PHYSICAL REVIEW D VOLUME 13, NUMBER 2 1 FEBRUARY 1976

Cabibbo angle and quark mass spectrum[*]

A. Zee[†]

Department of Physics, Princeton University, Princeton, New Jersey 08540

(Received 22 September 1975)

We propose that the light quarks $\mathcal{P}$, $\mathcal{N}$, and λ are massless when the weak interaction is switched off. Because of the mass degeneracy between the $\mathcal{N}$ and λ fields the Cabibbo angle θ is undefined. When the weak interaction is switched on the light quarks manage to acquire masses by undergoing weak radiative processes involving heavy quarks (such as the charm quark), thus lifting the mass degeneracy. By identifying appropriate linear combinations of the $\mathcal{N}$ and λ fields as the physical $\mathcal{N}$ and λ states one in effect determines the Cabibbo angle. In this view, the orientation between the strong and weak interactions is controlled by the structure of the weak interaction. We assume strong interactions to be asymptotically free. The implementation of this program leads us to consider a "vector-like" weak interaction theory which involves six quarks and right-handed currents and which has been discussed on phenomenological grounds elsewhere. (One feature of this program is that the smallness of the neutral kaon mass difference is ensured without further adjustments.) The Cabibbo angle θ and the mass ratio $m_{\mathcal{N}}/m_{\lambda}$ (which measures the extent of chiral symmetry breaking) are thus fixed in terms of some of the other parameters appearing in the theory, namely the mass ratio of the two positively charged heavy quarks and various mixing angles, for instance, the analog of θ in the right-handed current. These parameters are all measurable in principle. Our knowledge of θ and $m_{\mathcal{N}}/m_{\lambda}$ then enables us to make various predictive statements about these ratios and consequently the structure of the weak interaction at higher energies. Our point of view also leads naturally to an intimate connection between zero θ and exact chiral SU(2) symmetry.

I. INTRODUCTION

The origin of the angle[1] between the strong and weak interactions has long been shrouded in mystery. Evidently, to understand this angle one must first construct reasonably good theories of both interactions. Happily, we have seen, within recent years, enormous progress in this direction so that we are now fairly well assured that the fundamental interactions of the world involve color[2] quarks and gauge gluons, with the strong interaction based on the color group[3] and with the weak (and electromagnetic) interaction[4] based on a subgroup of the flavor[5] group. How then can one understand the Cabibbo angle within this framework?

To begin with, one may trivially remark that if the $\mathcal{N}$ quark and the λ quark are equal in mass then the Cabibbo angle θ is undefined. Thus the origin of θ must be intimately intertwined with the origin of the quark mass spectrum. Whatever mechanism that splits the $\mathcal{N}$ and λ quark should also generate the Cabibbo angle. In the next section we will discuss our present knowledge of the quark mass spectrum.

The rest of this paper is conveniently subdivided into a number of sections. Our motivations and point of view are discussed in Secs. II through VI. These have already been partially explored in an earlier work[6] by this author. The details of our picture are given in Sec. VII while the consequences, including, we hope, experimentally accessible ones, are discussed in Sec. VIII. A conclusion follows in Sec. IX. The considerations of this paper are to a certain extent qualitatively summarized in Eq. (8.15).

II. THE QUARK MASS SPECTRUM

The success of chiral symmetry[7] informs us that the $\mathcal{P}$ and $\mathcal{N}$ quarks are almost massless and considerably less massive than the λ quark. The tremendous series of experimental discoveries[8] of the last months have provided strong evidence[9] that the long-sought charm (c) quark[10] does exist. Its mass appears to be substantially higher than that of the λ quark. Further experimental developments and theoretical considerations[11] have led to the suggestion that there exist more quarks, beyond charm, and somewhat more massive than the charmed quark. The reader should consult Ref. 11 for the arguments on which this conclusion is based.

The following picture appears to emerge: There are three quarks, almost massless, with mass differences among themselves, but well-separated[12] in mass from another group of quarks. One cannot rule out at this stage the possibility of yet further groups of quarks, with mass differences within each group but well separated in mass from each other. At present, we will work with just two groups of quarks. We will refer to the $\mathcal{P}$, $\mathcal{N}$, and λ quarks as light quarks.

Parenthetically, we should remark that by "mass" we mean the mass parameter that appears in the Lagrangian. Since the theory allegedly produces quark confinement[13] the physical mass of quarks

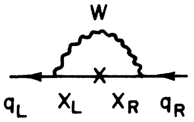


FIG. 1. In order for a massless quark q to acquire a mass from weak interaction, q_R has to transform into X by emitting an intermediate vector boson W and back into q_L by reabsorbing W. X is some massive quark. The cross denotes a mass insertion on the X propagator.

is a meaningless concept. These mass parameters depend on the renormalization point[14] but the dependence is weak enough to be neglected for the purposes of this paper as long as the renormalization point is well below the mass of the intermediate vector boson. In any case, it is not exactly clear how these mass parameters are to be related to observed hadronic masses. For the heavy quarks at least, one imagines that they form loosely bound mesons[9] and their masses could thus be identified. On the other hand, the light quarks form deeply bound relativistic systems and their actual masses are quite unknown.

