PHYSICAL REVIEW D VOLUME 13, NUMBER 3 1 FEBRUARY 1976

New resonances and $\Gamma(V \to e^+e^-)$

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(Received 12 May 1975; revised manuscript received 13 October 1975)

A new V-γ coupling constant relation is obtained. It rests on a previously derived relation and the modification involved in going from the Proca to the Duffin-Kemmer-Petiau formalism. This relation is validated by comparison of $\Gamma(V \to e^+e^-)$ for ψ-ψ' and ρ-ω-ϕ mesons, which groups' masses differ greatly. It shows that ψ-ψ' are consistent with a (modified) Han-Nambu model.

In exploring SU(3) symmetries among the coupling constants of particles, an ambiguity arises as to what mass weighting to assign. That is to say, given that a matrix element must have units of $(\text{mass})^4$, it is an assumption how many of these units are to be incorporated into the SU(3)-symmetric vertex function, and how many are to be incorporated in the physical-mass-valued quantities such as the wave function squared. Depending on which interpretation and which wave function one uses, different results will emerge.

This has been emphasized in discussions[1-3] of the Duffin-Kemmer-Petiau (DKP) formalism for pseudoscalar (P) and vector (V) mesons vs the normal Klein-Gordon (KG) or Proca formalisms. For example,[2] it was found that the DKP formalism with axial-vector coupling (preferred by some) for the baryon (B')-baryon (B)-meson (P) vertex gives a consistent value of the strong D/F ratio when compared to experiment, whereas pseudoscalar coupling ($g_{B'BP}$) with the KG formalism does not. [Specifically, one is saying that the coupling $\hat{g}_{B'BP} = g_{B'BP}(m_{B'} + m_B)(m_P)^{-1/2}$ is SU(3)-symmetric.] Another example is the work of Golowich and Kapila[3] in comparing the different predictions for SU(3) tensor (T)-P-P coupling constant sum rules from a number of Lagrangians with DKP or KG wave functions.

There is at this stage no *a priori* logical argument favoring one formalism over the other; given a specific interaction, the two prescriptions are simply different,[4] and nature happens to choose one of them (or perhaps another). One must appeal to experiment for the ultimate determination.

A similar situation obtains in coupling constant relations for the vector-dominance model (VDM). Usually the dimensionless vector-meson—photon coupling constants e/f_V are taken to follow the standard quark-model prediction for ρ, ω, and ϕ mesons, namely,

$$\frac{1}{f_\rho^2} : \frac{1}{f_\omega^2} : \frac{1}{f_\phi^2} = 9:1:2\,. \tag{1}$$

[This can also be obtained from the Lagrangian $V_\lambda(e^2/f_V)l_\lambda$, where V_λ is the Proca field and l_λ is the lepton current.]

Okubo has provided theoretical justification for U(3) symmetry of the relevant Schwinger terms in a derivation of the first Weinberg spectral-function sum rule.[5] On this basis one of us[6] has shown that the quark-model result should be modified to

$$\frac{m_\rho^2}{f_\rho^2} : \frac{m_\omega^2}{f_\omega^2} : \frac{m_\phi^2}{f_\phi^2} = 9:1:2\;, \tag{2}$$

where the Proca formalism was used. Feynman[7] has also suggested that this may be the correct modification to the quark-model prediction, although he gives no reason for his theoretical prejudice. [Equation (2) can come from the Lagrangian $(\hat{m}_{0p}{}^{-1}V_\lambda)(e^2/g_V)l_\lambda$, meaning $g_V{}^{-1} = m_V/f_V$ and $\hat{m}_{0p}$ is a mass operator.]

A major difficulty in determining which, if either, of these coupling-constant relations is correct is that the coupling constants have only been determined with an accuracy comparable to the order of SU(3)-symmetry breaking, say about 10–30%. The discovery of e^+e^- resonances in the 3–5-GeV mass range promises a solution to this problem; here the mass ratio of the resonances to the ordinary vector mesons is of order 4–5. This exceeds expected SU(3)-symmetry breaking by at least an order of magnitude.

