Diffractive channels in absorptive eikonal models*

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We have examined the effect of an arbitrary number of diffractive channels on absorptive eikonal elastic scattering amplitudes. Diffraction is assumed to be turned on by some coupling constant, well above threshold. We find that the absorption of the scatterer is decreased at all impact parameters, leading to a decrease in the elastic, nondiffractive inelastic, and total cross sections. Ratios of cross sections are also examined. Finally, we find that the peripheral nature of diffractive production directly affects the nature of elastic scattering amplitudes near the center of the scattering region.

I. INTRODUCTION

It has been known for some time that a complete description of high-energy scattering processes should include the existence of diffractive states.[1] Since the validity of the (s-channel unitary) absorptive eikonal model as a description of these processes is well established, it is of interest to investigate the effect of diffractive production processes on the model.

Much work has been done in this area, but two restrictive assumptions seem to be prevalent. Some authors limit the number of diffractive particles to one, and others choose particular functional forms for the high-energy elastic scattering profile function in the absence of diffraction. In this work we have lifted these restrictions.

One assumption underlying our calculation is that only terms of order $\sqrt{s}$ are kept in the numerators of fermion propagators. In particular, then, our results do not include the effects of the different masses of the diffractively produced particles, since all masses are assumed to be small in comparison to the center-of-mass energy. Therefore, the behavior we discuss is not expected to prevail near the threshold for the production of the diffractive states, a region of great interest in the interpretation of experimental data. Rather, we are examining the purely theoretical case in which diffractive channels are introduced at high energies via the turning on of some diffractive coupling constant. However, we feel that the present work is a small step towards the theoretical understanding of the full behavior of the cross sections.

The effect of diffractive channels is not intuitively obvious. The relation among the cross sections which must be satisfied is $\sigma_e + \sigma_d + \sigma_i = \sigma_t$. (These are, respectively, the elastic, diffractive, nondiffractive inelastic, and total cross sections.) As σ_d changes one could imagine many ways the other three cross sections could change in order to satisfy the equation. We find, among other results, that as σ_d increases just above zero, all three of the other cross sections will decrease. Such an effect can be attributed to s-channel unitarity.

The behavior that we predict for eikonal models modified to include diffraction is consistent with the features found by other authors using different models and approaches, where the results overlap. Blankenbecler showed that at high energies diffractively produced inelastic states will decrease the total cross section.[2] His proof used a Feynman diagram technique in which certain classes of diagrams were summed. Fried and Soh have illustrated this in the context of a particular field-theoretic eikonal model in which the existence of one diffractive state was postulated.[3] For the case of black disk scattering they find $\sigma_e/\sigma_t < 1/2$ and $(\sigma_e + \sigma_d)/\sigma_t = 1/2$. These results are also reported by Chang and Yan.[4] They used a simplified eikonal-Regge model in the black disk limit but included an infinite number of diffractive states. Pumplin has used a model similar to the one we describe in this paper and concluded that $(\sigma_e + \sigma_d)/\sigma_t \leq 1/2$ for black disk scattering.[5]

Skard and Fulco have considered a unitary multiperipheral model with diffractive production.[6] They find that the profile function is modified by an energy-dependent function $I(s) > 1$ which cannot grow faster than $\ln^2(s)$. For black disk scattering they find that $\sigma_e/\sigma_t = 1/2I(s)$. In the absence of diffraction the eikonal model with a grey disk profile of absorption $c \leq 1$ gives $\sigma_e/\sigma_t \leq c/2$.[7] Thus the interpretation of the function $I(s)$ is that diffractive channels will decrease the absorption of the profile.

In these papers two approaches have been followed. The first is explicit calculation of graphs in various field theories. The second is the inclusion of diffraction in absorptive eikonal models by making the S matrix $(1 - e^{i\chi})$ a true matrix of amplitudes, with off-diagonal elements describing diffractive production. Recent work has demonstrated that the matrix method is equivalent to the summation of certain contributing graphs in the eikonal approximation if the existence of only one resonant state is postulated.[8] The most straightforward way to include an arbitrary number of diffractive states would be to extend the size of the matrix describing scattering. In this paper we have made this generalization, calculated the cross sections, and then found limits on them.

We hasten to point out that the methods used here are not new. The matrix generalization of the eikonal amplitude has been described by other authors.[9] Also there exist excellent descriptions of the application of Lagrange extremization procedures to the solution of problems involving bounds on scattering amplitudes in the presence of various constraints.[10] Our work has been motivated in part by these earlier papers.

