SU(3)-symmetry breaking and the $K_2^0 \to \pi^0\gamma\gamma$ decay

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The parity-conserving decay $K_2^0 \to \pi^0\gamma\gamma$ is studied using vector gauge fields with current mixing. The effects of SU(3)-symmetry breaking are included in the *PVV* vertices and the "simplest" $\Delta S = 1$ Hamiltonian (with a minimum of neutral currents) consistent with $\Delta I = \frac{1}{2}$ is employed. The calculated decay rate is compared to previous theoretical estimates as well as to the current experimental upper limit for the decay.

I. INTRODUCTION

The weak radiative decays of the K meson have been theoretically studied in the last few years with the expectation that such an investigation might provide some important insight into the dynamics of weak-interaction processes.

Some time ago, a successful approach for describing vector- and pseudoscalar-meson decay was developed by Brown, Munczek, and Singer.[1] Their scheme, characterized by a phenomenological Lagrangian with gauge fields, current mixing, and octet breaking in the *PVV* vertices, led to a good fit of the observed strong and electromagnetic meson decays. A particularly interesting feature of this scheme is the prediction of large SU(3)-symmetry-breaking effects in strange-meson decays.

Recently, Moshe and Singer[2,3] have applied this model to a large class of radiative K-meson decays: $K^+ \to e\nu\gamma$, $K_2^0 \to \gamma\gamma$, $K_2^0 \to \pi^+\pi^-\gamma$, $K^+ \to \pi^+\gamma\gamma$, $K^+ \to \pi^+\pi^0\gamma$, and the vector part of K_{e4}. They were successful in obtaining a suitable description of these processes by assuming the "simplest" $\Delta S = 1$ nonleptonic weak Hamiltonian of the current-current form with both octet terms and a term belonging to the $\underline{27}$ representation of SU(3) with the restriction $\Delta I = \frac{1}{2}$.

In this paper we investigate within the framework of the Moshe-Singer model another interesting weak radiative K-meson decay, the parity-conserving transition $K_2^0 \to \pi^0\gamma\gamma$. One recalls that one of the difficulties in the *CP*-violation experiments involving $K_2^0 \to \pi^0\pi^0$ is the distinction of this decay from such modes as $K_2^0 \to \pi^0\gamma\gamma$. The presence of the latter mode would be particularly difficult to deal with, since it represents a four-γ event (just as in the case of $K_2^0 \to \pi^0\pi^0$), and the exact form of the matrix element for this decay is not known.

At the present time there has only been one experiment for which an estimate of the $K_2^0 \to \pi^0\gamma\gamma$ background has been attempted.[4] This experiment yields an upper limit for the branching ratio $\Gamma(K_2^0 \to \pi^0\gamma\gamma)/\Gamma(K_2^0 \to \pi^0\pi^0) < 0.24$ and an upper limit on the decay rate $\Gamma(K_2^0 \to \pi^0\gamma\gamma) < 4.6\times 10^3\,\text{sec}^{-1}$. Despite the limited experimental data, it is still theoretically desirable to have a reliable estimate for the frequency of the $K_2^0 \to \pi^0\gamma\gamma$ decay mode.

We present in this paper a theoretical estimate of the decay rate for $K_2^0 \to \pi^0\gamma\gamma$ decay based on the model of Refs. 1 and 2, taking into account the effects of SU(3)-symmetry breaking. In addition, from calculating the exact form of the matrix element for this process, we determine the pion energy spectrum and certain features of the Dalitz-plot density for this reaction.

We wish to point out that there have been a number of previous estimates for $K_2^0 \to \pi^0\gamma\gamma$ decay. One calculation[5] was based on the assumption that the decay is dominated by the η pole: $K_2^0 \to \eta^0 \to \pi^0\gamma\gamma$, thus requiring the amplitude to be related to the amplitude for $\eta^0 \to \pi^0\gamma\gamma$. This picture was especially appealing a number of years ago when it was believed that $\eta^0 \to \pi^0\gamma\gamma$ was one of the dominant modes of η decay. However, with recently improved experiments, it is now believed that this mode of η decay is a very rare one and thus the η-pole model has lost much of its original appeal.

Another estimate has been made recently by Sehgal[6] based on dispersion relations and a model which determines the amplitude for $K_2^0 \to \pi^0\gamma\gamma$ decay in terms of the amplitude for $K_2^0 \to \pi^+\pi^-\pi^0$. Finally, there has also been a calculation based on current-algebra and soft-pion techniques.[7]

The paper is arranged as follows: In Sec. II we discuss the SU(3)-symmetry-breaking model for radiative meson decays. In Sec. III we present our calculation of the matrix element and decay rate for $K_2^0 \to \pi^0\gamma\gamma$ based on the Moshe-Singer model. In addition, we determine the pion energy spectrum and the behavior of the Dalitz-plot density for the decay. Section IV is devoted to a summary of our results.