Now, this picture of quark mass spectrum can emerge within the framework of gauge theories in two different ways:

(a) All the quarks become massive "at tree level" from the vacuum expectation value of Higgs fields[4] in the theory.

(b) Only some of the quarks become massive "at tree level" from the vacuum expectation value of Higgs fields. The rest remain massless for some reason and become massive by dint of undergoing the radiative process depicted in Fig. 1.

We very much favor possibility (b) over (a). Firstly, one would have to resort to artifice to arrange the Higgs fields to produce widely different mass scales. Either the couplings or the vacuum expectation values have to be vastly different, or else some of the Clebsch-Gordon coefficients have to be abnormally large. Perhaps more importantly, we may argue that we should subscribe to the esthetically appealing view that Higgs fields are not elementary but are merely manifestations of dynamical symmetry breaking.[15] However, we expect that dynamical symmetry breaking to produce only one mass scale. The relatively small mass of the familiar $\mathcal{P}$, $\mathcal{N}$, and λ quarks can then only be the result of radiative correction.

We thus propose a picture in which the $\mathcal{N}$ and λ quarks are massless at tree level. Weak radiation then splits this degeneracy and picks out the eigenstates corresponding to the true $\mathcal{N}$ and λ quarks, thus defining a direction for strong interaction. At the same time, the relative strength of the weak couplings of $\mathcal{P}_L$ to $\mathcal{N}_L$ and λ_L become determined, by weak interaction itself, so to speak. This mechanism will be implemented in what follows.

III. MASS FROM WEAK RADIATION

In order for a massless fermion q to acquire a mass from weak and electromagnetic interactions, it will be necessary to have a gauge boson W which couples q_L to X_L and q_R to X_R, where X is some massive fermion (see Fig. 1). Hence, $\mathcal{P}_R$, $\mathcal{N}_R$, and λ_R must belong to some nontrivial representation of the weak-interaction gauge group. One is thus immediately threatened by the presence of right-handed current involving $\mathcal{P}$, $\mathcal{N}$, and λ quarks which may (i) void the success of the current-algebraic[7] treatment of nonleptonic decay[16] and (ii) ruin low-energy phenomenology. These pose severe constraints on model building.

We must now commit ourselves to a specific theory of weak interaction. The gauge group will be conventionally chosen to be[17] SU(2)×U(1). We insist, however, that all quark fields, left- and right-handed, be assigned to doublets. The attractiveness of this assumption has been emphasized elsewhere.[18] For instance, the neutral current in such a theory is a purely vector current and the hadronic anomalies cancel. It also provides a possible rationale for the central feature of weak interaction, namely parity violation. The underlying coupling is perfectly symmetrical between right-handed and left-handed fields. The fermion mass matrix happens to be such that the right-handed currents couple light quarks to heavy quarks only while the left-handed currents couple light quarks to light quarks.

We have nothing to say about the leptons except that we shall assume the known left-handed leptons are assigned to doublets, viz.,

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L. \tag{3.1}$$

It is easy to see that the design requirements mentioned cannot be met with only four quarks at one's disposal. We need at least six quarks. In this paper we will assume the existence of six quarks (although the ideas presented here are readily adaptable to the case of more than six quarks). Indeed, the present author has constructed some time ago a specific model[6] which realizes precisely the remarks made here and which involves six quarks. We will have more to say about this model later.

IV. ANGLES AND MASS RATIOS

For the moment let us anticipate what will emerge from the detailed analysis to be given in

Secs. VII and VIII. Our problem is a standard one in degenerate perturbation theory. After weak interaction is switched on, we will encounter a two-by-two mass matrix for the $\mathcal{N}$ and λ system. We diagonalize this matrix in order to pick out the physical $\mathcal{N}$ and λ state (and thus determine the Cabibbo angle). With six quarks in three left-handed and three right-handed doublets there will in general be a number of mixing angles. Indeed, the Cabibbo angle is just one among this multitude of angles. Thus, we can only expect to obtain relations between these angles and various quark mass ratios, including in particular $m_{\mathcal{N}}/m_{\lambda}$. These relations, to be given below in Sec. VIII, are not without predictive value, however; when heavy quark states are produced, their relative couplings and mass ratio should be accessible to measurement.