The remainder of this section is a presentation of our results. A complete description of the methods employed follows in the next section.

We find that when diffractive channels are turned on as indicated above, the absorption of the elastic profile is decreased at all impact parameters. Since the elastic and total cross sections are integrals of this profile function over all impact parameters, we find that both σ_e and σ_t are reduced in the presence of diffractive production. This decrease in absorption is also responsible for the decrease in σ_i.

The value of the ratios of cross sections is also of interest. We find, for real profile functions,

$$\frac{\sigma_e}{\sigma_t} \leqslant \frac{\sigma_e^{(0)}}{\sigma_t^{(0)}} \leqslant \frac{\sigma_e+\sigma_d}{\sigma_t} \leqslant \frac{1}{2}.$$

(The superscript zero refers to the case of zero coupling to the diffractive channels.)

The leftmost inequality is a consequence of the fact that the elastic cross section decreases faster than the total cross section as the profile function decreases. The middle inequality means that elastic plus diffractive production processes will make up more of the total cross section than will elastic processes in the absence of diffraction. The factor of $\frac{1}{2}$ is the familiar limit in the case of purely imaginary amplitudes.

It has been observed that diffraction is a peripheral process. For example, in Glauber theory one expects diffractively produced particles to come from collisions which occur at the edge of the nucleus.[11] Also one can indirectly observe the diffractive profile function, whose integral over impact parameters is σ_d. For p-p scattering it has been experimentally determined that this function peaks at an impact parameter of about 0.6 fermis.[12] This is to be contrasted to the elastic profile function which is largest at the origin.

Our calculation shows a relationship between the peripheral nature of diffractive production and the behavior of elastic scattering profiles near the center of the scattering region. We find that at impact parameters where diffractive production is least likely to occur, the two-body elastic scattering processes of any combination of states that exist in the problem are all described by the same profile function. For example, if diffractive production is zero out to some radius r_o, then there is one profile function which describes *all* elastic scattering processes for impact parameters less than r_o. Since the total scattering cross section is the integral of the elastic profile function, differences among the total scattering cross sections for the various states must be due to the nature of the interactions at large impact parameters.

II. CALCULATIONS

First we describe the incorporation of diffraction into an absorptive eikonal model. The elastic scattering amplitude is given by

$$M_e = 2is\int_0^\infty b\,db\,\Gamma(b)\,J_0(b\sqrt{-t}), \tag{1}$$

$$\Gamma(b) = 1 - e^{g(b)}, \quad 0 < \Gamma < 1. \tag{2}$$

Γ is the profile function, and $g(b)$ is the eikonal phase. The cross sections are

$$\sigma_e = 2\pi\int_0^\infty b\,db\,|\Gamma|^2, \tag{3}$$

$$\sigma_t = 4\pi\mathrm{Re}\int_0^\infty b\,db\,\Gamma, \tag{4}$$

$$\begin{aligned}\sigma_i = \sigma_t - \sigma_e &= 2\pi\int_0^\infty b\,db(2\mathrm{Re}\Gamma - |\Gamma|^2)\\ &= 2\pi\int_0^\infty b\,db(1-e^{2\mathrm{Re}\,g(b)}).\end{aligned} \tag{5}$$

This form arises from many sources, both classical multiple scattering theory in the optical limit[13] and field-theoretic calculations, for example, QED.[14] All these make different predictions for specific form of $g(b)$.

Diffractive channels are included by writing $g(b)$ as a matrix in the following way. Let N_1 be the "ground state" particle, and $N_2 \cdots N_m$ be the $m-1$ diffractive states. There are m^2 distinct two-particle states $|N_a N_b\rangle$. Then $g(b)$ is a square matrix of dimension m^2, whose elements $\langle N_c N_d | g(b) | N_a N_b\rangle$ are the eikonal phases which describe the elementary processes $N_a + N_b \to N_c + N_d$ in the absence of all other processes. Buried in the exponentiation of the matrix $g(b)$ is the description of the way these elementary processes combine to generate the physically observed amplitudes.

Applications of this exponentiation procedure to scattering processes with the inclusion of only one diffractive state exist in the literature. Diffraction was included in the Chou-Yang optical model in this way by Elitzur and Lipes.[15] In the field-theoretic eikonal model previously mentioned[8] the elementary amplitudes were assumed to be mediated by the exchange of some leading (e.g., vector) potential, with some diffractive coupling constant at production vertices. In this paper it was demonstrated that exponentiation of the matrix of elementary amplitudes was equivalent to summing of the leading behavior of all multiple-exchange graphs.

