PHYSICAL REVIEW D VOLUME 13, NUMBER 3 1 FEBRUARY 1976

Mass formulas for ψ particles

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(Received 16 June 1975)

Five classes of mass formulas in a color-quark model for ψ particles are presented. They include SU(3) and $SU(3)_c$ mass sum rules, $SU(3)\times SU(3)_c$ mass proportionalities and reciprocities, and $1+F_c{}^2$ mass relations. All the formulas that can be checked experimentally are satisfied within experimental errors of less than 2%. A unified mass formula is also proposed which explains the masses of ψ particles unambiguously and which predicts those of yet undiscovered color-octet vector mesons.

In previous papers[1,2] (I and II), Sanda and I have assigned the recently discovered vector-meson resonances,[3] $\psi(3.1)$, $\psi(3.7)$, and $\psi(4.1)$, and a predicted one, $\psi(4.9)$, to the states (ω,ρ^0), (ω,ω_8), (ϕ,ρ^0), and (ϕ,ω_8), respectively, in an irreducible representation of $SU(3)\times SU(3)_c$ symmetry, based on a color-quark model.[4] By introducing a single working hypothesis, we have explained the ratio of the observed partial decay widths into a lepton pair of $\psi(3.1)$, $\psi(3.7)$, and $\psi(4.1)$, the narrowness of $\psi(3.1)$ and $\psi(3.7)$, and the mass and broadness of $\psi(4.1)$, and have predicted the mass, the partial decay width, and the broadness of $\psi(4.9)$. We have also discussed various possible decay modes of ψ particles. In this paper, I shall propose a dynamical model in which the hypothesis is automatically satisfied and shall present five classes of mass formulas for new particles which can be derived from the model. Furthermore, I shall find that all the formulas that can be checked with the presently available data are remarkably well satisfied. Finally, I shall propose a unified mass formula which explains the masses of $\psi(3.1)$, $\psi(3.7)$, and $\psi(4.1)$ unambiguously and which predicts those of yet undiscovered color-octet vector mesons.

Let me summarize what we have assumed in I and II by the following three assumptions and one hypothesis:

Assumption 1: There exist three triplets of colored quarks, $q_{i,j}$ ($i=\mathcal{P},\mathfrak{N},\lambda$ and $j=1,2,3$, the masses $m_{i,j}$, and the charges $Q_{\mathcal{P},j}=\frac{2}{3}+\alpha_j$, $Q_{\mathfrak{N},j}=Q_{\lambda,j}=-\frac{1}{3}+\alpha_j$), which obey an approximate $SU(3)\times SU(3)_c$ symmetry and the sum of whose "excess charges," α_j, vanishes; hadrons are bound states of these quarks.

Assumption 2: Ordinary hadrons are very pure color-singlet states; color singlet-nonsinglet mixing is extremely small; and color quantum numbers are conserved by strong interactions to a very good accuracy.

Assumption 3: The color symmetry $SU(3)_c$ is broken to $SU(2)_{I_c}\times U(1)_{Y_c}$ which conserves the color isospin (I_c) and the color hypercharge (Y_c). It cannot be stressed too strongly that this $SU(3)_c$ breaking of the Gell-Mann–Okubo type does not generate any color singlet-nonsinglet mixing to the lowest order and, therefore, that assumption 3 has nothing in contradiction with assumption 2.

Hypothesis: Mass matrices between color-nonsinglet states, (a,b) and (a',b), are proportional, with coefficients independent of a and a', to those between the corresponding ordinary hadron states, $(a,1)$ and $(a',1)$.

Hereafter, I shall define "ψ particles" theoretically as color-octet vector mesons (ground states only) with the photon quantum numbers.

Let me also remind readers of an interesting work of Nambu and Han.[5] In their simplified model with coordinate-independent interactions between quarks in the color space, they have found that the total energy (E) of a hadron depends only on the number (N) of quarks in the hadron and on the two non-negative integers (l_1 and l_2) specifying an $SU(3)_c$ irreducible representation to which the hadron belongs:

$$E=N\mu_0+\tfrac{1}{2}g^2F_c{}^2$$

with (1)

$$F_c{}^2=\sum_{i=1}^{8}F_{c_i}{}^2=\tfrac{1}{3}(l_1{}^2+l_1l_2+l_2{}^2)+l_1+l_2\,,$$

where μ_0, g, and F_{c_i} are the effective quark mass, the coupling constant, and the $SU(3)_c$ generators, respectively. According to their interpretation, the large mass splitting, *which is consistent with the small mixing*, between a color-singlet state and color-nonsinglet states is due to a large ratio $r=\frac{1}{2}g^2/N\mu_0$.

