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Polarization calculations in a box-diagram model

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A box-diagram model is presented in which we assume that the exotic exchange reaction $\pi^- p \to \Sigma^- K^+$ occurs as a result of multiple nonexotic exchanges, e.g., $\pi^- p \to \rho^0 n \to \Sigma^- K^+$, and each step of this process is evaluated under t-channel single-pole approximations. The imaginary part thus obtained when multiplied by the real contributions of one-particle-exchange diagrams accounts for the main features of the polarization data of Σ^-.

We have calculated[1] the contributions of the rescattering box diagram to the differential cross section for the process

$$\pi^- + p \to \Sigma^- + K^+ . \tag{1}$$

In this model we have essentially evaluated the contribution to the imaginary part of the amplitude from a two-particle intermediate state $\rho^0 n$ by assuming that the processes $\pi^- p \to \rho^0 n$ and $\rho^0 n \to \Sigma^- K^+$ are approximated by one-meson-exchange contribution alone. We have shown that the main features of the observed production angular distributions and its strong energy dependence are reproduced by our model without using any arbitrary parameter. Recently, it has also been shown[2] that the imaginary contributions to the invariant amplitudes for the above reaction as calculated from our model show Odorico zeros[3] at small values of t, which has also been supported by the experimental analysis. The purpose of this paper is to point out the prediction of our model for the polarization of the final-state hyperon Σ^- in the process (1), because the polarization data supply valuable information concerning both the relative phase and the spin-flip and spin-nonflip structure of scattering amplitudes. In this way it serves as a more rigorous test of the model.

The experimental status for the polarization of Σ^- in reaction (1) is not satisfactory. The experiment with unpolarized protons fails to give any polarization information as $\Sigma^- \to n\pi^-$ decay[4] is essentially an S-wave process. The asymmetry parameter α thus comes out to be $\alpha \simeq -0.07 \pm 0.01$. Use of polarized targets has also been made to extract the polarization information[5,6] for process (1). Recently, Davies *et al.*[7] have made use of the charge symmetry to report Σ^- polarization with the help of the process

$$\pi^+ + n \to K^0 + \Sigma^+ . \tag{2}$$

We shall make use of this limited amount of information on Σ^- polarization to test our theoretical prediction.

In extracting the box-diagram contribution we assume[8] that the absorptive part of the diagram dominates over the dispersive part. We thus consider reaction (1) as a two-step process,

$$\pi^- + p \to \rho^0 + n \to \Sigma^- + K^+ , \tag{3}$$

where ρ^0 and n are placed on their mass shells. We propose that the imaginary part obtained from the s-channel discontinuity (3) can be coupled with the real part of the amplitude obtained from the nucleon pole in the s channel to yield the polarization calculation. In doing this, we completely ignore the one-baryon-exchange diagram[1,9,10] in the u channel as it fails to give a quantitative as well as a qualitative description of process (1). To be more explicit, we assume that in the intermediate energy region (i.e., below the incident meson momentum of 4 GeV/c) interaction (1) is mostly peripheral,[10] dominated by the nearby singularities, and hence the u-channel exchanges can be ignored. Here it should be added that the imaginary parts are always peripheral and, therefore, appreciable cuts are calculated when needed to enforce the peripherality of the imaginary function. This viewpoint is supported by the dual[11,12] model of Harari, where it is connected to the idea that peripheral resonances dominate at low energy. Thus we can say that the s-channel mechanism involved in our model calculation makes it equivalent to a model based on the superimposition of the s-channel resonances and both need not be mutually exclusive.[13] Moreover, a fit including many resonances does not appear to be meaningful[13] owing to the lack of much better knowledge of the resonance spectra at higher masses. Nevertheless, the structures in the production angular distributions in the energy region below the incident meson momentum of 4 GeV/c do not appear to be s-dependent[14] also.