V. RIGHT-HANDED CURRENTS

One necessary feature of the present scheme is the presence of right-handed currents. Recently, such currents have been extensively discussed by several groups of theorists.[19] In particular, Wilczek, Zee, Kingsley, and Treiman[11] have proposed a number of models, among which they favor one with six quarks.

We write the proposed model here with the Cabibbo angle set equal to zero for simplicity:

$$\begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} \mathcal{P} \\ \mathcal{N} \end{pmatrix}_L, \begin{pmatrix} r \\ \lambda \end{pmatrix}_L, \begin{pmatrix} \mathcal{P} \\ s \end{pmatrix}_R, \begin{pmatrix} c \\ \mathcal{N} \end{pmatrix}_R, \begin{pmatrix} r \\ \lambda \end{pmatrix}_R. \tag{5.1}$$

Here r and s denote two additional quarks beyond the charmed quark c, and the model is represented, as is customary, by the doublet content of the weak gauge group. We will take this opportunity to summarize briefly the features of this model as discussed by Wilczek *et al.*[11]

(a) *Copious production of dimuons.* The coupling of c_R to $\mathcal{N}_R$ allows for copious production of dimuons $\mu^+\mu^-$ pair in neutrino experiments.[20]

(b) K_L-K_S *mass difference and rare K decay.* λ_R couples to the same quark (namely r) as λ_L. This is necessary to avoid an excessively large mass difference between K_L and K_S.

(c) *Nonleptonic decay and V-A phenomenology.* The constraints (i) and (ii) mentioned in Sec. III are satisfied.

(d) *Wrong sign dimuons.* The neutral mesons D^0 and $\bar{D}^0$ can mix via the heavy s in order G^2. This may provide a possible explanation[21] for the production of "wrong" sign dimuon $\mu^-\mu^-$ pairs[22] in neutrino experiments.

Since there are now as many as six quarks and since the nomenclature is far from unified it may be convenient to provide a dictionary of quark names here:

$\mathcal{P}$: the up quark;
$\mathcal{N}$: the down quark;
λ: the strange quark;
c: the charm quark; however, it need not be coupled in precisely the same way as in Ref. 10.
r: a charge $+\frac{2}{3}$ quark;
s: a charge $-\frac{1}{3}$ quark coupled mainly to $\mathcal{P}_R$.

VI. CABIBBO ANGLE AND CHIRAL SYMMETRY

The model written in the previous section is evidently an idealization of the real world with various possible mixing angles all set equal to zero. Its very simplicity, however, allows us to have a preliminary glimpse of how the mechanism discussed here will actually function before we launch ourselves into a detailed investigation. Let us then imagine that for some reason the $\mathcal{P}$, $\mathcal{N}$, and λ quarks are all massless. Now switch on weak interaction. The λ quark acquires mass through its coupling to the massive r quark. In contrast the $\mathcal{P}$ and $\mathcal{N}$ quarks remain massless since $\mathcal{P}_L$ and $\mathcal{N}_L$ are coupled to each other. Thus the degeneracy between $\mathcal{N}$ and λ will be lifted but in a way such that the Cabibbo angle remains zero and such that chiral SU(2)×SU(2) is an exact symmetry. That an intimate connection exists between zero Cabibbo angle and exact chiral SU(2)×SU(2) has been suspected for a long time.[23,24] Within our present framework, it is a necessary consequence of the doublet assignments and has been discussed by us in Ref. 6. It holds, in particular, for the model to be discussed in the next section.

VII. WEAK INTERACTION WITH SIX QUARKS

Keeping all the preceding remarks in mind, we now assign the six quarks as follows:

$$\begin{aligned}
&\psi_{1R} = \begin{pmatrix} -\mathcal{P} \\ s \end{pmatrix}_R, \quad \psi_{2R} = \begin{pmatrix} c \\ \mathcal{N}_0 \end{pmatrix}_R, \\
&\psi_{3R} = \begin{pmatrix} r \\ \lambda_0 \end{pmatrix}_R, \\
&\psi_{1L} = \begin{pmatrix} -\mathcal{P}\sin\alpha + \cos\alpha c(\beta) \\ s \end{pmatrix}_L, \\
&\psi_{2L} = \begin{pmatrix} \mathcal{P}\cos\alpha + \sin\alpha c(\beta) \\ \mathcal{N}_0 \end{pmatrix}_L, \\
&\psi_{3L} = \begin{pmatrix} r(\beta) \\ \lambda_0 \end{pmatrix}_L,
\end{aligned} \tag{7.1}$$

where $c_L(\beta)$ and $r_L(\beta)$ are two orthogonal mixtures of c_L and r_L,

$$c_L(\beta) = \cos\beta\, c_L + \sin\beta\, r_L\,,$$

$$r_L(\beta) = -\sin\beta\, c_L + \cos\beta\, r_L\,.$$

Several remarks are in order here.

