

Models with more than four quarks*

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Several models with more than four quarks are reviewed and compared with the four-quark model. They generally have right- and left-handed currents and are of the $SU_2 \times U_1$ type. The lepton sectors are also discussed including the possibilities of heavy charged and neutral leptons. The weak phenomenology, triangle anomalies, degenerate quark masses, and radial excitations are examined.

I. INTRODUCTION

There has been considerable work in the last few months on models¹⁻¹² of the hadrons with more than four quarks, most of which involve right-handed currents in addition to the usual left-handed ones. Four-quark models with right-handed currents¹³ have significant problems^{6-8,14} explaining some data and do not have a cancellation of VVA triangle anomalies¹⁵⁻¹⁷ (discussed below).

The "standard" GIM model¹⁸⁻²⁰ with four quarks and four leptons predicts

$$R^{e^+e^-} \equiv \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{10}{3}, \quad (1.1)$$

and requires that the dominant decay of charmed mesons be into a K meson plus other hadrons or leptons. No evidence of such decays currently exists.^{21,22} This model has the somewhat artificial cancellation of triangle anomalies because of the sum of lepton and quark charges together being zero.

One can also consider a model with four quarks, but with six leptons,²³ where one of the new leptons is taken as charged and having a mass of about 1.8 GeV. Such a heavy lepton is consistent with the SPEAR μe results reported by Perl.²⁴ Since the heavy lepton frequently decays to hadrons, it contributes almost 1 to $R^{e^+e^-}$. The semileptonic decays of this lepton are dominantly to neutrino plus $u-d$ quark pairs (since it is presumed too light for decay to charm, i.e., through $c-s$ quark pairs), and this has the effect of decreasing the number of K mesons expected, thereby confusing the charm search.

Although two problems may have been solved with the inclusion of the heavy lepton, the fact that there are unequal numbers of quarks and leptons is not very appealing, and there is no cancellation of the triangle anomalies without adding more quarks. As will be discussed in Sec. VI, this model gives a rate for $\mu^+\mu^-$ production in neutrino

interactions which is below that found experimentally, and has no mechanism for $\mu^+\mu^-$ production.

As further data from neutrino interactions become available, they may provide further need for additional quarks, and the models discussed here consider a range of possibilities for the weak phenomenology. None of the authors, to my knowledge, feel that the models they are proposing are likely to be completely true, but rather that they are exploring the effects of models with more than four quarks since there are basic features in these models which are likely to be shared, in part, by future theories.

The problem mentioned above of the VVA triangle anomalies¹⁵ concerns the failure of renormalization in certain gauge theories due to the triangle diagram, Fig. 1. In "quasirenormalizable" gauge theories of the weak interactions, the VVA triangle diagram, which is associated with the axial-vector current, prevents renormalization unless its divergent contribution can be canceled. One means to effect this cancellation¹⁶ is to have the charges satisfy the equation

$$\sum (Q_{\text{quarks}} + Q_{\text{leptons}}) = 0, \quad (1.2)$$

as in the standard left-handed models^{18,25} (one must count each color of quarks).

A more "natural" method (since the anomaly is an axial-vector one) is to add to each left-handed current ($V-A$), a right-handed current ($V+A$) to form vectorlike theories in which the axial-vector triangle anomaly is clearly canceled.¹⁶

II. FOUR MODELS

All models discussed here are of the $SU_2^{\text{weak}} \times U_1$ type; however, this is not a necessary feature (although all models must satisfy the same criteria discussed here). It is always assumed here that all quarks come in three colors and have the usual fractional charges.

The Cabibbo angles (and other new angles) will be suppressed below so that

$$\bar{u}\gamma_\mu(1+\gamma_5)(d\cos\theta+s\sin\theta) + \bar{c}\gamma_\mu(1+\gamma_5)(s\cos\theta-d\sin\theta) \quad (2.1)$$

will be written as

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \quad (2.2)$$

where the subscript L means left-handed ($1+\gamma_5$) and R means right-handed ($1-\gamma_5$). The consequences of these models will be discussed in the later sections.

In the first model (which I proposed in part in Refs. 1 and 2), the u and c quarks appear in right-handed doublets with new "down" type quarks. These heavier quarks are indicated with primes.

