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Nonet boson-mass formula in SU(4) and the width of the $\eta_c(0^-)$

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It is argued that Schwinger's nonet mass formula is a more general mass relation than the ideal nonet mass formulas for any 16-plet bosons of the SU(4) group. The explicit limiting procedure by which the old SU(3) result is recovered from SU(4) is shown. The width of $\eta_c\,(0^-)$ could be large depending on the assignment of the ninth 0^{-+} meson.

We consider bosons, π_s, K_s, η_s, η_{cs}, D_s, F_s, and η_s', belonging to a $15\oplus 1$ representation of SU(4) group.[1] The subscript s denotes J^{PC}. η_s, η_{cs} and η_s' are the $I=0$ uncharmed members of the 15-plet and singlet, respectively. We have recently derived[2,3] general SU(4) sum rules which exhibit a remarkable interplay of the masses, SU(4) mixing angles, and axial-vector matrix elements. This *interplay* can produce selection rules which may explain the stability of the new narrow resonances.

In this paper we discuss the implications and the possible validity of Schwinger's nonet mass formula[4] (SNF) in the new SU(4) scheme. Applying the sum rules especially to the 0^- mesons, we point out that the width of $\eta_c(0^-)$ could be quite large, contrary to the usual expectation.[5]

Our only theoretical assumptions are[2,3] asymptotic SU(4), the chiral SU(4)$\otimes$SU(4) charge algebra, and the simple mechanism of symmetry breaking, characterized by the presence of the *exotic* commutation relations (CR's) of the form $[\dot{V}_\alpha, V_\beta]=0$ and $[\dot{V}_\alpha, A_\beta]=0$ ($\dot{V}_\alpha = dV_\alpha/dt$), where (α,β) stands for all the *exotic* combinations of the SU(4) indices.[2,3] The SU(4) mixing parameters are defined[2,3] for the physical annihilation operators $a^s_\alpha(\vec{k})$ ($\alpha=\eta, \eta_c, \eta'$) in our asymptotic limit $\vec{k}\to\infty$ (suppressing s) by

$$a_\eta = \alpha a_8 + \alpha_c a_{15} + \alpha' a_0,$$

$$a_{\eta'} = \beta a_8 + \beta_c a_{15} + \beta' a_0,$$

and

$$a_{\eta_c} = \gamma a_8 + \gamma_c a_{15} + \gamma' a_0.$$

α, etc. can also be parametrized[2,3] in terms of Euler angles (ϕ, θ, ψ). The SU(4) "ideal" configurations, $\eta^i = s\bar{s}$, $\eta_c^i = c\bar{c}$, and $\eta'^i = (u\bar{u}+d\bar{d})/\sqrt{2}$, correspond to $\phi^i = 30°$, $\theta^i \simeq 35°$ $[\sin\theta^i = (\frac{1}{3})^{1/2}]$, and $\psi^i = 0$. By realizing the CR's, $[\dot{V}_\alpha, V_\beta]=0$, at $\vec{k}\to\infty$, the four Gell-Mann–Okubo-type independent mass relations are obtained[2,3] as *exact* constraints. Two of them are ($\eta_s{}^2 \equiv m_{\eta_s}{}^2$, etc.)

$$(\alpha^s)^2\eta_s{}^2 + (\beta^s)^2\eta_s'^2 + (\gamma^s)^2\eta_{cs}{}^2 = \tfrac{1}{3}(4K_s{}^2 - \pi_s{}^2), \qquad (1)$$

$$\alpha^s\alpha^s_c\eta_s{}^2 + \beta^s\beta^s_c\eta_s'^2 + \gamma^s\gamma^s_c\eta_{cs}{}^2 = -\frac{\sqrt{2}}{3}(K_s{}^2 - \pi_s{}^2). \qquad (2)$$

