $f_2(p_1q_1,p_2q_2)$ and $\tilde{f}_2(p_1q_1,p_2q_2)$.

For $q_i\to 0$ we have the relation

$$2\omega_1 2\omega_2[\langle n(p_1)n(p_2) - \delta(\vec{p}_1-\vec{p}_2)n(p_1)\rangle - \langle n(p_1)\rangle\langle n(p_2)\rangle] = \frac{1}{(2\pi)^6}\tilde{f}_2(p_1q_1,p_2q_2)|_{q_1=q_2=0}, \tag{3.14}$$

from which follows

$$f_2 \equiv \int\frac{1}{(2\pi)^6}\frac{d^3p_1}{2\omega_1}\frac{d^3p_2}{2\omega_2}\tilde{f}_2(p_1q_1,p_2q_2)\Big|_{q_1=q_2=0}. \tag{3.15}$$

The construction of f_3 follows the usual semi-invariant structure:

$$\begin{aligned} f_3(p_1R_1,p_2R_2,p_3R_3) &= F_3(p_1R_1,p_2R_2,p_3R_3) - F_1(p_1R_1)F_2(p_2R_2,p_3R_3) \\ &\quad - F_1(p_2R_2)F_2(p_1R_1,p_3R_3) - F_1(p_3R_3)F_2(p_1R_1,p_2R_2) + 2F_1(p_1R_1)F_1(p_2R_2)F_1(p_3R_3). \end{aligned} \tag{3.16}$$

This can also be surmised from the connection of F_3 with $(d\sigma/d^3p_1d^3p_2d^3p_3)/\sigma_{\text{in}}$ and the integral relation $\sum_n n(n-1)(n-2)\sigma_n/\sigma_{\text{in}} = f_1{}^3 + 3f_1f_2 + f_3$.

The integrated correlation function is

$$\begin{aligned} f_3 &= \frac{1}{(2\pi)^9}\int\frac{d^3p_1}{2\omega_1}\frac{d^3p_2}{2\omega_2}\frac{d^3p_3}{2\omega_3}\tilde{f}_3(p_1q_1,p_2q_2,p_3q_3)\Big|_{q_i=0} \\ &= \langle N^3\rangle - 3\langle N^2\rangle\langle N\rangle + 2\langle N\rangle^3. \end{aligned} \tag{3.17}$$

In nth order we have

$$f_n = \int\prod_{i=1}^{n}\frac{d^4p_i}{(2\pi)^4}\,2\pi\delta_+(p_i{}^2-\mu^2)\tilde{f}_n\{p_i\,q_i\}\Big|_{q_i=0}. \tag{3.18}$$

The distribution functions F_n, or the correlation functions f_n, provide a detailed local description of the inclusive reactions. It would be attractive if the F_n sufficed to give a complete dynamical description of all the inclusive processes. This turns out not to be the case except in certain approximations. The origin of this situation is seen to arise from the disposition of the T products in Eq. (3.7). If we calculate the equation of motion of $F_i(pR)$ we are led to a hierarchy of correlation functions without any time ordering since no T product occurs in F_n. There is nothing intrinsically wrong with this since we can certainly construct the F_n given the set of all products of field operators. Alternatively, we can simply use "Green's functions," the simplest of which involves $\langle T\phi(x_1)\phi(y_1)\rangle\!\rangle$ in place of $\langle \phi(x_1)\phi(y_1)\rangle\!\rangle$ in Eq. (3.6). Now the hierarchy is essentially the usual one cast in transport form, from which one can also reconstruct the "physical" phase-space distributions F_n. Another feature of the F_n's is shown if we compute, for example, the equation of motion for F_2 in a theory with ϕ^4 coupling. Here one encounters expectation values of the form $\langle T\phi_1\phi_2\phi_3\phi_4T\phi_5\phi_6\rangle$ which also lie outside the set of quantities F_n. Hence although the quantities F_n defined above have a rather direct physical significance, it appears essential, in an exact treatment, to compute them by detouring to the set of all Green's functions or the set of all products.