Model 1:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} u \\ d' \end{pmatrix}_R, \begin{pmatrix} c \\ s' \end{pmatrix}_R + \text{singlets}, \quad (2.3)$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_e \\ E \end{pmatrix}_R, \begin{pmatrix} \nu_\mu \\ M \end{pmatrix}_R + \text{singlets}, \quad (2.4)$$

where E and M are new heavy leptons. The leptons shown (where the neutrinos need not have a non-zero mass) are only suggestive and other possibilities are allowed. Gürsey *et al.*³ considered a similar model (without right-handed currents)

Model 3:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} u' \\ d' \end{pmatrix}_L, \begin{pmatrix} c' \\ s' \end{pmatrix}_L, \begin{pmatrix} u \\ d' \end{pmatrix}_R, \begin{pmatrix} c \\ s' \end{pmatrix}_R, \begin{pmatrix} c' \\ s \end{pmatrix}_R, \begin{pmatrix} u' \\ d \end{pmatrix}_R, \quad (2.7)$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} N_e \\ E \end{pmatrix}_L, \begin{pmatrix} N_\mu \\ M \end{pmatrix}_L, \begin{pmatrix} N_e \\ e \end{pmatrix}_R, \begin{pmatrix} N_\mu \\ \mu \end{pmatrix}_R, \begin{pmatrix} \nu_e \\ E \end{pmatrix}_R, \begin{pmatrix} \nu_\mu \\ M \end{pmatrix}_R. \quad (2.8)$$

It can be argued that there are two unnecessary quarks in Model 3 although one motivation for keeping all eight quarks will be given in Sec. VIII.

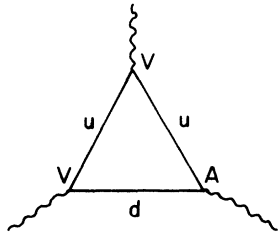


FIG. 1. Diagram for the VVA triangle anomaly. The solid lines are fermions.

from the point of view of exceptional groups and octonions.

In the second model (which Minkowski, Wilczek, and I have considered), the d and s quarks are the ones which appear in right-handed doublets with new "up" type quarks.

Model 2:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} u' \\ s \end{pmatrix}_R, \begin{pmatrix} c' \\ d \end{pmatrix}_R + \text{singlets}, \quad (2.5)$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} N_e \\ e \end{pmatrix}_R, \begin{pmatrix} N_\mu \\ \mu \end{pmatrix}_R + \text{singlets}, \quad (2.6)$$

where N_e and N_μ are heavy neutral leptons (see Sec. VII). Further heavy charged leptons may be found in doublets paralleling e and μ (just as there are several colors of quarks).

There are four other related models which can be obtained by giving (s and c), (s and u), (d and c), or (d and u) right-handed couplings; however, these present no new features which are not present in the above models and, therefore, are not discussed here.

The third model (which Fritzsche and Minkowski, Pati and Salam,¹⁰ and I have considered) is obtained by combining models 1 and 2, so that all quarks have right-handed couplings, and there are no singlets.

If the c' and s' quarks are dropped (requiring some rearrangement on the right-hand side), one obtains the fourth model (which has been proposed in Refs. 6-9).

Model 4:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} u' \\ d' \end{pmatrix}_L, \begin{pmatrix} u \\ d' \end{pmatrix}_R, \begin{pmatrix} c \\ s \end{pmatrix}_R, \begin{pmatrix} u' \\ d \end{pmatrix}_R, \quad (2.9)$$

$$\begin{pmatrix} \nu_e \\ E \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_E \\ E \end{pmatrix}_L, \begin{pmatrix} N_e \\ e \end{pmatrix}_R, \begin{pmatrix} N_\mu \\ \mu \end{pmatrix}_R, \begin{pmatrix} N_E \\ E \end{pmatrix}_R. \quad (2.10)$$

The consequences of these models will be discussed in the following sections.

III. EFFECTS OF DEGENERATE QUARK MASSES

A. With quarks of different charges

If

$$m(c) \approx m(d') \quad (3.1)$$

(where m = mass), a possible explanation (discussed in Ref. 2) for the narrow width²⁶ of ψ_J is obtained. This and other consequences of this approximate mass degeneracy are analogous to the consequences of

$$m(u) \approx m(\bar{d}) . \quad (3.2)$$

Just as

$$\rho = (u\bar{u} - d\bar{d})/\sqrt{2} \quad (3.3)$$

and

$$\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$$

we find

$$\psi_J(3.1) \equiv \rho_2 = (cc - d'\bar{d}')/\sqrt{2} \quad (3.4)$$

The production rates in e^+e^- annihilation and the leptonic widths for $\rho:\omega$ and $\rho_2:\omega_2$ are 9:1 (from the coherent addition and squaring of charges).

