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Phase transitions of second and zero kind in high-energy physics: A phenomenological field-theoretical approach. σ model and superfluidity of hadronic matter*

Shalom Eliezer

Theoretical Division, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87545 and Physics Department, University of Massachusetts, Amherst, Massachusetts† 01002

Richard M. Weiner

Theoretical Division, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87545 and Physics Department, Marburg University, 355 Marburg, Germany‡

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We introduce the temperature concept of Fermi, Landau, and Hagedorn associated with the energy of an elementary-particle reaction into the thermodynamics of field theory constructed by Weinberg for external temperature. For weak and electromagnetic interactions this implies that the phase transitions predicted within a unified gauge theory of electromagnetic and weak interactions should be looked for in elementary-particle interactions at very high energies (cosmic rays). The experimental observation of these effects which might include, e.g., conservation of strangeness in weak interactions will constitute one of the most clear-cut confirmations of the unified gauge theory. We formulate a phenomenological field theory at finite temperature and derive all the relevant thermodynamical quantities (thermodynamical potential, pressure, entropy, energy, specific heat, and velocity of sound). We consider two possible types of phase transitions, namely of second order and of zero order (Hagedorn type). We discuss the implications of phase transitions in strong interactions for the momentum distribution of secondaries. In the particular case of the σ model we find a phase transition of the second kind induced by the energy of the reaction and a phonon-like excitation spectrum for the pion cloud inside the nucleon in the spontaneously-broken-symmetry phase, giving support to the idea that hadronic matter has superfluid properties. This leads to scaling effects at low excitation energies in scattering reactions on nucleons. The broken symmetry is restored at a critical temperature T_c in agreement with previous phenomenological predictions based on a superfluid approach to strong interactions. Above T_c the parton masses vanish, which leads again to scaling, but this time in the high-energy-transfer domain. Conservation of axial-vector currents is found to hold in both phases. In the $T > T_c$ phase we expect chiral multiplets.

I. INTRODUCTION

For a long time it has been realized that there exist formal analogies between spontaneously broken symmetries and the phenomenon of superconductivity.[1,2] Recently the existence of phase transitions in gauge models for electromagnetic and weak interactions has been suggested,[3-6] starting from the analogy between the Landau-Ginsburg equation for superconductors and the Lagrangian of the Salam[7]-Weinberg[8] model. In a different context it has been suggested[9] that hadronic matter might display superfluid properties and undergo a phase transition of the second kind.

We define in general a phase transition (ph.t.) of order n as a transition in which the first $n-1$ derivatives of the thermodynamic potential ϕ with respect to temperature T exist and are continuous while the nth derivative has a discontinuity. In particular, mostly studied are ph.t. of the first kind when entropy, pressure, energy, etc. are discontinuous and ph.t. of the second kind when the above quantities are continuous but the specific heat C_p, the velocity of sound c_0, etc. are discontinuous. In all these cases the partition function

$$Z = \mathrm{Tr}\, e^{-H/T} \tag{1.1}$$

and

$$\phi = -\frac{T}{\Omega}\ln Z \tag{1.2}$$

(where H is the Hamiltonian of the system and Ω is the volume) are finite in the whole temperature range, including T_c (critical temperature), although ϕ is not necessarily analytic at T_c.

Hagedorn[10] has suggested the existence of another type of ph.t. for strong interaction in particle physics which is characterized by the existence of a maximum temperature T_0, beyond which hadronic matter cannot exist. Since in this theory it is not possible to go from $T < T_0$ to $T > T_0$ a proper name for this kind of phenomenon would be ph.t. of the zero kind (no transition takes place).

In this new type of transition the partition function (and hence the thermodynamical potential) diverges at $T = T_0$. The same happens for all the other thermodynamical observables, including energy and specific heat, which are defined at

$T = T_0$ by a limiting process ($T \to T_0$) from their expressions at $T < T_0$.

