Polarization correlations in electron-positron reactions*

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We study the angular dependence of one-particle inclusive reactions with polarized electron-positron beams. It is observed that spin-$\frac{1}{2}$ and spin-1 partons, with zero anomalous magnetic moments, give the same angular distribution. The production of ordinary vector mesons, with zero anomalous magnetic moments, has also the same angular dependence and provides a candidate for a new component to the annihilation cross section. Present observations do not exclude such a component. A separate section elaborates on general properties of the quasielastic reaction for beams with arbitrary polarizations.

I. INTRODUCTION

It has been anticipated[1] for quite some time that the present generation of storage rings will provide polarized electron-positron beams. The natural states of polarization are expected to be collinear with the magnetic field, but other configurations are also being contemplated. This experimental capability is now becoming a reality[2] and brings into focus several new studies in the field of weak and electromagnetic interactions. The advantages of observing neutral-current effects[3] with polarized beams have already been emphasized. There are in addition advantages[4] in using the polarization in order to analyze the hadronic final states and infer the underlying structure of the hadronic vertex. This article deals with two studies on polarization: (1) It analyzes one-particle inclusive reactions for naturally polarized beams, and (2) it discusses the quasielastic process for beams with arbitrary polarizations.

Among the numerous mechanisms which contribute to the final hadronic states, the parton model makes precise predictions testable in the immediate future. Assuming that an asymptotic region has been reached one can apply the understanding obtained in electron- and neutrino-induced experiments to predict specific features of inclusive reactions. Comparisons with available data indicate that the characteristics of the final states are still changing, suggesting that an asymptotic region has not been reached.[5] In fact it seems quite plausible, in view of the newly discovered resonances, that the available data are in a transitional region where new resonances are produced. It is thus natural to consider three regions for the continuum:

(i) an asymptotic region, where the parton model applies and where the distinct parton features of spin, charge, and other quantum numbers become evident;

(ii) a threshold region where new resonances, such as charged vector mesons, are produced and then fade away as the energy increases;

(iii) a transitional region where both of the above are important and where nonscaling corrections are also present.

Section II summarizes the results of the parton model for spin-0, $\frac{1}{2}$, and 1 partons. It is observed that spin-$\frac{1}{2}$ and spin-1 partons, with zero magnetic moments, give the same angular distribution. The same angular distribution is also predicted for the production of two charged vector mesons with zero magnetic moments. This in turn suggests a transitory behavior of the cross section for energies comparable to twice the mass of "produced" vector mesons. The energy dependence on the other hand is very model-dependent. For single-pole or dipole form factors one can obtain a slowly varying function $R(q^2)$ by introducing numerous pairs of vector mesons.

Section III considers beams with arbitrary polarizations and discusses elastic, as well as one-particle inclusive, processes. It is noted that for vector and axial-vector currents the term linear in the transverse polarization is always absent. For elastic scattering with naturally polarized beams there is a minimum which has been discussed in the literature.[6] Explicit calculation indicates that other configurations of polarization give a minimum at the same place and minima at other regions of phase space. Finally we describe measurements which are sensitive to the CP content of the hadronic current. Technical details on the notation are summarized in the Appendix.

II. ONE-PARTICLE INCLUSIVE REACTIONS

The physical process to be studied in this section is the annihilation of electron-positron pairs into a hadron with fixed momentum plus anything else; i.e., the process

$$e^- + e^+ \to h + x\,. \tag{2.1}$$

The detected hadron can be a meson or a baryon and x stands for any possible hadronic state. The initial electron and positron momenta are defined as $\vec{k}_- = -\vec{k}_+ = E\hat{n}$, $|\hat{n}| = 1$. The transverse polarization vectors are $\vec{P}_+ = P_+ \hat{l}$ and $\vec{P}_- = -P_- \hat{l}$, where $|\hat{l}| = 1$ and $\hat{l}\cdot\hat{n} = 0$ (natural polarization). The unit vector along the direction of the outgoing hadron is denoted by $\hat{m}$ and the scattering angles θ and φ are defined as

$$\cos\theta \equiv \hat{n}\cdot\hat{m} ,$$

$$\hat{l}\cdot\hat{m} \equiv \sin\theta\cos\varphi ,$$

$$(\hat{m}\times\hat{n})\cdot\hat{l} \equiv \sin\theta\sin\varphi .$$

The notation is convenient for transversely polarized beams, which is the only case to be considered in this section.