II. THE BROKEN SU(3) MODEL FOR MESON DECAYS

We begin by presenting a review of the scheme developed by Moshe and Singer[2] for investigating weak radiative K-meson decays. The nonleptonic

$\Delta S=1$, CP-conserving, weak Hamiltonian which is chosen has the current-current form

$$\mathcal{H}_w=\sqrt{2}\,G_{\rm NL}[J_\mu^1(x)J_\mu^4(x)+J_\mu^2(x)J_\mu^5(x)-J_\mu^3(x)J_\mu^6(x)], \quad (1)$$

where $J_\mu^a=j_\mu^{Va}+j_\mu^{Aa}$, $a=1,2,\ldots,8$ is an SU(3) index and we set[8] $G_{\rm NL}=1.1\times10^{-5}/m_p^2$. As the result of its form, $\mathcal{H}_w$ accommodates the $\Delta I=\frac{1}{2}$ rule with a minimum of neutral currents and transforms as members of an octet and a 27 representation of SU(3).

As suggested by Sakurai,[9] the currents appearing in (1) are assumed to be dominated by vector, pseudoscalar, and axial-vector fields, and this assumption can be stated by the field-current identities

$$j_\mu^{Va}=\frac{m_V^2}{f_V}V_\mu^a, \quad (2a)$$

$$j_\mu^{Aa}=\frac{m_A^2}{f_A}A_\mu^a-C_a\partial_\mu\phi^a, \quad (2b)$$

where C_a represents the pseudoscalar-meson decay constants and f_A, f_V are the current-field couplings.

Turning to the strong interactions, the effective Lagrangian responsible for the PVV and PPV interactions which emerge in the vector-meson-dominance picture is given by the general octet-broken form[1]

$$\mathcal{L}_{\rm int}=\tfrac{1}{4}\epsilon_{\alpha\beta\mu\nu}(hD^{abc}V_{\alpha\beta}^aV_{\mu\nu}^bP^c+\lambda D^{ab}V_{\mu\nu}^aP^bV_{\alpha\beta}^0)-gf^{abc}\partial_\mu P^aV_\mu^bP^c, \quad (3)$$

where

$$V_{\mu\nu}^a=\partial_\mu V_\nu^a-\partial_\nu V_\mu^a-gf^{abc}V_\mu^bV_\nu^c, \quad (4)$$

$$D^{abc}=d^{abc}+\sqrt{3}\,\epsilon_1 d^{abd}d^{48c}+\frac{\sqrt{3}}{2}\epsilon_2(d^{acd}d^{d8b}+d^{bcd}d^{d8a})+\frac{\epsilon_3}{\sqrt{3}}\delta^{ab}\delta^{c8}, \quad (5)$$

$$D^{ab}=\delta^{ab}+\sqrt{3}\,\epsilon_4 d^{ab8}. \quad (6)$$

V_μ^a, V_μ^0 represent the nine vector fields described by a Yang-Mills-type Lagrangian which when diagonalized in terms of the physical particles gives the relationships of the V_μ^a to the physical fields

$$V_\mu^{1,2,3}=\frac{1}{\sqrt{K_\rho}}\rho_\mu^{1,2,3},$$
$$V_\mu^{4,5,6,7}=\frac{1}{\sqrt{K_{K^*}}}K_\mu^{*4,5,6,7},$$
$$V_\mu^8=-\frac{\sin\theta}{\sqrt{K_\omega}}\omega_\mu+\frac{\cos\theta}{\sqrt{K_\phi}}\phi_\mu, \quad (7)$$
$$V_\mu^0=\frac{\cos\theta}{\sqrt{K_\omega}}\omega_\mu+\frac{\sin\theta}{\sqrt{K_\phi}}\phi_\mu,$$

with

$$K_i=\frac{m^2}{m_i^2}\ \ (i=\rho,K^*,\omega,\phi),\quad m=847\ \text{MeV},\quad \theta=27.5°. \quad (8)$$

For the electromagnetic interaction we adopt the formalism of vector-meson dominance of the hadronic electromagnetic current developed by Kroll, Lee, and Zumino.[10] This leads, in the absence of weak interactions, to the effective coupling

$$\mathcal{L}_{\rm em}=\frac{em^2}{g}\left(\frac{1}{\sqrt{K_\rho}}\rho_\mu^0+\frac{\sin\theta}{\sqrt{3K_\omega}}\omega_\mu-\frac{\cos\theta}{\sqrt{3K_\phi}}\phi_\mu\right)A_\mu^\gamma, \quad (9)$$

where A_μ^γ is the electromagnetic field.

The Lagrangians (3) and (9) were used by Brown, Munczek, and Singer[1] to calculate a large class of strong and electromagnetic meson decays. SU(3)-symmetry-breaking effects on the amplitudes for these decays are incorporated by the presence of factors containing the masses of the vector mesons, the current mixing angle θ, and the octet-breaking parameters ϵ_i. From fitting the available experimental data on these decays they obtained[1]

$$\lambda=\frac{2h(1+\epsilon_1)\cot\theta}{\sqrt{3}\,(1+\epsilon_4)},\quad \frac{g^2}{4\pi}=3.20,\quad \frac{m_\pi^2h^2}{4\pi}(1+\epsilon_1)^2=0.1, \quad (10)$$

and the two possible solutions for the ϵ_i:

$\epsilon_1=0.85$, $\epsilon_2+\epsilon_3=2.16$, $\epsilon_4=-0.32$ or 0.65 (solution I).