It is important to remember that this procedure is valid only at energies well above the threshold for diffraction. This is because the eikonal form and its matrix generalization are valid only when the masses of all particles can be neglected with respect to the center-of-mass energy. As previously discussed, when we refer to the onset of diffraction we imagine that this happens via some diffractive coupling constant which appears in the elementary production amplitudes.

The matrix generalizations of Eq. (1) and (2) are

$$M_{ij}=2is\int_0^\infty b\,db\,J_0(b\sqrt{-t}\,)\Gamma_{ij}\,, \tag{6}$$

$$\Gamma_{ij}=(1-e^{g(b)})_{ij}\equiv-\sum_{l=1}^{\infty}\frac{1}{l!}(g^l)_{ij}. \tag{7}$$

To exponentiate the matrix $g(b)$ and therefore find the elements Γ_{ij} one first diagonalizes g. The unitary transformation U does this.

$$g_D=U^{-1}gU\,, \tag{8}$$

$$(\Gamma_D)_{ij}=-\sum_{l=1}^{\infty}\frac{1}{l!}(g_D^l)_{ij}. \tag{9}$$

The subscript D denotes a diagonal matrix. The elements of g_D are the eigenvalues λ_k of g, so $(g_D^l)_{ij}=\lambda_i^l\,\delta_{ij}$, giving $(\Gamma_D)_{jj}=1-e^{\lambda_j}$.

Now we transform back to get Γ_{ij}, the matrix of profiles for the physical processes:

$$\Gamma_{ij}=(U\Gamma_D U^{-1})_{ij}=-\sum_{l=1}^{\infty}\frac{1}{l!}U_{ip}(g_D^l)_{pk}U^{-1}{}_{kj}\ .$$

Using the unitary properties of U,

$$\sum_{p=1}^{m^2}U_{ip}U_{pj}^{-1}=\delta_{ij}\,,$$

we find

$$\Gamma_{ij}=\delta_{ij}-\sum_{p=1}^{m^2}U_{ip}\,U^{-1}{}_{pj}e^{\lambda_p}. \tag{10}$$

This formalism is consistent with the ideas of Good and Walker.[1] The incident wave is written as a linear combination of the eigenstates of the matrix g. These eigenstates are only elastically scattered by the absorber, since g_D has no off-diagonal elements. The profile function $1-e^{\lambda_p}$ describes the absorption of the pth eigenstate. The eigenfunctions $\lambda_p(b)$ are in general different, so the scattered wave is different from the initial wave. The final wave is then written as a linear combination of the physical states $|N_aN_b\rangle$.

The profile function describing the elastic process $N_1+N_1\to N_1+N_1$ is Γ_{11} of Eq. (10). We insert the expression Γ_{11} into Eqs. (3) and (4), getting

$$\sigma_e=2\pi\int_0^\infty b\,db\left|1-\sum_{p=1}^{m^2}|U_{1p}|^2e^{\lambda_p}\right|^2, \tag{11}$$

$$\sigma_t=4\pi\,\mathrm{Re}\int_0^\infty b\,db\left(1-\sum_{p=1}^{m^2}|U_{1p}|^2e^{\lambda_p}\right). \tag{12}$$

The diffractive production cross section is calculated as follows. Label the states from $|1\rangle$ to $|m^2\rangle$. $|1\rangle$ is the state $|N_1N_1\rangle$. The profile Γ_{1l} describes the production of $|l\rangle$ from $|1\rangle$. The cross section for the production of this state is

$$\sigma_d^l=2\pi\int_0^\infty b\,db\,|\Gamma_{1l}|^2.$$

The total diffractive production cross section is the sum over all possible diffractive states;

$$\sigma_d=\sum_{l=2}^{m^2}\sigma_d^l$$
$$=2\pi\int_0^\infty b\,db\sum_{p=1}^{m^2}U_{1p}e^{\lambda_p}\sum_{j=1}^{m^2}U^{-1}{}_{j1}e^{\lambda_j^*}\sum_{l=2}^{m^2}U^{-1}{}_{pl}U_{lj}.$$