The isovector ρ can decay to two pions. The isoscalar ω with negative G parity should not decay to two pions; however, it has a width of 130 keV for that decay mode. This occurs because ω mixes electromagnetically with ρ (electromagnetism does not conserve isospin), since they are very close in mass, $\Delta m = 13$ MeV.

The ρ_2 and ω_2 both have isospin zero since they are not constructed of u and d quarks. But there is a new "charmed" isospin associated with the c and d' quarks, and ρ_2 has charmed isospin = 1. However, decay mechanisms such as those through gluons are charmed isoscalar, so that the decay through gluons is not allowed. Equivalently, the square of the coherent sum of gluon couplings for $\rho_2 = (c\bar{c} - d'\bar{d}')/\sqrt{2}$ is zero. However, in analogy with $\rho-\omega$, ρ_2 can mix electromagnetically with ω_2 , an isoscalar, and decay. The ρ_2 width to hadrons is, therefore, finite but can be very small.

The ω_2 which should be a few MeV in mass from ρ_2 is then much wider than ρ_2 (as are all other resonances without this mechanism) and is produced $\frac{1}{9}$ as much. As a result, it would be very difficult to observe in e^+e^- annihilation.

B. With quarks of the same charge

Wilczek has suggested⁵ that if

$$m(c) \approx m(u') \quad (3.5)$$

one new resonance will be hidden. One would find

$$\psi_J(3.1) = (c\bar{c} + u'\bar{u}')/\sqrt{2} \quad (3.6)$$

and

$$\psi_{\Psi} = (c\bar{c} - u'\bar{u}')/\sqrt{2} .$$

Here ψ_{Ψ} not only does not couple to gluons, but it does not couple to photons since c and u' have the same charge. As a result it is not produced in e^+e^- annihilation, and if produced in hadronic collisions, it does not decay to leptons pairs. In effect, one resonance is hidden under the other.

If one wishes to invoke a u' quark of "low" mass for purposes such as $\mu^+\mu^-$ production (as discussed in Sec. VI) without observing a new resonance, this is a useful mechanism. Although $R^{e^+e^-}$ would increase by $\frac{4}{3} + \frac{4}{3}$ with the c and u' quarks passing threshold, only one resonance would be observed. This is not, of course, a mechanism to make ψ_J narrow.

IV. $\psi(3.7)$ AND RADIAL EXCITATIONS

The models discussed above all assume that the narrow resonance at $\sqrt{s} = 3.7$ GeV is a radial excitation of the state at 3.1 GeV (the same is also true of the structure at 4.2 GeV). However, Harari has proposed⁴ a model in which the $\psi'(3.7)$ is a different particle:

$$\psi_J(3.1) = (c\bar{c} + d'\bar{d}' + u'\bar{u}')/\sqrt{3} , \quad (4.1)$$

$$\psi'(3.7) = (c\bar{c} + d'\bar{d}' - 2u'\bar{u}')/\sqrt{6} , \quad (4.2)$$

$$\psi''(4.2) = (c\bar{c} - d'\bar{d}')/\sqrt{2} . \quad (4.3)$$

The $\psi''(4.2)$ does not have noncharmed hadronic decay modes, but is (as in all models) presumed to be above threshold for decay into charmed mesons (not through gluons) and is then quite wide. The $\psi'(3.7)$ also lacks hadronic decay modes since it is not an SU_3^{charm} singlet (i.e., the square of the sum of couplings is zero). The observation of hadronic decays (5 pions and 2 pions plus 2 kaons) reported by Abrams²⁷ is difficult to explain in this model. Since these modes like leptonic modes are proportional to $|\psi(0)|^2$ and since the leptonic modes²⁶ for $\psi'(3.7)$ are 2.2 keV [compared to 4.8 keV for $\psi_J(3.1)$], the hadronic width is also expected to be smaller.