In order to satisfy assumptions 1–3 and the hypothesis and to produce the large color singlet-nonsinglet mass splitting, I propose that the Hamiltonian (H) of hadrons transforms, to the first order in strong interactions, as

$$H \sim [1 + fF_8 + dD'_8 + \text{SU(3)-singlet-nonsinglet mixing}] \times (1 + rF_c{}^2)(1 + f_c F_{c_8} + d_c D'_{c_8}) . \quad (2)$$

Here f (f_c) and d (d_c) are arbitrary constants, and D'_8 (D'_{c_8}) is the eighth SU(3) (SU(3)$_c$) operator from which an SU(3)- (SU(3)$_c$-) invariant part is subtracted. This Hamiltonian has the following properties: (a) For small f, d, f_c, and d_c, SU(3)×SU(3)$_c$ symmetry holds approximately so that assumption 1 is satisfied. (b) Singlet-nonsinglet mixing is present only in the SU(3) space and not in the SU(3)$_c$ space so that assumption 2 is satisfied. (c) For nonvanishing f_c and d_c, SU(3)$_c$ symmetry is broken down to $SU(2)_{I_c} \times U(1)_{Y_c}$ as SU(3) symmetry to $SU(2)_I \times U(1)_Y$ so that assumption 3 is satisfied. (d) The Hamiltonian is a product of the SU(3) and SU(3)$_c$ operators so that the hypothesis is satisfied. (e) For large r, the large color singlet-nonsinglet mass splitting occurs.

I am ready to present the following three classes (1–3) of mass formulas which can be derived from this model. For immediate experimental relevance, I present them only for vector-meson resonances including ψ particles although similar formulas can easily be written down for other particles such as pseudoscalar mesons and baryons.

1. SU(3) (Gell-Mann–Okubo) mass sum rules:

$$4M(K^*, b) = 3M(\omega_8, b) + M(\rho, b) \quad \text{for } b = 1, \rho, \omega_8, K^*, \quad (3)$$

where $M(a, b)$ is either the mass or the mass squared of (a, b). An easy (but not compelling) way to derive these is to imagine $m_{\mathcal{P},j} = m_{\mathfrak{N},j} \neq m_{\lambda,j}$ and to calculate the vector-meson masses in the free-quark model. Owing to the large ω-ϕ mixing, none of these sum rules can be checked experimentally. However, they can be used for determining the masses of ideal SU(3)-octet (ω_8, b) mesons. For example, from the original Gell-Mann–Okubo sum rule (3) for $b = 1$ and the data[6] on m_ρ and M_{K^*},

$$M(\omega_8, 1) = 933 \pm 4 \text{ MeV} \quad \text{or} \quad (929 \pm 4 \text{ MeV})^2 . \quad (4)$$

2. SU(3)$_c$ mass sum rules:

$$4M(a, K^*) = 3M(a, \omega_8) + M(a, \rho) \quad \text{for } a = \omega, \phi, \rho, K^* . \quad (5)$$

Again, an easy way to derive these is to imagine $m_{i,1} = m_{i,2} \neq m_{i,3}$. Since no color-nonsinglet particles other than $\psi(3.1) \equiv (\omega, \rho^0)$, $\psi(3.7) \equiv (\omega, \omega_8)$, and $\psi(4.1) \equiv (\phi, \rho^0)$ have yet been found, none of these sum rules can be checked. From (5) for $a = \omega$ and the data[3] ($m_{\psi(3.1)} = 3095 \pm 4$ MeV and $m_{\psi(3.7)} = 3684 \pm 5$ MeV), however, $M(\omega, K^*)$ can be predicted:

$$M(\omega, K^*) = 3537 \pm 5 \text{ MeV} \quad \text{or} \quad (3546 \pm 5 \text{ MeV})^2 . \quad (6)$$

3. SU(3)×SU(3)$_c$ mass proportionalities:

$$m(\omega, 1) : m(\phi, 1) : m(\omega, 1) : m(K^*, 1) = m(\omega, b) : m(\phi, b) : m(\rho, b) : m(K^*, b) \quad \text{for } b = \rho, \omega_8, K^* . \quad (7)$$

These can be derived from the SU(3)×SU(3)$_c$ factorization property in the model Hamiltonian (2). Notice that the proportionalities hold whether or not they are for masses or masses squared. Only a part of the proportionality (7) can be checked with the data[3,6] on m_ω, m_ϕ, $m_{\psi(3.1)}$, and $m_{\psi(4.1)}$:

$$m(\omega, 1) : m(\phi, 1) = m(\omega, \rho) : m(\phi, \rho) ,$$

with (8)

$$\text{left-hand side} = 1 : (1.303 \pm 0.001)$$

and

$$\text{right-hand side} = 1 : (1.32 \pm 0.02) .$$

It is remarkable that this proportionality holds within a 2% experimental error.

As is stressed in II, the Hamiltonian of hadrons seems to be symmetric under the exchange of the SU(3) and SU(3)$_c$ spaces except for the large ω-ϕ mixing in the SU(3) space and the large singlet-nonsinglet mass splitting in the SU(3)$_c$ space. It is, therefore, tempting to assume another approximate symmetry obeyed by quarks under the exchange of

$$\begin{pmatrix} \mathcal{P} \to 1 \\ \mathfrak{N} \to 2 \\ \lambda \to 3 \end{pmatrix} \text{ with } m_{\mathcal{P},j} : m_{\mathfrak{N},j} : m_{\lambda,j} = m_{i,1} : m_{i,2} : m_{i,3} . \quad (9)$$

This new symmetry (call it E symmetry) indicates in the model Hamiltonian (2) that

$$f = f_c \quad \text{and} \quad d = d_c . \quad (10)$$

If this is the case, one can derive the fourth class of mass formulas.