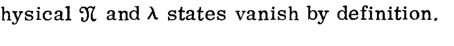
(1) This would have been the most general assignment if we had also put in an angle mixing p_R with c_R and r_R. However, we know from β decay and hyperon decay that this angle has to be very small.[25] Our analysis will not be substantially affected by carrying this very small angle along. In the interest of notational simplicity we will set this angle to zero.

(2) It is important to note that $\mathcal{N}_0$ and λ_0 are massless and degenerate at this stage. Thus, one may take arbitrary linear combinations of ψ_{2R} and ψ_{3R} and of ψ_{2L} and ψ_{3L}. In particular, we have chosen linear combinations such that p_L does not appear in ψ_{3L}.

(3) Also, at this stage $\mathcal{N}_{0L}$ and $\mathcal{N}_{0R}$ are merely symbols for two-component fields that may not have anything to do with each other. The same holds true for λ_{0L} and λ_{0R}. We should have put "primes," say, on the left-handed fields to emphasize this trifling failing of notation. What $\mathcal{N}_{0L}$ and λ_{0L} actually are in terms of the physical $\mathcal{N}_L$ and λ_L fields is, of course, for weak interaction to decide and for us to determine for the rest of this paper.

To be sure, it behooves one to exhibit a specific model which actually possesses all the desired features: the presence of three naturally massless quarks and the absence of right-handed currents coupling these quarks to each other. This state of affairs will have to come about in a natural way so that the masses acquired by the quarks are calculable. "Naturalness" is a powerful constraint on model building and has been particularly emphasized by Weinberg in a series of papers.[26]

Here we only wish to know that such a model in fact exists. In Ref. 6 we have constructed precisely a model of this kind. Unfortunately, in the present state of the art, symmetry breaking[15] has to be induced by elementary Higgs fields. In the model of Ref. 6 we had to introduce, besides a multitude of Higgs fields, three sets of SU(2) gauge fields. Finite and calculable masses result from cancellation among these gauge fields.



(a) (b)

FIG. 2. (a) High-frequency contribution to K_L-K_S mass difference from second-order weak interaction. (b) Off-diagonal elements of the mass matrix for the physical $\mathcal{N}$ and λ states vanish by definition.

In this paper we will not be concerned with such details. We will be agnostic on the behavior of weak interaction for energy much larger than ~100 GeV. We will not inquire into the origin of the heavy quark masses. In fact, all we need to know is that the mass acquired by q in the process depicted in Fig. 1 is proportional to the mass of X, the coupling of q_R to X_R, and the coupling of q_L to X_L. The proportionality factor is a numerical constant of order α times a logarithmic factor of a large mass of the order m_W.

Our assumption that the strong interaction is asymptotically free[27] enables one to neglect strong interactions entirely, at least in calculating the contribution of the high-frequency region. We assume that this contribution dominates.

An important remark is the following. As discussed in Ref. 11 and as mentioned in Sec. V the K_L-K_S mass difference imposes a severe constraint on the way right-handed currents can be introduced. The diagram in Fig. 2(a) gives a large high-frequency contribution. We note, however, that within the program outlined here the leading contribution cancels to zero since the physical $\mathcal{N}$ and λ states are diagonal to this order of weak interaction. In other words, the diagram in Fig. 2(b) is zero by construction and the diagrams in Fig. 2(a) and Fig. 2(b) involve the same combinations of couplings and masses. This illustrates a particular mechanism[28] for suppressing the leading "left-right" contribution to the K_L-K_S mass difference, namely by cancellation among the contributions of various charge $+\frac{2}{3}$ quarks.

VIII. RELATIONS BETWEEN ANGLES AND MASS RATIOS

Having said all this we can now write down the masses acquired by the low-lying quarks merely by inspecting Eq. (7.1). The $\mathcal{P}$-quark mass is

$$m_{\mathcal{P}} = \xi m_S \sin\alpha\,. \tag{8.1}$$

Here ξ denotes all those factors which were mentioned and which do not concern us, including a factor which may depend on the nature of weak interaction at high frequencies. The mass acquired by the $\mathcal{N}$-λ system may be described by the following contribution to the effective Lagrangian:

$$\mathcal{L}_{\rm eff} = -(\bar{\mathcal{N}}_0,\ \bar{\lambda}_0)_L\, m \begin{pmatrix} \mathcal{N}_0 \\ \lambda_0 \end{pmatrix} + \text{H.c.}, \tag{8.2}$$

where m denotes a two-by-two mass matrix

$$m=\xi\begin{pmatrix} m_c\sin\alpha\cos\beta & m_r\sin\alpha\sin\beta \\ -m_c\sin\beta & m_r\cos\beta \end{pmatrix}. \tag{8.3}$$

