How strong are the Pomeron decoupling theorems?*

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We investigate how close to one a Pomeron pole can be before the decoupling mechanisms make the total pion-pion cross section too small to be consistent with that deduced from experiment by means of factorization. Our method employs energy-momentum sum rules for singly and doubly inclusive cross sections. By assuming pion pole dominance in the forward direction we are able to eliminate the triple-Pomeron coupling, which leaves an inequality involving only the Pomeron-pole location and the pion-Pomeron coupling. From this we are able to conclude that the decoupling mechanism is exceedingly weak.

I. INTRODUCTION

The simplest concept of the Pomeron is that of a simple pole with $\alpha_P(0)<1$.[1] This would provide an immediate explanation for the observed factorizability of high-energy processes. It would also simplify theoretical discussions by suppressing cut contributions. Of course, if $\alpha_P(0)<1$, then the total cross section must ultimately fall at high energy as $\sigma \propto (s/s_0)^{\alpha_P(0)-1}$. Such a picture is at odds with current experimental data which show the total cross section rising gently with energy. To pursue the idea of a Pomeron less than one, it is necessary to assume that the observed rise in cross sections is due to transient subdominant effects coming from lower-lying Regge singularities. However disagreeable such an assumption might seem, a Pomeron below one, in addition to giving a rationale for factorizability and cut suppression, also appears to possess a philosophical advantage: One can imagine a theoretical explanation for the value of $\alpha_P(0)$ coming from, say, self-consistency arguments in which various effects add and subtract giving $\alpha_P(0)$ less than, but close to, unity.[2] Other approaches which give a Pomeron that saturates the Froissart bound or that give the Pomeron singularity at exactly one—whether poles or cuts—do not give a real explanation of the location of $\alpha_P(0)$. In these approaches, some assumption must be made *ab initio* about an "unperturbed" Pomeron which is usually assumed to either be at one or greater than one.

Given these advantages it seems sensible to keep alive the picture of Pomeron with $\alpha_P(0)<1$ at least until it is conclusively ruled out either by better theoretical models or by experiment. What we show in this paper is that general theoretical considerations such as unitarity enforced in a fairly model-independent way by means of the energy-momentum sum rules do not in fact rule out a Pomeron less than one. More detailed models which make additional assumptions could possibly be in conflict with current experiments if $\alpha_P(0)<1$.

We know that if $\alpha_P(0)$ is close enough to one we must necessarily get a contradiction, for general principles show that the Pomeron must decouple from total cross sections when $\alpha_P(0)=1$. The essential point of this paper is to examine how close to one $\alpha_P(0)$ must be before these decoupling mechanisms limit the size of cross sections to a point where they are in contradiction with present experiments. We shall see that the decoupling mechanism is exceedingly weak and that no direct contradiction with experiment arises unless $1-\alpha_P(0)\lesssim \frac{1}{2}e^{-2100}$.

Our approach is to employ the energy-momentum sum rules for singly and doubly inclusive cross sections for pion-pion scattering. By using these two sum rules in the form of inequalities, and also making the assumption of pion pole dominance in the forward direction, we can eliminate the triple-Pomeron coupling from these relations. This yields an upper bound on $\gamma_{\pi\pi P}(0)$, the pion-pion-Pomeron coupling, in terms of $\alpha_P(0)$. We then estimate $\gamma_{\pi\pi P}(0)$ from experiments to determine an upper bound on $\alpha_P(0)$.

This paper modifies and corrects some of the conclusions reached in a report circulated about a year ago by J. B. Bronzan and C. E. Jones.

In a subsequent paper we shall investigate the constraints on the Pomeron couplings for $\alpha_P(0)<1$ arising from a new set of cross-section inequalities deduced by Brower, Mueller, Sen, and Weis.[3] Preliminary analysis shows that such inequalities yield stronger results than those discussed here, but they do not appear to rule out the possibility of the Pomeron pole being below one.