IV. THE PAIRING APPROXIMATION; ϕ^4 COUPLING

Let us now return to the example with $\mathcal{L}_I = \frac{1}{4}\lambda\phi^4$, mentioned briefly in Sec. III, and use this theory as an illustration of how one might employ the machinery we have set up. The transport equation for the one-particle distribution is

$$2ip\cdot\frac{\partial}{\partial R}F(p,R)=\lambda\int d^4r\,e^{ip\cdot r}\langle\phi^3(R-\tfrac{1}{2}r)\phi(R+\tfrac{1}{2}r)-\phi(R-\tfrac{1}{2}r)\phi^3(R+\tfrac{1}{2}r)\rangle\!\rangle\,. \tag{4.1}$$

We may also write the transport equation obeyed by F_2:

$$\begin{aligned}\sum_{i=1}^{2}2p_i\cdot q_i F_2=\lambda\prod_{i=1}^{2}\int d^4x_i\int d^4y_i\exp\{i[(p_i+\tfrac{1}{2}q_i)\cdot y_1-(p_i-\tfrac{1}{2}q_i)\cdot x_1]\}\\ \times[\langle T\phi^3(x_1)\phi(x_2)T\phi(y_1)\phi(y_2)-T\phi(x_1)\phi(x_2)T\phi^3(y_1)\phi(y_2)\rangle\!\rangle+(1\longleftrightarrow 2)]\\ -2i\delta(p_1-\tfrac{1}{2}q_1+p_2-\tfrac{1}{2}q_2)G^*(p_1+\tfrac{1}{2}q_1,p_2+\tfrac{1}{2}q_2+2i\delta(p_1+\tfrac{1}{2}q_1+p_2+\tfrac{1}{2}q_2)G(p_1-\tfrac{1}{2}q_1,p_2-\tfrac{1}{2}q_2),\end{aligned} \tag{4.2}$$

where we define

$$G(p,p')=\int d^4x\int d^4x' e^{-ip\cdot x}e^{-ip'\cdot x'}\langle T\phi(x)\phi(x')\rangle\!\rangle\,. \tag{4.3}$$

The last two terms here arise from the action of the $\Box$ operators on the time-ordering symbols in F_2. The expectation value on the right-hand side of (4.1) is, as we have noted above, not expressible exactly in terms of the set of F_n's. We are, however, not going to attempt to solve the field theory exactly, but will instead invoke the analog of the conventional Hartree, or random-phase, approximation which is commonly exployed in nonrelativistic transport theory. We shall not attempt to justify this approximation here, other than to remark that it is based on the idea that correlations are weak and that this in fact seems to be the case experimentally in high-multiplicity high-energy collisions.

The approximation consists of writing the expectation value of a product of fields as the sum of the expectation values of all possible pairings. Thus, in Eq. (4.1), we write

$$\langle\phi^3(R-\tfrac{1}{2}r)\phi(R+\tfrac{1}{2}r)-\phi(R-\tfrac{1}{2}r)\phi^3(R+\tfrac{1}{2}r)\rangle\!\rangle\approx3[\langle\phi^2(R-\tfrac{1}{2}r)\rangle\!\rangle-\langle\phi^2(R+\tfrac{1}{2}r)\rangle\!\rangle]\langle\phi(R-\tfrac{1}{2}r)\phi(R+\tfrac{1}{2}r)\rangle\!\rangle\,. \tag{4.4}$$

With this replacement, the transport equation closes. Expressed in terms of the Fourier transform of the distribution function, we find

$$2p\cdot qF(pq)=3\lambda\int\frac{d^4p'}{(2\pi)^4}\int\frac{d^4q'}{(2\pi)^4}F(p'q')[F(p-\tfrac{1}{2}q',q-q')-F(p+\tfrac{1}{2}q',q-q')]. \tag{4.5}$$

In writing the pairing approximation for F_2, it is convenient to modify our notation, let $P=p-\frac{1}{2}q$, $P'=p+\frac{1}{2}q$ and write F as a function of P and P'. Thus P and P' are the initial and final momenta in the scattering process of which F is the absorptive part. Then, in the pairing approximation, we find from the definition of F_2 that

$$\begin{aligned}F_2(P_1P_2,P_1'P_2')=F(P_1P_1')F(P_2P_2')\\ +F(P_1P_2')F(P_2P_1')\\ +G(P_1P_2)G^*(P_1'P_2')\end{aligned} \tag{4.6}$$

(the Bose symmetry, by which F_2 is symmetric in either P_1 and P_2 or P_1' and P_2', is retained by

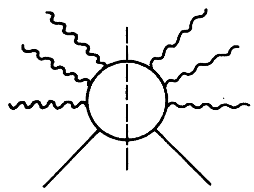

FIG. 2. Graphical representation of the distribution function $F_3(p_1q_1,\ p_2q_2,\ p_3q_3)$.

the approximation), and it may be directly verified that this form is consistent with the transport equation, Eq. (4.2). Thus the approximation is a consistent one.