The (asymmetric) mass matrix m will now reveal to us which linear combinations of $\mathfrak{N}_0$ and λ_0 are in fact the physical $\mathfrak{N}$ and λ states if we will bring it to diagonal form by performing independent rotations on the left and on the right:

$$R_L^T(\theta)\,mR_R(\varphi)=\begin{pmatrix} m_{\mathfrak{N}} & 0 \\ 0 & m_\lambda \end{pmatrix}. \tag{8.4}$$

The rotation angle θ is the Cabibbo angle, of course, viz.,

$$\begin{pmatrix} \mathfrak{N}_0 \\ \lambda_0 \end{pmatrix}_L = R_L(\theta)\begin{pmatrix} \mathfrak{N} \\ \lambda \end{pmatrix}_L = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \mathfrak{N} \\ \lambda \end{pmatrix}_L. \tag{8.5}$$

The angle ϕ is its right-handed analog and measures the relative strength of c_R coupling to $\mathfrak{N}_R$ and λ_R.

Equation (8.4) informs us that there exist three relations enabling us to determine the three parameters $\{\theta, \phi, m_{\mathfrak{N}}/m_\lambda\}$ in terms of the three parameters $\{\alpha, \beta, m_c/m_r\}$ or vice versa. We may perhaps list these relations at this point. A certain amount of arithmetic yields

$$\sin^2\alpha=(I_2I_3)/(I_1I_4)\,, \tag{8.6}$$

$$(m_c/m_r)^2=(I_2I_4)/(I_1I_3)\,, \tag{8.7}$$

$$\tan^2\beta=(I_3I_4)/(I_1I_2)\,, \tag{8.8}$$

where

$$\begin{aligned} I_1 &=\cos\theta\,\cos\varphi+\sin\theta\,\sin\varphi(m_{\mathfrak{N}}/m_\lambda)\,,\\ I_2 &=\sin\theta\,\sin\varphi+\cos\theta\,\cos\varphi(m_{\mathfrak{N}}/m_\lambda)\,,\\ I_3 &=\sin\theta\,\cos\varphi-\cos\theta\,\sin\varphi(m_{\mathfrak{N}}/m_\lambda)\,,\\ I_4 &=-\cos\theta\,\sin\varphi+\sin\theta\,\cos\varphi(m_{\mathfrak{N}}/m_\lambda)\,. \end{aligned} \tag{8.9}$$

In principle, all of these parameters can be measured and the three relations contained in Eqs. (8.6)–(8.8) can be checked. In practice, however, we know little or nothing about most of these parameters. The angle θ is quite well measured, and is of order $\frac{1}{4}$. The ratio $m_{\mathfrak{N}}/m_\lambda$ measures the extent of chiral-symmetry breaking and is known to be rather small from the analysis of Gell-Mann *et al.*[29] Its precise value, however, is not known. The numerical results usually quoted[30,24] correspond to $m_{\mathfrak{N}}/m_\lambda\sim\frac{1}{20}$ and $m_{\mathcal{P}}/m_{\mathfrak{N}}\sim\frac{1}{2}$. The uncertainties on these numbers are very large and difficult to estimate, particularly since one has to invoke strange PCAC to obtain these numbers. Our feeling is that the ratio $m_{\mathfrak{N}}/m_\lambda$ is probably not as small as $\frac{1}{20}$. Other considerations, based on current-algebra and/or quark-model analysis, suggest that $m_{\mathfrak{N}}/m_\lambda$ may be more like $\sim\frac{1}{4}$. In any case, we believe that these two values probably bracket the correct value. With our assumption about the lepton sector [Eq. (3.1)] we must take the angle α to be quite small. (This assumption may well be wrong, but so far there is no evidence to contradict it.) Define $\delta\equiv1-G_\beta/G_\mu=$ the deviation from unity of the ratio of coupling strengths in β decay and μ decay. In our theory $\delta=1-\cos\alpha\cos\theta$. The determination of δ suffers from uncertainty in calculating the radiative corrections[31] to β decay. A typical value cited is $\delta\sim2\%$. The uncertainty in θ as determined from K decay (typically $0.21\lesssim\theta\lesssim0.27$) certainly allows a value for α of the order of $\frac{1}{10}$. As has already been explained, the deviations of the Cabibbo angle from zero and of chiral symmetry from being exact depend on the deviation of α from zero. What we have to show, in fact, is that a relatively small α can generate a relatively large θ.

The remaining parameters are, of course, totally unknown. However, we can, and will, use Eqs. (8.6)–(8.8) to determine ϕ, β, and m_c/m_r. Thus, when and if the new quark r is discovered we will have ready-made predictions about its mass and its couplings on the left (β) and on the right (ϕ).