In this paper we construct the thermodynamics of a renormalizable field theory guided by Laudau's[11] phenomenological approach to ph.t. of the second kind and Weinberg's[4] field theory at finite temperature. However, we introduce the temperature concept in accordance with the statistical approach to high-energy reactions where it is a measure of the center of mass of energy of the reaction, in contradistinction to the conventional approach where T is an external parameter. For the uninitiated reader this difference might appear to be of minor importance. That this impression is misleading can be seen from the fact that new and striking consequences, which could not have been foreseen previously, become now an obvious possibility. From a purely theoretical point of view *in this way high-energy physics becomes a field theory at finite temperature rather than at zero temperature*. This implies that the phase transitions predicted within a unified gauge theory of electromagnetic and weak interactions should not be looked for only in the astrophysical domain but also in elementary-particle interactions at very high energies. In a unified theory of weak and electromagnetic interactions we might expect, e.g., that in reactions induced by very energetic neutrinos parity, strangeness, CP, etc. are conserved. For hadron physics the implications have presumably already been seen in the cutoff of transverse momenta. In other words, the whole discussion of phase transitions in field theories, which has had so far an academic character, becomes a subject of immediate experimental interest.

We consider two possible types of transitions, namely of the second and zero kinds with a critical temperature $T_c \sim m_\sigma/\sqrt{\lambda}$, where m_σ is the mass of the scalar meson and λ is the four-point coupling constant. We discuss the possible implications of the discontinuity of C_p for the transverse momentum distributions of secondaries in strong interactions. In the particular case of the σ model we find a phase transition of the second kind induced by the energy of the reaction and a phonon-like excitation spectrum for the pion cloud inside the nucleon in the spontaneously-broken-symmetry phase giving support to the idea that hadronic matter has superfluid properties. This leads to scaling effects at low excitation energies in scattering reactions on nucleons. Above T_c the chiral symmetry is restored but the parton masses vanish, which leads again to scaling, but this time in the high-energy-transfer domain. CAC (conservation of axial-vector current) is found to hold in both phases. Moreover, in the $T > T_c$ phase we expect chiral multiplets. The organization of this paper is as follows: In Sec. II we recapitulate the main results of Ref. 4 with emphasis on the derivation of the critical temperature. We derive the expressions of certain physical quantities such as entropy, energy, specific heat, and velocity of sound. This derivation is quite elementary but the expressions obtained might be useful in phenomenological applications. Section III sketches briefly Landau's theory of phase transitions of the second kind and its relation to field theory at finite temperature. A short discussion of the place of Hagedorn's theory within this context is given. In Secs. IV and V some new phenomenological results are analyzed. These results appear as a natural consequence of the formalism of Secs. II and III and the interpretation of the temperature as an internal parameter.

II. PHASE TRANSITIONS OF THE SECOND KIND IN FIELD THEORY

Let us consider a renormalizable field theory which contains among other fields a scalar field σ (or a scalar multiplet of σ fields) the Lagrangian of which reads

$$L = T - V, \tag{2.1}$$

where V is the scalar potential and T contains the kinetic terms and other interaction terms. The renormalizability condition constrains V to be a polynomial in σ of order not higher than 4. Since we are interested in spontaneously broken symmetries V will be chosen to be

$$V = \tfrac{1}{2}\mu^2\sigma^2 + \tfrac{1}{4}\lambda\sigma^4, \tag{2.2}$$

where μ^2 and λ are parameters. In order to have a spectrum bounded from below λ has to be positive. For the moment we assume λ to be small: $\lambda \ll 1$. The generalization to $\lambda \sim 1$ will be discussed in the next section. The Lagrangian L is invariant under a group G. We consider the case where the vacuum expectation value

$$\langle\sigma\rangle \equiv \eta \neq 0 \tag{2.3}$$

so that the symmetry G is spontaneously broken, and we construct the corresponding field theory at finite temperature as suggested recently by Weinberg.[4] In Ref. 4 it was shown that the leading terms in any group for small coupling constant λ

and large temperature T are given by the quadratically divergent loops. These quadratic divergences can be absorbed by adding a counterterm to the Lagrangian and by a redefinition of the quadratic terms in $V(\sigma)$. The corresponding effective potential is

$$V_{\rm eff} = V_r(\sigma) + \tfrac{1}{2} Q_{ij}(T)\sigma_i \sigma_j \ , \qquad (2.4)$$

where $V_r(\sigma)$ is given by Eq. (2.2) with μ^2 replaced by its renormalized value μ_r^2. $Q_{ij}(T)$ is calculated by using the fact that the quadratic divergences in our renormalizable theory are given by the tadpole and the boson self-energy graphs. This formalism gives

$$V_{\rm eff}(\sigma) = \tfrac{1}{2}(\mu_r^2 + \lambda N^2 T^2)\sigma^2 + \tfrac{1}{4}\lambda\sigma^4 \, , \qquad (2.5)$$

where N^2 is a parameter of order 1 which depends on the group G and the multiplets considered in L. The minimum of $V_{\rm eff}$ is given by

$$\frac{\partial V_{\rm eff}}{\partial \sigma} = 0 \quad \text{at } \sigma = \eta \, . \qquad (2.6)$$