New dependence on the azimuthal angle φ appears when one considers the inclusive reaction (2.1). The leptonic tensor is the same for any final state and in the one-photon approximation has the form

$$\tau^{\mu\nu} = (1 - P_+ \cdot P_-)(k_-^\mu k_+^\nu + k_-^\nu k_+^\mu - g^{\mu\nu} k_- \cdot k_+) - 2P_+^\mu P_-^\nu k_- \cdot k_+ . \qquad (2.2)$$

Terms proportional to the square of the electron mass are being neglected. The hadronic tensor for reaction (2.1) has the general form

$$W_{\mu\nu} \equiv (2\pi)^3 \sum_x \langle 0|J_\mu(0)|h, x\rangle\langle h, x|J_\nu(0)|0\rangle\,\delta^4(q - p - p_x)$$
$$= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)W_1(q^2, q\cdot p) + \frac{1}{M^2}\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)W_2(q^2, q\cdot p), \qquad (2.3)$$

where M and p are, respectively, the mass and momentum of the detected hadron and $q = k_+ + k_-$. In an alternative notation one can define structure functions corresponding to the polarization states of the virtual photon. In the center-of-mass frame a coordinate system is introduced with the z axis along the direction of the momentum of the produced hadron. In this frame the polarization vectors are defined by

$$\epsilon_\mu^S = (0, 0, 0, 1), \qquad (2.4)$$

$$\epsilon_\mu^{R,L} = \frac{1}{(2)^{1/2}}(0, 1, \pm i, 0), \qquad (2.5)$$

and they satisfy $\epsilon_i^2 = -1$, $\epsilon_{S,R,L}\cdot q = 0$. The structure functions corresponding to the polarization vectors are related to W_1 and W_2 by

$$W^S = \epsilon_\mu^S W^{\mu\nu}\epsilon_\nu^S = W_1 + \frac{(p\cdot q)^2}{q^2}\frac{W_2}{M^2}, \qquad (2.6)$$

$$W^T = \epsilon_\mu^{*R} W^{\mu\nu}\epsilon_\nu^R + \epsilon_\mu^{*L} W^{\mu\nu}\epsilon_\nu^L = 2W_1 . \qquad (2.7)$$

The differential cross section is given in terms of either set of structure functions, and in the scaling limit ($q^2/\nu^2 \to 0$; $\nu = p\cdot q/M$) has the form[7]

$$E_p \frac{d\sigma}{d^3p} = \frac{8M\alpha^2}{q^4}\left\{2W_1(Q^2, \nu) + [1 + |P_+||P_-|\cos 2\varphi]\sin^2\theta\,\frac{(p\cdot q)^2}{q^2}\frac{W_2(Q^2, \nu)}{M^2}\right\}$$
$$= \frac{8M\alpha^2}{q^4}\{2(1 + |P_+||P_-|\cos 2\varphi)\sin^2\theta W^S + [2 - (1 + |P_+||P_-|\cos 2\varphi)\sin^2\theta]W^T\} . \qquad (2.8)$$

The new dependence on the azimuthal angle φ is valuable in separating the structure functions and distinguishing between models.

In the parton model[8] and in the scaling limit the structure functions satisfy constraint equations which depend on the spin of the constituents.