Using the fact that

$$\sum_{l=2}^{m^2}U^{-1}{}_{pl}U_{lj}=\delta_{pj}-U^{-1}{}_{p1}U_{1j},$$

we can simplify the expression for σ_d:

$$\sigma_d=2\pi\int_0^\infty b\,db\sum_{p=1}^{m^2}U_{1p}e^{\lambda_p}\sum_{j=1}^{m^2}U^{-1}{}_{j1}e^{\lambda_j^*}(\delta_{pj}-U^{-1}{}_{p1}U_{1j})$$
$$=2\pi\int_0^\infty b\,db\left(\sum_{p=1}^{m^2}|U_{1p}|^2e^{2\mathrm{Re}\lambda_p}-\left|\sum_{p=1}^{m^2}|U_{1p}|^2e^{\lambda_p}\right|^2\right). \tag{13}$$

The inelastic cross section is calculated from the expression $\sigma_i=\sigma_t-(\sigma_e+\sigma_d)$.

$$\sigma_i=2\pi\int_0^\infty b\,db\left(1-\sum_{p=1}^{m^2}|U_{1p}|^2e^{2\mathrm{Re}\lambda_p}\right). \tag{14}$$

The cross sections are formally functions of the elements of the unitary transformation $|U_{1p}|^2$ and the eigenvalues λ_p of the matrix g. We vary the cross sections with respect to these parameters to find the extrema. The constraints which are used in the Lagrange procedure are those of the unitarity of the transformation U; that is, the rows and columns of U are orthonormal and the trace of g is invariant under the rotation. This gives us the two constraint equations;

$$\sum_{p=1}^{m^2} |U_{1p}|^2 = 1, \tag{15}$$

$$\mathrm{Tr}\, g = \mathrm{Tr}\, g_D \Rightarrow \sum_{p=1}^{m^2} g_{pp} \equiv \Lambda(b) = \sum_{p=1}^{m^2} \lambda_p(b). \tag{16}$$

In principle it is the elements of the matrix $g(b)$ which are known. The diagonal elements describe all elementary elastic scattering processes, and the off-diagonal elements describe elementary diffractive production processes. As we stated before, elementary elastic and diffractive processes are assumed independent of one another. It is the exponentiation that contains the physics of the way they interfere. Therefore we assume that the elastic eikonal phases $g_{jj}(b)$ are independent of the diffractive production coupling constant, so the function $\Lambda(b)$ in Eq. (16) is not altered as the strength of the diffractive production is changed. Therefore we may use Λ as a constant constraint in the Lagrangian.

Up to this point the eigenvalues $\lambda_p(b)$ have been assumed to be complex. However, at high energies one expects diffractive amplitudes (both elastic and production) to be imaginary, meaning that the eikonal phases are real. For the rest of this paper we make this simplifying assumption.

We are now ready to calculate the bounds. We start with consideration of the elastic scattering profile function.

Equation (10) with $i = j = 1$ is the function we need. It is

$$\Gamma_{11} = 1 - \sum_{p=1}^{m^2} u_p e^{\lambda_p}, \tag{17}$$

where $u_p \equiv |U_{1p}|^2 \geq 0$ for simplicity. The Lagrangian is, with multipliers α and β and the constraints from Eqs. (15) and (16),

$$L = 1 - \sum_{p=1}^{m^2} u_p e^{\lambda_p} + \alpha\left(1 - \sum_{p=1}^{m^2} u_p\right) + \beta\left(\Lambda - \sum_{p=1}^{m^2} \lambda_p\right). \tag{18}$$

The first and second derivatives of L are

$$\frac{\partial L}{\partial u_p} = -e^{\lambda_p} - \alpha, \quad p = 1, \cdots, m^2 \tag{19}$$

$$\frac{\partial L}{\partial \lambda_p} = -u_p e^{\lambda_p} - \beta, \quad p = 1, \cdots, m^2 \tag{20}$$

$$\frac{\partial^2 L}{\partial u_q \partial u_p} = 0, \quad q, p = 1, \cdots, m^2 \tag{21}$$

$$\frac{\partial^2 L}{\partial \lambda_q \partial \lambda_p} = -u_p e^{\lambda_p}\delta_{pq} \leq 0, \quad q, p = 1, \cdots, m^2 \tag{22}$$

$$\frac{\partial^2 L}{\partial u_q \partial \lambda_p} = -e^{\lambda_p}\delta_{pq} \leq 0, \quad q, p = 1, \cdots, m^2. \tag{23}$$

Normally we would set the first derivatives of the Lagrangian to zero in order to find the local extrema. However, it is also necessary that all the eigenvalues of the matrix of second derivatives have the same sign and be nonzero. This is not the case here, so there are no local extrema.