We will proceed by first requiring that α almost vanishes. From Eq. (8.6) we see that this implies either $I_2\approx0$ or $I_3\approx0$. These two possibilities are, in fact, equivalent and trivially related by a renaming. It suffices to consider $I_2\approx0$ which in fact fixes ϕ:

$$\tan\varphi=-m_{\mathfrak{N}}/(m_\lambda\tan\theta)\,. \tag{8.10}$$

(By a γ_5 transformation we can always choose θ to be positive. The relative sign between θ and ϕ is then fixed.) Eliminating ϕ from Eqs. (8.7) and (8.8) we then obtain

$$\begin{aligned} m_c/m_r &\approx \frac{(1/\sin^2\theta)(m_{\mathfrak{N}}/m_\lambda)\sin\alpha}{1+(1/\tan^2\theta)(m_{\mathfrak{N}}/m_\lambda)^2}\\ &\approx \frac{16(m_{\mathfrak{N}}/m_\lambda)\sin\alpha}{1+16(m_{\mathfrak{N}}/m_\lambda)^2}\,, \end{aligned} \tag{8.11}$$

$$\begin{aligned} \tan\beta &\approx \frac{\tan\theta[1+(1/\tan^2\theta)(m_{\mathfrak{N}}/m_\lambda)^2]}{[1-(m_{\mathfrak{N}}/m_\lambda)^2]\sin\alpha}\\ &\approx \frac{1}{4}\left[\frac{1+16(m_{\mathfrak{N}}/m_\lambda)^2}{1-(m_{\mathfrak{N}}/m_\lambda)^2}\right](1/\sin\alpha)\,. \end{aligned} \tag{8.12}$$

We have inserted the value $\tan\theta\sim\frac{1}{4}$. Combining these two equations we obtain a relation independent of $\sin\alpha$ (provided that it is small)

$$(m_c/m_r)\tan\beta \approx \frac{(m_{\mathcal{N}}/m_\lambda)}{1-(m_{\mathcal{N}}/m_\lambda)^2}\;\frac{1}{\sin\theta\cos\theta} \approx m_{\mathcal{N}}/(m_\lambda \sin\theta\cos\theta)\,. \tag{8.13}$$

To illustrate these relations numerically, let us take $\alpha \sim \frac{1}{10}$ and $m_{\mathcal{N}}/m_\lambda \sim \frac{1}{4}$. Then

$$\begin{aligned} &\tan\varphi \approx -1\,, \\ &m_c/m_r \approx \tfrac{1}{5}\,, \\ &\tan\beta \approx 5\,. \end{aligned} \tag{8.14}$$

In other words, the structure of the weak interaction looks like (approximately)

$$\begin{pmatrix} -\mathcal{P} \\ s \end{pmatrix}_R, \begin{pmatrix} c \\ \frac{\mathcal{N}-\lambda}{\sqrt{2}} \end{pmatrix}_R, \begin{pmatrix} r \\ \frac{\mathcal{N}+\lambda}{\sqrt{2}} \end{pmatrix}_R, \\ \begin{pmatrix} -\frac{1}{10}\mathcal{P}+r \\ s \end{pmatrix}_L, \begin{pmatrix} \mathcal{P}+\frac{1}{10}r \\ \mathcal{N}+\frac{1}{4}\lambda \end{pmatrix}_L, \begin{pmatrix} -c \\ -\frac{1}{4}\mathcal{N}+\lambda \end{pmatrix}_L. \tag{8.15}$$

Experimental implications can be read off in the usual way. For example, c quark states and r quark states are produced off $\mathcal{N}$ quarks by neutrinos with roughly equal strength. The r-quark analog of the 3.1 GeV narrow resonance should occur at ~15 GeV. Also noteworthy is the fact that in the left-handed sector the r quark prefers not to couple to the light quarks.

So far, we have not been concerned with the $\mathcal{P}$ quark. In this scheme it requires mass from a different heavy quark(s) than $\mathcal{N}$ and λ. Thus we can only use the near equality of $m_{\mathcal{P}}$ and $m_{\mathcal{N}}$ to estimate m_s. Taking the determinant of (8.3) we find the simple relation

$$(m_{\mathcal{P}}^2/m_{\mathcal{N}}m_\lambda) = (m_s^2/m_c m_r)\sin\alpha, \tag{8.16}$$

which may be rewritten as

$$(m_s/m_c) \approx (m_{\mathcal{P}}/m_{\mathcal{N}})[1+16(m_{\mathcal{N}}/m_\lambda)^2]^{1/2}/(4\sin\alpha)\,. \tag{8.17}$$

Again, with $m_{\mathcal{N}}/m_\lambda \sim \frac{1}{4}$ and $\alpha \sim \frac{1}{10}$ we find

$$(m_s/m_c) \sim 3.5(m_{\mathcal{P}}/m_{\mathcal{N}})\,. \tag{8.18}$$

Thus, an $s\bar{s}$ resonance should exist around 5 or 6 GeV.

