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On the energy dependence of charged-particle multiplicity

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Within the framework of the charge-conserving hadronic-bremsstrahlung model and using the lower-regime-of-high-energy collisions approximation, we derive a new expression for the integral mean multiplicities of produced charged particles. Although in pp collisions the lower-energy regime extends to about $s \approx 60$ GeV^2 [s = the c.m. (center of mass) energy squared], the expressions for the integral mean charged-particle multiplicities describe the trend of the experimental data suprisingly well through the CERN-ISR (intersecting-storage-ring) energies ($s \approx 2000$ GeV^2). This, according to our formalism, is an indication that mean multiplicities are rather smooth functions of t (the negative square of the c.m. momentum transfer of protons) for all s.

I. INTRODUCTION

Some time ago Białas and Zalewski[1] showed that in the case of pp collisions, the integral two-, three-, and four-negative-pion correlation parameters undergo a dramatic behavior change when going from the lower to higher energies. On the other hand, Berger and Krzywicki[2] have been speculating that $\langle n_{ch}(s)\rangle$ (the average multiplicity of produced charged particles) should have a power dependence on s at lower energies and a logarithmic dependence on s at higher energies. This, therefore, would indicate that it might be useful to introduce a notion of two energy regimes for high-energy collisions.

Within the framework of the charge-conserving hadronic-bremsstrahlung model we have split the whole energy region into the lower- and higher-energy regimes.[3] In the lower-energy regime we have successfully fitted the derived multiplicity distribution functions[3-5] and the dispersion for the total number of charged particles (see Ref. 6) up to $p_{lab} \approx 27$ GeV/c (the laboratory momentum of the incident proton). This would suggest that the lower-energy regime extends to about $p_{lab} \approx 27$ GeV/c ($s \approx 60$ GeV^2). The same theory describes for pp and π^+p reactions the two-, three-, and four-negative-pion correlation parameters as functions of $\langle n_{\pi^-}(s)\rangle$ in the lower-energy regime[7] consistent with the experimental data compiled by Białas and Zalewski.[1] These authors estimate the border between the lower- and higher-energy regimes to be $p_{lab} \approx 20$ GeV/c (for pp collisions), which is fairly consistent with our estimate of $p_{lab} \approx 27$ GeV/c.

It is quite clear that we cannot expect every quantity to exhibit as dramatically different behavior in each of the energy regimes as, for example, the integral two-negative-pion correlation parameters $f_2^-(s)$. While, according to Białas and Zalewski[2] and Harari and Rabinovici,[8] one has for pp collisions that

$$f_2^- < 0 \text{ for } p_{lab} < 50 \text{ GeV}/c,$$

$$f_2^- > 0 \text{ for } p_{lab} \gtrsim 50 \text{ GeV}/c,$$

it is a well-known experimental fact that the integral average multiplicities are positive and rather smooth functions of s throughout all energies. Thus different integral quantities may have various degrees of different behaviors in the two energy regimes, of which probably the behaviors of the integral many negative-pion correlation parameters are most dramatic.

Clearly, the behavior of one quantity may directly influence the behavior of some other quantity. In particular, the observed behavior of $f_2^-(s)$ for pp collisions suggests that at lower energies ($p_{lab} \lesssim 50$ GeV/c) the integral multiplicity distribution function for negative pions is narrower than Poisson and at higher energies ($p_{lab} \gtrsim 50$ GeV/c) is broader than or perhaps equal to Poisson. Now $f_2^-(s)$ will quite strongly reflect the conservation laws, in particular the charge-conservation laws. At lower energies where $f_2^-(s) < 0$, we have anticorrelations between produced negative pions, which means that one cannot expect them to appear in clusterlike formations. At higher energies ($p_{lab} \gtrsim 50$ GeV/c) the constraint of charge conservation on f_2^- is relaxed. This does not mean that we do not have charge-conservation constraints any more. They are shifted toward higher integral correlation parameters. From Białas and Zalewski's paper[1] we see that in particular (see also Ref. 8)

$$f_3^-(s) > 0 \text{ for } \langle n_{\pi^-}(s)\rangle < 1.5,$$

$$f_3^-(s) < 0 \text{ for } \langle n_{\pi^-}(s)\rangle \gtrsim 1.5;$$

$$f_4^-(s) < 0 \text{ for } \langle n_{\pi^-}(s)\rangle < 1.5,$$

$$f_4^-(s) > 0 \text{ for } \langle n_{\pi^-}(s)\rangle \gtrsim 1.5.$$

We see that while f_4^- behaves very similarly to f_2^-, f_3^- becomes negative at higher energies, which we can take to be a reflection of the charge-conservation law [it can be argued that in the presence of conservation laws at least some of the higher integral correlation parameters should be negative (see, for example, Ref. 9)].