$\epsilon_1=1.18$, $\epsilon_2+\epsilon_3=-2.54$, $\epsilon_4=1.53$ or -3.12 (solution II).

By extending this model to a class of radiative K-meson decays, using the weak Hamiltonian (1), Moshe and Singer[2] were able to rule out solution I from the current experimental upper limit on the decay $K_2^0\to\pi^+\pi^-\gamma$. In addition, from these decays they succeeded in establishing limits on the range of ϵ_2, namely, $-10<\epsilon_2<-4.5$. In a subsequent work[3] the same authors have been able, by applying their model to $K^+\to\pi^+\pi^0\gamma$ decay, to fix the value of the parameter at $\epsilon_2=-0.9^{+2.0}_{-1.1}$ on the basis of the recently measured[11] rate for $K^+\to\pi^+\pi^0\gamma$. As a result, the presently acceptable solutions for the SU(3)-breaking parameters ϵ_i consistent with experiment are

$$\epsilon_1 = 1.18,$$
$$\epsilon_2 + \epsilon_3 = -2.54,$$
$$\epsilon_2 = -0.9^{+2.0}_{-1.1}, \qquad (11)$$
$$\epsilon_4 = 1.53 \text{ or } -3.12.$$

Thus, the broken-SU(3) model, described by the Lagrangians (3) and (9), the weak Hamiltonian (1), and the values of the parameters as stated in Eqs. (10) and (11), is successful in giving an accurate description of a large class of strong and electromagnetic decays of vector and pseudoscalar mesons as well as various weak radiative decays of K mesons.

III. THE DECAY $K_2^0 \to \pi^0\gamma\gamma$

A. General Considerations

In the limit of CP invariance, $K_2^0 \to \pi^0\gamma\gamma$ involves only the parity-conserving part of the nonleptonic weak Hamiltonian. Thus, the general Lorentz- and gauge-invariant form of its matrix element is given by

$$M = F_1[(k\cdot k')(\epsilon\cdot\epsilon') - (k\cdot\epsilon')(k'\cdot\epsilon)] + F_2[(p\cdot k)(p\cdot k')(\epsilon\cdot\epsilon') + (p\cdot\epsilon)(p\cdot\epsilon')(k\cdot k') - (p\cdot k')(k\cdot\epsilon')(p\cdot\epsilon) - (p\cdot k)(k'\cdot\epsilon)(p\cdot\epsilon')], \qquad (12)$$

where p, k, and k' are the momentum four-vectors of K_2^0 and the two photons, respectively, and ϵ and ϵ' are the polarization vectors of the two photons. F_1 and F_2 are form factors which are, in general, functions of the kinematical invariants.

On the experimental side, the present upper limit set for this process[4] gives the branching ratio

$$\frac{\Gamma(K_2^0 \to \pi^0\gamma\gamma)}{\Gamma(K_2^0 \to \text{all})} < 2.4\times 10^{-4}, \qquad (13)$$

or the decay rate

$$\Gamma(K_2^0 \to \pi^0\gamma\gamma) < 4.6\times 10^3\ \text{sec}^{-1}. \qquad (14)$$

In Sec. IIIB we calculate the matrix element and decay rate for $K_2^0 \to \pi^0\gamma\gamma$ based on the model presented in Sec. II.

B. Calculation of decay rate

In Fig. 1 we display the three classes of diagrams contributing to the decay in the Moshe-Singer model under the assumption of CP invariance. We wish to point out the absence of a class of four-particle-vertex diagrams which Moshe and Singer found to be the dominant contributors to the amplitude for $K^+ \to \pi^+\gamma\gamma$ decay.[2] Such diagrams with $PVVV$ vertices are generated through the use of a phenomenological Lagrangian like Eq. (3) with with vector-gauge particles. However, in the case of $K_2^0 \to \pi^0\gamma\gamma$ decay, a parity-conserving transition, such four-particle-vertex diagrams are forbidden by parity conservation and thus can give no contribution to this decay.

By applying the weak Hamiltonian defined by Eq. (1), the detailed structure of the weak interaction vertices are obtained as shown in Fig. 2.

The various diagrams appearing in Fig. 1 can be classified into groups according to their vertices. The detailed diagrams are displayed in Fig. 3. Certain diagrams (a, b, c, e, f) are characterized by two PVV vertices, whereas other diagrams (d, g) contain one PVV vertex and one PPV vertex. We have included the contributions of the axial-vector mesons to the axial-vector current using a procedure developed by Rockmore.[12] In this procedure one exploits partial conservation of the axial-vector current in order to express the combined contribution of the pseudoscalar and axial-vector poles in terms of the pseudoscalar-pole contribution only.