Consider that all the λ_p are given, so the sum in the expression for Γ_{11} [Eq. (17)] is a linear function of the u_p. Since the relationship among all the u_p themselves is linear [Eq. (15)], the sum remains a linear function of each of the u_p when the constraints are taken into account. Thus the extrema of Γ_{11} must occur when the u_p assume their endpoint values. The only way for this to occur and simultaneously satisfy Eq. (15) is to choose one of them equal to one and the rest zero. Therefore, to extremize Γ_{11} we choose $u_1 = 1$ and all other $u_p = 0$.

The minimum value Γ_{11} can have is zero. This corresponds to no interactions, since all the cross sections would then be zero. The physically interesting extreme of Γ_{11} is its maximum value, which means we are looking for the minimum value of e^{λ_1} subject to the constraint of Eq. (16). Since all the λ_p are negative, choose $\lambda_1 = \Lambda$ and all other $\lambda_p = 0$. Then we have $(\Gamma_{11})_{\max} = 1 - e^{\Lambda}$.

This can be seen more simply in the following way. The sum in the expression for Γ_{11} can be written with the constraints on the u_p included;

$$\begin{aligned} S &= \sum_{p=1}^{m^2} u_p e^{\lambda_p} \\ &= \sum_{p=1}^{m^2-1} u_p e^{\lambda_p} + e^{\lambda_{m^2}}\left(1 - \sum_{p=1}^{m^2-1} u_p\right) \\ &= \sum_{p=1}^{m^2-1} u_p\left(e^{\lambda_p} - e^{\lambda_{m^2}}\right) + e^{\lambda_{m^2}}; \quad \sum_{p=1}^{m^2-1} u_p \leq 1. \end{aligned}$$

To minimize S, choose the λ_p so that $e^{\lambda_{m^2}}$ is the smallest of all the e^{λ_p}. Then the first term in S is positive, so we can minimize it by choosing $u_p = 0$, $p \leq m^2 - 1$. Then $S_{\min} = e^{\lambda_{m^2}}$ where $\lambda_{m^2} = \Lambda$, the most negative value it can have.

Therefore we are let to the conclusion that the elastic scattering profile function is decreased at the onset of diffraction. Since the eigenvalues λ_p are functions of b, this decrease occurs at all impact parameters. As we stated before, this means that σ_e, σ_t, and σ_e/σ_t are all decreased.

The reader can easily see that σ_i is also decreased. Replace the function Γ_{11} in Eq. (18) by the integrand of Eq. (14), which determines the inelastic cross section. The proof goes as before, and we conclude that the contributions to the nondiffractive inelastic cross section are reduced at all impact parameters.

We wish to consider the ratio $(\sigma_e + \sigma_d)/\sigma_t$. The easiest way to approach this is to examine the function $\sigma_e+\sigma_d-\frac{1}{2}\sigma_t$. From Eqs. (11)–(13) we find

$$\sigma_e+\sigma_d-\tfrac{1}{2}\sigma_t=2\pi\int_0^\infty b\,db\sum_p u_p e^{\lambda_p}(e^{\lambda_p}-1). \quad (24)$$

This expression is a linear function of the u_p, exactly as we found when we considered the elastic scattering profile function. Therefore the extremum exists when $u_1=1$ and all other $u_j=0$. As before, this condition on the u_j makes U the unit matrix, and this in turn makes the physical and diagonal spaces the same; the off-diagonal elements of g are zero. Thus the extremum exists when there is no coupling among the different channels, or equivalently when $\sigma_d=0$ [Eq. (13)]. The extremum is $2\pi\int_0^\infty b\,db\; e^{\lambda_1}(e^{\lambda_1}-1)=\sigma_e^{(0)}-\frac{1}{2}\sigma_t^{(0)}$.

The trivial extremum is a maximum, when $\lambda_1=0$. This corresponds to all cross sections being zero. Thus we are interested in the minimum value, and we have shown that

$$\sigma_e+\sigma_d-\tfrac{1}{2}\sigma_t\geq\sigma_e^{(0)}-\tfrac{1}{2}\sigma_t^{(0)}. \quad (25)$$

If we define the ratios $R=(\sigma_e+\sigma_d)/\sigma_t$, $R_0=\sigma_e^{(0)}/\sigma_t^{(0)}$, we may write Eq. (25) as

$$(R-\tfrac{1}{2})\sigma_t\geq(R_0-\tfrac{1}{2})\sigma_t^{(0)}. \quad (26)$$

This inequality, plus the two relations $R_0\leq\frac{1}{2}$ and $0<\sigma_t<\sigma_t^{(0)}$, allows us to write this as $R_0<R<\frac{1}{2}$, which is the result discussed in the first section of this paper.