If, however, the $\psi'(3.7)$ is not a radial excitation, one may ask where the radial excitations are. Harari argued that they are at higher masses (and above threshold for decay to charm) and similarly for p -wave states. The $\rho'(1.6)$ (assuming it is a radial excitation) is not so far above the ρ in mass. It has been argued in nonrelativistic potential models²⁸ and in the MIT bag model²⁹ that ψ' should appear below 4 GeV, and that the p -wave states are expected to lie between ψ_J and ψ' . With the apparent discovery^{30,31} of p -wave states at

3.4 and/or 3.5 GeV, some doubt is cast on models which put radial excitations above 3.7 GeV although more definitive data is still needed.

V. WEAK PHENOMENOLOGY

The neutrino interactions provide a sensitive test^{6,7,32-34} of models of the weak interactions. I will concentrate here on inclusive interactions. The exclusive channels such as

$$\begin{aligned} \nu p &\rightarrow \nu n \pi^+ , \\ \nu p &\rightarrow \nu p , \\ \bar{\nu}_\mu e &\rightarrow \bar{\nu}_\mu e \end{aligned} \quad (5.1)$$

put further limitations on models and are discussed in Refs. 7 and 33.

The charged-current interactions are

$$\bar{\nu} d \rightarrow \mu^+ X \text{ and } \nu d \rightarrow \mu^- X , \quad (5.2)$$

where $X \equiv$ anything and d indicates that we consider the sum of neutron and proton cross sections. The variable y is defined as the fractional energy loss of the leptons, $(E - E')/E$. It is assumed that the neutron and proton contain only u and d quarks (sea quarks are ignored). If we then assume that the weak interactions of u and d quarks are given by

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \quad (5.3)$$

it follows that the $\bar{\nu}$ interaction with u quarks via W^- exchange has a distribution

$$\left(\frac{dN}{dy}\right)_{\bar{\nu}} \propto (1-y)^2, \quad (5.4)$$

which when integrated over y gives a factor (for the cross section) of $\frac{1}{3}$ (there is, of course, no $\bar{\nu}$ interaction with d quarks since the W^- is exchanged). The ν interacts with d quarks giving a constant distribution

$$\left(\frac{dN}{dy}\right)_\nu \propto 1 \quad (5.5)$$

and an integrated factor of 1.

If, in addition to the left-handed interaction, Eq. (5.3), we give the u quark a right-handed interaction with some quark, then the ν has a distribution

$$\left(\frac{dN}{dy}\right)_{\bar{\nu}} \propto [(1-y)^2 + 1] \quad (5.6)$$

and an integrated factor of $\frac{4}{3}$. Similarly, if d quarks also have a right-handed interaction, the

TABLE I. Cross sections for $(\nu d \rightarrow \mu + \text{anything})$ in the models of Sec. II and in the standard four-quark model. R_c is the ratio of the $\bar{\nu}$ to ν cross sections integrated over y .

Model	Asymptotic $(dN/dy)_{\bar{\nu} \rightarrow \mu^+}$	Asymptotic $(dN/dy)_{\nu \rightarrow \mu^-}$	Asymptotic R_c
Weinberg-Salam	$(1-y)^2$	1	$\frac{1}{3}/1 = 1/3$
1	$1 + (1-y)^2$	1	$\frac{4}{3}/1 = 4/3$
2	$(1-y)^2$	$1 + (1-y)^2$	$\frac{1}{3}/\frac{4}{3} = 1/4$
3 and 4	$1 + (1-y)^2$	$1 + (1-y)^2$	$\frac{4}{3}/\frac{4}{3} = 1$

ν has a distribution

$$\left(\frac{dN}{dy}\right)_\nu \propto [1 + (1-y)^2] \quad (5.7)$$

and a factor of $\frac{4}{3}$.

However, without further experimental or theoretical limitations (discussed later), we are free to give the new quarks, with which u and d have right-handed interactions, as large a mass as we wish, thereby maintaining the original distributions and integrated cross sections until higher energies. These results are summarized in Table I where

$$R_c \equiv \frac{\sigma(\bar{\nu} d \rightarrow \mu^+ X)}{\sigma(\nu d \rightarrow \mu^- X)}. \quad (5.8)$$