Spin-0 partons.

$$W_1(Q^2, \nu) = 0$$

and

$$\frac{d\sigma}{d\Omega} \propto (1 + |P_+||P_-|\cos 2\varphi)\sin^2\theta\, W_2(q^2, \nu) . \qquad (2.9)$$

Spin-$\frac{1}{2}$ partons (with zero anomalous magnetic moment).

$$MW_1 = F_1(x) = \frac{x}{2}\sum_i Q_i^2 f_i(x) ,$$

$$\nu W_2 = F_2(x) = -\sum_i Q_i^2 f_i(x) ,$$

where $x = 2q\cdot p/q^2 = 2M\nu/q^2$, Q_i is the charge of the quark, and $f_i(x)$ is a momentum distribution for the ith-type partons. Now

$$W_S/W_T = 0 \quad \text{or} \quad \frac{F_1}{F_2} = -\frac{x}{2} \qquad (2.10)$$

and

$$\frac{d\sigma}{dx d\Omega} = \frac{4\alpha^2}{q^4} q\cdot p\, x\{1 - \tfrac{1}{2}[1 + |P_+||P_-|\cos 2\varphi]\sin^2\theta\} \times \sum_i Q_i^2 f_i(x)\,. \tag{2.11}$$

Spin-1 partons. The angular distribution in this case depends on the magnetic moment coupling. For zero magnetic moment ($\kappa = 0$)

$$\frac{W_2}{M_V^2} \propto -\frac{4q\cdot px}{M_V^2} + 12x^2$$

and (2.12)

$$W_1 \propto \frac{2(q\cdot px)^2}{M_V^2} - 4q\cdot px\,,$$

where M_V is the mass of the vector particle. In the limit $q^2, q\cdot p \gg M_V^2$ it reduces to the relation obtained for spin-$\frac{1}{2}$ partons. Thus the angular dependence for single-particle inclusive reactions is identical for spin-$\frac{1}{2}$ and spin-1 partons. There is a difference, however, arising from the energy dependence of the cross section. For spin-1 partons with $\kappa = 0$ the total annihilation cross section[9] is

$$\sigma(e^+e^- \to \text{hadrons}) = \frac{\pi\alpha^2}{3M_V^2}\left(1 - \frac{M_V^2}{4E^2} - \frac{3M_V^4}{4E^4}\right)\left(1 - \frac{M_V^2}{E^2}\right)^{1/2}, \tag{2.13}$$

leading to the ratio

$$R = \frac{\sigma(e\bar{e} \to h)}{\sigma(e\bar{e} \to \mu\bar{\mu})} = \frac{E^2}{M_V^2}\left(1 - \frac{M_V^2}{4E^2} - \frac{3M_V^4}{4E^4}\right)\left(1 - \frac{M_V^2}{E^2}\right)^{1/2}, \tag{2.14}$$

which rises with energy. For moderate values of $M_V < 1$–2 GeV/c^2 the observed energy dependence excludes the spin-1 possibility. But this constraint disappears if the pointlike structure of the hadronic coupling is abandoned. In the energy range under consideration, $E_{\text{cm}} < 8$ GeV, the characteristic features of the final hadronic states are still changing and could be attributed to the production of new pairs of charged vector mesons with conventional form factors.

This intriguing possibility is expected in many of the gauge models which are now under consideration. Most of the models introduce new quantum numbers and consequently new vector mesons whose masses lie in the 2–4-GeV/c^2 region. We discuss here the production of such states through the reaction

$$e^+ + e^- \to V^+ + V^-, \tag{2.15}$$

and point out that they could account for a transitory behavior of the annihilation data. In fact there are two other indirect experimental indications which point to the production of such states. The dimuon events observed in neutrino-induced reactions[10] indicate that one of the leptons is associated with the hadronic vertex. The production of a vector meson and its subsequent leptonic or semileptonic decay is consistent with the data. Similarly the (μe) pairs[11] observed in electron-positron collisions are also consistent with the production of vector states and their subsequent decays.

The expected angular dependence for vector mesons with zero magnetic moments is the same as in Eq. (2.11). The energy dependence of R, on the other hand, is a sensitive function of the number of states produced and of the functional form of the form factors. At the present time, there is no solid theoretical understanding of the electromagnetic form factors of vector mesons and the experimental evidence on their structure is even more limited. Intuitively one expects that the effect of each pair is over a limited region of q^2. Consequently, a constant value for $R(q^2)$ implies a large number of vector pairs. This is in fact the case if we assume simple pole or dipole form factors. In general, one expects the effect of vector mesons to be transitory and vanish at very high energies, but this expectation is not on firm theoretical ground.