The commutation relation (CR) $[\dot{V}_\alpha, A_\beta]=0$ produces, among others, another mass constraint

$$\alpha^s X^s_\beta X^s_\gamma(\eta_s{}^2 - \pi_s{}^2)(\eta_s'^2 - \eta_{cs}{}^2) + \beta^s X^s_\alpha X^s_\gamma(\eta_s'^2 - \pi_s{}^2)(\eta_{cs}{}^2 - \eta_s{}^2) + \gamma^s X^s_\alpha X^s_\beta(\eta_{cs}{}^2 - \pi_s{}^2)(\eta_s{}^2 - \eta_s'^2) = 0 \qquad (3)$$

(with $X^s_\alpha \equiv \alpha^s - \sqrt{2}\alpha^s_c$, $X^s_\beta \equiv \beta^s - \sqrt{2}\beta^s_c$, and $X^s_\gamma \equiv \gamma^s - \sqrt{2}\gamma^s_c$), *together* with the following general constraints upon the particular matrix elements of the axial charges $A_{\pi^\pm}$ (in the limit $\vec{k}\to\infty$):

$$R^s(\pi_u) \equiv \frac{\langle \eta_s | A_{\pi^-} | \pi_u^+(\vec{k})\rangle}{\langle \eta_s' | A_{\pi^-} | \pi_u^+(\vec{k})\rangle} = -\left(\frac{X^s_\beta}{X^s_\alpha}\right)\left(\frac{\eta_{cs}{}^2 - \eta_s'^2}{\eta_{cs}{}^2 - \eta_s{}^2}\right), \qquad (4)$$

$$R^s_c(\pi_u) \equiv \frac{\langle \eta_{cs} | A_{\pi^-} | \pi_u^+(\vec{k})\rangle}{\langle \eta_s' | A_{\pi^-} | \pi_u^+(\vec{k})\rangle} = -\left(\frac{X^s_\beta}{X^s_\gamma}\right)\left(\frac{\eta_s{}^2 - \eta_s'^2}{\eta_s{}^2 - \eta_{cs}{}^2}\right). \qquad (5)$$

Both s and u are completely arbitrary, provided $C_sC_u = 1$.

I. SCHWINGER'S NONET MASS FORMULA

We now comment on the possible role played by the SNF. The heavy masses of the new resonances suggest that η_{cs}'s are predominantly $c\bar{c}$ states. Let us, for the moment, *assume* that η_{cs} is a *pure* $c\bar{c}$, i.e., $\phi_s = 30°$ and $\psi_s = 0°$. Then from Eqs. (1) and (2) we obtain, by eliminating the still unspecified angle θ_s, a relation

$$(3\eta_s^2+\pi_s^2-4K_s^2)(3\eta_s'^2+\pi_s^2-4K_s^2)=-8(K_s^2-\pi_s^2)^2\,, \quad (6)$$

which coincides with the SNF.[4] Although we discover the SNF, with $\eta_c=c\bar{c}$ and $[\dot{V}_\alpha, V_\beta]=0$, the imposition of Eq. (3) (i.e., $[\dot{V}_\alpha, A_\beta]=0$) introduces a further constraint $(\eta_s^2-\pi_s^2)(\eta_s'^2-\pi_s^2)=0$. The constraint $\eta_s'^2=\pi_s^2$ is, therefore, imposed[6] in addition, which produces $\sin\theta_s=(\frac{1}{3})^{1/2}$ (ideal angle), $X_\beta=0$ and the "ideal" nonet mass formula (INF)

$$\eta_s'^2=\pi_s^2 \text{ and } \eta_s^2-K_s^2=K_s^2-\pi_s^2. \quad (7)$$

With $X_\beta=0$, Eqs. (4) and (5) yield remarkable selection rules, $R^s(\pi_u)=0$ and $R_c^s(\pi_u)=0$, valid for any u.