On the other hand, if $(m_{\mathcal{N}}/m_\lambda)$ is much smaller, of the order $\frac{1}{10}$ or $\frac{1}{20}$, then (m_c/m_r) also becomes very small, of the order $16\sin\alpha(m_{\mathcal{N}}/m_\lambda)$. In that case, hadronic states composed of r quarks will be too massive to be seen in the current round of experiments. In contrast, the mass ratio (m_s/m_c) and the angle β are relatively insensitive to the precise value of $(m_{\mathcal{N}}/m_\lambda)$.

For illustrative purposes let us take $m_{\mathcal{N}}/m_\lambda \sim \frac{1}{20}$ and $\alpha \sim \frac{1}{10}$ again. Then the structure of the weak interaction looks as follows:

$$\begin{pmatrix} -\mathcal{P} \\ s \end{pmatrix}_R, \begin{pmatrix} c \\ \mathcal{N}-\frac{1}{5}\lambda \end{pmatrix}_R, \begin{pmatrix} r \\ \frac{1}{5}\mathcal{N}+\lambda \end{pmatrix}_R, \\ \begin{pmatrix} -\frac{1}{10}\mathcal{P}+\frac{2}{5}c+r \\ s \end{pmatrix}_L, \begin{pmatrix} \mathcal{P}+\frac{1}{10}(\frac{2}{5}c+r) \\ \mathcal{N}+\frac{1}{4}\lambda \end{pmatrix}_L, \begin{pmatrix} -c+\frac{2}{5}r \\ -\frac{1}{4}\mathcal{N}+\lambda \end{pmatrix}_L.$$

Unfortunately, our ignorance of the precise value of $m_{\mathcal{N}}/m_\lambda$ prevents us from making more definite statements.[32]

IX. CONCLUSION

We have proposed a mechanism for generating the Cabibbo angle. In our picture the light quarks, $\mathcal{P}$, $\mathcal{N}$, and λ, acquire their masses via weak radiation which in turn fixes the orientation between the weak and the strong interactions. From this point of view the Cabibbo angle is merely one particular angle among many. These angles, however, are not arbitrary but are constrained, together with mass ratios, by a number of relations. These relations can hopefully be tested experimentally in the near future.

This picture leads us naturally to a model of six quarks, distributed symmetrically among three left-handed and three right-handed doublets. Besides being anomaly free and having a vector neutral current, it also provides a possible explanation for the glaring parity asymmetry of weak interaction. Various recent experimental observations can be accommodated and the leading contribution to neutral kaon mass difference is suppressed.

Our point of view is clearly based crucially on our prejudices about the quark mass spectrum. It would be incorrect if the light quarks were already massive at tree level. In that case weak radiation will only re-orient the interactions slightly.

We have nothing to say about leptons. However, people have speculated[33] that the electron also acquires its mass from weak radiation via the muon and/or perhaps heavy leptons. It could be that virtually all hitherto known masses come from weak interaction. If that is indeed so then we have only begun to explore the structure of weak interaction.

ACKNOWLEDGMENT

Part of this work was performed at the Aspen Center for Physics, Aspen, Colorado, and at the École de Physique, Les Houches, France. We thank these institutions for their hospitality.

*Work supported in part by the National Science Foundation under Grant No. MPS73-04997.

†A. P. Sloan Fellow.

[1] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).

[2] M. Gell-Mann (unpublished); H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. 47B, 365 (1973).

[3] D. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973).

[4] S. Weinberg, in proceedings of the Second International Conference on Elementary Particles, Aix-en-Provence, 1973 [J. Phys. (Paris) Suppl. 34, C1-45 (1973)]; E. Abers and B. W. Lee, Phys. Rep. 9C, 1 (1973); M. A. B. Bég and A. Sirlin, Annu. Rev. Nucl. Sci. 24, 379 (1974).

[5] The term "flavor" has been introduced to denote generically the various quark species.

[6] A. Zee, Phys. Rev. D 9, 1772 (1974).

[7] S. L. Adler and R. Dashen, *Current Algebra* (Benjamin, 1968).

[8] J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974); J.-E. Augustin *et al.*, *ibid.* 33, 1407 (1974); G. S. Abrams *et al.*, *ibid.* 33, 1453 (1974).