The calculation of the diagrams in Fig. 3 using Eqs. (1)–(9) yields for the matrix element for $K_2^0 \to \pi^0\gamma\gamma$ decay

$$M = \sqrt{2}\,G_{\text{NL}}\frac{(\hbar em)^2}{g^2} \times \sum_{i=1}^{10} g_i[(P_iA + P_i'A')B + (P_i + P_i')CD], \qquad (15)$$

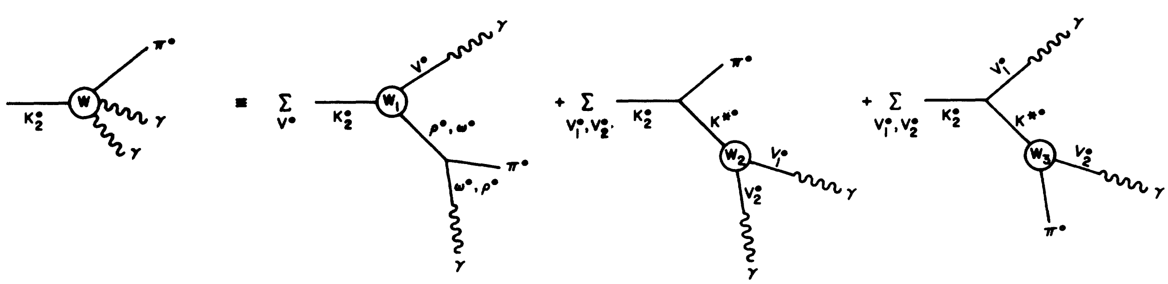

FIG. 1. The three classes of diagrams contributing to $K_2^0 \to \pi^0\gamma\gamma$.

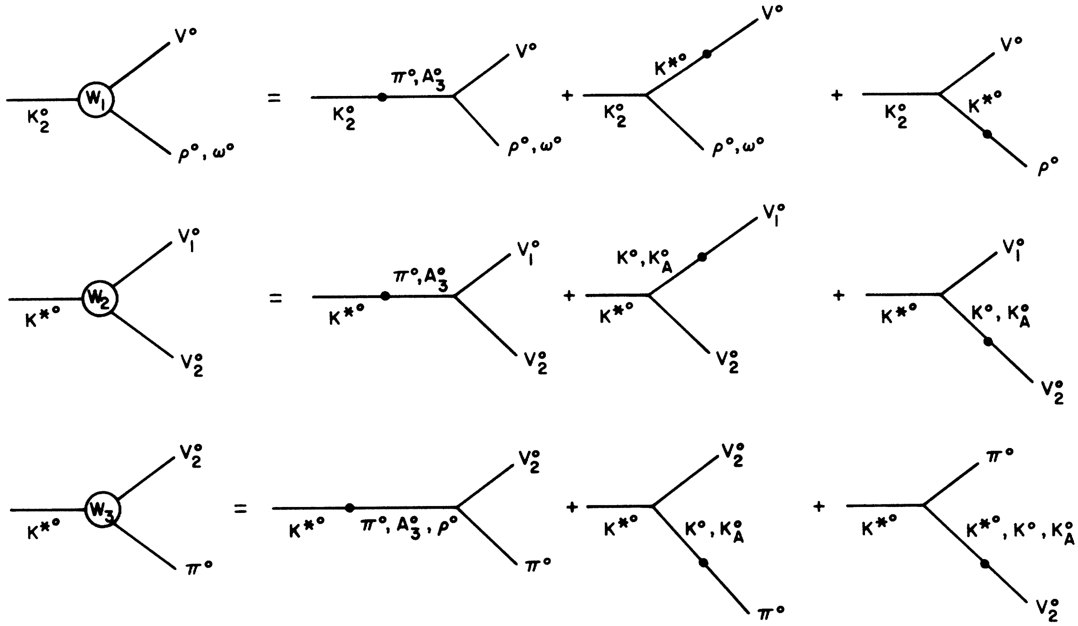

FIG. 2. Nonvanishing contributions to the structure of the weak-interaction vertices W_i.

where

$$g_1 = \frac{4}{9} C_K C_\pi \frac{m_\pi^2}{(m_K^2 - m_\pi^2)} \frac{(1+\epsilon_1)^2}{K_\rho^2 m_\rho^2}, \tag{16a}$$

$$g_2 = \frac{4}{3} C_K C_\pi \frac{m_\pi^2}{(m_K^2 - m_\pi^2)} \frac{(1+\epsilon_1)^2}{K_\omega^2 m_\omega^2 \sin^2\theta}, \tag{16b}$$

$$g_3 = -\frac{\sqrt{2}}{3} \frac{(1+\epsilon_1)(1-\frac{1}{2}\epsilon_1+\frac{1}{4}\epsilon_2)}{f_\rho f_{K^*}\sqrt{K_{K^*}}\sqrt{K_\rho}\, K_\rho^2}, \tag{16c}$$

$$g_4 = \frac{\sqrt{2}}{3}(1+\epsilon_1)\left[2\cos^2\theta \frac{(1+\epsilon_1)}{(1+\epsilon_4)}(1-\tfrac{1}{2}\epsilon_4) - \sin^2\theta(1-\tfrac{1}{2}\epsilon_1-\tfrac{3}{4}\epsilon_2)\right] \tag{16d}$$
$$\times (f_\rho f_{K^*} K_\omega \sin^2\theta \sqrt{K_{K^*}}\sqrt{K_\rho})^{-1},$$