The Caltech-Fermilab collaboration^{34,35} finds no significant indications of deviations from the distributions of the Weinberg-Salam model or from $R_c = \frac{1}{3}$ (they report $R_c = 0.33 \pm 0.08$). The Harvard-Pennsylvania-Wisconsin-Fermilab (HPWF) collaboration^{36,37} report $R_c = 0.34 \pm 0.03$ and a flat distribution for ν scattering. But at small x where

$$x \equiv \frac{-k^2}{2\nu}, \quad \nu = k \cdot p_N, \quad k = p - p' \quad (5.9)$$

(and only at small x) they report a flat distribution for $\bar{\nu}$ scattering above $E_{\bar{\nu}} = 30$ GeV. If such an effect exists for $\bar{\nu}$ on u quarks without any equivalent effect for ν on d quarks, it would be a violation of charge-symmetry invariance. A violation is predicted for very high energies in model 1; however, R_c should begin to rise above $\frac{1}{3}$ when this threshold is reached. While this discrepancy between these groups exists, no conclusion can be reached on the basis of these data.

For the neutral-current interactions

$$\bar{\nu} d \rightarrow \bar{\nu} X \text{ and } \nu d \rightarrow \nu X, \quad (5.10)$$

the distributions and the cross sections integrated over y are dependent on the Weinberg angle. The asymptotic value of R_n for the models for $\sin^2 \theta_w$

TABLE II. The ratio of $(\bar{\nu}d \rightarrow \bar{\nu} + \text{anything})$ to $(\nu N \rightarrow \nu + \text{anything})$ cross sections integrated over y for the models of Sec. II and for the standard four-quark model. The range of values shown are for $\sin^2\theta_w$ from 0 to 1.

Model	Asymptotic R_n
Weinberg-Salam	0.3-2.4
1	0.6-1.5
2	0.7-2.3
3 and 4	1.0

from 0 to 1 are given in Table II, where

$$R_n \equiv \frac{\sigma(\bar{\nu}d \rightarrow \bar{\nu}X)}{\sigma(\nu d \rightarrow \nu X)}. \quad (5.11)$$

The values of R_n may change in these models as the thresholds for new quark production are reached. The Gargamelle and HPWF groups report³⁸ respectively $R_n = 0.5 \pm 0.2$, ($E_\nu \sim 2$ GeV) and $R_n = 1.0 \pm 0.2$ ($E_\nu \sim 30$ GeV). The Caltech-Fermilab group emphasizes that these neutral-current re-

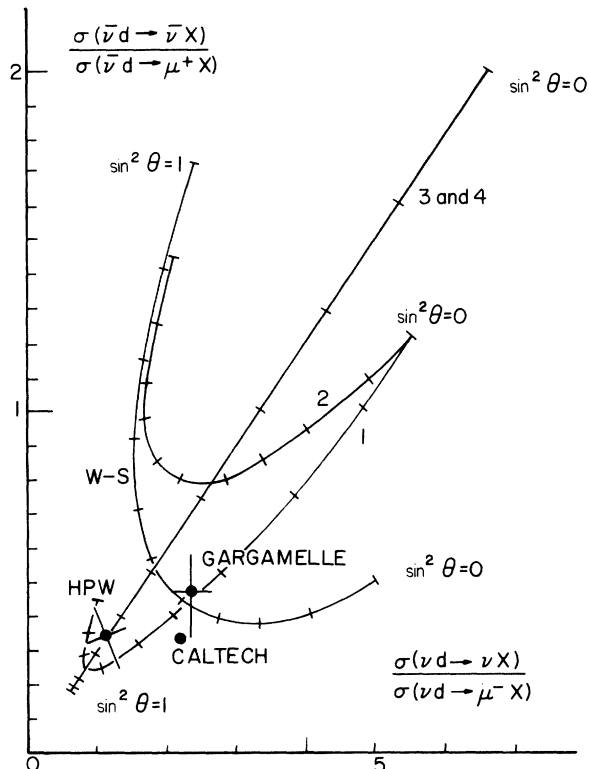


FIG. 2. The ratio of neutral to charged currents for antineutrinos plotted against that ratio for neutrinos. The cross marks on the curves indicate tenths of $\sin^2\theta$ where θ is the Weinberg angle. The curves are numbered 1-4 referring to the models of Sec. II and W-S refers to the standard Weinberg-Salam model. Data are from Refs. 34 and 38.

sults are dependent on the assumed weak couplings. From their raw data one obtains³⁴ $R_n \sim 0.65$ (by contrast, if one assumes pure $V-A$ coupling one can obtain³⁴ $R_n \sim 0.75$).