Therefore, once the η_{cs} takes a *pure* $c\bar{c}$ configuration, the multiplet is forced in our scheme to take the SU(4) "ideal" configuration, in which the INF is satisfied and the selection rules $R^s(\pi_u)=R_c^s(\pi_u)=0$ emerge. The SNF is *trivially* satisfied due to the INF in the "ideal" case. The converse may be more useful, i.e., if the 16-plet satisfies the INF, the η_{cs} must be a *pure* $c\bar{c}$. The ground state 1^{--} which may involve $\psi(3105)$ and the 2^{++} mesons satisfy (but only *approximately*) these "ideal" constraints. In the real world, the "ideal" configuration is certainly violated, the η_{cs} deviates to some extent from a pure $c\bar{c}$ state and the situation is more complex.

We now assert that the study of a limit $\eta_{cs}^2\to\infty$ in our scheme (which is not unrealistic in view of the high masses of the new resonances) may give us a good tool to study the complex situation. The fact that we have lived long in the SU(3) world without noticing, for example, the effect of the η_{cs}, suggests that the couplings between the c quark and the ordinary quarks are such that the world of SU(3) of the ordinary quarks is not disturbed in the limit of infinite charmed-quark mass. We now demonstrate in our scheme that this situation is indeed realized in the following limit: $\eta_{cs}^2\to 0$ and $\gamma^s\to 0$ (i.e., $\psi_s\to 0$), but $\gamma^s\eta_{cs}^2\to 0$.

Assuming $K_s^2\neq\pi_s^2$, Eqs. (1), (2), and (3) lead in our limit to $\phi_s\to 30°$, i.e., $\eta_{cs}\to c\bar{c}$ as is expected. However, θ_s is not constrained and tends to the SU(3) value [from Eq. (1)], $\sin^2\theta_s=\frac{1}{3}(4K_s^2-3\eta_s^2-\pi_s^2)(\eta_s'^2-\eta_s^2)^{-1}$. However, the elimination of the θ_s from these sum rules *yields* in our limit Eq. (6), the celebrated SNF. Equation (5) gives $R_c^s(\pi_u)\to 0$ and Eq. (4) reduces, after using the now obtained SNF, to $R^s(\pi_u)\to\tan\theta_s(\eta_s'^2-\pi_s^2)(\eta_s^2-\pi_s^2)^{-1}$. This is exactly the result previously obtained[7] in the *pure SU(3)* framework. Therefore in our limit the η_{cs} tends to a pure $c\bar{c}$ state, SU(3) is then restored and the nonet $(\pi_s, K_s, \eta_s, \eta_s')$ tends to satisfy the SNF. If the real mass of the η_{cs} is sufficiently large, i.e., it is close to a $c\bar{c}$ state (which we believe to be the case), to *justify* the use of the above limiting procedure the SNF thus emerges as a nonet mass constraint.

To sum up, we assert that the SNF may be valid[8] for any SU(4) 16-plet if the η_{cs} is heavy (i.e., it is close to a $c\bar{c}$). The INF is valid only when the η_{cs} is *very* close to a pure $c\bar{c}$. Indeed, even for the 1^{--} and 2^{++} which are approximately "ideal", the SNF is better satisfied than the INF. A relatively small violation of the pure $c\bar{c}$ configuration of the η_{cs} (measured by the deviations of ϕ_s and ψ_s from 30° and 0°, respectively) leads to a significant violation of the INF, although the SNF may be well satisfied. The 0^- raises a problem. Assigning $\eta_c(1^{--})\equiv\psi(3105)$, we have computed[3] the mixing angles and the mass of $\eta_c(0^-)$ from our sum rules, Eqs. (1)–(3), for the two popular assignments[9] $\eta'\equiv X(958)$ or $\eta'\equiv E(1420)$; $\phi\simeq 21.9°$, $\theta\simeq -10.4°$, $\psi\simeq 3.4°$, and $\eta_c\simeq 2.72$ GeV for $\eta'\equiv X$, while $\phi\simeq 36.1°$, $\theta\simeq -6.2°$, $\psi\simeq -0.20°$, and $\eta_c\simeq 3.04$ GeV for $\eta'\equiv E$.