V. COLLECTIVE MODES AND CLUSTERS

The next question, if we accept the pairing, or Hartree, or random-phase, approximation and thereby Eq. (4.5), is what can we use it for? In this section we shall describe an interesting application.

First let us modify Eq. (4.5) by adding to the interaction Lagrangian a term $\frac{1}{2}V\phi^2$, so that $\mathcal{L}_I=\frac{1}{2}V\phi^2+\frac{1}{4}\lambda\phi^4$. Then (4.5) is modified to read

$$2p\cdot qF(pq)=\int dq'\left[V(q')+3\lambda\int dp'F(p'q')\right]\times[F(p-\tfrac{1}{2}q,\,q-q')-F(p+\tfrac{1}{2}q,\,q-q')],\qquad(5.1)$$

where we use the shorthand $\int dp'$ for $\int d^4p'/(2\pi)^4$. We may think of this Lagrangian as an approximation to an interaction $\mathcal{L}_I=\frac{1}{2}g\bar{\psi}\psi\phi^2+\frac{1}{4}\lambda\phi^4$, in which the field $\bar{\psi}\psi$ is treated like an external classical source. One could, for example, choose V to imitate the two incident particles in a collision process. The picture would then be one in which two sources (the two incident particles) travel along spewing out particles, which interact among themselves with a $\lambda\phi^4$ coupling as well as back on the sources, and eventually radiate giving rise to inclusive cross sections. Equation (5.1) is reminiscent of the classical Vlasov equation for an electron plasma, so the terminology of a plasma of particles being created by the sources, and then radiating, is appropriate. Once one thinks of a plasma analog, it is natural to search for collective modes among the produced particles in the collision. That Eq. (5.1) in fact contains such modes is readily apparent. To exhibit them, let us imagine that a plasma with distribution function $F^{(0)}$ has been created around each of the two sources separately. The other source then travels through this plasma, disturbs it, and causes it to radiate particles. We can calculate the inclusive cross section for this to happen from Eq. (5.1) if we assume the disturbance to be weak, and the unperturbed plasma to be spatially large compared to the perturbing source V. (Neither of these is a plausible assumption for a real high-energy collision, so the calculation we are about to describe is to be taken as an illustration, and not necessarily as a description of nature.)

We thus want to solve (5.1) in a power series in V, where the zeroth-order term $F^{(0)}$ is of the form $F^{(0)}(p,q)=F^{(0)}(2\pi)^4\delta^4(q)$ reflecting the large space-time extent of the undisturbed plasma. To first order in V, the equation reads

$$2p\cdot qF^{(1)}(p,q)=\left(V(q)+\int\frac{d^4p'}{(2\pi)^4}F^{(1)}(p',q)\right)\times[F^{(0)}(p+\tfrac{1}{2}q)-F^{(0)}(p-\tfrac{1}{2}q)],\qquad(5.2)$$

whose solution is

$$F^{(1)}(p,q)=\frac{X(p,q)V(q)}{1-\lambda\int[d^4p/(2\pi)^4]X(p,q)}\equiv\frac{X(p,q)V(q)}{D(q)},\qquad(5.3)$$

where

$$X(p,q)\equiv\frac{F^{(0)}(p+\frac{1}{2}q)-F^{(0)}(p-\frac{1}{2}q)}{(p+\frac{1}{2}q)^2-(p-\frac{1}{2}q)^2}.\qquad(5.4)$$

The condition that a collective mode exists in the absence of an external stimulus $V(q)$ is clearly the vanishing of D, which occurs if

$$1=\lambda\int\frac{d^4p}{(2\pi)^4}\left[\frac{F^{(0)}(p+\frac{1}{2}q)-F^{(0)}(p-\frac{1}{2}q)}{(p+\frac{1}{2}q)^2-(p-\frac{1}{2}q)^2}\right].\qquad(5.5)$$

We note that classical stability considerations require the constant λ to be negative.