Now the integral quantities like $\langle n_{\pi^-}(s)\rangle$ and $f_2^-(s)$ are appropriate integrals in variable t of (nonintegral) quantities $\langle n_{\pi^-}(s,t)\rangle$ and $f_2^-(s,t)$. A sudden change of $f_2^-(s)$ from negative to positive values at about $s \approx 60$ GeV^2 we shall find to be compatible with nonsmooth t dependence of $f_2^-(s,t)$ for $s \gtrsim 60$ GeV^2. This would then suggest that the change of constraints due to charge conservation affects the t behavior of nonintegral quantities in the higher-s region. Now the integral average multiplicities seem to be quite insensitive to the constraints of conservation laws at higher energies in view of the fact that their experimentally acceptable smooth s dependences [such as ln(s)] apply to all kinds of produced secondary particles irrespective of what the primary colliding particles are (see Ref. 10). Thus we expect $\langle n_{\rm ch}(s,t)\rangle$ to be rather smooth in t also for $s \gtrsim 60$ GeV^2. Consequently, we expect the expression for $\langle n_{\rm ch}(s)\rangle$ computed formally in the lower-energy regime ($s \lesssim 60$ GeV^2) (where, by definition, all nonintegral quantities have smooth t behavior) to be describing the trends of experimental data also at higher energies. In other words, as far as the average multiplicities are concerned, the "lower-energy regime" extends to much higher s values than 60 GeV^2. With this in mind the question to be answered is actually where $\langle n_{\rm ch}(s)\rangle$ behaves differently than ln(s), which, according to Berger and Krzywicki,[2] should hold only at very high energies. We shall find that $\langle n_{\rm ch}(s)\rangle$ in the lower-energy regime is basically still a logarithmic function, but not as simple as ln(s). Only in a very low s region can $\langle n_{\rm ch}(s)\rangle$ be approximated as some combination of terms, each having appropriate power dependence on s. From a general point of view this agrees with Berger and Krzywicki.[2] However, the power dependence of $\langle n_{\rm ch}(s)\rangle$ on s is in a much smaller region that that claimed in Ref. 2.

An interesting result is that in the limit of very high s, low-energy $\langle n_{\rm ch}(s)\rangle$ not only behaves as ln(s), but also describes surprisingly well the trends of experimental data.

In Sec. II we give a collection of necessary formulas from the charge-conserving hadronic-bremsstrahlung model and a brief discussion of lower- and higher-energy regimes.

Section III is devoted to deriving the mean multiplicities (at fixed s and t) of produced charged particles in terms of $d\sigma^T/dt$ and $d\sigma^{\rm el}/dt$, the total and elastic differential cross sections, respectively, for the quasi-elastic scattering of two primary particles (protons).[3,7] In this section we also relate $d\sigma^{\rm el}/dt$ for pp collisions to the effective low- and high-energy effective Regge trajectories,[11] and using the lower-regime-of-high-energies approximation,[3] derive the integral mean multiplicities as a function of s, formally only valid in the lower-energy regime. Finally, the results are compared with experiments.

In Sec. IV we summarize the results and compare them with some other approaches in the literature.

II. HADRONIC-BREMSSTRAHLUNG MODEL AND TWO ENERGY REGIMES

A. Preliminaries

In this section we wish to discuss briefly the assumptions and main results from the charge-conserving hadronic-bremsstrahlung model (for details see Ref. 3 and 7).

In general, the hadronic-bremmstrahlung models describe the quasi-elastic collisions of two primary particles (protons) accompanied by the emission of secondary particles (pions) whose energies are limited to values much less than those of primary particles. It is assumed that as compared to the secondary pions, the production of secondary strange particles and antinucleons is small. We shall also assume that to a good approximation the productions of secondary neutral and secondary charged pions are independent from each other and concentrate on the production of secondary charged pions only.

The generalized hadronic-bremsstrahlung model gives the following expression for the multiplicity distribution function of m secondary pions[3,7]:

$$
\begin{aligned}
&W_m(s,t;\epsilon) = W_0(s,t;\epsilon)\left\{\frac{b^m(s,t)}{m!} + \epsilon[\sinh b(s,t)]_m\right\},\\
&W_0(s,t;\epsilon) = [\exp b(s,t) + \sinh b(s,t)]^{-1},\\
&\qquad m = 0,1,2,\ldots,
\end{aligned} \tag{1}
$$

where $b(s,t)$ $[b(s,t) \geq 0]$ is some function of s and t. In relation (1), ϵ is some constant assumed to be independent of s and t, and (for $l = 0,1,2,\ldots$)

$$
\begin{aligned}
&[\sinh b(s,t)]_{2l} = 0,\\
&[\sinh b(s,t)]_{2l+1} = \frac{b^{2l+1}(s,t)}{(2l+1)!}.
\end{aligned} \tag{2}
$$

Setting arbitrarily in (1) $\epsilon = 0$, we get for W_m the Poisson multiplicity distribution function as described within the ordinary hadronic-bremsstrahlung model by Gemmel and Kastrup in Ref. 12. However, in the ordinary hadronic-bremsstrahlung model we neither have charge nor parity conservation, which accounts for the fact that the secondary pions are emitted in a statistically independent way.