The SNF predicts[7] the mass of $\eta'(0^-)$ around $\eta'^2\simeq 2.25$ compared with $X^2\simeq 0.92$ and $E^2\simeq 2.02$ GeV^2. Our above result is in line with our observation here. The assignment $\eta'\equiv E$, for which the SNF is *better* satisfied, indeed predicts a *higher* mass of η_c, 3.04 GeV, comparable to that of the $\psi(3105)$, and a *smaller* deviation of the η_c from a pure $c\bar{c}$ state compared with the case $\eta\equiv X$. The difference in the configurations of the η, η', and η_c in the assignments $\eta'\equiv X$ or E produces some marked differences in the decays of the η_c which will be discussed in Secs. II and III.

II. WIDTH OF $\eta_c(0^-)$

Some of the important decays of $\eta_c(0^-)$ involving a pseudoscalar meson P_γ can be estimated starting from Eqs. (4) and (5). With PCAC (partial conservation of axial-vector current) (assuming $f_\pi\simeq f_K\simeq f_\eta$) we obtain[10] in our theoretical framework (in the $\bar{k}\to\infty$ limit)

$$\Gamma(\alpha\to\beta+P_\gamma)\propto f_\gamma^{-2}(2J+1)^{-1}p_\beta^3\sum_{\text{spin}}|\langle\alpha|A_\gamma|\beta\rangle|^2\,, \quad (8)$$

where J is the spin of α and p_β is the c.m. momentum. $\langle\alpha|A_\gamma|\beta\rangle$ can be parametrized by the usual prescription of exact SU(4) plus mixing.[10]

We first consider the $\eta_c\to A_2\pi$ decay. Our parameters of the 0^- mentioned in Sec. I give

$$R(A_2) \equiv \frac{\langle \eta | A_{\pi^-} | A_2^+ \rangle}{\langle \eta' | A_{\pi^-} | A_2^+ \rangle} = \begin{cases} 0.64 & \text{for } \eta' \equiv X, \\ 0.84 & \text{for } \eta' \equiv E, \end{cases}$$

$$R_c(A_2) \equiv \frac{\langle \eta_c | A_{\pi^-} | A_2^+ \rangle}{\langle \eta' | A_{\pi^-} | A_2^+ \rangle} = \begin{cases} -0.04 & \text{for } \eta' \equiv X, \\ -0.16 & \text{for } \eta' \equiv E. \end{cases} \tag{9}$$

If the 0^- is "ideal," $R(A_2) = R_c(A_2) = 0$. The much smaller value of R_c than R reflects the fact that the η_c is still dominantly a $c\bar{c}$ (i.e., η_c is heavy) but the configuration $\eta = s\bar{s}$ is strongly violated. Equation (9) then predicts from Eq. (8)

$$\Gamma(\eta_c \to A_2\pi) \simeq \begin{cases} 2.2 \text{ MeV} & \text{for } \eta' \equiv X, \\ 102 \text{ MeV} & \text{for } \eta' \equiv E, \end{cases} \tag{10}$$

with the experimental input[9] $\Gamma(A_2 \to \eta\pi) \simeq 15$ MeV. Equation (8) also gives with the same input

$$\Gamma(\eta_c \to K^{**}K) \simeq \begin{cases} 2.9 \text{ MeV} & \text{for } \eta' \equiv X, \\ 104 \text{ MeV} & \text{for } \eta' \equiv E. \end{cases} \tag{11}$$

Assuming an "ideal" structure for the 2^{++} we also estimate,[11] for example (assuming $\eta \equiv \eta_8$),

$$\Gamma(\eta_c \to f\eta) \simeq \begin{cases} 0.14 \text{ MeV} & \text{for } \eta' \equiv X, \\ 5.6 \text{ MeV} & \text{for } \eta' \equiv E, \end{cases}$$

$$\Gamma(\eta_c \to f'\eta) \simeq \begin{cases} 0.04 \text{ MeV} & \text{for } \eta' \equiv X, \\ 3.6 \text{ MeV} & \text{for } \eta' \equiv E. \end{cases} \tag{12}$$