[9] T. Appelquist and H. D. Politzer, Phys. Rev. Lett. 34, 38 (1975); A. De Rújula and S. Glashow, *ibid.* 34, 46 (1975); C. G. Callan, R. L. Kingsley, S. B. Treiman, F. Wilczek, and A. Zee, *ibid.* 34, 52 (1975); T. Appelquist, A. De Rújula, H. Politzer, and S. Glashow, *ibid.* 34, 365 (1975); E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K. D. Lane, and T.-M. Yan, *ibid.* 34, 363 (1975).

[10] S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1972).

[11] F. Wilczek, A. Zee, R. L. Kingsley, and S. Treiman, Phys. Rev. D 12, 2768 (1975). See also R. L. Kingsley, F. Wilczek, and A. Zee (unpublished).

[12] This is more or less an assumption depending on what one imagines the λ-quark mass to be.

[13] K. Wilson, Phys. Rev. D 10, 2445 (1974).

[14] S. Weinberg, Phys. Rev. D 8, 3497 (1973); C. Callan (unpublished).

[15] Dynamical symmetry breaking [Y. Nambu and G. Jona-Lasinio, Phys. Rev. 124, 246 (1961)] is far from understood. In particular, one must contend with various (pseudo-) Goldstone bosons that may appear. See for example, S. Weinberg, talk at the Philadelphia Conference, 1975 (unpublished).

[16] This has been remarked by a number of authors, including those of Ref. 11 and M. A. B. Bég and A. Zee, Phys. Rev. D 8, 1480 (1973). A more quantitative discussion has been given recently by E. Golowich and B. Holstein, Phys. Rev. Lett. 35, 831 (1975).

[17] J. Schwinger, Ann. Phys. (N.Y.) 2, 407 (1957); S. Glashow, Nucl. Phys. 22, 579 (1961); S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity* (Nobel Symposium No. 8), edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[18] M. Gell-Mann (unpublished); M. A. B. Bég and A. Zee, Ref. 16.

[19] F. Wilczek, A. Zee, R. Kingsley, and S. Treiman, Ref. 11; A. De Rújula, H. Georgi, and S. Glashow, Phys. Rev. Lett. 35, 69 (1975); H. Fritszch, M. Gell-Mann, and P. Minkowski (unpublished).

[20] A. Benvenuti *et al.*, Phys. Rev. Lett. 34, 419 (1975).

[21] The possibility that D-$\bar{D}$ mixing may provide a suitable mechanism was proposed by A. De Rújula *et al.*, Ref. 19; see also R. Kingsley, S. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D 11, 1919 (1975); T. Appelquist, H. Georgi, S. Glashow, and H. Politzer, Phys. Rev. Lett. 34, 365 (1975).

[22] D. Cline, talk at Washington APS meeting, 1975 (unpublished).

[23] R. Gatto, G. Sartori, and M. Tonin, Phys. Lett. 28B, 128 (1968); N. Cabibbo and L. Maiani, *ibid.* 28B, 131 (1968).

[24] H. Pagels, Phys. Rev. D 11, 1213 (1975); see also R. J. Oakes, Phys. Lett. 29B, 683 (1969); W. Palmer, Phys. Rev. D 6, 2897 (1972).

[25] The Adler-Weisberger relation (Ref. 7) would suggest that this angle is no more than ~0.05.

[26] S. Weinberg, Phys. Rev. Lett. 29, 388 (1972); 29, 1698 (1972); Phys. Rev. D 7, 2887 (1973); see also H. Georgi and S. Glashow, *ibid.* 6, 2977 (1972); R. N. Mohapatra and P. Vinciarelli, *ibid.* 8, 481 (1973); G.'t Hooft, Nucl. Phys. B35, 167 (1971).

[27] D. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); H. Politzer, *ibid.* 30, 1346 (1973); G. 't Hooft (unpublished).

[28] This mechanism had also been mentioned in passing in Ref. 11 (in Sec. III, in connection with model E).

[29] M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968); S. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1968).

[30] For example, M. Gell-Mann, lectures delivered at the Summer School of Theoretical Physics, University of Hawaii (Caltech Report No. CALT-68-224) (unpublished).

[31] M. A. B. Bég, J. Bernstein, and A. Sirlin, Phys. Rev. D 6, 2597 (1973); A. Sirlin, Nucl. Phys. B71, 29 (1974) and NYU/TR6/75 (unpublished).

[32] There is an interesting speculation that the c and r quarks are degenerate in mass. (F. Wilczek, unpublished; see also footnote 15 of Ref. 6). In that limit one obtains from Eqs. (8.6)–(8.8) that $m_{\mathfrak{N}}/m_\lambda = \sin\alpha$ and $\tan\phi = \tan\beta$.

[33] H. Georgi and S. Glashow, Phys. Rev. D 7, 2457 (1973).