Another way to look at the neutrino data is to plot $R_{\bar{\nu}}$ vs R_ν , where

$$R_{\bar{\nu}} \equiv \frac{\sigma(\bar{\nu}d \rightarrow \bar{\nu}X)}{\sigma(\bar{\nu}d \rightarrow \mu^+X)}, \quad (5.12)$$

$$R_\nu \equiv \frac{\sigma(\nu d \rightarrow \nu X)}{\sigma(\nu d \rightarrow \mu^-X)}. \quad (5.13)$$

These ratios³⁸ of neutral to charged currents are shown in Fig. 2. The Caltech-Fermilab point is again raw data. In a later run with a different configuration,³⁴ they obtain a point which lies near the Gargamelle point. Final determinations of R_ν and $R_{\bar{\nu}}$ (for all groups) depend on more complete data for which fewer assumptions are needed.

Since there is some freedom to adjust the mass of the Z^0 boson, one can slide the curves in Fig. 2 for each model along the direction defined by the line for models 3 and 4. As a result it may be difficult to distinguish between models on the basis of this graph alone (although the line corresponding to models 3 and 4 obviously cannot be adjusted significantly if the data do not lie on the line shown). However, the Weinberg angle can be fixed here and must agree with other determinations.

VI. CHARM PHENOMENOLOGY

These models do not necessarily have a solution to the problem of the decay of charmed mesons to a K meson plus other particles. If another heavy quark is close in mass to the c quark, mesons containing that quark will not decay in general to a K meson.

However, all such models would benefit from the existence of a heavy lepton of mass ~ 1.8 GeV as discussed in Secs. I and VIII; this is the most plausible solution.

Another type of solution to this problem, the inclusion of a $(c, d)_R$ term,^{12, 13} raises problems with the phases in isospin amplitudes of $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ decays,¹⁴ and with the GIM mechanism.¹⁸ Since a $(u, s)_R$ term is certainly not allowed, there is no cancellation of the new contribution due to $(c, d)_R$ in the $dd \rightarrow ss$ diagrams (two- W exchange with L and R vertices) leading to possible problems in the K_L-K_S mass difference. There is still debate on this point.^{6, 7, 11} However, all models here may avoid this term by an appropriate choice of a Cabibbo-type angle to obtain the forms shown in Sec. II.

Another possibility is that the ψ_J is not constituted of c but of, say, u' quarks which might

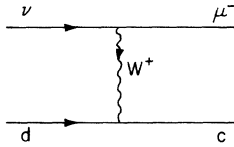


FIG. 3. Production by neutrinos of a charmed quark c which in a charmed meson can decay to a μ^+ plus other particles.

couple with d quarks, although one must keep in mind the limits on the c mass set by Gaillard *et al.*³⁹ The $c\bar{c}$ meson, if it is not ψ_J , might be quite wide if the narrowness of ψ_J is due to the mechanism described in Sec. III A; it could then lie near ψ_J .

The problem can only be resolved by the observation of invariant-mass peaks in some multiparticle channel which should be present at some level irrespective of the presence of heavy leptons or of the weak coupling. Some discussion of this observation appears in Sec. VIII.

The recent discovery of dimuon events in neutrino interactions^{34, 35, 40-42} can be interpreted as evidence for charmed-meson production (an alternative possibility, heavy-lepton production, has also been considered).⁴³⁻⁴⁵ In the standard four-quark model this occurs as in Fig. 3. The W boson converts the d quark into a c quark with a $\sin^2\theta_{\text{Cabibbo}}$ suppression. The c quark is contained in a charmed meson which can decay through channels such as $K\mu^+\nu$. Single-muon events, of course, occur by converting the d into a u quark without any suppression. Using a Cabibbo suppression of 20 and a branching ratio of charm to modes with muons of 10%, the ratio of double- to single-muon events (ignoring threshold and efficiency effects which would lower the predicted ratio) should be less than 0.5%. Experimentally,^{34, 35, 37, 40} the number is about or above 1%. Therefore, without an unrealistic branching ratio to muons, this explanation of dimuons is in trouble.