If we wish to have charge conservation, then it can be shown that $\epsilon = -1$, and we get the charge-conserving hadronic-bremsstrahlung model, which, incidentally, also obeys parity conservation (for details see Ref. 3 and 6). Denoting now the number of produced secondary charged pions with m_{ch}, we get from (1) and (2) the multiplicity distribution function and the average multiplicity to be

$$W_{m_{ch}}(s,t) = \frac{1}{\cosh b(s,t)} \frac{b^{m_{ch}}(s,t)}{m_{ch}!}, \quad m_{ch} = 0, 2, \ldots, \tag{3}$$

$$\langle m_{ch}(s,t) \rangle = b(s,t) \tanh b(s,t). \tag{4}$$

Now one thinks of hadronic-bremsstrahlung models as models where the primary particles in the course of collision "shake off" the secondary particles. This view is definitely quite appropriate for the ordinary hadronic-bremsstrahlung model. However, it should be supplemented somewhat for the charge-conserving one. Namely, within the charge-conserving hadronic-bremsstrahlung model, the multiplicity distribution function fits the data (up to 30 GeV/c) not only for

$$p + p \to p + p + m_{ch}(\text{pions}),$$

$$\pi^{\pm} + p \to \pi^{\pm} + p + m_{ch}(\text{pions}),$$

$$n + n \to n + n + m_{ch}(\text{pions}),$$

but also for

$$\pi^- + p \to \pi^- + (\Lambda + K^0 + \pi^+) + m_{ch}(\text{pions}),$$

where we formally consider $(\Lambda + K^0 + \pi^+)$ as a "primary" particle in the final state (see Ref. 3). In view of this we wish to think of the charge-conserving hadronic-bremsstrahlung model as one in which the colliding primary particles provide a space-time interaction region (of about one fm in size and lasting about 10^{-23} sec) from which primary and secondary particles emerge with the property of conserving charge and the "identities" of primaries [the "primary" particle $(\Lambda + K^0 + \pi^+)$ in the final state of the $\pi^- p$ reaction has the same charge, baryon number, and strangeness as the proton in the initial state]. The charge conservation here is global rather than local; i.e., the charged pions can be produced in such a way that we can associate the production of a single charged pion rather than a pair of charged pions with a space-time point within the interaction region. In view of this, one may think of this model as the ordinary hadronic-bremsstrahlung model in which the charge conservation is achieved by allowing only the "right" number of secondary charged pions to be emitted.

It does not seem to be possible to describe the model with a local Lagrange function (see Ref. 3). This makes a detailed dynamical description quite difficult, although on the phenomenological level we hope to establish a formal link with some models in the literature. Consequently, the charge-conserving hadronic-bremsstrahlung model does not belong to a class of "complete theories," since it does not intend to describe every detail of high-energy processes. Rather, it has qualities of semiphenomenological theories, such as statistical models, droplet models, and diffractive models.

Since m_{ch} is even and the "identities" of primary particles conserved, we have the number of positive (m_+) and negative (m_-) secondary pions to be equal:

$$m_{ch} = 2m_+ = 2m_- . \tag{5}$$

Consequently, $W_{m_{ch}}$ is numerically equal to the multiplicity distribution functions for the positive and negative secondary charged pions

$$W_{m_\pm}(s,t) = \frac{1}{\cosh b(s,t)} \frac{b^{2m_\pm}(s,t)}{(2m_\pm)!} \equiv W_{m_{ch}}(s,t), \quad m_\pm = 0, 1, 2, \ldots, \tag{6}$$

and the corresponding mean multiplicity is

$$\langle m_+(s,t) \rangle = \langle m_-(s,t) \rangle = \tfrac{1}{2} \langle m_{ch}(s,t) \rangle . \tag{7}$$

Let us now briefly analyze the multiplicity distribution functions $W_{m_{ch}}(s,t)$ and $W_{m_-}(s,t)$. According to Mueller,[13] from some multiplicity distribution function $W_m(s,t)$ one can form the generating function defined as

$$I(s,t;h) = \sum_m W_m(s,t)(1+h)^m = \langle (1+h)^m \rangle . \tag{8}$$

Now Mueller showed that the generating function is simply related to the correlation parameters f_n, as

$$\ln I(s,t;h) = \sum_n \frac{h^n}{n!} f_n(s,t), \tag{9}$$

and the binomial moments F_r,

$$I(s,t;h) = \sum_r \frac{h^r}{r!} F_r(s,t), \tag{10}$$

where the comparison of (10) with (8) gives

$$F_r(s,t) = \langle m(m-1) \cdots (m-r+1)(s,t)\rangle . \qquad (11)$$

From (8) we see that each of the multiplicity disbribution functions $W_{m_{ch}}(s,t)$ and $W_{m_-}(s,t)$ corresponds to the generating functions $I^{ch}(s,t;h)$ and $I^-(s,t;h)$, respectively:

$$I^{ch}(s,t;h) = \frac{\cosh[b(s,t)(1+h)]}{\cosh[b(s,t)]}, \qquad (12)$$

$$I^-(s,t;h) = \frac{\cosh[b(s,t)(1+h)^{1/2}]}{\cosh[b(s,t)]}, \qquad (13)$$

The first thing that we notice is different behaviors of the corresponding two-particle correlation parameters

$$f_2^{ch}(s,t) = b^2(s,t)[1-\tanh^2 b(s,t)] \geq 0, \qquad (14)$$

$$f_2^-(s,t) = \frac{b(s,t)}{4}\{b(s,t)[1-\tanh^2 b(s,t)] - \tanh b(s,t)\} \leq 0. \qquad (15)$$