Finally we wish to discuss the implications of the peripheral nature of diffractive production. As previously noted, the finction $\Gamma_p=1-e^{\lambda_p}$ describes the elastic scattering of the pth eigenchannel. The elements of the unitary transformation $|U_{1p}|^2=u_p$ can be interpreted as the fraction of the pth eigenchannel which is in the initial state. Thus the elastic scattering profile in the physical system, Γ_{11}, is just the average value of the elastic scattering eigenprofiles. That is, $\Gamma_{11}=\sum_p u_p\Gamma_p\equiv\overline{\Gamma}$, by Eq. (10). Similarly, we note that σ_d is the integral of the dispersion of the Γ_p;

$$\sigma_d=2\pi\int_0^\infty b\,db\,\langle(\Gamma_p-\overline{\Gamma})^2\rangle_{av}. \quad (27)$$

Since diffraction is expected to be peripheral, contributions to σ_d are expected to be small in the central region and largest near the edge of the scattering region. Thus Eq. (27) means that the $\Gamma_p(b)$ all behave alike near the center ($\Gamma_p\simeq\overline{\Gamma}$ for all p). Now

$$\begin{aligned}\Gamma_{ij}(b)&=(U\Gamma_D U^{-1})_{ij}\\&=\sum_{k,l}U_{ik}(\Gamma_D)_{kl}U^{-1}{}_{lj}\\&=\sum_k U_{ik}(\Gamma_D)_k U^{-1}{}_{kj}.\end{aligned}$$

If all $\Gamma_k\equiv(\Gamma_D)_k\simeq\overline{\Gamma}(b)$, then $\Gamma_{ij}(b)\simeq\overline{\Gamma}(b)\sum_k U^{-1}{}_{kj}=\overline{\Gamma}(b)\delta_{ij}$.

In other words, all elastic scattering profiles $\Gamma_{ii}(b)$ are approximately the same in the central region. Thus around the center of the scatterer the profile for the process $N_a+N_b\to N_a+N_b$ is nearly independent of the particular particles N_a and N_b which are being scattered, and this behavior is a consequence of the experimentally observed peripheral nature of diffractive production. Whether the correct handling of thresholds will affect the strong result expressed here is not known to us.

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[1]M. L. Good and W. D. Walker, Phys. Rev. 120, 1857 (1960).
[2]R. Blankenbecler, Phys. Rev. Lett. 31, 964 (1973).
[3]H. M. Fried and K. S. Soh, Phys. Rev. D 10, 636 (1974).
[4]S. J. Chang and T. M. Yan, Phys. Rev. D 10, 1531 (1974)
[5]J. Pumplin, Phys. Rev. D 8, 2899 (1973).
[6]J. A. J. Skard and J. R. Fulco, Phys. Rev. D 8, 312 (1973).
[7]P. J. Camillo, P. M. Fishbane, and J. S. Trefil, Phys. Rev. D 10, 3022 (1974).
[8]J. G. Schaffner, P. J. Camillo, P. M. Fishbane, and J. S. Trefil, Phys. Rev. D 12, 244 (1975).
[9]R. Blankenbecler, J. R. Fulco, and R. L. Sugar, Phys. Rev. D 9, 736 (1974).
[10]M. B. Einhorn and R. Blankenbecler, Ann. Phys. (N. Y.) 67, 480 (1971); R. Savit, R. Blankenbecler, and M. B. Einhorn, J. Math. Phys. 12, 2092 (1971).
[11]R. J. Glauber, in *High Energy Physics and Nuclear Structure*, edited by G. Alexander (North-Holland, Amsterdam, 1967), pp. 311 ff.
[12]L. Caneschi, P. Grassberger, H. I. Miettinen, and F. Henyey, Phys. Lett. 56B, 359 (1975).
[13]W. Czyż and L. C. Maximon, Ann. Phys. 52, 59 (1969).
[14]H. Cheng and T. T. Wu, Phys. Rev. Lett. 24, 1456 (1970).
[15]M. Elitzur and R. G. Lipes, Phys. Rev. D 7, 1420 (1973).