The absence of spontaneous symmetry breaking at any temperature is determined by this equation (i.e., $\eta \neq 0$ implies spontaneous breakdown of symmetry).

The solution of Eq. (2.6) which corresponds to a minimum is

$$\eta^2 = \begin{cases} -\dfrac{\mu_r^2 + \lambda N^2 T^2}{\lambda} & \text{for } T \leq T_c \, , \\ 0 \quad \text{for } T > T_c \, . \end{cases} \qquad (2.7)$$

Since N^2 is positive definite,[4] in order to have $\eta \neq 0$ one must have $\mu_r^2 < 0$. The critical temperature T_c is defined by the equation

$$\mu_r^2 + \lambda N^2 T_c^2 = 0 \, . \qquad (2.8)$$

The physical mass of the scalar particle is defined by

$$M^2 = \frac{\partial^2 V_{\rm eff}}{\partial \sigma^2} \quad \text{at } \sigma = \eta \, , \qquad (2.9)$$

which leads to

$$M^2(T) = \begin{cases} 2\lambda\eta^2 \quad \text{for } T \leq T_c \, , \\ \mu_r^2 + \lambda N^2 T^2 \quad \text{for } T > T_c \, . \end{cases} \qquad (2.10)$$

In order to exhibit explicitly the dependence on T_c we rewrite Eqs. (2.7) and (2.10):

$$\eta^2 = \begin{cases} N^2(T_c^2 - T^2) \quad \text{for } T \leq T_c \, , \\ 0 \quad \text{for } T > T_c \, , \end{cases} \qquad (2.11)$$

$$M^2(T) = \begin{cases} 2\lambda N^2(T_c^2 - T^2) \quad \text{for } T \leq T_c \, , \\ \lambda N^2(T^2 - T_c^2) \quad \text{for } T > T_c \, . \end{cases} \qquad (2.12)$$

From Eq. (2.12) we get T_c as a function of the coupling constant λ and the physical mass of the scalar particles:

$$T_c = \frac{M(o)}{\sqrt{2\lambda}\, N} \sim \frac{M(o)}{\sqrt{\lambda}} \, . \qquad (2.13)$$

The partition function Z defined in (1.1) becomes in the "no-loop" approximation[4]

$$Z = \exp\left(-\frac{\Omega}{T} V_{\rm eff}(\eta)\right), \qquad (2.14)$$

where Ω is the volume of the system. This approximation is valid as long as we are interested only in the phase transition, since it gives the nonanalytic parts at T_c. In order to get also the analytic contributions one has to supplement $V_{\rm eff}(\eta)$ by other terms which, e.g. for non-gauge field theories, contain the Stefan-Boltzmann term plus higher-order corrections in λ which we shall neglect. The complete $V_{\rm eff}$ is now

$$\tilde{V}_{\rm eff} = V_{\rm eff} - N_0 T^4 + O(\lambda^2) \, , \qquad (2.15)$$

where $N_0 = \pi^2/90$ for one scalar boson. From (2.15) we get for the pressure p

$$p = -\tilde{V}_{\rm eff} = \tfrac{1}{4}\lambda N^4 (T_c^2 - T^2)^2 + N_0 T^4 \quad \text{for } T \leq T_c \qquad (2.16)$$

and

$$p = N_0 T^4 \quad \text{for } T > T_c \, , \qquad (2.17)$$

for the entropy density s

$$s = -\frac{\partial \tilde{V}_{\rm eff}}{\partial T} = 4N_0 T^3 - \lambda N^4 (T_c^2 - T^2) T \quad \text{for } T \leq T_c \, , \qquad (2.18)$$