From (14) and (15), we see that $W_{m_{ch}}(s,t)$ and $W_{m_-}(s,t)$, although numerically equal, are qualitatively quite different, $W_{m_{ch}}(s,t)$ being wider and $W_{m_-}(s,t)$ being narrower than the Poisson multiplicity distribution function. This is, of course, a consequence of charge conservation. Namely, what $f_2^-(s,t)<0$ tells us is that for fixed t ($t\neq 0$) we have anticorrelations between negative pions, or equivalently, they are not emitted in clusterlike formations. On the other hand, $f_2^{ch}(s,t)$, although generally positive, is actually small most of the time. As $t\to 0$, $b(s,t)\to 0$,[12,14] and consequently $f_2^{ch}(s,t)\to 0$. As $|t|$ increases, from the increase of $\langle m_{ch}(s,t)\rangle$, we expect $b(s,t)$ to increase also. Relation (14) tells us then that $f_2^{ch}(s,t)$ gets close to zero, since $\tanh b(s,t)$ approaches unity rather fast as $b(s,t)$ increases. Thus, for fixed rather large $|t|$ we expect the secondary charged pions to be emitted with small correlations, which implies that the correlations between pions of different charge slightly outweigh the anticorrelations between pions of the same charge.

The differential cross section for the quasi-elastic scattering of two primaries accompanied by the emission of m_{ch} secondary charged pions is[3,7]

$$\frac{d\sigma^{(m_{ch})}(s,t)}{dt} = \frac{d\sigma^T(s,t)}{dt} W_{m_{ch}}(s,t), \qquad (16)$$

where $d\sigma^T/dt$ (the total differential cross section) can be connected to the "potential" scattering of two primaries and it is assumed to be a smooth function of t.[3,7,12] From (16) we have that

$$\frac{d\sigma^T(s,t)}{dt} = \sum_{m_{ch}} \frac{d\sigma^{(m_{ch})}(s,t)}{dt}. \qquad (17)$$

In the limit $t\to 0$, no secondary particles can be produced,[12,14] thus

$$\begin{aligned} &b(s,0)=0\\ &W_{m_{ch}}(s,0)=\delta_{0m_{ch}}. \end{aligned} \qquad (18)$$

Now, since

$$\frac{d\sigma^{(m_{ch}=0)}}{dt} = \frac{d\sigma^{el}}{dt}, \qquad (19)$$

from (17) we get

$$\left.\frac{d\sigma^T(s,t)}{dt}\right|_{t=0} = \left.\frac{d\sigma^{el}(s,t)}{dt}\right|_{t=0}. \qquad (20)$$

The integral cross section for the quasi-elastic scattering of two primary particles accompanied by the emission of m_{ch} secondary charged pions is

$$\begin{aligned} \sigma_{m_{ch}}(s) &= \int d|t| \frac{d\sigma^{(m_{ch})}(s,t)}{dt}\\ &= \int d|t| \frac{d\sigma^T(s,t)}{dt} W_{m_{ch}}(s,t). \end{aligned} \qquad (21)$$

The integral multiplicity distribution function and the corresponding mean multiplicity for the emitted secondary charged pions follow in the usual way

$$\begin{aligned} P_{m_{ch}}(s) &= \frac{\sigma_{m_{ch}}(s)}{\sigma_T(s)}\\ &= \int d|t|\, \rho(s,t) W_{m_{ch}}(s,t), \end{aligned} \qquad (22)$$

and

$$\langle m_{ch}(s,t)\rangle = \int d|t|\, \rho(s,t)\langle m_{ch}(s,t)\rangle, \qquad (23)$$

where $\sigma_T(s)$ is the total cross section of two primaries and $\rho(s,t)$, itself the distribution function, is defined as[3]

$$\begin{aligned} &\rho(s,t) = \frac{1}{\sigma_T(s)}\frac{d\sigma^T(s,t)}{dt},\\ &\int d|t|\rho(s,t) = 1, \end{aligned} \qquad (24)$$

where in view of the assumed smoothness in t of $d\sigma^T/dt$, $\rho(s,t)$ is also assumed to be smooth in t. In analogy to (6) and (7), the integral multiplicity distribution function for negative secondary pions and the corresponding mean multiplicity are given as

$$P_{m_-}(s) = \int d|t|\,\rho(s,t)W_{m_-}(s,t)$$
$$\equiv P_{m_{\rm ch}}(s), \tag{25}$$

$$\langle m_-(s)\rangle = \int d|t|\,\rho(s,t)\langle m_-(s,t)\rangle$$
$$= \tfrac{1}{2}\langle m_{\rm ch}(s,t)\rangle. \tag{26}$$

We can also define the integral generating functions

$$I^{\rm ch}(s,h) = \int d|t|\,\rho(s,t)I^{\rm ch}(s,t;h), \tag{27}$$

$$I^-(s,h) = \int d|t|\,\rho(s,t)I^-(s,t;h), \tag{28}$$

corresponding to each of the integral multiplicity functions $P_{m_{\rm ch}}(s)$ and $P_{m_-}(s)$, respectively.