II. UPPER BOUND ON THE POMERON POLE INTERCEPT

The two energy-momentum sum rules we use are discussed in Ref. 4, to which we refer the reader for background and notation. These sum rules are used to develop inequalities (i) between the total and single-particle inclusive cross sections and (ii) between the single-particle inclusive and two-particle inclusive cross sections. We use these inequalities in the high-energy triple-Regge region where we assume that a simple Pomeron pole with $\alpha_P(0)$ close to one dominates over secondary cut terms. In inequality (ii) we take all external particles to be pions. In our calculation we integrate over only the region of phase space where Pomeron-Pomeron and pion-Pomeron cuts are subordinant to the Pomeron and pion Regge poles, respectively, and where the pion particle pole term can be assumed to dominate.

The pion-pion total cross section is given in our notation by

$$\sigma_{\pi\pi} \underset{s\to\infty}{=} \frac{1}{s}\gamma_{\pi\pi P}{}^2(0)\left(\frac{s}{s_0}\right)^{\alpha_P(0)} = \frac{1}{s_0}\gamma_{\pi\pi P}{}^2(0)\left(\frac{s}{s_0}\right)^{\alpha_P(0)-1}, \tag{1}$$

where we take $s_0 \sim 1\ \mathrm{GeV}^2$ and $\gamma_{\pi\pi P}$ is the pion-pion-Pomeron-Regge coupling. The two-Pomeron cut at $2\alpha_P(0)-1$, which is the next correction to (1), is down by a factor R (assuming pole and cut strengths to be comparable) where

$$R \sim \left(\frac{s}{s_0}\right)^{\alpha_P(0)-1}\frac{1}{\ln(s/s_0)}. \tag{2}$$

As $s\to\infty$, the cut contribution can be made arbitrarily small.

We now write down an inequality between the coupling $\gamma_{\pi\pi P}(0)$ and the triple-Pomeron coupling $\Gamma_{PPP}(0,0,0)$ which follows from the sum rule relating the total and single-particle inclusive cross sections (see Sec. III of Ref. 4):

$$\gamma_{\pi\pi P}{}^2(0) > \frac{\pi}{2}\,\frac{\Gamma_{PPP}(0,0,0)\gamma_{\pi\pi P}{}^3(0)}{2\alpha_P'(0)(2\pi)^3} \times\{-\mathrm{Ei}(-[1-\alpha_P(0)]\ln(1/\epsilon))\}(1-r), \tag{3}$$

where

$$-\mathrm{Ei}(-z) = \int_z^\infty du\,\frac{e^{-u}}{u}. \tag{4}$$

The small number ϵ in (3) represents the cutoff in the phase-space integral over the triple-Regge region of the variable $1-x$, where x is the standard Feynman variable for the observed particle. The number r represents the magnitude of the fractional error made in neglecting subdominant terms in the triple-Regge region. Estimating r from the leading subdominant correction, which comes from the iteration of a Pomeron pole and cut, we find

$$\frac{1}{r} = \left(\frac{1}{\epsilon}\right)^{1-\alpha_P(0)}\ln\left(\frac{1}{\epsilon}\right). \tag{5}$$

Since $\epsilon<1$ it follows that $\ln(1/\epsilon)<1/r$ and (4) shows that the inequality (3) is only strengthened by setting $\ln(1/\epsilon)=1/r$, so we have the following inequality expressed as a restriction on the triple-Pomeron coupling:

$$\Gamma_{PPP}(0,0,0) < \frac{4\alpha_P'(0)(2\pi)^3}{\pi\gamma_{\pi\pi P}\{-\mathrm{Ei}(-[1-\alpha_P(0)]/r)\}(1-r)}. \tag{6}$$

From (6) we note that the decoupling theorem[4,5] for $\Gamma_{PPP}(0,0,0)$ is recovered as $\alpha_P(0)\to 1$. We see that the decoupling of the triple-Pomeron vertex holds even in the presence of cuts. As we have formulated it here this is because the cut is suppressed by a logarithmic factor and the pole is thus distinguishable because it dominates. Of course, the logarithmic suppression is not present in all models when $\alpha_P(0)=1$,[6] but is always present for $\alpha_P(0)<1$, where the pole and cuts are separated.[7]