$$s = 4N_0 T^3 \quad \text{for } T > T_c \, , \qquad (2.19)$$

and

$$\epsilon = -p + Ts = 3N_0 T^4 - \lambda N^4 (T_c^2 - T^2)[\tfrac{1}{4}(T_c^2 - T^2) + T^2] \quad \text{for } T \leq T_c \, , \qquad (2.20)$$

$$\epsilon = 3N_0 T^4 \quad \text{for } T > T_c \, , \qquad (2.21)$$

where ϵ is the energy density. From Eqs. (2.16) and (2.20) we get for the velocity of sound c_0 defined by

$$c_0^{-2} = \frac{\partial \epsilon}{\partial p} \qquad (2.22)$$

the values

$$c_0^2 = \frac{1}{3} - \frac{1}{18}\frac{N^4}{N_0}\lambda \quad \text{for } T \leq T_c\,, \tag{2.23}$$

$$c_0^2 = \frac{1}{3} \quad \text{for } T > T_c\,. \tag{2.24}$$

For the specific heat C_p we get

$$C_p = \frac{\partial \epsilon}{\partial T} = 12 N_0 T^3 + 2\lambda N^4 T^3 - \lambda N^4 (T_c^2 - T^2) T \quad \text{for } T \leq T_c\,, \tag{2.25}$$

$$C_p = 12 N_0 T^3 \quad \text{for } T > T_c\,. \tag{2.26}$$

We observe that the first derivatives of V_{eff} (e.g., s, p, ϵ) are continuous at $T = T_c$, while the second derivatives C_p and c_0 are not, which proves that we have a phase transition of the second kind. It is also remarkable that the equation of state which follows from a general field theory with spontaneous breakdown is in a first approximation of Stefan-Boltzmann type with corrections which can be computed explicitly for any given global symmetry. As expected for a Stefan-Boltzmann-type equation of state the velocity of sound turns out to be $1/\sqrt{3}$. The corrections for $T \leq T_c$ are negative and can in principle be checked experimentally.[12] The deviations of the equation of state from the Stefan-Boltzmann form are also testable.[12] On the other hand it should be clear that the results given in this section apply at best only in a perturbation approach for small λ. (Indeed it is not excluded that higher-order corrections might lead to weak phase transitions of first kind.) This of course is the case of weak and electromagnetic interactions. For strong interactions when $\lambda \sim 1$ no reliable field-theoretical approach seems to exist so far. Therefore we shall proceed phenomenologically, suggesting a generalization of these results for large coupling constants.

III. PHASE TRANSITIONS OF THE SECOND KIND FOR STRONG INTERACTIONS

A. Landau's theory of phase transitions of the second kind

In Landau's theory of phase transitions the thermodynamic potential ϕ is expanded in terms of an order parameter η:

$$\phi = \frac{1}{2} a\eta^2 + \frac{1}{4} b\eta^4 + \phi_0\,, \tag{3.1}$$

where a and b are analytic functions of temperature T and pressure p. ϕ_0 is the analytic part of ϕ (e.g., Stefan-Boltzmann term). η vanishes at $T \geq T_c$. In order to describe a phase transition at $T = T_c$ from a less symmetric phase to a phase with higher symmetry, a and b have to satisfy the following conditions:

$$a < o \quad \text{for } T < T_c\,, \qquad a > o \quad \text{for } T \geq T_c\,, \tag{3.2}$$

$$b > o \quad \text{for any } T\,. \tag{3.3}$$

Condition (3.2) implies that a vanishes at T_c,

$$a(T_c) = 0\,. \tag{3.4}$$

This equation can be used to determine T_c (cf. Sec. II). Landau assumes that a can be expressed near T_c as

$$a = A(T - T_c)\,. \tag{3.5}$$

The equilibrium condition

$$\frac{\partial \phi}{\partial \eta} = 0 \tag{3.6}$$

implies

$$\eta^2 = \begin{cases} 0 \text{ for } T \geq T_c\,; & a > 0 \\ -a/b \text{ for } T < T_c\,; & a < 0\,. \end{cases} \tag{3.7}$$

Assuming Eq. (3.5) the second half of Eq. (3.7) becomes

$$\eta^2 = -\frac{A}{b}(T_c - T)^{2\beta}; \quad \beta = \frac{1}{2} \text{ for } T < T_c\,, \tag{3.8}$$

which coincides with Eq. (2.11) obtained in Sec. II from a scalar field theory.

