Isoscalar $\pi\pi$ resonances and current-algebra constraints in $\psi' \to \psi + 2\pi$

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The difficulty of classifying three $I=0$, low-lying scalar resonances [in particular $\epsilon(700)$] within an SU(4) framework is stressed. Current-algebra constraints are worked out for the decay $\psi' \to \psi + 2\pi$; these lead to a matrix element disfavoring small pion momenta. A companion discussion is given of the decays $\eta' \to \eta\pi\pi$ and $\omega' \to \omega\pi\pi$.

We regard the narrow resonances[1,2] $\psi(3095)$ and $\psi'(3684)$ as members of ideally mixed[3] SU(4) multiplets[4] consisting of ρ, ω, ϕ, D, F, ..., ψ and $\rho'(1600)$, ω', ..., ψ; i.e., ψ and ψ' are pure $c\bar{c}$ states where c is the charmed quark. The decay $\psi' \to \psi + 2\pi$ is known[5] to be the dominant decay mode of ψ'. In the following we consider the constraints provided by current algebra or chiral symmetry for this amplitude.

To see qualitatively the applicability of chiral-symmetry considerations, it is useful as in the successful quark-model discussion[6] of the decays $\Delta \to N\pi$, $\rho \to \pi\pi$, $\omega \to 3\pi$, etc., to regard the pion as the divergence of the isovector axial-vector current rather than as a quark-antiquark bound state. Since the c quark is an isoscalar, the axial-vector current does not couple and therefore we expect the Adler zero to be more effective in $\psi' \to \psi + 2\pi$ than, for example, in $\omega' \to \omega + 2\pi$. For the same reason there can be no $\psi\pi$ resonance. For the current-algebra constraints to be useful it is also necessary that $\pi\pi$ scattering is not large in the region of interest. We would like to make the following observations in this connection.

Some of the $\pi\pi$ phase-shift analyses[7] require the existence of three scalar $I=0$ resonances $\epsilon(700)$, $S^*(997)$, and $\epsilon'(1240)$. If all three are substantiated by further analysis there will be considerable difficulty in classifying them from a symmetry point of view.[8] If the SU(3) classification is correct one expects only two $I=0$ resonances which in the quark model are the two linearly independent possible combinations of the $^3P_0$ states of $(\mathcal{P}\bar{\mathcal{P}} + \mathcal{N}\bar{\mathcal{N}})$ and $\lambda\bar{\lambda}$. Further, one expects [in analogy with the $^3P_2$ states $f(1270)$ and $A_2(1310)$] the lower of these states to be approximately degenerate with $\delta(970)$ ($I=1, \eta\pi$ resonance) and presumably to be identified with $S^*(997)$, and the other a few hundred MeV above. This view is further supported by the fact that $K\pi$ phase-shift analyses[9] rule out the existence of a broad scalar $K\pi$ resonances below 1 GeV.

If $\epsilon(700)$ exists one can still include it in the SU(3) scheme[8] by making it an SU(3) singlet $(\mathcal{P}\bar{\mathcal{P}} + \mathcal{N}\bar{\mathcal{N}} + \lambda\bar{\lambda})/3$. A favorite candidate for identification of such a state is the "dilaton" or the Nambu-Goldstone boson associated with conformal transformation.[10] However, such a possibility disappears when one enlarges strong-interaction symmetry to SU(4) in which case $\epsilon(700)$ must be an SU(4) singlet if it is to be identified with the dilaton. Presence of such a state is incompatible with the observed decay characteristics ψ and ψ'. This may be seen as follows. For an SU(4) singlet state, ϵ has the wave function $\frac{1}{4}(\mathcal{P}\bar{\mathcal{P}} + \mathcal{N}\bar{\mathcal{N}} + \lambda\bar{\lambda} + c\bar{c})$. We then expect strong radiative decays $\psi \to \epsilon + \gamma$, $\psi' \to \epsilon + \gamma$, etc., proceeding through the $c\bar{c}$ part of the ϵ wave function. Further, it is also extremely difficult to understand how a state containing 25% $c\bar{c}$ ($^3P_0$ state) in its wave function can have such a low mass as 700 MeV.

At present, the main source of experimental information on $\pi\pi$ scattering comes from reactions such as $\pi N \to \pi\pi N$. The phase-shifts analyses based on these data are by no means unique.[11] It is possible to fit the $\pi\pi$ data without[12] the $\epsilon(700)$. In view of these remarks we shall disregard the $\epsilon(700)$.