To check for the existence of a collective mode and to examine its nature we must specify the unperturbed distribution function. As an example, take $F^{(0)}(p)$ to be that appropriate to a system of free bosons in thermal equilibrium[13] (remembering to subtract out the vacuum expectation value):

$$F^{(0)}(p)=2\pi\delta(p^2-\mu^2)\left(\frac{\epsilon(p_0)}{\exp(\beta p_0)-1}-\theta(-p_0)\right).\qquad(5.6)$$

Here $\epsilon(p_0)$ is $+1$ or 1 according to whether p_0 is >0 or <0. Evaluation of the integral (5.5) now leads to the eigenvalue condition

$$\frac{8\pi^2}{\lambda}=\frac{1}{q}\int_\mu^\infty\frac{dx}{e^{\beta x}-1}\times\ln\left|\frac{[\frac{1}{2}q^2-\frac{1}{2}q_0{}^2+q(x^2-\mu^2)^{1/2}]^2-x^2q_0{}^2}{[\frac{1}{2}q^2-\frac{1}{2}q_0{}^2-q(x^2-\mu^2)^{1/2}]^2-x^2q_0{}^2}\right|,\qquad(5.7)$$

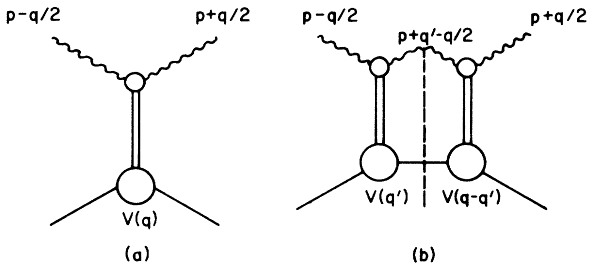

FIG. 3. Diagrams representing $F_1(a)$ and $F_2(b)$ with the double lines standing for the "propagator" $D^{-1}(q)$ of the collective mode.

where here q means $|\vec{q}|$.

We now examine solutions $q_0 = q_0(q)$ of Eq. (5.7). It is simplest to study the long-wavelength ($q \to 0$) and the low-temperature ($\beta\mu \gg 1$) limits. (It is not necessary to take these limits simultaneously.) In either case the quantity $q(x^2 - \mu^2)^{1/2}$ in the logarithm is small so that one can expand the logarithm. For finite q it is not difficult to solve (5.7) for large $\beta\mu$ since the contributing values of x are close to μ. The solution is

$$q_0^2 = q^2 + \tfrac{1}{2}(4\mu^2 - f(T) + \{[4\mu^2 - f(T)]^2 + 16\mu^2 q^2\}^{1/2}), \tag{5.8}$$

where the function $f(T)$ is

$$f(T) = \frac{\lambda}{\pi^2}\int_\mu^\infty dx (x^2 - \mu^2)^{1/2}/(e^{\beta x} - 1), \tag{5.9}$$

and vanishes exponentially as $T \to 0$. As q becomes large $q_0 \cong q$. The sound velocity $\partial q_0/\partial q$ is less than unity, but approaches unity for large q. In the long-wavelength limit ($q \to 0$) for arbitrary β it is easy to see that no root exists for $q_0 < 2\mu$; the $q_0 > 2\mu$ root agrees with the expression (5.8).

The example system, infinite in extent and at constant temperature T, does not accurately imitate the initial state in a multiparticle production process. Nor does the model Lagrangian represent the real world. However, the existence of collective excitations is evidently possible in the real world. It is tempting to speculate that such excitations are at the root of the phenomenon of clustering.

To explore this possibility let us carry the calculation of F to second order in V, the lowest order in which there is a contribution to the inclusive cross section. The equation for $\tilde{F}^{(2)}$, the quantity of physical interest, is

$$\tilde{F}^{(2)}(p, q) = \int dq' \left[V(q') + 3\lambda \int dp' F^{(1)}(p', q')\right] \frac{[(p+\frac{1}{2}q)^2 - \mu^2][(p-\frac{1}{2}q)^2 - \mu^2]}{[(p+\frac{1}{2}q)^2 - \mu^2] - [(p-\frac{1}{2}q)^2 - \mu^2]}$$
$$\times [F^{(1)}(p - \tfrac{1}{2}q', q - q') - F^{(1)}(p + \tfrac{1}{2}q', q - q')]. \tag{5.10}$$

Using the expression (5.3) for $F^{(1)}$, this can be simplified to

$$\tilde{F}^{(2)}(p, q) = \int dq' \frac{V(q')}{D(q')} \frac{V(q-q')}{D(q-q')} F^{(0)}(p + q' - \tfrac{1}{2}q) 2\pi\delta((p + q' - \tfrac{1}{2}q)^2 - \mu^2). \tag{5.11}$$

Graphically, $\tilde{F}^{(2)}$ can be represented as in Fig. 3.