B. Definition of two energy regimes

If we look at the integral multiplicity distribution function $P_{m_-}(s)$, we see that it is essentially given as an average of $W_{m_-}(s,t)$ with respect to $\rho(s,t)$:

$$P_{m_-}(s) = \langle W_{m_-}(s,t)\rangle$$
$$\equiv \int d|t|\rho(s,t)W_{m_-}(s,t). \tag{29}$$

The same is true for any other integral quantity. Now, we can expand $W_{m_-}(s,t)$ around $\langle t\rangle$ and get

$$P_{m_-}(s) = W_{m_-}(s,\langle t\rangle)$$
$$+\frac{\langle t^2\rangle - \langle t\rangle^2}{2}\left(\frac{d^2W_{m_-}(s,t)}{dt^2}\right)_{t=\langle t\rangle} + \cdots, \tag{30}$$

where

$$\langle t\rangle = \int d|t|\; t\rho(s,t),$$
$$\langle t^2\rangle = \int d|t|\; t^2\rho(s,t). \tag{31}$$

Clearly, $\langle t\rangle$ is the average of the negative invariant momentum transfer squared of primary particles, and $\langle t^2\rangle - \langle t\rangle^2$ is the square of the dispersion of variable t for given s. How each of these quantities depends on s is an open question. However, because $\rho(s,t)$ is a slowly varying function in t (Ref. 12) and since it is also a reasonable assumption that it varies slowly with s [at $t=0$, $\rho\approx\sigma_T$, due to (20)], from relation (31) one then concludes that $\langle t\rangle$ and $\langle t^2\rangle$ should not vary too strongly with s. This we shall assume throughout this paper.

Now we give a mathematical definition of the lower-energy regime as

$$\langle t^2\rangle - \langle t\rangle^2 \ll 2\left|\frac{W_{m_-}(s,\langle t\rangle)}{[d^2W_{m_-}(s,t)/dt^2]_{t=\langle t\rangle}}\right|, \tag{32}$$

with a similar relation involving $W_{m_{\rm ch}}$. Of course, in the lower-energy regime we can write for $P_{m_{\rm ch}}(s)$ and $P_{m_-}(s)$

$$P_{m_{\rm ch}}(s) = W_{m_{\rm ch}}(s,\langle t\rangle), \tag{33}$$

$$P_{m_-}(s) = W_{m_-}(s,\langle t\rangle), \tag{34}$$

i.e., in the lower-energy regime all integral quantities are obtained from corresponding nonintegral quantities by replacing t with $\langle t\rangle$, which can be equivalently stated by writing the integral generating functions as

$$I^{\rm ch}(s;h) = I^{\rm ch}(s,\langle t\rangle;h), \tag{35}$$

$$I^-(s;h) = I^-(s,\langle t\rangle;h). \tag{36}$$

Now, let us try to see the physical meaning of the lower-energy regime. First of all, let us show that specifically in the lower-energy regime $\langle t^2\rangle - \langle t\rangle^2$ should indeed be weakly dependent on s and very small. Putting $m_-=0$ in (32) and replacing W_0 by $d\sigma^{\rm el}/dt$ because of the assumed smoothness of $d\sigma^T/dt$ in t, we get

$$\langle t^2\rangle - \langle t\rangle^2 \ll \frac{2}{|a_{\rm el}^2(s,\langle t\rangle) + [da_{\rm el}(s,t)/dt]_{t=\langle t\rangle}|}, \tag{37}$$

where $a_{\rm el}$ is the slope of the elastic differential cross section defined as

$$a_{\rm el}(s,t) = \frac{d}{dt}\ln\left[\frac{d\sigma^{\rm el}(s,t)/dt}{[d\sigma^{\rm el}(s,t)/dt]_{t=0}}\right]. \tag{38}$$

Now at lower energies slope $a_{\rm el}(s,t)$ varies slowly with t ($da_{\rm el}/dt\approx 0$ as compared with $a_{\rm el}^2$), and, furthermore, because of the near constancy of the diffraction peak, $a_{\rm el}(s,t)$ changes very slowly with s. Thus, $\langle t^2\rangle - \langle t\rangle^2$ is much smaller than a quantity which is almost constant in s. Consequently, we have an effective characterization of the lower-energy regime as $\langle t^2\rangle - \langle t\rangle^2 \approx 0$. This is nothing else but the fact that in the lower-energy regime t's from various samples of the same reaction with fixed s are confined to a small region around $\langle t\rangle$, which itself, due to kinematics, is small.