It is then reasonable to expand the matrix element of $\psi' \to \psi + 2\pi$ in powers of pion momenta and study the constraints provided by current algebra.[13]

Consider the transition amplitude $\psi'(\epsilon^A, P_A) \to \psi(\epsilon^B, P_B) + \pi^0(q_1) + \pi^0(q_2)$, where ϵ^A and ϵ^B are the polarization vectors with $\epsilon^A \cdot p_A = \epsilon^B \cdot p_B = 0$ and p_A, p_B, q_1, q_2 are the momenta as indicated. Introducing the combinations $Q = (q_1 + q_2)$, $\Delta = (q_1 - q_2)$ of the pion momenta, the most general form of the matrix element is

$$T(\nu, \nu_B, q_1^2, q_2^2) = \epsilon^A \cdot Q \epsilon^B \cdot Q A(\nu, \nu_B, q_1, q_2) + \epsilon^A \cdot Q \epsilon^B \cdot \Delta B(\nu, \nu_B, q_1, q_2) + \epsilon^A \cdot \Delta \epsilon^B \cdot Q C(\nu, \nu_B, q_1, q_2)$$
$$+ \epsilon^A \cdot \Delta \epsilon^B \cdot \Delta D(\nu, \nu_B, q_1, q_2) + \epsilon^A \cdot \epsilon^B E(\nu, \nu_B, q_1, q_2), \quad (1)$$

where $\nu = -\Delta\cdot(p_A + p_B)$, $\nu_B = -q_1\cdot q_2$. The amplitudes A, B, C, D, and E are free of kinematic singularities and will be expanded in powers of q_1 and q_2. Bose statistics implies that A, D, and E are symmetric in $q_1 \leftrightarrow q_2$ while B and C are antisymmetric. Following standard procedure[14] the Adler conditions read

$$(A - B - C + D)|_{q_{1\mu}=0} = 0\,, \tag{2}$$

$$(A + B + C + D)|_{q_{2\mu}=0} = 0\,, \tag{3}$$

$$E(\nu, \nu_B, q_1^2, q_2^2)|_{q_{1\mu}=0} = 0\,, \tag{4}$$

$$E(\nu, \nu_B, q_1^2, q_2^2)|_{q_{2\mu}=0} = 0\,, \tag{5}$$

while current algebra gives

$$\epsilon^A\cdot\epsilon^B E(\nu, \nu_B, q_1^2, q_2^2)|_{q_{1\mu}=0, q_{2\mu}=0} = -\frac{2}{F_\pi^2}(2\pi)^3(4p_A^0 p_B^0)^{1/2}\langle\psi(\epsilon^B, p_B)|\Sigma|\psi'(\epsilon^A, p_A)\rangle \tag{6}$$

$$= -\frac{2}{F_\pi^2}\epsilon^A\cdot\epsilon^B\sigma_{\psi\psi'}\,, \tag{7}$$

where

$$\Sigma = i[F_3^5, \partial_\mu A_3^\mu(0)] \tag{8}$$

is the commutator of the third component of the isovector axial-vector charge F_3^5 with the corresponding axial-vector current $A_{3\mu}(x)$. $F_\pi \simeq 0.97 m_\pi$ is the pion decay constant. Using Bose statistics we expand E as follows:

$$E(\nu, \nu_B, q_1^2, q_2^2) = -e_0 + e_1(q_1^2 + q_2^2) + e_2\nu_B + \cdots\,. \tag{9}$$

Then using Eqs. (4)–(7) we find on the mass shell

$$E(\nu, \nu_B) = +e_0 + e_2\nu_B + \cdots\,, \tag{10}$$

where

$$e_0 = \frac{2\sigma_{\psi\psi'}}{F_\pi^2} \tag{11}$$

with e_2 undetermined.[15]