B Generalization of Landau's theory of phase transition

It is now more or less accepted that Landau's theory of phase transitions as well as any perturbative field theory at finite temperature can yield only qualitative results at T_c (power behavior in $T - T_c$, existence of an order parameter, etc.). In order to get quantitatively correct results, e.g. the actual values of critical indexes, a nonperturbative procedure seems necessary. Such a procedure is provided by the bootstrap approach used in the renormalization-group method. This suggests that in order to investigate phase transitions in strong interactions of elementary particles, where no perturbation theory is expected to work, a similar procedure has to be applied. This is a very difficult and as yet unaccomplished task. On the other hand, it is known from the physics of condensed matter that the renormalization-group method recovers the main qualitative results of Landau's theory as well as those of a perturbative field theory at finite temperature but provides new (and better) values for β and the other critical indices. This leads us to the natural conjecture that the same is expected to happen with a future theory of strong interactions. This conjecture has some independent support also from the observation that the bootstrap approach, especially its thermodynamical (Hagedorn) version, has already been applied with apparent success to hadron physics. In the following we shall therefore adopt the above conjecture and assume that for phase transitions in strong interactions the relations given above are still valid, but with a modified value of β. Equations (3.1), (3.5), and (3.7) lead to

$$\phi = \phi_0 + \tfrac{1}{4} a \eta^2$$
$$= -\tfrac{1}{4}\frac{A^3}{b^2}(T_c - T)^{\gamma+2} + \phi_0 \quad \text{for } T \leq T_c , \qquad (3.9)$$

where

$$\gamma = 2\beta - 1 . \qquad (3.10)$$

$\gamma = 0$ is the "canonical" value corresponding to the value $\beta = \frac{1}{2}$ obtained in Sec. II as well as in Landau's original theory. Depending on the value of γ we get a classification into different types of phase transitions (ph.t) as can be seen below:

(i) $\gamma < -2$: zero-order ph.t.

(ii) $\gamma = -2$: no ph.t.

(iii) $-1 > \gamma > -2$: "unphysical" first-order ph.t.

(iv) $0 \geq \gamma \geq -1$: second-order ph.t.

(v) $\gamma > 0$: higher-order ph.t.

We give here also the expressions near T_c of the most interesting thermodynamic quantities: pressure p, entropy density s, energy density ϵ, velocity of sound c_0, and specific heat C_p for arbitrary γ:

$$p = \Gamma(T_c - T)^{\gamma+2} + N_0 T^4 , \qquad (3.11)$$

$$s = 4N_0T^3 - \Gamma(\gamma+2)(T_c - T)^{\gamma+1} , \qquad (3.12)$$

$$\epsilon = 3N_0T^4 - \Gamma(T_c - T)^{\gamma+1}[T_c - T(\gamma+1)] , \qquad (3.13)$$

$$c_0^2 = \frac{4N_0T^3 - \Gamma(\gamma+2)(T_c - T)^{\gamma+1}}{12N_0T^3 + \Gamma T_c(T_c - T)^{\gamma}} , \qquad (3.14)$$

$$C_p = 12N_0T^3 + \Gamma T_c (T_c - T)^{\gamma} , \qquad (3.15)$$

where

$$\Gamma = \lambda N^4 T_c^{2-\gamma} . \qquad (3.16)$$

In Eq. (3.9) we neglected the logarithmic divergences which exist in field theories. These terms might change the classification given above for $\gamma = -2$ or for $\gamma \geq 0$. For $-1 > \gamma > -2$ the potential is finite at T_c and the first derivatives are discontinous as in ph.t. of the first kind, but they diverge at T_c. This could be interpreted as an unphysical case.

For ph.t. of the second kind we observed that we have an infinite jump in C_p at T_c and the velocity of sound $c_0(T_c) = 0$ (if $\gamma \neq 0$). c_0 vanishes for a zero-order ph.t.