The F-type coupling, such as $\eta_c \to K^*K$, is less important,[3] since its amplitude is suppressed by the factor $\gamma = \sin\theta \sin\psi$ compared with its SU(4) counterpart, $K^* \to K\pi$, i.e.,

$$\Gamma(\eta_c \to K^*K) \simeq \begin{cases} 4.4 \text{ MeV} & \text{for } \eta' \equiv X, \\ 4.6 \text{ keV} & \text{for } \eta' \equiv E. \end{cases} \tag{13}$$

Another possibly important mode is the $\eta_c \to \delta\pi$, provided $\delta(970)$ exists. Taking $\pi_u^+ \equiv \delta^+$, Eqs. (4) and (5) produce

$$\frac{\Gamma(\eta_c \to \delta\pi)}{\Gamma(\delta \to \eta\pi)} \simeq \begin{cases} 0.6 & \text{for } \eta' \equiv X, \\ 8.7 & \text{for } \eta' \equiv E, \end{cases} \tag{14}$$

$$\Gamma(\eta' \to \delta\pi) \simeq 6.5\ \Gamma(\delta \to \eta\pi) \quad \text{for } \eta' \equiv E. \tag{15}$$

$\Gamma(\delta \to \eta\pi)$ is not well known. An educated guess listed[9] is 50 ± 20 MeV. Therefore, we see that if such a δ exists, $\Gamma(\eta_c \to \delta\pi)$ can be quite large for $\eta' \equiv E$. In this case $\Gamma(E \to \delta\pi)$ is also large [see Eq. (15)] and our E may be hidden as a broad resonance including the region of $E(1422)$, if the δ exists and its width is sizable.

Summing up our above result we find

$$\Gamma(\eta_c \to \text{hadrons}) \gtrsim \begin{cases} 9.7 \text{ MeV} + \Gamma(\eta_c \to \delta\pi) & \text{for } \eta' \equiv X, \\ 215 \text{ MeV} + \Gamma(\eta_c \to \delta\pi) & \text{for } \eta' \equiv E. \end{cases} \tag{16}$$

For the assignment $\eta' \equiv X$, if we neglect the $\eta_c \to \delta\pi$ mode, our estimate is close to the ones obtained in Ref. 5. Even if the δ exists, we find $\Gamma(\eta_c \to \delta\pi) \simeq 18$ MeV for the choice of the δ width, $\Gamma(\delta \to \eta\pi) \simeq 30$ MeV. Therefore, even in this case the η_c is a relatively *narrow* resonance. On the other hand, for the assignment $\eta' \equiv E$, the η_c will be a broad resonance ($\Gamma > 200$ MeV), even if the δ meson does not exist. The broad width of the η_c will make the detection of the η_c more difficult.

III. DECAY $\psi \to \eta_c\gamma$

In the framework of asymptotic SU(4) and SU(4) charge-current CR's, the rate $\psi(3105) \to \eta_c\gamma$ is given,[12] to a reasonable approximation, by a simple broken-SU(4) formula [note the factor $(m_\alpha^2 - m_\beta^2)^2$ in $A(\alpha \to \beta\gamma)$]

$$A(\psi \to \eta_c\gamma) \simeq |(\tfrac{2}{3} + 2(\tfrac{2}{3})^{1/2}x)[A(\omega \to \pi\gamma)]^{1/2} - (\tfrac{3}{2})^{1/2}[A(\phi \to \eta\gamma)]^{1/2}|^2, \tag{17}$$

where

$$A(\alpha \to \beta\gamma) = \Gamma(\alpha \to \beta\gamma)(m_\alpha^2 - m_\beta^2)^2 \times [(m_\alpha^2 - m_\beta^2)/(2m_\alpha)]^{-3}.$$