The approach to an asymptotic limit is now of interest. Dispersion relations for the vacuum-polarization function give an indication for the spacelike region. Dispersion-relation[12] estimates indicate that the approach to the asymptotic value is slow and the limit is approached from below. Information in the timelike region is obtained from asymptotically free theories where the limit is approached from above but the rate of approach is now arbitrary. We feel that the case for the production of vector mesons introduces a new component which could account for one to two units of R. There is no experimental evidence at this time which rules them out. In fact there is some positive albeit indirect evidence pointing to this direction.

III. $e^+e^- \to \mu^+\mu^-$

The quasielastic process

$$e^+e^- \to \mu^+\mu^- \tag{3.1}$$

has been discussed in numerous articles,[3] but the emphasis has been on beams with natural or longitudinal polarizations. Anticipating the possibility that other configurations of polarized beams will also be achieved we elaborate on some general properties.

Considering arbitrarily polarized beams, one can, without loss of generality, decompose the polarization vectors into transverse and longitudinal components denoted by

$P_\perp, P_\parallel$ for the electron,

$P'_\perp, P'_\parallel$ for the positron.

The remaining notation is as follows.

η denotes the longitudinal polarization of the muon,

ξ', ξ the unit polarization four-vectors,

k_+, k_- the momentum four-vectors for e^+, e^-,

q_+, q_- the momentum four-vectors for μ^+, μ^-.

The subsequent discussion will involve weak interactions and we state some general properties concerning the space-time structure[13] of the interactions. When the lepton masses are neglected, scalar (S), pseudoscalar (P), and tensor (T) vertices couple electron-positron pairs of same helicities. This is in contrast to vector (V) and axial-vector (A) vertices which couple pairs of opposite helicities. Consequently, for longitudinally polarized beam(s) there is no interference between (S, P, T) interactions and the electromagnetic current. In particular, for beams of the same helicity only S, P, T terms contribute. For transversely polarized beams interference between S, P, T and V, A currents can arise. However, for the reaction (3.1) and for transversely polarized beams, the above interference again vanishes, provided that a transverse polarization of the muon is not measured. In the following considerations the S, P, T contributions will be neglected, because they are of order $(Gs)^2$, which are too small relative to the interference terms for the energy ranges contemplated. For the remaining vector (V) and axial-vector (A) couplings we consider, in addition to the electromagnetic contribution, an effective Lagrangian of the form

$$\mathcal{L} = \frac{G}{(2)^{1/2}}\,[\bar{e}\gamma_\mu(g_V + g_A\gamma_5)e\bar{\mu}\gamma^\mu(g_V + g_A\gamma_5)\mu + \bar{e}\gamma_\mu(g_V + g_A\gamma_5)e(A_3^\mu + V_3^\mu + yJ_S^\mu)]\,, \tag{3.2}$$

where $J_i = V_i + A_i$ is one of the isospin components of the usual $V-A$ currents and J_S is an isoscalar current. In the simple model[14]

$$g_V = \frac{1}{(2)^{1/2}}(1 - 4\sin^2\theta_W),\quad g_A = -\frac{1}{(2)^{1/2}}\,,\qquad x = 1 - 2\sin^2\theta_W,\quad y = -2\sin^2\theta_W\,, \tag{3.3}$$

with $J_S = V_8/\sqrt{3}$, where θ_W is the Weinberg mixing angle. Assuming μ-e universality we obtain the general formula[15] for the production of longitudinally polarized muons