Another aspect of the lower-energy regime is that in it every nonintegral quantity is sufficiently smooth in t so that its integral counterpart is given by relations similar to (33) and (34). As s increases, some nonintegral quantities will remain smooth in t longer than the others. Clearly, the border of the lower-energy regime then will be determined by the nonintegral quantity which is first to violate the inequality analogous to (32). Of course, of all possible nonintegral quantities it is difficult to find precisely the one which is first to violate the inequality analogous to (32). However, as discussed in the introduction, because of the relaxation of charge-conservation constraints on $f_2^-(s)$, we expect $f_2^-(s,t)$ to be a good example of a nonintegral quantity showing nonsmoothness in t rather quickly as s increases. Namely, for sufficiently high s, $f_2^-(s)$ will not be given any more as $f_2^-(s,\langle t\rangle)$ $[f_2^-(s,\langle t\rangle)<0]$, but rather as

$$f_2^-(s)\approx f_2^-(s,\langle t\rangle)+\frac{\langle t^2\rangle-\langle t\rangle^2}{2}\left(\frac{d^2f_2^-(s,t)}{dt^2}\right)_{t=\langle t\rangle}. \tag{39}$$

This means that at sufficiently high s, as far as $f_2^-(s,t)$ is concerned, one does not have an inequality analogous to (32) but rather

$$\langle t^2\rangle-\langle t\rangle^2\gtrsim 2\left|\frac{f_2^-(s,\langle t\rangle)}{[d^2f_2^-(s,t)/dt^2]_{t=\langle t\rangle}}\right|, \tag{40}$$

where, of course, for sufficiently high s, $\langle t^2\rangle-\langle t\rangle^2\neq 0$. Relation (40) is the equality-inequality characterizing the higher-energy regime, at least as far as the two-negative-pion correlation parameter is concerned.

Experimentally we can easily see how for sufficiently high s negative $f_2^-(s)$ goes into positive $f_2^-(s)$, thus requiring the second term in (39). First of all, if we denote

$$b(s)=b(s,\langle t\rangle), \tag{41}$$

then in the lower-energy regime

$$\begin{aligned} f_2^-(s)&=f_2^-(s,\langle t\rangle)\\ &=\frac{b(s)}{4}\{b(s)[1-\tanh^2 b(s)]-\tanh b(s)\}\\ &\leqslant 0. \end{aligned} \tag{42}$$

In Fig. 1 the pp and π^+p data for $f_2^-(s)$, as compiled by Białas and Zalewski,[1] are plotted against $\langle n_{\pi^-}(s)\rangle$ [for pp and π^+p collisions the number of all negative pions in final state n_{π^-} equals the number of secondary negative pions m_-, thus also $\langle n_{\pi^-}(s)\rangle=\langle m_-(s)\rangle$]. The solid line is $f_2^-(s)$ from (42) where $\langle m_-(s)\rangle=\frac{1}{2}b(s)\tanh b(s)$ was taken into account. We see that $f_2^-(s)$ from (42) cannot describe the pp data for $\langle n_{\pi^-}(s)\rangle>1$. Consequently, for $\langle n_{\pi^-}(s)\rangle>1$ (which according to Ref. 1 corresponds to $p_{\rm lab}\approx 20$ GeV/c), we must have the second term in expression (39) for $f_2^-(s)$. As a matter of fact, here not only

$$\left(\frac{d^2f_2^-(s,t)}{dt^2}\right)_{t=\langle t\rangle}$$

should be different from zero, but also should be sufficiently large and positive so as to compensate the first term, $f_2^-(s,\langle t\rangle)$, which is negative, thus making the whole expression positive. This indicates that in pp collisions for $p_{\rm lab}\gtrsim 20$ GeV/c, $f_2^-(s,t)$ should show rather strong t dependence. This indicates that the border between the lower- and higher-energy regimes as far as $f_2^-(s)$ is concerned (for pp collisions) is at about $p_{\rm lab}\approx 20$ GeV/c (perhaps even at 30 GeV/c). It is amusing to note that this example actually implies the smooth s dependence of $\langle m_-(s)\rangle$ for $p_{\rm lab}<20$ GeV/c and $p_{\rm lab}\gtrsim 20$ GeV/c. The surprising result from Fig. 1 is the fact that all available π^+p data are well described by the "lower-energy regime" expression for $f_2^-(s)$, indicating that in π^+p collisions