The parameter x depends on the charge assignment of quarks. The usual choice[13] corresponds to $x = +(\tfrac{3}{2})^{1/2}$. $x = -2(\tfrac{2}{3})^{1/2}$ is the alternative one.[14] With the input $\Gamma(\omega \to \pi\gamma) = 800$ keV and $\Gamma(\phi \to \eta\gamma) = 200$ keV, we obtain

$$\Gamma(\psi \to \eta_c\gamma) \simeq \begin{cases} 67 \text{ keV } [x = +(\tfrac{3}{2})^{1/2}], & 384 \text{ keV } [x = -2(\tfrac{2}{3})^{1/2}] \text{ for } \eta' \equiv X, \\ 12 \text{ keV } [x = +(\tfrac{3}{2})^{1/2}], & 66.5 \text{ keV } [x = -2(\tfrac{2}{3})^{1/2}] \text{ for } \eta' \equiv E. \end{cases} \tag{18}$$

In the above estimate the η_c is assumed stable. The choice of $\eta' \equiv X$ [especially $x = -2(\tfrac{2}{3})^{1/2}$] tends to predict larger rates, which are harder to reconcile with the observed narrow width of the ψ. The main cause is the relatively *low* mass of the η_c predicted[3,15] around 2.72 GeV for $\eta' \equiv X$. Also the assignment $x = -2(\tfrac{2}{3})^{1/2}$ does not seem to be favored. Another but again weak evidence against

$\eta' \equiv X$ is the prediction [based on $R(A_2)$ in Eq. (9)], $r \equiv \Gamma(A_2 \to X\pi)/\Gamma(A_2 \to \eta\pi) \simeq (1/2.8)$, in contrast to the present experimental value[9] $r < \frac{1}{15}$.

On the other hand, the assignment $\eta' \equiv E$ gives smaller widths[3,16] consistent with present experiment[17] because of the larger predicted mass[3] of the η_c. Furthermore, the $\psi \to \eta_c\gamma$ will not exhibit a monochromatic photon energy because of the broad width of the η_c. This may be especially relevant to the possible $\psi(3695) \to \eta_c\gamma$ decay. Since we do not know the SU(4) counterparts of this decay, we are not able to predict its rate contrary to the case of $\psi(3105) \to \eta_c\gamma$. However, even if this decay mode constitutes a sizable fraction of the $\psi(3695)$ decays it may not be detected by looking for the energetic monochromatic photon.[18]

Added note. It has come to our attention that although the method used is very different from ours, the interesting approximate relations between the leakage coefficients from a pure $c\bar{c}$ state and the nonet masses obtained in Ref. 15 may also suggest the possible validity of the SNF. For example, we observe that if there is an argument that $c \to 0$, in the limit of large charmed-quark mass ($y \to \infty$), Eq. (2.10) of Ref. 15 implies the validity of the SNF in the limit $y \to \infty$.

Recently the DESY group reported[19] a possible candidate for the $\eta_c(0^{-+})$ at 2.75 GeV which is *close* to our prediction of 2.72 GeV for the assignment $\eta' \equiv X(958)$. If this is established, the $\eta_c(0^{-+})$ deviates significantly from a pure $c\bar{c}$ state and SNF is significantly violated. Our crude estimate of $\Gamma(\psi \to \eta_c\gamma)$ with $x = (\frac{3}{2})^{1/2}$ in Eq. (19) will then deserve a close scrutiny. Our above calculation cannot claim an accuracy within a factor of 2 in the amplitude. We have neglected inter-16-plet SU(4) mixings, and the G-forbidden vertex $\langle \pi_s | A_{\pi^-} | \pi_u^+ \rangle$ is neglected compared with the G-allowed ones, $\langle \eta_s | A_{\pi^-} | \pi_u^+ \rangle$ and $\langle \eta_{cs} | A_{\pi^-} | \pi_u^+ \rangle$, which, however, vanishes in the ideal limit.

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