$$\begin{aligned}\frac{d\sigma_\eta}{d\Omega} = \frac{\alpha^2}{s^3}\Big[&(1 + P_\parallel P'_\parallel)[(k_+\cdot q_+)^2 + (k_-\cdot q_+)^2] - (P_\parallel + P'_\parallel)\eta\,[(k_+\cdot q_+)^2 - (k_-\cdot q_+)^2]\\ &- P_\perp P'_\perp\{\xi'\cdot\xi[(k_-\cdot q_+)^2 + (k_+\cdot q_+)^2] + q_+\cdot q_-[(\xi'\cdot q_+)(\xi\cdot q_-) + (\xi'\cdot q_-)(\xi\cdot q_+) - \xi'\cdot\xi\, q_+\cdot q_-]\}\\ &+ \epsilon(s)\big([g_V(1 + P_\parallel P'_\parallel) + g_A(P_\parallel + P'_\parallel)](g_V + \eta g_A)[(k_+\cdot q_+)^2 + (k_-\cdot q_+)^2]\\ &\quad - [g_A(1 + P_\parallel P'_\parallel) + g_V(P_\parallel + P'_\parallel)](\eta g_V + g_A)[(k_+\cdot q_+)^2 - (k_-\cdot q_+)^2]\\ &\quad - P_\perp P'_\perp g_V(g_V + \eta g_A)\{\xi'\cdot\xi[(k_-\cdot q_+)^2 + (k_+\cdot q_+)^2]\\ &\qquad + q_+\cdot q_-[(\xi'\cdot q_+)(\xi\cdot q_-) + (\xi'\cdot q_-)(\xi\cdot q_+) - \xi'\cdot\xi\, q_+\cdot q_-]\}\big)\Big]\,.\end{aligned} \tag{3.4}$$

The dots here indicate the inner product between four-vectors and $\epsilon(s) = (2)^{1/2}Gs/4\pi\alpha$, with $s = q^2 = (k_- + k_+)^2$. Detailed definitions of the angles and explicit expressions for the inner products are given in the Appendix. It is instructive to work out explicit formulas for some specific cases.[16]

Longitudinal polarization $P_\perp = P'_\perp = 0$: In this case we have

$$\begin{aligned}\frac{d\sigma_\eta}{d\Omega} = \frac{\alpha^2}{8s}\{&(1 + P_\parallel P'_\parallel)(1 + \cos^2\theta) + 2(P_\parallel + P'_\parallel)\eta\cos\theta + \epsilon(s)[g_V(1 + P_\parallel P'_\parallel) + g_A(P_\parallel + P'_\parallel)](g_V + \eta g_A)(1 + \cos^2\theta)\\ &+ 2\epsilon(s)[g_A(1 + P_\parallel P'_\parallel) + g_V(P_\parallel + P'_\parallel)](\eta g_V + g_A)\cos\theta\}\,.\end{aligned} \tag{3.5}$$

Transverse polarization $P_\parallel = P'_\parallel = 0$. Here we consider the case where polarization vectors are perpendicular to the beam direction but not antiparallel to each other. In fact let us denote by ϕ_1 and ϕ_2 the angles which the polarization four-vectors $\vec{\xi}'$ and $\vec{\xi}$ make with their respective projections into the production plane, as defined in Fig. 1. Then we have

$$\frac{d\sigma_\eta}{d\Omega} = \frac{\alpha^2}{8s}\left(\{2 - \sin^2\theta[1 - P_\perp P'_\perp\cos(\phi_1 + \phi_2)]\}[1 + \epsilon(s)(g_V + \eta g_A)g_V] + 2\epsilon(s)g_A(\eta g_V + g_A)\cos\theta\right). \tag{3.6}$$