We can exploit our assumption for nearby singularities in ending the uncertainty involved in the choice of the t-channel exchanges in the box graph. Thus, we assume that the graph with π and K exchange in the t channel will dominate over the graph with ρ and K^* exchanges in the energy region of our interest. Related to this point, we find from the simple Regge-pole model that the cross section for the forward hemisphere can be expressed as $\sigma \sim s^{2\alpha_0 - 2}$, where α_0 is the value of a Regge trajectory at $t = 0$ in the Chew-Frautschi plot. For two-particle exchanges in the t channel, α_0 assumes the form[15] $\alpha_0^{\rm I} + \alpha_0^{\rm II} - 1$, where $\alpha_0^{\rm I}$ and $\alpha_0^{\rm II}$ are the values of two possible trajectories at $t = 0$. The values of α_0 are experimentally[16] given as follows:

$$\alpha_0(\pi, K) \simeq -1.25$$

and

$$\alpha_0(\rho, K^*) \simeq -0.4.$$

Thus the fall of the differential cross section at $0°$ given by the graph with π, K exchange would be more rapid ($s^{-4.5}$) than the cross section given by ρ, K^* exchange ($s^{-2.8}$). Thus we find support for our assumption, as the experimental power[17] law behavior is around $s^{-9.6}$ below the incident meson momentum of 4 GeV/c, and above this it changes markedly to $s^{-1.1}$, showing the dominance of ρ, K^* Regge cut prescriptions at higher energies. In our model we must admit that the π, K exchange graph gives an explanation for the forward peak and its energy dependence present in the *octet*-production reactions which are exotic, but quantitatively the agreement[1,9] is not very good in the forward direction.

On separating[2] the box-diagram amplitude into the invariant amplitudes A and B, we get (following the notations used in Fig. 1 of Ref. 1)

$$\mathrm{Im}A_{\rm box} = \frac{YZ|\vec{q}\,'|}{4\pi\sqrt{s}}\, g_{\rho\pi^+\pi^-} g_{\rho K^+K^-} g_{K^-n\Sigma^-} g_{\pi^-pn}\left[m_p \frac{|\vec{q}\,'|}{|\vec{q}_1|} + \tfrac{1}{2}(m_p + m_\Sigma)\left(p_0' - p_{10}\frac{|\vec{q}\,'|}{|\vec{q}_1|}\right) + m_p\right], \tag{4}$$

$$\mathrm{Im}B_{\rm box} = -\frac{YZ|\vec{q}\,'|}{4\pi s}\, g_{\rho\pi^+\pi^-} g_{\rho K^+K^-} g_{K^-n\Sigma^-} g_{\pi^-pn}\left(p_0' - p_{10}\frac{|\vec{q}\,'|}{|\vec{q}_1|}\right), \tag{5}$$

where

$$Y = \frac{1}{4|\vec{q}_1||\vec{q}\,'|^2|\vec{q}_2|}\,\frac{1}{\sqrt{-\beta}}\ln\frac{\alpha_1\alpha_2 - \cos\theta + \sqrt{-\beta}}{\alpha_1\alpha_2 - \cos\theta - \sqrt{-\beta}},$$

$$Z = q_2 \cdot q_1 + \left(\vec{q}_2\cdot\vec{q}_1\frac{|\vec{q}\,'|}{|\vec{q}_1|} - q_{20}q_0'\right) \times (|\vec{q}_1||\vec{q}\,'| - q_{10}q_0')/m_\rho^2,$$

$$\beta = 1 - \cos^2\theta - \alpha_1^2 - \alpha_2^2 + 2\alpha_1\alpha_2\cos\theta,$$

$$\alpha_1 = (2q_{10}q_0' - m_\rho^2)/(2|\vec{q}_1||\vec{q}\,'|),$$

$$\alpha_2 = (2q_{20}q_0' - m_\rho^2)/(2|\vec{q}_2||\vec{q}\,'|),$$

$$\cos\theta = \vec{q}_2\cdot\vec{q}_1/(|\vec{q}_2||\vec{q}_1|).$$

Here $\vec{q}_1, \vec{q}\,', \vec{q}_2$ give the direction and magnitude of the c.m. momenta for π^-, ρ^0, K^+ and q_{10}, q_0', q_{20} stand for their c.m. energies, respectively. Similarly p_{10} and p_0' denote the c.m. energies of the initial proton and intermediate neutron, respectively.