Whether the amplitude T in Eq. (1) has a constant term independent of pion momenta then depends on the magnitude of e_0 in Eq. (11) which is determined by the nature of chiral-symmetry breaking. The decay $\eta'(958) \to \eta\pi\pi$ which is similar to $\psi' \to \psi + 2\pi$ is relevant in this context. It was studied by Riazuddin and Oneda[16] using the $(3, 3^*) + (3^*, 3)$ model of chiral-symmetry breaking proposed by Gell-Mann, Oakes, and Renner[17] and Glashow and Weinberg.[18] For $\eta' \to \eta + 2\pi$ the matrix element of the Σ term analogous to our Eq. (11) is calculable in terms of the known pseudoscalar masses and leads to $\sigma_{\eta\eta'} \approx 0.01$ (GeV)^2 or $(e_0)_{\eta\eta'} \approx 1$. It gives too small a value for $\Gamma(\eta' \to \eta + 2\pi)$ leading one to doubt the validity of the $(3, 3^*) + (3^*, 3)$ model and the introduction of complicated schemes of chiral-symmetry breaking. However, Singh and the present author[19] have shown that the presence of the $\eta\pi$ resonance $\delta(970)$ approximately degenerate with $\eta'(958)$ leads to strong variation of the matrix element of $\eta' \to \eta\pi\pi$ with pion momenta near the soft-pion limit and therefore invalidates current-algebra calculations based on linear extrapolation. Therefore, at least at the SU(3) level, there is no need to abandon the $(3, 3^*) + (3^*, 3)$ model. A natural generalization at the SU(4) level is then a $(4, 4^*) + (4^*, 4)$ model, which will tend to suggest that $\sigma_{\psi\psi'}$ and therefore e_0 as given by Eq. (11) are negligible. Accepting this it follows that the matrix element T in Eq. (1) has no constant term independent of pion momenta or, in other words, the matrix element disfavors low-momentum pions.

Since we are assigning ρ' and ω' to the same multiplet as ψ' the question of comparison of $\rho' \to \rho\pi\pi$ and $\omega' \to \omega\pi\pi$ with $\psi' \to \psi\pi\pi$ arises. Recall that the ideal mixing ansatz[3] leads to $m_\omega = m_\rho$. If we momentarily disregard the widths of ρ and ω then it is easy to see that in the amplitude $\omega' \to \omega\pi\pi$ the presence of the ρ pole in $\omega' \to \pi(q_1) + \rho \to \pi(q_1) + \omega + \pi(q_2)$ leads to a nonvanishing value when $q_{2\mu} \to 0$ if $m_\omega = m_\rho$, so that the Adler conditions Eqs. (2)–(5) will have to be modified. If we take the effect of finite widths of ρ and ω, the Adler zeros will be present, but the matrix element will vary rapidly around the soft-pion limit. Similar considerations apply to the ρ' intermediate state also since $m_{\omega'} \simeq m_{\rho'}$. Further, it is known experimentally that the $B(1235)$ which is a $\omega\pi$ resonance is present in the final state.[20] We cannot expand the matrix element in powers of pion momenta and current algebra does not provide useful constraints. It is therefore best to proceed as in the Gell-Mann–Sharp–Wagner[21] model for $\omega \to 3\pi$; that is, $\omega' \to B\pi$, $\rho\pi$, $\rho'\pi \to \omega\pi\pi$ should account for most of the observed decay characteristics.[22]

As we have already emphasized, the fact that the c quark is an isoscalar means that the axial-vector current does not couple and there can be no pole terms in $\psi' \to \psi\pi\pi$ or a $\psi\pi$ resonance. Thus there is no inconsistency in using chiral symmetry for $\psi' \to \psi\pi\pi$ on the one hand and not applying it to $\omega' \to \omega\pi\pi$, although both ω' and ψ' belong to the same multiplet.[23]

Returning to Eq. (1) we see that the amplitudes A, B, C, D, and E are free of kinematical singularities and therefore are suitable for use in dispersion relations. From the standard Regge-pole analysis of vector-pseudoscalar scattering amplitude one gets for the asymptotic behavior in

ν at fixed ν_B

$$A(\nu, \nu_B) \underset{\nu\to\infty}{\sim} \nu^{\alpha-2},$$
$$B, C \sim \nu^{\alpha-1},$$
$$D, E \sim \nu^{\alpha}.$$

Since the Pomeron is the leading singularity we see that A satisfies an unsubtracted dispersion relation. Using the antisymmetry property in ν, we can write unsubtracted dispersion relations also for B/ν and C/ν. Since ψ and ψ' decouple with ordinary hadrons, to a good first approximation, we need keep only charmed hadron intermediate states in the dispersion integrals.[24] Quantitative estimates based on these will be provided elsewhere.

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