$$g_5 = \frac{8}{9} \frac{(1-\frac{1}{2}\epsilon_1)(1+\epsilon_1)}{f_\rho f_{K^*}\sqrt{K_\rho}\sqrt{K_{K^*}}K_{K^*}}, \tag{16e}$$

$$g_6 = -\frac{1}{2\sqrt{2}} \frac{C_K g}{f_\rho h} \frac{(1-\frac{1}{2}\epsilon_1+\frac{1}{4}\epsilon_2)}{m_K^2\sqrt{K_\rho}\sqrt{K_{K^*}}}, \tag{16f}$$

$$g_7 = \frac{1}{6\sqrt{2}} \frac{C_K g}{f_\rho h m_K^2 \sqrt{K_\rho}\sqrt{K_{K^*}}} \times \left(2\cos^2\theta \frac{(1+\epsilon_1)}{(1+\epsilon_4)}(1-\tfrac{1}{2}\epsilon_4) - \sin^2\theta(1-\tfrac{1}{2}\epsilon_1-\tfrac{3}{4}\epsilon_2)\right), \tag{16g}$$

$$g_8 = \frac{16}{9} C_\pi C_K \frac{m_K^2}{(m_K^2 - m_\pi^2)} \frac{(1-\frac{1}{2}\epsilon_1)^2}{m_{K^*}^2 K_{K^*}^2}, \tag{16h}$$

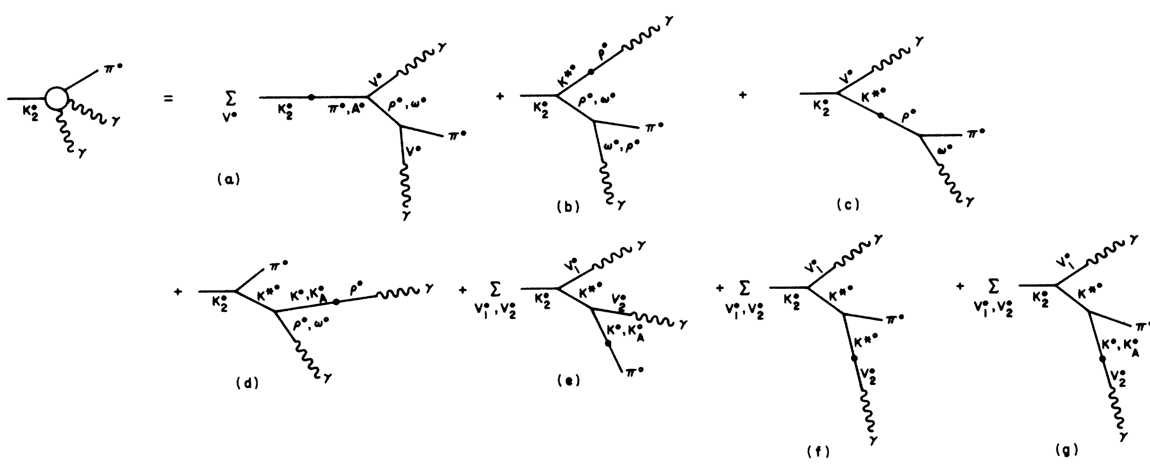

FIG. 3. Feynman diagrams contributing to parity-conserving $K_2^0 \to \pi^0\gamma\gamma$.

$$g_9=\frac{8}{3}\,\frac{(1-\frac{1}{2}\epsilon_1)(1+\epsilon_1-\frac{1}{2}\epsilon_2)}{f_\rho f_{K^*}\sqrt{K_\rho}\,\sqrt{K_{K^*}K_{K^*}}}, \tag{16i}$$

$$g_{10}=\frac{4}{3}\,\frac{C_K g(1-\frac{1}{2}\epsilon_1)}{f_\rho h m_K{}^2\sqrt{K_{K^*}}\sqrt{K_\rho}}, \tag{16j}$$

and

$$A=m_K{}^2-m_K k, \tag{17a}$$

$$A'=m_K{}^2-m_K k', \tag{17b}$$

$$B=(\vec{k}\cdot\vec{\epsilon}\,')(\vec{k}'\cdot\vec{\epsilon})-(k\cdot k')(\vec{\epsilon}\cdot\vec{\epsilon}\,'), \tag{17c}$$

$$C=m_K{}^2 k k', \tag{17d}$$

$$D=\vec{\epsilon}\cdot\vec{\epsilon}\,', \tag{17e}$$

$$P_1=P_3=(m_\rho{}^2-m_K{}^2+2m_K k)^{-1}, \tag{17f}$$

$$P_2=P_4=(m_\omega{}^2-m_K{}^2+2m_K k)^{-1}, \tag{17g}$$

$$P_6=P_7=(m_{K^*}{}^2-m_K{}^2-m_\pi{}^2+2m_K E_\pi)^{-1}, \tag{17h}$$

$$P_8=P_9=P_{10}=(m_{K^*}{}^2-m_K{}^2-2m_K k)^{-1}, \tag{17i}$$

$$P_5=m_\rho{}^2 P_1 P_9, \tag{17j}$$

where E_π represents the pion energy and the expressions for the P_i' are obtained from those of P_i by replacing k by k'.