In some models one has the process shown in Fig. 4 where the d - u' coupling is right-handed and has no Cabibbo suppression. It is, therefore, easily capable of explaining the single- to double-muon ratio. If such a threshold has been reached,

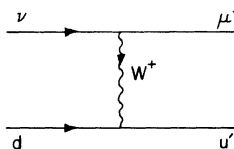


FIG. 4. Production by neutrinos of a heavy u' quark which in a meson can decay to a μ^+ plus other particles.

then R_c (see Sec. V) should approach the value of $\frac{1}{4}$.

A very serious and important problem is presented by the recent results of the HPWF collaboration on dimuon production by antineutrinos.⁴⁶ They report observing

$$\sigma(\bar{\nu} \rightarrow \mu\mu)/\sigma(\bar{\nu} \rightarrow \mu) = (2 \pm 1) \times 10^{-2}, \quad (6.1)$$

$$\sigma(\bar{\nu} \rightarrow \mu\mu)/\sigma(\nu \rightarrow \mu\mu) = 0.8 \pm 0.6. \quad (6.2)$$

Despite the large error bars, the group argues⁴⁶ that the data "indicate unambiguously that dimuon events are indeed produced by $\bar{\nu}$."

In the standard four-quark model, the exchanged W^- can only change a u quark into a d or s quark, so no charm production is possible at all (except through suppressed mechanisms discussed below). In a model with a d' (or s') quark, the u quark can be changed to a d' quark; however, this would eventually require $R_c \rightarrow 1$ and there is no indication of that in the data. If dimuons have as their source d' quarks which are being produced at a fraction of their asymptotic rate, then perhaps the value of R_c does not yet reflect the presence of the $(u, d')_R$ coupling. It should be noted from Sec. V that while $\sigma(\nu \rightarrow \mu^-) = 3\sigma(\bar{\nu} \rightarrow \mu^+)$ for left-handed couplings only, one obtains $\sigma(\bar{\nu} \rightarrow \mu^+\mu^-) \approx 3\sigma(\nu \rightarrow \mu^-\mu^+)$ if u and d both have right-handed couplings, which account for dimuon production.

If the dimuon rate rises without a change in R_c , then there may be flaw in the discussion given above of obtaining dimuons through charm production. Among alternative possibilities are charm production by a different mechanism or off "sea" \bar{s} quarks or simply another source for dimuons; but there is experimental evidence against all of these. In any case this $\bar{\nu}$ experiment is of crucial importance to these models and more extensive results are needed.

The HPWF collaboration has also observed $\mu^-\mu^-$ events.⁴⁰ These can occur if charm-changing neutral currents⁶ are allowed, which is possible in some models depending on details of Cabibbo mixing not shown here. If $D^0 - \bar{D}^0$ mixing results (where D^0 is a charmed meson), then the decay to μ^- rather than μ^+ is possible although the

$$\sigma(\nu \rightarrow \mu^-\mu^-)/\sigma(\nu \rightarrow \mu^-\mu^+) \quad (6.3)$$

ratio observed may be hard to obtain by this method.

VII. MASSIVE NEUTRAL LEPTONS

The remarks in this section, which are applicable to models 2, 3, and 4, are due to Fritzsche *et al.*⁸ They argue that in order to give a mass to the neutral gauge bosons in the $SU_2^{\text{weak}} \times U_1$ the-

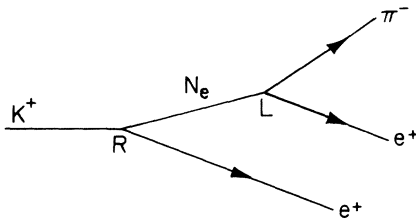


FIG. 5. An example of neutrinoless double β decay (violating lepton-number conservation). N_e is a heavy neutral lepton.

ory, one can violate lepton number with massive neutral leptons (which are then Majorana spinors). With the lepton doublets shown in models 2–4, one must give N_e and N_μ masses greater than the K meson mass or

$$K \rightarrow \mu N_\mu \quad (7.1)$$

(right-handed) would have been observed. Once one gives a mass to neutral leptons, one runs into problems with neutrinoless double β decay (an example of which is shown in Fig. 5, although the most stringent bounds come from nuclear double β decay).

The lepton-number-violating processes which are not observed can be avoided in two ways. One is by making the neutral lepton so light that its left-handed (or its right-handed) component is very small, thereby making the contribution of diagrams such as Fig. 5 small (as for the $\bar{\nu}_e$). The

other way is by making N_e so heavy that the propagator is very small. Since N_e cannot be made light enough, it must be made very heavy, on the order of 100 GeV. This mass is of the same order as the gauge bosons and can cause couplings of fermions to Higgs fields to be very large (of the same order as electromagnetic couplings). This is a serious problem, and Fritzsch *et al.* suggest it may be necessary that e_R and μ_R be singlets (as in model 1) although there are other possible solutions.

