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Present status of the Cabibbo-Radicati sum rule*

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We have reevaluated the Cabibbo-Radicati sum rule using recent data and analyses of pion photoproduction in the resonance region. As an alternate method we have applied the extended vector-dominance model together with the quark model and the latest pion-nucleon total cross sections. Both procedures consistently indicate a breakdown of the sum rule; the disagreements are 14% and 12%, respectively.

I. INTRODUCTION

Historically the concept of scaling in deep-inelastic electron-nucleon scattering[1] was motivated by current algebra and in particular by the Adler sum rule (ASR).[2] Soon afterwards the SLAC-MIT experiments[3] provided a strong confirmation of the scaling hypothesis. However, more recent experiments at Fermilab[4] have shown some evidence for a violation of scaling at large momentum transfers. Some speculations about the breakdown of the ASR have been advanced in the past.[5,6] However, a true separation of the neutrino and antineutrino deep-inelastic structure functions on protons has not been performed yet. Therefore a numerical evaluation of the ASR cannot yet be carried out in a model-independent way.[7]

It is well known that the vector part of the ASR, i.e.,

$$\int d\nu [W_2^{\bar\nu p}(\nu, q^2) - W_2^{\nu p}(\nu, q^2)]^{VV} = 1, \tag{1}$$

is exactly satisfied at $q^2 = 0$. On the other hand, if one differentiates Eq. (1) with respect to q^2 at $q^2 = 0$ and uses conserved vector current (CVC), one obtains the Cabibbo-Radicati sum rule[8]

$$\tfrac{1}{6}\langle r_V^2\rangle = \tfrac{1}{2}\left(\frac{\mu_p - \mu_n}{2M}\right)^2 + \frac{1}{4\pi^2\alpha}\int_{\nu_0}^{\infty}\frac{d\nu}{\nu}[\sigma_T(\gamma^- p) - \sigma_T(\gamma^+ p)]. \tag{2}$$

If both the ASR and CVC are correct in the neighborhood of $q^2 = 0$ one expects Eq. (2) to be satisfied. Hence, a careful evaluation of the Cabibbo-Radicati sum rule is of the utmost importance in connection with the possible breakdown of the ASR. Such an evaluation was done by Gilman and Schnitzer[9] and Adler and Gilman,[10] who found agreement between both sides of Eq. (2) within 4%. However, new data and analyses of pion photoproduction in the resonance region[11-18] have become available recently. In addition, pion-nucleon total cross sections[19] are now measured up to $P_{\rm lab} = 200$ GeV/c.

Considering the importance of the Cabibbo-Radicati sum rule we believe that a reevaluation of Eq. (2) in the light of the new data is in order, and indeed we now find agreement only within 12–14%.

In Sec. II we present the results of the calculation of the integral in Eq. (2) assuming saturation with resonances. In Sec. III we compare the contributions of the resonances above the $\Delta(1232)$ with the prediction of the extended vector-dominance model (EVDM) in conjunction with the quark model.

II. SATURATION WITH RESONANCES

A. First resonance region

Performing an isospin rotation on $\sigma_T(\gamma \pm p)$ one can relate them to the cross sections σ_T(isovector $\gamma^0 + p \to I = \frac{1}{2}$) and σ_T(isovector $\gamma^0 + p \to I = \frac{3}{2}$), and the sum rule Eq. (2) then reads

$$\tfrac{1}{6}\langle r_V^2\rangle = \tfrac{1}{2}\left(\frac{\mu_p - \mu_n}{2M}\right)^2 + \frac{1}{4\pi^2\alpha}\int_{\nu_0}^{\infty}\frac{d\nu}{\nu}[2\sigma_T(\gamma^0 p \to I = \tfrac{1}{2}) - \sigma_T(\gamma^0 p \to I = \tfrac{3}{2})]. \tag{3}$$

It is well known that in the first resonance region (from ν_0 to 500 MeV) the cross section is predominantly $I=\frac{3}{2}$. In addition, $\sigma_T(\gamma p\to\pi^0 p)$ is dominated by the $\Delta(1232)$ resonance (M_{1+} multipole), while $\sigma_T(\gamma p\to\pi^+ n)$ receives contributions from both the $\Delta(1232)$ and the E_{0+} multipole (nonresonant s wave).[20]