Squaring M and summing over photon polarizations gives

$$\sum|M|^2=m_K{}^2k^2k'^2\{[2g_i{}^2P_i{}^2(k-m_K)k+2g_i{}^2P_i'^2(k'-m_K)k'] +2g_i g_j P_i P_j'[2(k-m_K)(k'-m_K)+m_K(k-m_K)+m_K(k'-m_K)](1-\cos\psi)^2 +m_K{}^2g_i{}^2(P_i+P_i')^2(1+\cos^2\psi)\}, \tag{18}$$

where ψ is the angle between the photon three-momentum vectors $\vec{k}$ and $\vec{k}'$.

Integrating Eq. (18) over the three-particle phase space yields for the decay rate

$$\Gamma(K_2^0\to\pi^0\gamma\gamma)=\frac{\alpha^2 m_K{}^5 G_{\rm NL}{}^2}{16\pi}\left(\frac{hm}{g}\right)^4 G, \tag{19}$$

where $\alpha=e^2/4\pi$ and G is a dimensionless integral ($m_K=1$) given by

$$G=\sum_{i,j} g_i g_j I_{ij}, \tag{20}$$

with

$$I_{ij}=-\int_{m_\pi}^{(1+m_\pi{}^2)/2}dE_\pi \times\int_{[1-E_\pi-(E_\pi{}^2-m_\pi{}^2)^{1/2}]/2}^{[1-E_\pi+(E_\pi{}^2-m_\pi{}^2)^{1/2}]/2} M_{ij}\,dk, \tag{21}$$

and

$$\begin{aligned} M_{ij}&=(\mu\beta^2-\rho)P_iP_j+(\delta\beta^2-\rho)P_i'P_j' +(\sigma\beta^2-\rho)P_iP_j',\\ \mu&=2k(1-k),\\ \beta&=(1+m_\pi{}^2-2E_\pi),\\ \delta&=2(1-E_\pi-k)(E_\pi+k),\\ \rho&=4k^2(1-E_\pi-k)^2+[2k^2-2k(1-E_\pi)+\beta]^2,\\ \sigma&=-2(1-k)(E_\pi+k)+1+E_\pi. \end{aligned} \tag{22}$$

The I_{ij} can be numerically evaluated, and we find

$I_{11}=I_{33}=I_{13}=I_{31}=5.78\times10^{-4}$,

$I_{22}=I_{44}=I_{24}=I_{42}=5.31\times10^{-4}$,

$I_{55}=4.55\times10^{-4}$,

$I_{66}=I_{77}=I_{67}=I_{76}=2.33\times10^{-4}$,

$I_{88}=3.28\times10^{-4}$,

$I_{99}=I_{1010}=I_{910}=I_{109}=2.81\times10^{-4}$,

$I_{15}=I_{51}=I_{35}=I_{53}=5.11\times10^{-4}$,

$I_{18}=I_{81}=I_{38}=I_{83}=4.35\times10^{-4}$,

$I_{12}=I_{21}=I_{23}=I_{32}=I_{14}=I_{41}=I_{34}=I_{43}=5.54\times10^{-4}$,

$I_{16}=I_{61}=I_{36}=I_{63}=I_{17}=I_{71}=I_{37}=I_{73}=3.65\times10^{-4}$,

$I_{19}=I_{91}=I_{39}=I_{93}=I_{110}=I_{101}=I_{310}=I_{103}=4.04\times10^{-4}$,

$I_{26}=I_{62}=I_{46}=I_{64}=I_{27}=I_{72}=I_{47}=I_{74}=3.50\times10^{-4}$,

$I_{29}=I_{92}=I_{49}=I_{94}=I_{210}=I_{102}=I_{410}=I_{104}=3.87\times10^{-4}$,

$I_{69}=I_{96}=I_{79}=I_{97}=I_{610}=I_{106}=I_{710}=I_{107}=2.55\times10^{-4}$,

$I_{25}=I_{52}=I_{45}=I_{54}=4.90\times10^{-4}$,

$I_{28}=I_{82}=I_{48}=I_{84}=4.18\times10^{-4}$,

$I_{56}=I_{65}=I_{57}=I_{75}=3.22\times10^{-4}$,

$I_{58}=I_{85}=3.85\times10^{-4}$,

$$I_{59} = I_{95} = I_{510} = I_{105} = 3.56 \times 10^{-4},$$

$$I_{68} = I_{86} = I_{78} = I_{87} = 2.76 \times 10^{-4},$$

$$I_{89} = I_{98} = I_{810} = I_{108} = 3.04 \times 10^{-4}.$$

Using these numerical results, the decay rate for $K_2^0 \to \pi^0\gamma\gamma$ can be calculated using Eqs. (8), (10), (11), (16), (19), and (20) along with the identity

$$\left(\frac{m_\rho}{f_\rho}\right)^2 = \left(\frac{m_{K^*}}{f_{K^*}}\right)^2 = \left(\frac{m}{g}\right)^2, \tag{23}$$

the relation[13] $C_K = 1.08\, C_\pi$, and the tabulated values[14] for C_π and the various particle masses. The result for the decay rate depends on the SU(3)-breaking parameters ϵ_1, ϵ_2, and ϵ_4, whose values are given by Eq. (11).