The inclusive cross section is proportional to $\tilde{F}^{(2)}(p, q=0)$ evaluated for p on mass shell,

$$\omega \frac{d\sigma}{d^3p} \propto \int dq' \left|\frac{V(q')}{D(q')}\right|^2 F^{(0)}(p + q') 2\pi\delta(q'^2 + 2p\cdot q'). \tag{5.12}$$

The presence of the function D, which contains the collective mode, obviously influences the cross section, but one cannot directly see the connection between the collective mode and correlations until one looks at the function F_2. Within the pairing approximation, F_2 is given by Eq. (4.6). The term of interest in connection with clustering is the last one, $G(p_1 - \frac{1}{2}q_1, p_2 - \frac{1}{2}q_2)G^*(p_1 + \frac{1}{2}q_1, p_2 + \frac{1}{2}q_2)$, since this term couples together the two initial and the two final particles in the scattering process. Its contribution to the two-particle inclusive cross section will be proportional to

$$\omega_1\omega_2 \frac{d\sigma}{d^3p_1 d^3p_2} \propto (p_1^2 - \mu^2)^2 (p_2^2 - \mu^2)^2 |G(p_1 p_2)|^2. \tag{5.13}$$

Now G is not F, because of the time-ordering symbol in its definition, Eq. (4.3), so we cannot directly use our solution (5.11). But we can express G in terms of F, through

$$G(p_1 p_2) = i\int_{-\infty}^{\infty} \frac{dp_0}{2\pi}\left(\frac{F((\frac{1}{2}[\vec{p}_1 - \vec{p}_2], p_0), p_1 + p_2)}{\frac{1}{2}(p_{10} - p_{20}) + p_0} - \frac{F((-\frac{1}{2}[\vec{p}_1 - \vec{p}_2], p_0), p_1 + p_2)}{\frac{1}{2}(p_{10} - p_{20}) - p_0}\right)$$
$$\approx \tfrac{1}{2}[F(\tfrac{1}{2}(p_1 - p_2), p_1 + p_2) + F(\tfrac{1}{2}(p_2 - p_1), p_1 + p_2)]. \tag{5.14}$$

Thus, approximately,

$$\omega_1\omega_2 \frac{d\sigma}{d^3p_1 d^3p_2} \propto |\tilde{F}(\tfrac{1}{2}(p_1-p_2), p_1+p_2)|^2$$

$$= \left| \int dq' \frac{V(q')}{D(q')} \frac{V(p_1+p_2-q')}{D(p_1+p_2-q')} \tfrac{1}{2}[F^{(0)}(p_1+q')2\pi\delta((p_1+q')^2-\mu^2)+F^{(0)}(p_2+q')2\pi\delta((p_2+q')^2-\mu^2)]\right|^2. \quad (5.15)$$

Thus there is an enhancement of the two-particle inclusive cross section for values of $(p_1+p_2)^2$ near twice the mass of the collective mode, and hence it does indeed suggest the existence of clustering.

VI. DISCUSSION

The present paper has been concerned with setting up the formal transport theory and exploring its physical content in simplified models involving external potentials. In addition we have considered possible collective behavior in uniform systems of large spatial extent. The real problem, the collision of two structured particles interacting via local field interactions, was not addressed except in outline. (In the first reference of footnote 1 we gave, but did not solve, a pair of coupled integral equations describing the production of particles in the collision of two "scalar protons.") The principal problems are to extract the geometrical information about the collision, both the structure of the colliding particles and the "trajectory" information contained in the initial wave packets. From a classical point of view the physical content of the approach is rather simple, although mathematically very difficult even for a system of charged particles interacting and radiating through the electromagnetic field. The field radiated away by the colliding particles is determined by the acceleration of the sources. The latter in turn depends on the energy-momentum lost to the radiated field. In the particle-physics case we have the additional complications that the radiated fields are strongly self-interacting, and that sources are structured.

Clearly, considerable effort is required to address such basic phenomena as leading-particle effects, bounded transverse momentum, etc. The dynamical complexity of the problem at hand seems to preclude an elementary solution. Further research on this problem will be reported elsewhere.

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