IV. DEFINITION OF TEMPERATURE AND EXPERIMENTAL CONSEQUENCES

A. Weak and electromagnetic interactions

Field theories at finite temperature considered so far[4-6] treated the temperature as an external parameter. The values of temperature involved are of the order of hundreds of GeV and according to these theories are relevant in astrophysical domains. Our philosophy is different since we consider temperature as an internal parameter connected with the center-of-mass energy or energy transfer as suggested by Fermi,[13] Landau,[14] Hagedorn,[10] *et al.* In any reaction the available center-of-mass energy or the energy loss of the projectile is transformed partially or totally into heat raising the temperature of the system (fireball). This system is the medium in which the elementary fields (e.g. parton fields, scalar fields, etc.) propagate and interact. This could lead to two most important consequences. For weak and electromagnetic interactions this implies that the critical temperatures[3-5] predicted by the unified gauge theories can be reached through scattering experiments. Furthermore, in these theories it follows that at $T \geq T_c$ the symmetries broken by weak interactions are restored, which could imply that in a reaction induced by neutrinos parity, strangeness, *CP*, etc. are conserved. Moreover, in unified theories of weak, electromagnetic, and strong interactions this might imply that in these reactions all quantum numbers are conserved. As a consequence of this only neutral weak currents could exist since from the generalized Gell-Mann–Nishijima relation

$$\Delta Q_{\text{hadrons}} = \Delta I_3 + \tfrac{1}{2}\Delta Y + \sum_i \Delta C_i , \qquad (4.1)$$

where Q, I_3, Y, and C_i stand for electric charge, isospin, hypercharge, and charmed quantum numbers, and from the separate conservation of these quantum numbers it follows that

$$\Delta Q_{\text{hadrons}} = 0 . \qquad (4.2)$$

This could be checked in reactions at very high energies (cosmic rays), where temperatures of order 300 GeV might be reached. The search for these effects should be one of the most urgent tasks of cosmic-ray physics, since the observation of these effects will constitute a clear test of the unified gauge theories and at the same time of the correctness and usefulness of the temperature concept as an internal parameter. Another aspect of these effects is that weak interactions at high energies might become strong in the sense that they would conserve all the symmetries characteristic for strong interactions.

B. Strong interactions and cutoff of transverse momenta

For strong interactions the consequences of the fact that temperature is an internal rather than an external parameter have already been seen, presumably, with present accelerators. The main implication is the shape of the $p_\perp$ distribution of secondaries in high-energy strong interactions. It is known[15] that as long as the incident momentum p_i in p-p collision, e.g., is below ~100 GeV/c the inclusive cross section $d\sigma/dp$ for the reaction

$$p + p \to \pi + \text{anything}$$

has the form

$$\frac{d\sigma}{dp_\perp} \sim e^{-p_\perp/T_0}, \tag{4.3}$$

where $T_0 \sim m_\pi$. This is interpreted by assuming that an instantaneous thermodynamical equilibrium is reached after the collision and the observed pions are emitted according to this equilibrium distribution which is of the Bose-Einstein type:

$$n \propto \{\exp[(m^2 + p_\perp^2)^{1/2}/T] - 1\}^{-1}. \tag{4.4}$$

[The longitudinal momentum $p_\parallel$ is not exhibited in Eq. (4.4) because the longitudinal (incoming) direction is a privileged direction which has to be eliminated from equilibrium considerations.] For $p_\perp > m_\pi \sim T$ Eq. (4.4) reduces to (4.3). In this way one has a device for the measurement of temperature in strong interactions, and this is also the definition of temperature used in statistical models of strong interactions.[10,13,14] It is one of the most remarkable facts of high-energy physics that in the momentum range 10 GeV/$c < p_i <$ 100 GeV/c T_0 turns out to be roughly constant, independent of p_i. This is very surprising indeed, since in a straightforward statistical approach[13] one would expect (at least for central collisions, where one can assume that the whole available center-of-mass energy is transformed into thermal energy, i.e., heat) T to be an increasing function of p_i according to the equation of state of hadronic matter.

An explanation for this observational fact is given by the statistical bootstrap model[10] which predicts the existence of a maximum temperature T_0 of hadronic matter. This prediction is based on the fact that in the bootstrap model the thermodynamical distribution function diverges for $T \geq T_0$, i.e., that a ph.t. of zero kind takes place at T_0. As pointed out in Sec. III, such a ph.t. corresponds in a phenomenological field-theoretical approach to $\gamma < -2$. We turn now to the region above 100 GeV. There deviations from the form (4.3) have been observed[16] (cf. Fig. 1) which manifest themselves in (among other ways) an effective increase of T with p_i [moreover, the possibility of a fit to the data with an equation of type (4.3) seems questionable]. There is so far no satisfactory explanation for this deviation, and this makes the search for an alternative explanation for the observed cutoff in $p_\perp$ desirable.