The real part of the amplitude can be calculated from the direct nucleon pole as

$$\mathrm{Re}A_n = -\frac{g_{K^-n\Sigma^-} g_{\pi^-pn}(m_p - m_\Sigma)/2}{m_p^2 - s}F(n), \tag{6}$$

$$\mathrm{Re}B_n = \frac{g_{K^-n\Sigma^-} g_{\pi^-pn}}{m_p^2 - s}F(n), \tag{7}$$

where $F(n)$ is a suitable form factor to account for the off-mass-shell nature of the intermediate particle in the Born approximation. We have chosen

$$F(n) = \exp[-1.5(-m_p^2 + s)] \tag{8}$$

as suggested by Chan and Liu.[18] In the absence of an established alternative, we consider this form-factor modification to account for the overestimation of the Born approximation for one-particle pole contribution. It should be mentioned here that in writing these invariant amplitudes, we have taken due care for the negative eigenparity of the mesons.

The polarization of Σ^- in reaction (1) is given as

$$P_\Sigma \frac{d\sigma}{d\Omega} = |\vec{q}_1|^2\frac{\sin\theta}{16\pi^2\sqrt{s}}\mathrm{Im}(A'B^*), \tag{9}$$

where

$$A' = A + \frac{s - u + \Delta}{4\bar{m}^2 - t}\bar{m}B,$$

$$\Delta = \frac{(m_p - m_\Sigma)(m_\pi^2 - m_K^2)}{2\bar{m}},$$

$$\bar{m} = \frac{m_p + m_\Sigma}{2}.$$

Equation (9) gives the polarization of Σ^- along the normal to the production plan given by

$$\vec{n} \propto \frac{\vec{q}_1 \times \vec{q}_2}{|\vec{q}_1||\vec{q}_2|}. \tag{10}$$

For our case,

$$A = \mathrm{Re}A_n + i\,\mathrm{Im}A_{\mathrm{box}},$$
$$B = \mathrm{Re}B_n + i\,\mathrm{Im}B_{\mathrm{box}}. \tag{11}$$

Substituting these values in (9), we finally get

$$P_\Sigma \frac{d\sigma}{d\Omega} = |\vec{q}_1|^2 \frac{\sin\theta}{16\pi^2\sqrt{s}}(\mathrm{Re}B_n \mathrm{Im}A_{\mathrm{box}} - \mathrm{Re}A_n \mathrm{Im}B_{\mathrm{box}}). \tag{12}$$

The result of our calculation for $P_\Sigma\, d\sigma/d\Omega$, with the values of the coupling constants[1]

$$g_{\rho\pi\pi}^2/4\pi = 2.4, \quad g_{K^-n\Sigma^-}^2/4\pi = 1.5,$$
$$g_{\rho KK}^2/4\pi = 1.2, \quad g_{\pi^-pn}^2/4\pi = 15. \tag{13}$$

is shown in Fig. 1. Our model predicts that the polarization changes sign near $\cos\theta \simeq 0.8$ at the incident meson momentum of 1.5 GeV/c. The calculated curve appears to be in reasonable agreement with the experimental data of Davies *et al.*[7] It may be noted that the change in the sign of the polarization observed in reaction (1) is automatically built in the contribution of the box-diagram amplitude and we do not need to invoke the cross-over factor. The change in sign in our amplitudes (4) and (5) arises due to the factor Z which appears due to the sum over spin and polarization of the ρ meson in our box diagram. We find that this property is always present whenever a particle of spin one decaying into two spin-zero particles is included in the box graph as an s-channel intermediate state. It should be interjected at this point that as the energy increases, other kinds of intermediate states are also expected to contribute in the box-diagram model. For example, as the c.m. energy increases above 2 GeV, another box diagram involving s-channel intermediate states K^* and Λ become effective. Above this energy region, the two box mechanisms are competitive and should be combined in a more rigorous investigation of the problem. This possibility would again increase the polarization for process (1) above 2.1 GeV. This trend is again supported by the experimental data of Davies *et al.*[7] We thus hope that a model incorporating several box diagrams could conceivably account for all the dynamical properties of reaction (1).