VIII. SUMMARY

In this section the four models of Sec. II and the standard Weinberg-Salam-GIM model^{18, 25} will be compared with no effort to rate them since it is unlikely that any of them are completely reasonable. The model of Harari⁴ can be included with model 4 with the exception that in Harari's model the $\psi'(3.7)$ is not a radial excitation.

In model 4 the right-handed couplings shown are the only ones allowed if one excludes $(c, d)_R$. Model 3 has more flexibility in this respect.

The problem of charm decay to K mesons is not solved convincingly in any model without invoking the ameliorating effect of a heavy charged lepton. The heavy lepton contributes approximately 1 to $R^{e^+e^-}$ and has a very small fraction of K mesons in its decays. It is also likely to have a small charged multiplicity, thereby allowing the charm

TABLE III. Summary of the properties of the models of Sec. II and of the standard four-quark model.

	W-S	Models			
		1	2	3	4
$R^{e^+e^-}$ (quarks)	$\frac{10}{3}$	4	6	$\frac{20}{3}$	5
$R^{e^+e^-}$ (total)	$\frac{13}{3}$	6	6	$\frac{26}{3}$	6
Number of charged heavy leptons	1	2	0 or more	2	1
Is $\psi'(3.7)$ a radial excitation?	yes	yes	yes	yes	yes
Is $c-d'$ mass degeneracy possible?	no	yes	no	yes	yes
Is $c-u'$ mass degeneracy possible?	no	no	yes	yes	yes
R_c (asymptotic)	1/3	4/3	1/4	1	1
R_n (asymptotic)	0.3–2.4	0.6–1.5	0.7–2.3	1.0	1.0
Has new quark mechanism for $\nu \rightarrow \mu^+ \mu^-$?	no	no	yes	yes	yes
Has new quark mechanism for $\bar{\nu} \rightarrow \mu^- \mu^+$?	no	yes	no	yes	yes
Anomalies cancel?	yes	yes	yes	yes	yes
Massive neutral lepton necessary?	no	no	yes	yes	yes
Any singlets?	yes	yes	yes	no	no

decays to have a somewhat larger average multiplicity (given the experimental average multiplicity for all events at those energies). This has the effect of increasing the number of available decay channels and making the search for charmed mesons in invariant-mass peaks more difficult. However, with adequate statistics these invariant-mass peaks should appear, most likely in channels such as $K\pi\pi$.

No model discussed here has a clearly correct explanation (with new quarks) for $\mu^+\mu^-$ production by antineutrinos (if those results are confirmed), although models 1, 3, and 4 do have a potential explanation. The explanations of $\mu^+\mu^-$ production by neutrinos are directly correlated with the values of $R^{e^+e^-}$ (if a u' quark is invoked) and R_c , and with the antineutrino results. This area should be watched closely.

The plot of $R_{\bar{\nu}}$ vs R_{ν} (Fig. 2) is a good test of the models, although the data is not yet reliable and the Z^0 and Weinberg angle must have other independent determinations for this graph to achieve greater usefulness. However, models 3 and 4 can be eliminated if the final data do not lie on their line, whereas the other models have greater flexibility. Models 3 and 4 agree with the point of the HPWF collaboration, and the naive versions of the Weinberg-Salam model and model 1 agree with the Gargamelle and Caltech-Fermilab points.

Many of the results discussed here are sum-

marized in Table III. In that table there are ranges of R_n accounting for all Weinberg angles (although the allowed range may be limited by other data). Included in $R^{e^+e^-}$ (total) are heavy charged leptons although there may be additional such leptons without changing the form of the models. The values of $R^{e^+e^-}$ are, of course, asymptotic values; it is expected that $R^{e^+e^-}$ will overshoot that value, as it apparently has for $\sqrt{s} < 3.6$ GeV.

The Weinberg-Salam-GIM model referred to in Table III includes the u , d , s , and c quarks, but also has six leptons rather than the original four. Without the heavy leptons there would be a cancellation of triangle anomalies (quarks cancel with leptons).

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