It is anyway remarkable that the increase of the effective T with s ($s = 2mp_i = E_{\text{c.m.}}^2$) at large $p_\perp$ is in a first approximation of the form

$$T \propto s^{1/8} = E_{\text{c.m.}}^{1/4}, \tag{4.5}$$

which coincides with what one would expect from a Stefan-Boltzmann-type equation of state [cf. Eq. (2.21)] lending support to a statistical interpretation of the effect.

It was suggested some time ago[9] that hadronic matter might display superfluid properties and undergo a phase transition of the second kind. This might explain the above-mentioned deviations from $T_0 =$ const, since one could assume that even below T_c there is a slow increase of T with $E_{\text{c.m.}}$ which has not been observed because the data are not precise enough or rather because an inclusive measurement is not an ideal tool in order to get the energy dependence of T. [In an inclusive re-

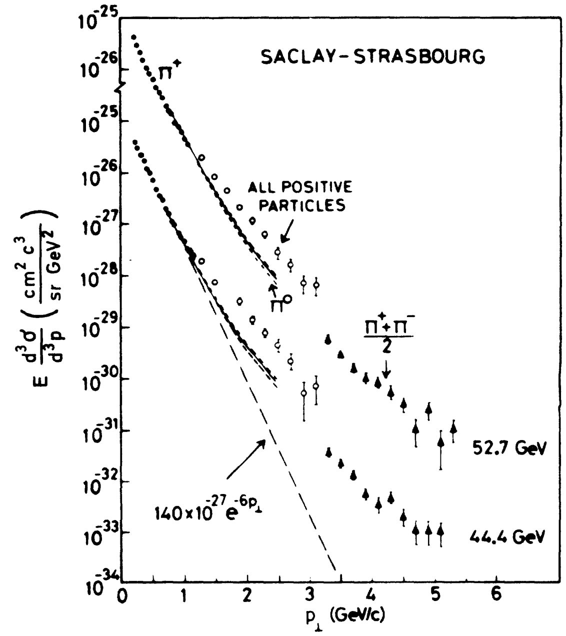

FIG. 1. A typical transverse-momentum distribution in particle physics. It is seen that small $p_\perp$ the slope corresponds to $T \sim m_\pi$, while at large $p_\perp$ the slope increases. The reaction is $p + p$ inclusive; $p_\perp$ is measured at $\theta_{\text{c.m.}} = 90°$ (from Ref. 16).

action we have a superposition of various impact parameters, i.e. momentum transfers (q, q_0), and it is to be expected that T depends on q_0 rather than on p_i.] On the other hand, near T_c if one has a ph.t. of the second kind there is a (possibly infinite) jump in C_p so that the transition beyond T_c might be inhibited as long as $E_{\text{c.m.}}$ does not exceed a certain value. The constancy of T_0 with $E_{\text{c.m.}}$ in the 10–100 GeV/c range is thus explained by the divergence of

$$C_p \sim \frac{dE_{\text{c.m.}}}{dT}, \quad \lim_{T \to T_c} \frac{dT}{dE_{\text{c.m.}}} = 0 . \tag{4.6}$$

V. THE σ MODEL AND SUPERFLUIDITY OF HADRONIC MATTER

A simple and well-known illustration of a field-theoretical model of strong interaction with spontaneous breakdown of symmetries is the σ model.[17] In the following we shall discuss some implications of our approach within the context of this model applied to quarks and boson fields.