We find that, depending on the solution for ϵ_4, the Moshe-Singer model predicts for the decay rate

$$\Gamma(K_2^0 \to \pi^0\gamma\gamma) = 2.7^{+2.3}_{-2.2}\ \text{sec}^{-1} \quad (\epsilon_4 = 1.53), \tag{24}$$

$$\Gamma(K_2^0 \to \pi^0\gamma\gamma) = 0.7^{+1.5}_{-0.6}\ \text{sec}^{-1} \quad (\epsilon_4 = -3.12), \tag{25}$$

which are three orders of magnitude below the present experimental upper limit.

In the SU(3)-symmetric limit ($\epsilon_i \to 0$) we find that

$$\Gamma^{\text{SU}(3)}(K_2^0 \to \pi^0\gamma\gamma) = 50\ \text{sec}^{-1}. \tag{26}$$

The above results are displayed graphically in Fig. 4. As can be seen from the graph, the results are not very sensitive to the value of ϵ_4 chosen. It is also clear that there are sizable SU(3)-symmetry-breaking effects present in this decay mode.

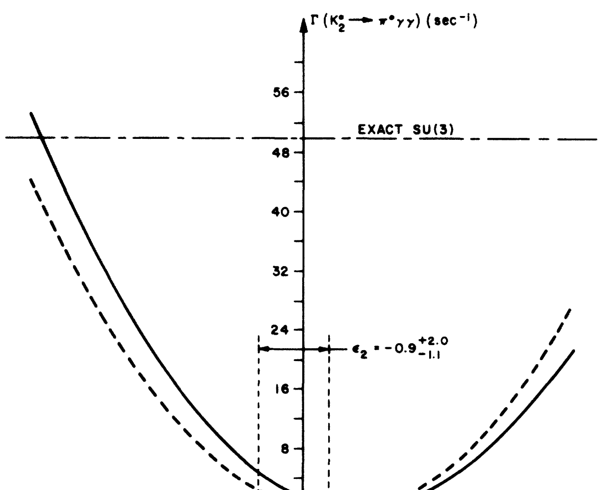

FIG. 4. The decay rate of $K_2^0 \to \pi^0\gamma\gamma$ as a function of the SU(3)-symmetry-breaking parameter ϵ_2. The allowed value for ϵ_2 is determined from the recently measured rate for $K^+ \to \pi^+\pi^0\gamma$. The calculation is made for two possible solutions for ϵ_4: $\epsilon_4 = 1.53$ (unbroken line); $\epsilon_4 = -3.12$ (broken line).

We have exhibited a comparison of our results with those based on other models in Table I. Our predicted values for the decay rate are about one order of magnitude below the estimate obtained from the dispersion-relation model but is consistent with the prediction of current algebra. On the other hand, our estimate is about three orders of magnitude below that predicted by the η-pole model, which is tentatively ruled out by the current experimental upper limit.[15]

We have also calculated the pion-energy spectrum for various values of ϵ_2. These results are shown in Fig. 5. We have included, for the purposes of comparison, the spectrum which is predicted by the η-pole model. The striking differences between the shapes of the two spectra are evident.

Further information concerning the exact form of the matrix element for $K_2^0 \to \pi^0\gamma\gamma$ decay can be extracted from its Dalitz plot. We recall that the general form of the matrix element is given by Eq. (12), where F_1 and F_2 are functions of the kinematical invariants s and t, where

$$\begin{aligned} s &= (k + k')^2, \\ t &= (p - k)^2, \\ u &= (p - k')^2, \\ s + t + u &= m_K^2 + m_\pi^2. \end{aligned} \tag{27}$$

The physical region of the invariants is shown in the Dalitz plot of Fig. 6 and is bounded by the curves

$$\begin{aligned} s &= 0, \\ (t - u)^2 &= (s - m_K^2 - m_\pi^2)^2 - 4 m_K^2 m_\pi^2, \end{aligned} \tag{28}$$

with s, the invariant mass squared of the photon pair, having values in the interval

$$0 < s < (m_K - m_\pi)^2. \tag{29}$$

From examining the structure of our matrix ele-

TABLE I. Comparison of various model predictions for the decay rate of $K_2^0 \to \pi^0\gamma\gamma$ with the present experimental data.