Yennie has pointed out[8] that if the $\psi(3.1) \to e^+e^-$ decay rate is compared with that for ρ, ω, φ (the e^+e^- decay rate being proportional to $m_V/f_V{}^2$) then, assuming that $\psi(3.1)$ is charmonium, one concludes that the SU(4) quark-model prediction

$$\frac{1}{f_\rho^2} : \frac{1}{f_\omega^2} : \frac{1}{f_\phi^2} : \frac{1}{f_\psi^2} = 9:1:2:8 \tag{3}$$

is not well satisfied. To a quite reasonable approximation, however, Yennie's[8] empirical relation

$$\frac{m_\rho}{f_\rho^2}:\frac{m_\omega}{f_\omega^2}:\frac{m_\phi}{f_\phi^2}:\frac{m_\psi}{f_\psi^2}=9:1:2:8 \tag{4}$$

is fulfilled. (This has also been independently observed by one of us.[9])

We suggest that this empirical fact be regarded as evidence for the correctness of the modified quark-model predictions, provided they are derived within the framework of the DKP formalism rather than the Proca formalism. The reason for this is that the primary effect of using the DKP formalism[10,11] is to replace the coupling constant $g_V^{-2}=m_V^2/f_V^2$ by $h_V^{-2}=m_V/f_V^2$ in the modified quark-model relations.

Once a mass weighting is established, the over-all coupling constants can then be used to study symmetry rules and their breaking. Reference 8 already shows on the basis of Eq. (4) that $\psi(3.1)\to e^+e^-$ is in good agreement with SU(4); we show below that the Han-Nambu model[12] or a modification with color-symmetry breaking[13] also accounts within experimental error for $\Gamma(V\to e^+e^-)$ among the new e^+e^- resonances.[14]

In the Han-Nambu model the $\psi=\psi(3.1)$ and $\psi'=\psi(3.7)$ are denoted[15] by $\psi(\omega,\rho^0)$ and $\psi(\phi,\rho^0)$ and are related by the same ideal mixing angle as the ordinary ω and ϕ. The predictions of this model, with the mass weighting suggested above and neglecting at this stage any color-octet admixture to ρ, ω, and ϕ, are[16]

$$\frac{m_\rho}{f_\rho^2}:\frac{m_\omega}{f_\omega^2}:\frac{m_\phi}{f_\phi^2}:\frac{m_\psi}{f_\psi^2}:\frac{m_{\psi'}}{f_{\psi'}^2}=9:1:2:6:3\ . \tag{5}$$

Now $\Gamma(V)=\Gamma(V\to e^+e^-)$ is proportional to m_V/f_V^2, so if we take[17] $\Gamma(\rho^0)=6.5\pm0.8$ keV, then the predicted and experimental[17,18] leptonic decay widths for the other vector mesons are as shown in Table I. The errors are large but the over-all agreement for such a wide range of masses is good.

The modified Han-Nambu model[13] introduces symmetry breaking in the color group SU(3)″. This does not affect the resonances $\psi(\omega,\rho^0)$ and $\psi(\phi,\rho^0)$ discussed above. It does, however, introduce mixing between the nonets $\psi(i,0)$ and $\psi(i,8)$, so that the ordinary neutral nonstrange mesons are no longer color singlets and are denoted by $\psi(\rho^0,\omega)$, $\psi(\omega,\omega)$, and $\psi(\phi,\omega)$. The corresponding $\psi(\omega,\phi)$, $\psi(\rho^0,\phi)$, $\psi(\phi,\phi)$ we associate with the complex of relatively broad resonances now discernible[19] near 4.2 GeV: the strongest at 4.1, another at 4.4, and perhaps a weaker one at 4.0 GeV. Any bump around 4.9 GeV may be associated with heavy-lepton production.[19]

In terms of respective mixing angles θ' and θ'' for SU(3)′ and SU(3)″

$$\Gamma(\rho^0,\omega)=9\Gamma\cos^2\theta''\ , \tag{6a}$$

$$\Gamma(\phi,\omega)+\Gamma(\omega,\omega)=3\Gamma\ , \tag{6b}$$

$$\Gamma(\phi,\omega)-\Gamma(\omega,\omega)=3\Gamma\cos2\theta'\cos2\theta''\ , \tag{6c}$$

where Γ is a fixed parameter characterizing $\Gamma(V\to e^+e^-)$. Since Eq. (6b) is independent of θ' and θ'', we find[17]

$$\Gamma=\tfrac{1}{3}[\Gamma(\omega\to e^+e^-)+\Gamma(\phi\to e^+e^-)]$$
$$=0.70\pm0.06\text{ keV}. \tag{7}$$

From this it appears that $\cos\theta''$ is very close to unity: i.e., that the ordinary vector mesons show no color admixture observable by this means. The corresponding $\cos2\theta'=0.3\pm0.1$ encompasses the value of $\frac{1}{3}$ expected for ideal mixing.