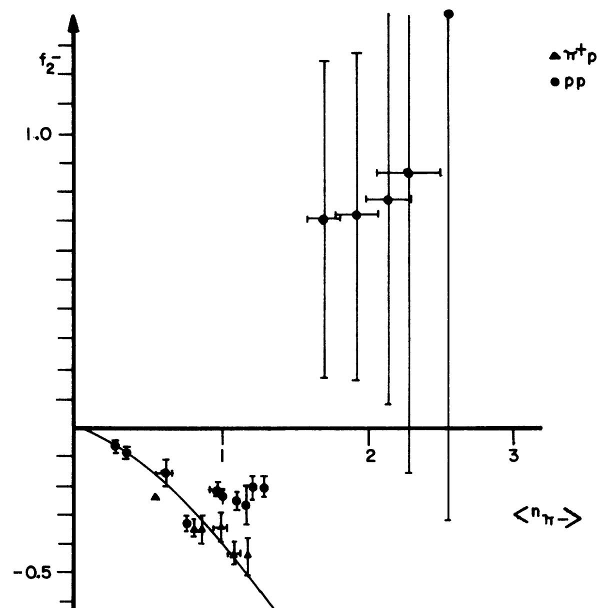

FIG. 1. Integral two-particle correlation parameter $f_2^-(s)$ for negative pions against the mean multiplicity $\langle n_{\pi^-}(s)\rangle$ of produced negative pions. The pp data show that the constraints of charge conservation of $f_2^-(s)$ are relaxed in the higher-energy regime. The solid curve is the prediction for $f_2^-(s)$ in the lower-energy regime where the constraints of charge conservation make $f_2^-(s)$ negative. The π^+p data are practically all in the lower-energy regime.

the border between the two energy regimes for $f_2^-(s)$ is much higher than the same border in pp collisions.

The question now is whether the border between the two energy regimes for $f_2^-(s)$ in pp collisions determines the common border of the two energy regimes for pp collisions; i.e., the laboratory momentum below which every integral quantity is given by relations similar to (33) and (34). We shall see this to be so and call the two energy regimes corresponding to $f_2^-(s)$ simply the two energy regimes for the whole collision. Actually, we can easily argue this point because the behavior of $f_2^-(s)$ influences the behavior of $P_{m_-}(s)$ which in turn influences the behavior of every other integral quantity. Namely, in the lower-energy regime of $f_2^-(s)$ ($s < 60$ GeV^2) where $f_2^-(s) < 0$ we have that $P_{m_-}(s)$ [given by (34)] is narrower than Poisson distribution. In the higher-energy regime of $f_2^-(s)$ ($s \gtrsim 60$ GeV^2) where $f_2^-(s) > 0$ we see that $P_{m_-}(s)$ is wider than Poisson distribution and will have to be described as

$$P_{m_-}(s) \approx W_{m_-}(s,\langle t\rangle) + \frac{\langle t^2\rangle - \langle t\rangle^2}{2}\left(\frac{d^2W_{m_-}(s,t)}{dt^2}\right)_{t=\langle t\rangle}. \tag{43}$$

Since $P_{m_{\rm ch}}(s)$ is numerically equal to $P_{m_-}(s)$ ($m_{\rm ch} = 2m_-$), we then expect $P_{m_{\rm ch}}(s)$ to behave similarly. Thus in the lower-energy regime of $f_2^-(s)$, $P_{m_{\rm ch}}(s)$ is simply given by expression (33), while in the higher-energy regime as

$$P_{m_{\rm ch}}(s) \approx W_{m_{\rm ch}}(s,\langle t\rangle) + \frac{\langle t^2\rangle - \langle t\rangle^2}{2}\left(\frac{d^2W_{m_{\rm ch}}(s,t)}{dt^2}\right)_{t=\langle t\rangle}. \tag{44}$$

Thus we conclude that the border between the lower and higher-energy regimes ($s \approx 60$ GeV^2) for $f_2^-(s)$ is the universal border between two energy regimes for pp collisions; i.e., every integral quantity for $s < 60$ GeV^2 to a good approximation is given by relations similar to (33) and (34). Equivalently, this means that every nonintegral quantity for $s < 60$ GeV^2 is a relatively smooth function of t.

It is quite clear that while in pp collisions every nonintegral quantity is expected to be a rather smooth function of t for $s < 60$ GeV^2, some nonintegral quantities may actually be quite smooth functions of t also for $s \gtrsim 60$ GeV^2. A trivial case is $\sum_{m_{\rm ch}} W_{m_{\rm ch}}(s,t)$, which is identically equal to 1 for every s and t. Another less trivial case is $\langle m_{\rm ch}(s,t)\rangle$. The experimentally determined slow variation with s of the integral $\langle m_{\rm ch}(s)\rangle$ indicates that $\langle m_{\rm ch}(s,t)\rangle$ should be a rather smooth function of t also for high s.