In order to get an inequality on $\gamma_{\pi\pi P}$ directly we need to eliminate $\Gamma_{PPP}(0,0,0)$ from (6). To do this we employ the sum rule which relates the double- and single-particle inclusive cross sections in the triple-Pomeron limit. In this limit an inequality is achieved for Γ_{PPP} by relating it to an integral over the Mueller discontinuity of the eight-point function shown in Fig. 1. The Regge $\alpha_R(\bar{t})$ is that of the pion. General Steinmann requirements show that the asymptotic formula corresponding to Fig. 1 consists of several terms. However, only one of them contains the pion pole at $\bar{t}=\mu^2$ (see Fig.

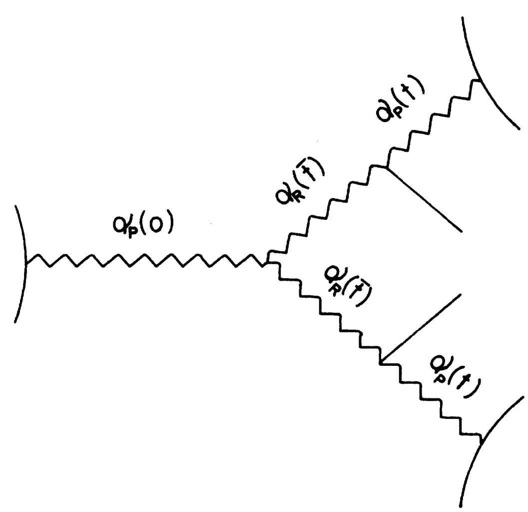

FIG. 1. Eight-point function.

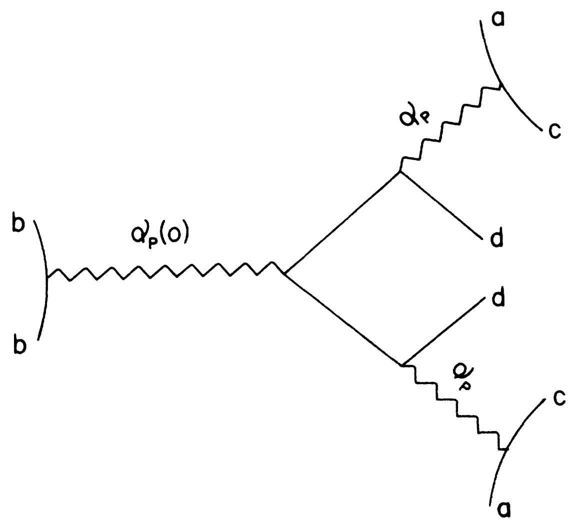

FIG. 2. Eight-point function near pion pole.

2). The region near $\bar{t} \simeq 0$, where we shall assume that the pion pole dominates, is the only one that will be included in the sum rule inequality. Thus we focus our attention on Fig. 2.

Using the normalization given by (1), we can write the contribution of Fig. 2 to the two-particle inclusive cross section near $\bar{t} \simeq 0$ as[8]

$$\frac{d\sigma}{dp_c\,dp_d} = \frac{1}{(2\pi)^6}\frac{1}{s}\left(\frac{\bar{M}^2}{s_0}\right)^{\alpha_P(0)} \frac{\gamma_{\pi\pi P}{}^6}{(\bar{t}-\mu^2)^2} \times \left(\frac{s_{cd}}{s_0}\right)^{2\alpha_P(t)} \left(\frac{ss_0}{\bar{M}^2 s_{cd}}\right)^{2\alpha_R(\bar{t})}, \qquad (7)$$

where

$$\bar{M}^2 = (p_a+p_b-p_c-p_d)^2,$$
$$s=(p_a+p_b)^2,$$
$$s_{cd}=(p_c+p_d)^2.$$

In (7) we have kept not only the pole behavior in $\bar{t}$ but also the exponential Regge behavior. We also note that there are no cuts in overlapping channel invariants in (7) as required by Steinmann.