For the lowest ψ mesons

$$\begin{aligned}&\Gamma(\omega,\rho^0)+\Gamma(\phi,\rho^0)=9\Gamma\ ,\\ &\Gamma(\omega,\rho^0)-\Gamma(\phi,\rho^0)=9\Gamma\cos2\theta_1'\ ,\end{aligned} \tag{8}$$

where θ_1' emphasizes that this mixing angle need not be the same as θ' in Eq. (6) for ordinary mesons. With the assignments indicated before Eq. (5) the experimental[18] values give

$$\Gamma=0.76\pm0.09\text{ keV},\quad \cos2\theta_1'=0.35\pm0.12. \tag{9}$$

The value of Γ agrees with that in Eq. (7), and θ_1' again centers about the ideal mixing angle.

For the ψ mesons around 4.1 GeV

$$\begin{aligned}&\Gamma(\rho^0,\phi)=9\Gamma\sin^2\theta_2'',\\ &\Gamma(\omega,\phi)+\Gamma(\phi,\phi)=3\Gamma\ ,\\ &\Gamma(\omega,\phi)-\Gamma(\phi,\phi)=3\Gamma\cos2\theta_2'\cos2\theta_2''.\end{aligned} \tag{10}$$

TABLE I. Comparison of the predicted and experimental leptonic decay widths assuming $\Gamma(\rho\to e^+e^-)=6.5\pm0.8$ keV.

Particle	Γ(predicted) (keV)	Γ(experimental) (keV)
ω	0.72 ± 0.09	0.76 ± 0.17
ϕ	1.45 ± 0.18	1.35 ± 0.08
ψ	$4.4\ \pm0.5$	$4.6\ \pm0.5$
ψ'	$2.2\ \pm0.3$	$2.2\ \pm0.6$

Here the mixing angle θ_2'' is expected not to be vanishingly small, as was the case with θ_2. For the ordinary vector mesons $\psi(i, \omega)$ the nearest color states for admixture are about 3 GeV away; the uncolored states for admixture to $\psi(i, 8)$ are in the same energy range as the $\psi(i, \phi)$ themselves. Accordingly, we can say only that

$$\Gamma(\rho^0, \phi) + \Gamma(\omega, \phi) + \Gamma(\phi, \phi) = 3(1 + 3\sin^2\theta_2'')\Gamma. \tag{11}$$

The current experimental value[19] for this quantity is 2.2–4.1 MeV, which yields no particular information on θ_2''.

Although the experimental information on $\Gamma_{ee}(V)$ in the 4.2 GeV region needs more definition for comparison here, that for the $\psi(3.7)$ and $\psi(3.1)$ is already precise enough to indicate a Γ_{ee} ratio of $\frac{1}{2}$ rather than $\frac{1}{3}$ for these states. This provides a preference for the assignment $\psi(3.1)$, $\psi(3.7) = \psi(\omega, \rho^0)$, $\psi(\phi, \rho^0)$ instead of the alternative[20] $\psi(\omega, \rho^0)$, $\psi(\omega, 8)$.

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†Work supported by the United States Energy Research and Development Administration.

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[11]A more ambitious and perhaps profound question is: What happens to the sum rules if one calculates them in a DKP formalism? That would depend on the type of $\vec{p}$ limiting rules used. For example, the zero at $t = t_0 = -(m+\mu)^2$ in $\partial_\lambda \langle V_\lambda \rangle$ for the DKP (but not the KG) description of K_{l3} decay (see the first article of Ref. 1) can be described in a current-algebra calculation as the difference between taking $\vec{p} \to 0$ and $\vec{p} \to \infty$ sum rules. See, footnote (X) on p. 244 of S. Fubini and G. Furlan, Physics (N.Y.) 1, 229 (1965), and especially the Appendix, point (iii), of E. Fischbach, C. K. Scott, and L. D. Simmons, LASL Report No. LA-UR-75-614 (unpublished).

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