The Lagrangian of the σ model is

$$L = \bar{\psi}[i\gamma_\mu \partial_\mu - g(\sigma - i\vec{\pi} \cdot \vec{\tau}\gamma_5)]\psi + \tfrac{1}{2}[(\partial_\mu \sigma)^2 + (\partial_\mu \vec{\pi})^2]$$
$$- \tfrac{1}{2}\mu^2(\sigma^2 + \vec{\pi}^2) - \tfrac{1}{4}\lambda^2(\sigma^2 + \vec{\pi}^2)^2 , \tag{5.1}$$

where ψ, $\vec{\pi}$, and σ are the quark, pion, and scalar fields, respectively, and g and μ are parameters. In (5.1) $(\sigma, \vec{\pi})$ transform as the $(\frac{1}{2}, \frac{1}{2})$ representation of chiral SU(2)×SU(2) so that L is invariant under this group. Because of parity conservation the vacuum expectation value

$$\langle \vec{\pi} \rangle_0 = 0 \tag{5.2}$$

so that the spontaneous breakdown of this symmetry is again achieved by the condition

$$\langle \sigma \rangle_0 \neq 0 . \tag{5.3}$$

Introducing temperature as discussed in Sec. II we get

$$T \leq T_c , \ \langle \sigma \rangle_0 \neq 0, \ m_\pi = 0, \ m = g\langle \sigma \rangle_0 \neq 0 , \tag{5.4}$$

$$T > T_c , \ \langle \sigma \rangle_0 = 0, \ m_\pi = m_\sigma \neq 0, \ m = 0 , \tag{5.5}$$

where m, m_π, and m_σ are the masses of the quarks, pions, and σ, respectively. It is natural to identify the spontaneously-broken-symmetry phase $T < T_c$ with the superfluid phase of hadronic matter as suggested already in Ref. 18 and the $T > T_c$ phase with the normal phase of hadronic matter. This follows from the important result of Eq. (5.4) $m_\pi = 0$ (Goldstone boson), which implies a phonon spectrum for the pion cloud of a nucleon

$$\epsilon = p . \tag{5.6}$$

Indeed, this is Landau's condition for superfluidity, which was found also to be consistent with experimental high-energy data.[19] This also implies scaling for scattering on nucleons, since the properties of a superfluid are determined only by the collective excitations of the system which are represented by the excitation function (5.6) where no mass scale is present. (The scattering occurs essentially on the pion cloud and not on the individual partons.) This could possibly explain the scaling observed in deep-inelastic e-p scattering even at low energies where the superfluid properties of the target are not affected by the energy transfer and no Bjorken scaling is expected. At the same time, at very high energies where one could expect the system to be in the phase $T > T_c$ similar scaling properties might appear (although the system is not superfluid anymore) because the scattering objects which are seen now by the virtual photon are massless partons[20] [cf. Eq. (5.5)]. Furthermore, it is interesting to point out that from this model it follows that CAC should hold at low energies (because $m_\pi = 0$) as well as at high energies (because the chiral symmetry is restored). Furthermore, at $T > T_c$ this model predicts the existence of chiral multiplets and all the other consequences of an exact chiral symmetry, such as $g_A/g_V = 1$, etc. Last but not least, within this model one can estimate using Eq. (2.13) the critical temperature

$$T_c = \frac{M_\sigma(o)}{\sqrt{2\lambda}\, N} \tag{5.7}$$

of the second-order phase transition in strong interaction connected with the superfluid properties of hadronic matter. Using the Weinberg formalism of Ref. 4 for our SU(2)×SU(2) σ model (including the quark, the pion, and the scalar fields ψ, π, and σ) we get an effective potential

$$V_{\text{eff}} = \tfrac{1}{2}[\mu_R^2 + (\lambda - \tfrac{1}{3}g)T^2](\sigma^2 + \vec{\pi}^2)$$
$$+ \tfrac{1}{4}\lambda(\sigma^2 + \vec{\pi}^2)^2 , \tag{5.8}$$

where μ_R is the renormalized value of μ in Eq. (5.1), and λ and g are defined in Eq. (5.1). Comparing Eqs. (2.5) and (5.8) we obtain

$$N^2 = 1 - \frac{g}{3\lambda} , \tag{5.9}$$

so that the critical temperature in Eq. (5.7) is

$$T_c = \frac{m_\sigma}{\lambda^{1/2}[2(1-g/3\lambda)]^{1/2}}, \tag{5.10}$$

where $m_\sigma = M_\sigma(0)$ is the mass of the σ particle at $T=0$. A numerical estimation of T_c would be possible if we had experimental knowledge about the scalar meson σ.

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† Present address.

‡ Permanent address.

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