The resulting inequality which follows from (4.9) of Ref. 4 at $t=0$ is

$$\Gamma_{PPP}(0,0,0) > \frac{1}{(2\pi)^3}\frac{\pi}{2}\gamma_{\pi\pi P}{}^3(0) \times\int d\bar{t}\int_{1-\bar{\epsilon}}^{1} dy\,\frac{(1-y)^{\alpha(0)-2\alpha_R(\bar{t})}}{(\bar{t}-\mu^2)^2} \times\left(\frac{M^2 s_{cd}}{ss_0}\right)^{2[\alpha_P(0)-\alpha_R(\bar{t})]}(1-\bar{r}), \qquad (8)$$

where $M^2=(p_a+p_b-p_c)^2$. The variable y is a Feynman (x-type) variable for the final-state pion in the pion-Pomeron inclusive process. Here, as before, $\bar{\epsilon}$ is a small number specifying the cutoff in the variable $1-y$ and $\bar{r}$ is the fractional error. To estimate $\bar{r}$ we take the ratio of the leading to the subdominant term (which is now the pion trajectory iterated with the pion-Pomeron cut) which gives

$$\frac{1}{\bar{r}} \approx \frac{1}{\alpha_R'\mu^2}\left(\frac{1}{\epsilon}\right)^{1-\alpha_P(0)}\ln\left(\frac{1}{\bar{\epsilon}}\right). \qquad (9)$$

The extra factor of $(\alpha_R'\mu^2)^{-1}$ as compared with (5) comes from the fact that the pion-Pomeron cut does not have the pion pole. This further suppresses the cut and

$$\ln\left(\frac{1}{\bar{\epsilon}}\right) \lesssim \frac{\mu^2\alpha_R'}{2\bar{r}}.$$

We call attention to the next-to-last factor in (8) which was absent in the report of a previous version of this work. The presence of such a factor and its importance have been stressed by Moen, Parry, and Zakrzewski.[9] For t near zero in the physical region, this factor has the form

$$\frac{M^2 s_{cd}}{ss_0} = \frac{\mu^2-\bar{t}}{s_0} + \frac{t-2(t\bar{t})^{1/2}\cos\omega}{s_0} + (\text{higher powers of } t), \qquad (10)$$

where ω is the relevant Toller angle for the Pomeron-Reggeon-particle vertex in Fig. 1. For finite energies, t cannot vanish in the physical region. However, once the sum rules are formulated and the external energy dependence cancels from each side of the inequality, we can take $t=0$.

The resulting factor of $[(\mu^2-\bar{t})/s_0]^{2[\alpha(0)-\alpha_R(\bar{t})]}$ in (8) serves to suppress the pion pole. One might wonder whether the suppression invalidates the assumption of pion pole dominance. Careful analysis of the Regge pole expansion of the eight-point function in this asymptotic region, including signature factors and Steinmann requirements, shows that other terms are down by a factor of at least

$$\{1-[\alpha_P(0)-\alpha_R(\bar{t})]\} \simeq 1-\alpha_P(0) \text{ or } (\mu^2-\bar{t})^2.$$

[The basic pole dominance we discuss is equivalent to that occurring in the five-point function and can be understood from Eq. (8) of Brower and Weis.[10]] Similar reasoning justifies the factor $(\alpha_R'\mu^2)^{-1}$ in (9).