It would be worthwhile to see the prediction of our model for the reaction

$$\pi^- + p \to \Sigma^0 + K^0, \tag{14}$$

which is related to (1) by charge independence. Here a vector-meson exchange diagram is also allowed and should be included in the peripheral interactions. The contributions of K^* exchange should be added[19] in the real parts of the invariant amplitudes. Again we have used a form factor suggested by Chan and Liu for the vector-meson exchange,[18]

$$F(V) = \exp[-1.75(m_{K^*}^2 - t)]. \tag{15}$$

The result of our calculation is shown in Fig. 2

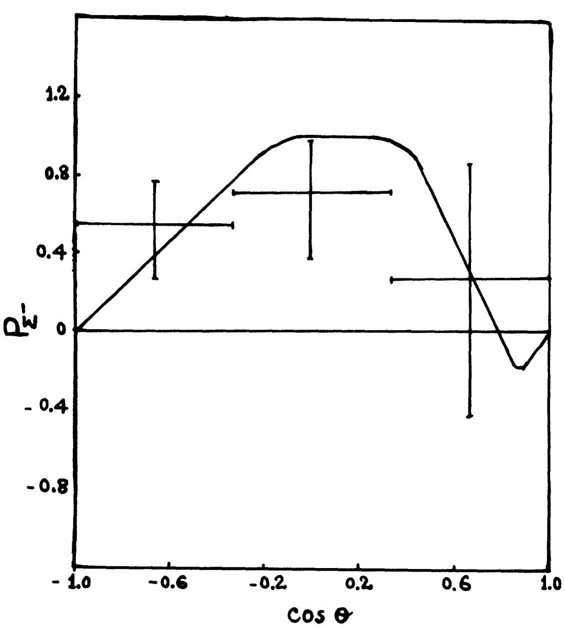

FIG. 1. The polarization of Σ^- at a π^- momentum of 1.5 GeV/c in the process $\pi^- p \to \Sigma^- K^+$ as calculated from Eq. (12) is shown by a solid line. The experimental data have been taken from Davies *et al*.

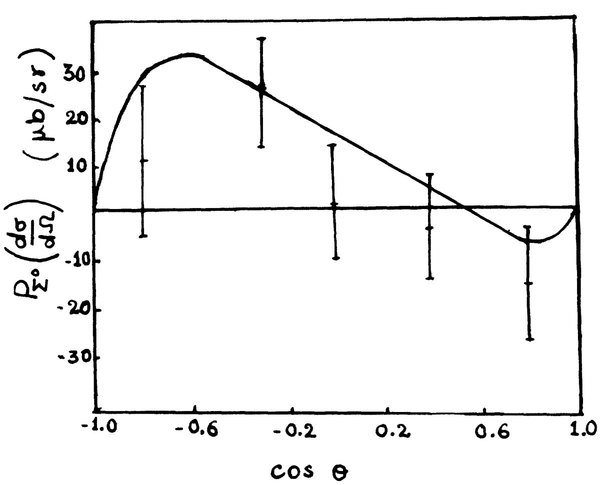

FIG. 2. The polarization of Σ^0 in the reaction $\pi^- p \to \Sigma^0 K^0$ as calculated by our model is shown by a solid line. The experimental data have been taken from Anderson *et al*. (see Ref. 20). The difference of over-all minus sign arising due to the difference in the definition of the normal has been taken into consideration.

at the incident meson momentum of 1.17 GeV/c, at which the experimental data are available.[20] Our calculation gives a close qualitative as well as quantitative agreement with the experiment. The sign change of the polarization is again the most distinct experimental feature which we theoretically reproduce.

In conclusion, these calculations make it amply clear that the box-diagram amplitudes for the exotic exchange processes constructed from an iteration of amplitudes without exotic exchange connected by on-mass-shell two-body or quasi-two-body intermediate states play a dominant role in hadron-hadron scattering in the intermediate energy region where the theoretical understanding is too vague. In this energy region, the extrapolations of Regge-pole and -cut model[21] descriptions from higher energies appear to be notoriously unreliable. Under such circumstances, we feel that our model, which takes into consideration the peripheral effects alone, gives close agreement in the production angular distributions, energy dependence of the cross section, and polarization calculations for process (1). Our theoretical analysis poses an important question[22] for reexamination: How far is it justified to ignore the u-channel effects completely in the intermediate energy region?

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