Source	$\Gamma(K_2^0 \to \pi^0\gamma\gamma)$ (sec^{-1})
η-pole model	6.3×10^3
Dispersion-relation model	1.3×10^1
Current algebra	4.2×10^{-2}–8.5×10^3
Present calculation	$2.7^{+2.3}_{-2.2}$ or $0.7^{+1.5}_{-0.6}$
Experiment	$< 4.6 \times 10^3$

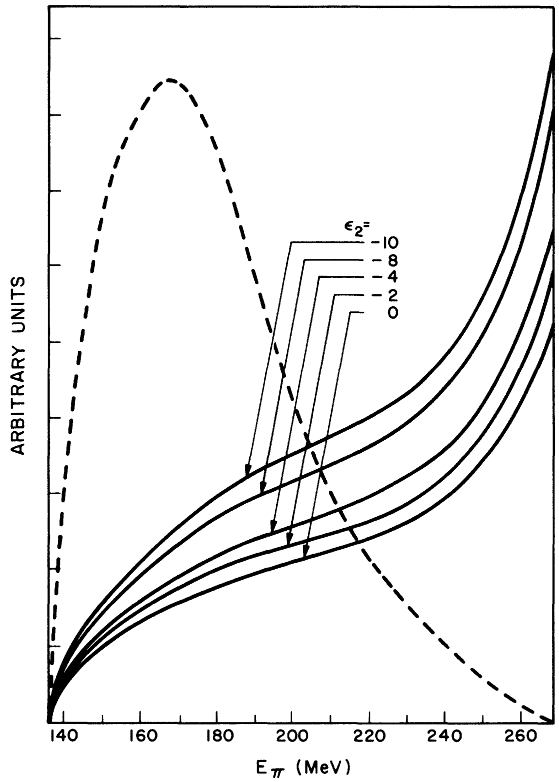

FIG. 5. Comparison of pion-energy spectrum for different models of $K_2^0 \to \pi^0\gamma\gamma$. Solid lines: present calculation (for different values of ϵ_2); dashed line: η-pole model (Ref. 4).

ment in Eq. (15) it can be seen that its form is such that both F_1 and F_2 appear and are both dependent on s and t. Thus the Moshe-Singer model predicts that the Dalitz-plot density for $K_2^0 \to \pi^0\gamma\gamma$ should vary in both the vertical and horizontal directions. This behavior of the Dalitz-plot is also a feature of the η-pole model. However, in the model by Sehgal, based on dispersion relations, the matrix element which is calculated contains only the F_1 function which depends only on s, thus implying a density variation only in the vertical direction. Hence, both the shape of the pion-energy spectrum and the behavior of the Dalitz-plot density can, in principle, provide ways of experimentally testing some of the various models proposed for $K_2^0 \to \pi^0\gamma\gamma$ decay.

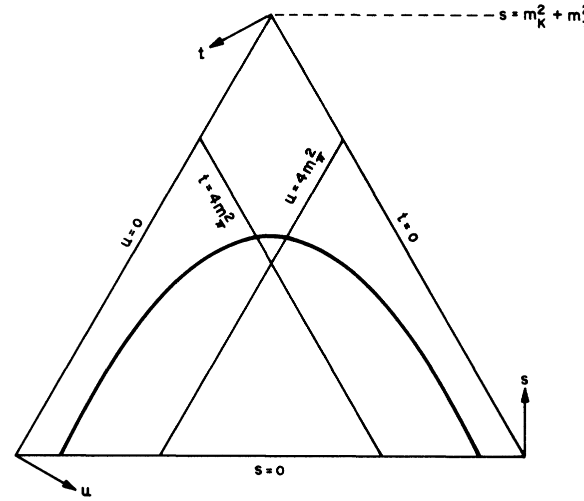

FIG. 6. Dalitz plot for $K_2^0 \to \pi^0\gamma\gamma$. The physical region is bounded by the curve and the line $s = 0$.

IV. SUMMARY AND CONCLUSIONS

We have examined the parity-conserving, weak radiative meson decay $K_2^0 \to \pi^0\gamma\gamma$. Using a Hamiltonian with a minimum of neutral currents but consistent with $\Delta I = \frac{1}{2}$ to describe the weak interaction and employing a phenomenological Lagrangian with gauge fields and SU(3) breaking for the strong interactions, we have calculated the decay rate for $K_2^0 \to \pi^0\gamma\gamma$. We find that the decay rate depends on three SU(3)-breaking parameters ϵ_1, ϵ_2, and ϵ_4. By using the solutions for these parameters as deduced from previous work on radiative K-meson decays, we arrive at predictions for the decay rate given by

$$\Gamma(K_2^0 \to \pi^0\gamma\gamma) = 2.7^{+2.3}_{-2.2}\ \mathrm{sec}^{-1} \quad (\epsilon_4 = 1.53),$$

$$\Gamma(K_2^0 \to \pi^0\gamma\gamma) = 0.7^{+1.5}_{-0.6}\ \mathrm{sec}^{-1} \quad (\epsilon_4 = -3.12).$$

It will be most interesting to see whether or not future experiments find any significant enhancement over these very low values for the decay rate.

We have also calculated the pion-energy spectrum and have studied the behavior of the Dalitz-plot density for $K_2^0 \to \pi^0\gamma\gamma$. We find that these features of the decay may prove useful in distinguishing the various models which have been proposed and in experimentally determining the correct mechanism for this decay mode.

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