Performing the y integration in (8) gives

$$\Gamma_{PPP}(0,0,0) > \frac{\pi}{2}\frac{1}{(2\pi)^3}\frac{\gamma_{\pi\pi P}{}^3}{s_0{}^2} \times \int_{-a\mu^2}^{0} d\bar{t}\,\frac{\bar{\epsilon}^{1+\alpha_P(0)-2\alpha_R(\bar{t})}}{1+\alpha_P(0)-2\alpha_R(\bar{t})} \times \left(\frac{\mu^2-\bar{t}}{s_0}\right)^{2[\alpha(0)-1-\alpha_R(\bar{t})]}(1-\bar{r}), \tag{11}$$

where a shall be picked to ensure pion pole dominance. All factors in the integrand of (11) vary slowly with $\bar{t}$ and $\alpha_R(\bar{t})$ is approximately zero over the integration range. Therefore, we have

$$\Gamma_{PPP}(0,0,0) \gtrsim \frac{\pi}{4}\frac{1}{(2\pi)^3}\frac{\gamma_{\pi\pi P}{}^3}{s_0{}^2}\bar{\epsilon}^{1+\alpha_P(0)} \times a\mu^2(1-\bar{r}), \tag{12}$$

where we have taken $\alpha(0)\simeq 1$.

Combining (12) with (6) we can eliminate Γ_{PPP} to derive an inequality on $\gamma_{\pi\pi P}$ alone:

$$\gamma_{\pi\pi P}{}^4 < \frac{16}{\pi^2}\frac{s_0{}^2\alpha_P'(0)(2\pi)^6}{a\mu^2\{-\mathrm{Ei}([1-\alpha_P(0)]/r)\}} \times \left(\frac{1}{\bar{\epsilon}}\right)^{1+\alpha_P(0)}\frac{1}{(1-r)(1-\bar{r})}. \tag{13}$$

We note in (13) that as $\alpha_P(0)\to 1$, the right side of the inequality vanishes due to a divergence of the denominator and thus $\gamma_{\pi\pi P}=0$ which corresponds to the Brower-Weis decoupling theorem[10] for the Pomeron. We shall now use (13) to deduce an approximate upper limit on $\alpha_P(0)$. To do this we first must estimate $\gamma_{\pi\pi P}$. At currently available high energies, assuming $1-\alpha_P(0)\lesssim 0.1$, the factor $(s/s_0)^{\alpha_P(0)-1}$ in Eq. (1) is never smaller than $\frac{1}{2}$ or as great as unity. The expression for R in (2) represents the fractional correction to (1) due to the next term which is the two-Pomeron cut. From (2) we can see that it is possible to make $R<\frac{1}{2}$ no matter how closely $\alpha_P(0)$ approaches one. Thus to an accuracy of at least 50% we can take $\sigma_{\pi\pi} \simeq \gamma_{\pi\pi P}{}^2(0)/s_0$. Using factorization, we can estimate $\sigma_{\pi\pi}$ from the observed high-energy total cross sections for nucleon-nucleon and pion-nucleon scattering as

$$\sigma_{\pi\pi}\approx\sigma_{\pi N}{}^2/\sigma_{NN}\approx 16 \text{ mb},$$

or

$$\gamma_{\pi\pi P}{}^2(0)\approx 40, \tag{14}$$

if we take $s_0=1$ (GeV)^2. Using (14) we can investigate the constraint placed on $\alpha_P(0)$ by inequality (13). We can replace $1/\bar{\epsilon}$ in (13) by $e^{\mu^2\alpha_R'/2\bar{r}}$ as discussed earlier. Taking $1/r=1/\bar{r}=2$, $\alpha_P'=\frac{1}{3}$ (GeV)$^{-2}$, $\alpha_R'=\frac{1}{2}$ (GeV)$^{-2}$, $a=2$, we arrive at the result

$$1-\alpha_P(0) > \tfrac{1}{2}e^{-2100}. \tag{15}$$

The question arises as to how good the inequality (15) is. We note that in arriving at (13) we have neglected contributions from the pionization and fragmentation regions and assumed pion pole dominance in the forward directions. Unless these effects imply that the right side of (13) is as great as several hundred times the left side, there will be no qualitative change in our result, which is that the Pomeron can be immeasurably close to one without violating the decoupling theorems. Experimental analyses usually proceed by assuming $\alpha_P(0)=1$ and the data indicate that high-energy processes factorize. The theoretical picture given here could provide a rationale for this.

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