The contribution of the M_{1+} multipole to the sum rule can be written as

$$I(M_{1+})=\frac{1}{4\pi^2\alpha}\int_{\nu_0}^{500\text{ MeV}}\frac{d\nu}{\nu}\left[-\tfrac{3}{2}\sigma_T(\gamma p\to\pi^0 p)\right], \tag{4}$$

while for E_{0+} one has

$$I(E_{0+})=\frac{1}{4\pi^2\alpha}\int_{\nu_0}^{500\text{ MeV}}\frac{d\nu}{\nu}\left[\sigma_T(\gamma p\to\pi^+ n) - \tfrac{1}{2}\sigma_T(\gamma p\to\pi^0 p)\right]. \tag{5}$$

In order to compute these integrals we have used the detailed multipole analyses of Refs. 11 and 12. These analyses agree with the gross features of previous ones[13-18] and have taken into account the most recent data on pion photoproduction. We have fitted the total cross sections $\sigma_T(\gamma p\to\pi^0 p)$ and $\sigma_T(\gamma p\to\pi^+ n)$ and obtained

$$I(M_{1+})=-\frac{0.033}{\mu^2} \tag{6}$$

and

$$I(E_{0+})=\frac{0.017}{\mu^2} \tag{7}$$

where $\mu^2\sim 0.019$ GeV^2 is the pion mass squared.

B. Second and third resonance regions

It has been established that both the $N^{**}(1520)$ and $N^{***}(1690)$ are excited by isovector photons[16] and are $I=\frac{1}{2}$ resonances. The contributions of these resonances to the sum rule Eq. (3) can be written as

$$I(N^{**})=\frac{1}{4\pi^2\alpha}\int_{500\text{ MeV}}^{800\text{ MeV}}\frac{d\nu}{\nu}3\sigma_T(\gamma p\to N^{**}\to\pi^+ n) \tag{8}$$

and

$$I(N^{***})=\frac{1}{4\pi^2\alpha}\int_{900\text{ MeV}}^{1100\text{ MeV}}\frac{d\nu}{\nu}3\sigma_T(\gamma p\to N^{***}\to\pi^+ n). \tag{9}$$

Using the latest pion-photoproduction analyses of Ref. 15 we obtain

$$I(N^{**})=\frac{0.009}{\mu^2} \tag{10}$$

and

$$I(N^{***})=\frac{0.005}{\mu^2}, \tag{11}$$

where we have already taken into account the fact that these resonances are roughly 50% inelastic.[21] We would like to mention that if one uses other photoproduction analyses the results are very similar, e.g., with the resonance parameters of Ref. 17 we find that $I(N^{**})=0.010/\mu^2$ and $I(N^{***})=0.004/\mu^2$.

In Table I we show the results of the present evaluation compared to the results obtained by Gilman and Schnitzer[9] using older pion-photoproduction data.[22-25] It can be seen that the individual contributions from the higher resonances are not falling off as fast as it was previously determined.[9]

C. Sum Rule

Using[26] $(\langle r_V^2\rangle)^{1/2}=0.90$ F and[27] $\mu_p-\mu_n=4.706$, we find for the Cabibbo-Radicati sum rule, Eq. (3),

$$\begin{aligned}\frac{0.066}{\mu^2}&=\frac{\langle r_V^2\rangle}{6}\\ &\overset{?}{=}\frac{0.059}{\mu^2}+\frac{0.017}{\mu^2}-\frac{0.033}{\mu^2}+\frac{0.009}{\mu^2}+\frac{0.005}{\mu^2}\\ &\quad+(\text{contributions from above 1100 MeV})\\ &=\frac{0.057}{\mu^2}\\ &\quad+(\text{contributions from above 1100 MeV}).\end{aligned} \tag{12}$$

Hence, if the contributions from above 1100 MeV are neglected altogether we obtain a disagreement between both sides of the sum rule of 14% instead of the 4% reported in Ref. 9.