4. SU(3)×SU(3)$_c$ mass reciprocities:

$$m(\rho, 1) : m(\omega_8, 1) : m(K^*, 1) = m(a, \rho) : m(a, \omega_8) : m(a, K^*) \quad \text{for } a = \omega, \phi, \rho, K^* . \quad (11)$$

Only a part of the reciprocity (11) can be checked with the data[3,6] on m_ρ, $m_{\psi(3.1)}$, and $m_{\psi(3.7)}$ and the previously estimated $M(\omega_8, 1)$ in (4):

$$m(\rho,1):m(\omega_8,1)=m(\omega,\rho):m(\omega,\omega_8)$$

with

$$\text{left-hand side}=1:(1.20\pm0.02) \qquad (12)$$

and

$$\text{right-hand side}=1:(1.190\pm0.004).$$

This reciprocity is satisfied again within a 2% experimental error.

The fifth (and last) class of mass formulas can be derived from the ansatz (call it the $1+F_c^2$ ansatz) on the constant r in the model Hamiltonian (2):

$$r=1. \qquad (13)$$

In fact, since $1+F_c^2=1$ and 4 for an $SU(3)_c$ singlet ($l_1=l_2=0$) and octet ($l_1=l_2=1$), respectively, this ansatz leads to the following mass formulas.

5. $1+F_c^2$ mass relations:

$$4=m(a,\rho)/m(a,1)\quad \text{for } a=\omega,\phi,\rho,K^*. \qquad (14)$$

Two of these relations can be checked with the data[3,6] on m_ω, m_ϕ, $m_{\psi(3.1)}$, and $m_{\psi(4.1)}$, and are found to hold again within 2% experimental errors:

$$m(\omega,\rho)/m(\omega,1)=3.954\pm0.008$$

and (15)

$$m(\phi,\rho)/m(\phi,1)=4.02\pm0.05.$$

Combining these five classes of mass formulas, I finally propose the following unified mass formula for a vector meson with the quantum numbers (I,Y) and (I_c,Y_c):

$$M((I,Y),(I_c,Y_c))=M_0\left\{1-a\frac{\langle F^2\rangle}{3}\left[(I-1)(I+2)-\frac{Y^2}{4}\right]\right\}(1+\langle F_c^2\rangle)\left\{1-a\frac{\langle F_c^2\rangle}{3}\left[(I_c-1)(I_c+2)-\tfrac{1}{4}Y_c^2\right]\right\}, \qquad (16)$$

with

$$\langle F^2\rangle,\langle F_c^2\rangle=\begin{cases}0 & \text{for a singlet}\\ 3 & \text{for an octet.}\end{cases}$$

By using this mass formula and by taking the data[6] on m_ω, m_ϕ, m_ρ, and m_{K^*} as input, one can predict the masses of all $9\times8=72$ color-octet vector mesons without any arbitrary parameter. The result is shown in Table I. Comparing the predicted and experimental values for $\psi(3.1)$, $\psi(3.7)$, and $\psi(4.1)$, one can find that this unified mass formula is perfectly valid within the small experimental errors of at most 3%.

I thank Dr. H. Sugawara at National Laboratory for High Energy Physics and also many colleagues in our theory group of Institute for Nuclear Study for useful discussions.

TABLE I. The experimental values for the masses of ω, ϕ, ρ, and K^* mesons taken as input and the values predicted from the unified mass formula for the masses of all color-octet vector mesons. The values in parentheses are experimental ones for already discovered ψ particles.

Input: experimental values (GeV)			
$m_\omega=0.7827\pm0.0006$	$m_\phi=1.0197\pm0.0003$	$m_\rho=0.77\pm0.01$	$m_{K^*}=0.8922\pm0.0005$
Output: predicted values (GeV) (experimental values)			
$m(\omega_8,1)=0.931\pm0.004$			
$m(\omega,\rho)=3.13\pm0.03$	$m(\phi,\rho)=4.08\pm0.04$	$m(\rho,\rho)=3.08\pm0.07$	$m(K^*,\rho)=3.57\pm0.04$
$(m_{\psi(3.1)}=3.095\pm0.004)$	$(m_{\psi(4.1)}\cong4.1)$		
$m(\omega,\omega_8)=3.79\pm0.12$	$m(\phi,\omega_8)=4.94\pm0.15$	$m(\rho,\omega_8)=3.73\pm0.15$	$m(K^*,\omega_8)=4.32\pm0.13$
$(m_{\psi(3.7)}=3.684\pm0.005)$			
$m(\omega,K^*)=3.63\pm0.08$	$m(\phi,K^*)=4.73\pm0.10$	$m(\rho,K^*)=3.57\pm0.11$	$m(K^*,K^*)=4.14\pm0.08$

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