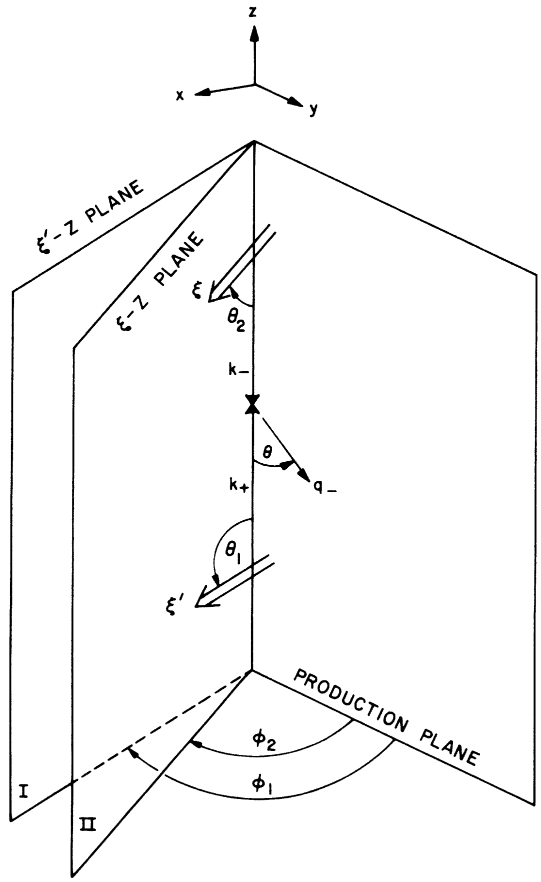

FIG. 1. Azimuthal angles for arbitrary polarizations of the beams.

It is evident that the only change from the natural state of polarization is the dependence on the azimuthal angles ϕ_1 and ϕ_2. For the natural state of polarization $\phi_1 = \phi$ and $\phi_2 = \phi + \pi$ and (3.6) reduces to the well-known formula.[3]

There are several properties of the formulas (3.4)–(3.6) which are of physical interest and we remark on them.

(1) For the V, A couplings considered there is a general result concerning the dependence of the cross section on beam polarization.

Theorem. For arbitrarily polarized beams and in the limit $m_e = 0$, the dependence of the cross section on the polarization vectors has the general form

$$\sigma = A + BP_\perp P'_\perp + CP_\parallel P'_\parallel + D(P_\parallel + P'_\parallel), \qquad (3.7)$$

where A, B, C, and D are Lorentz scalars independent of the beam polarization. The theorem is obviously satisfied for reaction (3.1), but it also holds for any final state and also for restricted regions of phase space. The dependence on the transverse polarization is quadratic and does not provide evidence for parity violation. The presence of a linear term in $P_\parallel$ and $P'_\parallel$, on the other hand, is an unambiguous indication of parity violation, provided that the number of independent vectors in the problem is limited so that another pseudoscalar cannot be formed. This observation can be employed to distinguish weak effects in reactions with final hadronic states.

(2) For the leptonic reaction it is desirable to find regions of phase space, where the electromagnetic (one-photon) contribution has minima so that one can search there for small, weak, and/or higher-order electromagnetic effects. The electromagnetic contribution alone gives the simple expression

$$\frac{d\sigma_\eta}{d\Omega} = \frac{\alpha^2}{8s}\{2(1 + P_\parallel P'_\parallel) - \sin^2\theta[1 + P_\parallel P'_\parallel - P_\perp P'_\perp \cos(\phi_1 + \phi_2)] + 2(P_\parallel + P'_\parallel)\eta \cos\theta\}\,. \qquad (3.8)$$

Two well-known zeros are evident for

(1) $P_\perp = P'_\perp = 0$ and $P_\parallel = -P'_\parallel = 1$;

(2) $P_\parallel = P'_\parallel = 0$, $P_\perp = P'_\perp = 1$, and $\phi_1 + \phi_2 = \pi$, $\theta = \frac{1}{2}\pi$.

In fact at $\theta = \frac{1}{2}\pi$, the cross section in the one-photon approximation has a minimum for any configuration of beams where their spins are collinear and antiparallel. There are other minima of the cross section in the one-photon approximation. The functional form of (3.8) is simple enough and we leave it for the interested reader to investigate the minima for each particular case of polarization.