III. VECTOR-MESON DOMINANCE

It has been suggested by Gilman and Schnitzer[9] that an alternate method of estimating the contributions to the sum rule from above 500 MeV could be to use ρ dominance and the simple quark model of scattering of Lipkin and Scheck.[28] In this case

TABLE I. Contributions to the Cabibbo-Radicati sum rule from the different resonance regions.

	$\mu^2 I(M_{1+})$	$\mu^2 I(E_{0+})$	$\mu^2 I(N^{**})$	$\mu^2 I(N^{***})$
This paper	−0.033	0.017	0.009	0.005
Reference 9	−0.028	0.016	0.016	0.002

one has that

$$\frac{1}{4\pi^2\alpha}\int_c \frac{d\nu}{\nu}[\sigma_T(\gamma^- p)-\sigma_T(\gamma^+ p)]$$

$$=\frac{1}{4\pi\gamma_\rho^2}\int_c \frac{d\nu}{\nu}[\sigma_T(\rho^- p)-\sigma_T(\rho^+ p)]$$

$$=\frac{1}{4\pi\gamma_\rho^2}\int_c \frac{d\nu}{\nu}[\sigma_T(\pi^- p)-\sigma_T(\pi^+ p)]. \quad (13)$$

However, it has been now established that the extended vector-dominance model[29] provides a much better description of the electromagnetic interactions of hadrons. In both the diagonal[29] and off-diagonal[30] versions of this model, the right-hand side of Eq. (13) should be multiplied by[31]

$$\lambda(2)=\sum_{n=0}^{\infty}(1+2n)^{-2}=1.2337.$$

In order to evaluate Eq. (13) we have used[32] $\gamma_\rho^2/4\pi=0.65$ and the latest pion-nucleon total cross section compilations[19] up to $\nu=5$ GeV. Above 5 GeV we have taken the fit of Ref. 33, i.e.,

$$\sigma_T(\pi^- p)-\sigma_T(\pi^+ p)=(5.24\pm0.10)\text{ mb}\times P_{\text{lab}}^{\;-(0.426\pm0.01)}, \quad (14)$$

which takes into account the available data up to 200 GeV, and extrapolated it up to infinity.

We then find that

$$\frac{1}{4\pi^2\alpha}\int_c \frac{d\nu}{\nu}[\sigma_T(\gamma^- p)-\sigma_T(\gamma^+ p)]$$

$$=\frac{\lambda(2)}{4\pi\gamma_\rho^2}\int_c \frac{d\nu}{\nu}[\sigma_T(\pi^- p)-\sigma_T(\pi^+ p)]$$

$$=\frac{0.015}{\mu^2}, \quad (15)$$

which is in good agreement with the contributions of the second and third resonances ($0.014/\mu^2$). In contrast, naive ρ dominance would have given a value of only $0.012/\mu^2$.

Using Eq. (15) and the result already found for the first resonance region, Eq. (12) now becomes

$$\frac{0.066}{\mu^2}=\frac{\langle r_V^2\rangle}{6}=\frac{0.058}{\mu^2}, \quad (16)$$

showing a disagreement of 12%.

IV. DISCUSSION

We have found that the disagreement between both sides of the Cabibbo-Radicati sum rule has now become larger. It should be stressed that a similar disagreement has been found after evaluating the sum rule by two different procedures.

If one would insist in the validity of the Cabibbo-Radicati sum rule, and grant that the present data will remain essentially unchanged, then the missing contributions should come either from extra resonances at higher masses in pion photoproduction or from a different asymptotic behavior of $[\sigma_T(\pi^- p)-\sigma_T(\pi^+ p)]$.[34] If this were not the case one would be led to expect a breakdown of CVC and/or the Adler sum rule.

Finally we would like to point out that if the ASR is not correct, then the present disagreement in the Cabibbo-Radicati sum rule is indicating that for small spacelike q^2 the integral in Eq. (1) is less than one. Such a behavior is opposite to predictions based on modified current-algebra sum rules due to quark structure.[6]

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