(3) The CP content of the weak current can be determined by selecting first a term of definite parity and then studying its properties under charge conjugation. One-particle inclusive experiments with a longitudinally polarized beam provide such an opportunity. The dependence on the polarization of the beam is given by

$$P^l = \frac{d\sigma'(+) - d\sigma'(-)}{d\sigma'(+) + d\sigma'(-)}$$

$$= \epsilon(s)\,\frac{g_A[W_1^{ew} + (M\nu/q^2)(1 - q^2/\nu^2)(\nu W_2^{ew}/2M)\sin^2\theta] + g_V(1 - q^2/\nu^2)^{1/2}(\nu W_3^{ew}/M)\cos\theta}{W_1^e + (M\nu/q^2)(1 - q^2/\nu^2)(\nu W_2^e/2M)\sin^2\theta}\,, \qquad (3.9)$$

where W_1, W_2, W_3 are the structure functions of the hadronic vertex with the superscripts e and ew denoting electromagnetic and electromagnetic-weak interference contributions, respectively. This quantity is obviously parity violating. If the axial-vector and the vector currents have opposite charge conjugation properties, then νW_3 should change sign when h is replaced by its charge conjugate particle. For self-conjugate particles such as $\pi^0, \rho^0, \ldots$ this term should be absent.

In models of the strict Weinberg type[14] the structure functions in the numerator and denominator are related because they involve vector currents whose relative strengths are known:

$$W_{1,2}^{ew} = (1 - 2\sin^2\theta_W) W_{1,2}^{e} \,. \tag{3.10}$$

Such a relation holds in other models as well, provided that the isoscalar contributions are neglected. Thus the contribution to $|P^I|$ from the W_1 and W_2 terms alone is $\gtrsim +4\%$ at 30-GeV center-of-mass energy.

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APPENDIX

Here we define the variables occurring in Sec. III and simplify some of the expressions. We define two planes I and II formed by the beam direction (z axis) $\hat{n}$ and the polarization vectors $\vec{\xi}$ and $\vec{\xi}\,'$, respectively. The angle formed between the plane I (II) and the production plane is denoted by ϕ_1 (ϕ_2) as shown in Fig. 1. The four-vectors in the relativistic limit have the components

$$k_- = (1, 0, 0, -1)E, \quad k_+ = (1, 0, 0, 1)E \,,$$

$$q_-^\mu = (1, 0, \sin\theta, -\cos\theta)E \,,$$

$$q_+^\mu = (1, 0, -\sin\theta, \cos\theta)E \,,$$

$$\xi^\mu = (\cot\theta_2, \sin\phi_2, \cos\phi_2, \cot\theta_2) \,,$$

$$\xi'^\mu = (\cot\theta_1, \sin\phi_1, \cos\phi_1, -\cot\theta_1) \,.$$

The inner products are given by

$$\xi'\cdot\xi = 2\cot\theta_1\cot\theta_2 - \sin\phi_1\sin\phi_2 - \cos\phi_1\cos\phi_2 \,,$$

$$\xi\cdot k_- = 2E\cot\theta_2, \quad \xi\cdot k_+ = 0 \,,$$

$$\xi\cdot q_- = E(\cot\theta_2 - \sin\theta\cos\phi_2 + \cos\theta\cot\theta_2) \,,$$

$$\xi\cdot q_+ = E(\cos\theta_2 + \sin\theta\cos\phi_2 - \cos\theta\cot\theta_2) \,.$$

The remaining inner products in Eq. (3.4) are expressible in a similar manner in terms of laboratory variables.

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[15]To my knowledge the general case did not appear in the literature. Specific cases have been discussed and we mention them below.

[16]The general formula for the single-photon term was given by G. V. Grigoryan and V. A. Khoze, Yad. Fiz. 16, 1078 (1972) [Sov. J. Nucl. Phys. 16, 592 (1973)]. A formula for arbitrarily polarized initial beams but unpolarized muons was given by R. Budny [Phys. Lett. 45B, 340 (1973)] and A. McDonald [Nucl. Phys. B75, 343 (1975)]. Longitudinally polarized beams were discussed by K. C. Mikaelian, Phys. Lett. 55B, 221 (1975).