III. MEAN MULTIPLICITIES

Setting in (18) $m_{\rm ch} = 0$, we get

$$\frac{1}{\cosh b(s,t)} = \frac{d\sigma^{\rm el}(s,t)/dt}{d\sigma^T(s,t)/dt} \equiv h(s,t). \tag{45}$$

Relation (45) will be a starting relation from which we shall try to deduce mean multiplicities. By noting that

$$\tanh b(s,t) = [1 - h^2(s,t)]^{1/2},$$

we get from (4)

$$\langle m_{\rm ch}(s,t)\rangle = [1-h^2(s,t)]^{1/2} b(s,t) = [1-h^2(s,t)]^{1/2}\tfrac{1}{2}\ln\left\{\frac{1+[1-h^2(s,t)]^{1/2}}{1-[1-h^2(s,t)]}\right\}. \tag{46}$$

Suppose that $d\sigma^T/dt \gg d\sigma^{\rm el}/dt$ [which corresponds to highly inelastic scattering of two primary particles], then $\langle m_{\rm ch}(s,t)\rangle$ becomes rather large. If we take a formal limit, $t \to 0$, then in view of (20) we have

$$h(s,0) = 1. \tag{47}$$

We see that relation (47) always holds, no matter what s is, meaning that unless the primary particles suffer some momentum transfer there is no production of secondaries. On the other hand, if we fix t and vary s, it is clear that as s increases, $h(s,t)$ will decrease and, consequently, $\langle m_{\rm ch}(s,t)\rangle$ will increase. Conversely, for fixed rather small t, if s is very small (not smaller than $|t| + 4M_p^2$, where M_p is a mass of a proton), $h(s,t)$ will be close to 1; consequently $\langle m_{\rm ch}(s)\rangle \approx 0$. The main point is that $h(s,t)$ may also be close to 1 for $t \neq 0$ if s is sufficiently small.

Next we wish to arrive at the integral average multiplicity $\langle m_{\rm ch}(s,t)\rangle$ of produced charged secondary pions in pp collisions, confining ourselves, however, to the lower-energy regime. To begin, it is possible to describe quite well the elastic pp differential cross section by means of an effective Regge trajectory $\alpha_{\rm eff}$ (see Ref. 11):

$$\ln\frac{d\sigma^{\rm el}(s,t)}{dt} = 2[\alpha_{\rm eff}(t) - 1]\ln s + F(t). \tag{48}$$

Now we are primarily interested in the lower-energy regime ($s \lesssim 60$ GeV^2). Here we can write[11]

$$\alpha_{\rm eff}(t) \approx 1.06 + 0.4t. \tag{49}$$

For the sake of completeness we also mention that in the CERN-ISR region (which definitely is a part of our higher-energy regime)

$$\alpha_{\text{eff}}(t) \approx 1.06 + 0.25t, \quad |t| < 1.2\ (\text{GeV}/c)^2; \tag{50}$$

$$\alpha_{\text{eff}}(t) \approx 1.0 \text{ (with large errors)}, \quad |t| > 1.2\ (\text{GeV}/c)^2.$$

In the lower-energy regime the integral average multiplicity $\langle m_{\text{ch}}(s)\rangle$ is simply given as

$$\langle m_{\text{ch}}(s)\rangle = \langle m_{\text{ch}}(s,\langle t\rangle)\rangle. \tag{51}$$

Consequently, we need [see (46)]

$$h(s,\langle t\rangle) = \left(\frac{d\sigma^{\text{el}}(s,t)/dt}{d\sigma^{T}(s,t)/dt}\right)_{t=\langle t\rangle}. \tag{52}$$

Now $h(s,\langle t\rangle)$ is only a function of s, since according to (31) $\langle t\rangle$ also depends on s. As s becomes smaller, $\langle t\rangle$ becomes smaller and $h(s,\langle t\rangle)$ eventually becomes 1. Unfortunately, since we do not know precisely the functional dependence of $\langle t\rangle$ on s, we shall replace $\langle t\rangle$ with $\langle t\rangle'$, which is more or less an average of $\langle t\rangle$ over the whole lower-energy regime and as such is independent of s. Such an approximation is certainly not bad if we do not expect $\langle t\rangle$ to reach excessively large values throughout the lower-energy regime. Thus, for the lower-energy regime we write

$$h(s,\langle t\rangle) \approx h(s,\langle t\rangle') = \left(\frac{d\sigma^{\text{el}}(s,t)/dt}{d\sigma^{T}(s,t)/dt}\right)_{t=\langle t\rangle'}. \tag{53}$$

In this approximation we have to take into account the fact that for some $s_{\min} \gtrsim \langle |t|\rangle' + 4M_p^2$ we should have [see the discussion after relation (47)]

$$h(s_{\min},\langle t\rangle') \approx 1. \tag{54}$$

Of course, from the physical point of view $s_{\min}$ has to be given in such a way that

$$\langle m_{\text{ch}}(s_{\min})\rangle \approx 0. \tag{55}$$

In order to proceed we have to approximate $d\sigma^T/dt$ at $t=\langle t\rangle'$. As mentioned before, $d\sigma^T/dt$ is smooth in t, and, since $\langle t\rangle'$ is not expected to be very large, we can approximate [see relation (20)]

$$\left.\frac{d\sigma^T}{dt}\right|_{t=\langle t\rangle'} \approx \left.\frac{d\sigma^T}{dt}\right|_{t\approx 0} \approx \sigma_T^{\,2}(s),$$

where, in using the optical theorem, we have neglected the real part of the scattering amplitude at $t=0$. Thus, we write

$$h(s,\langle t\rangle') \approx \frac{K}{\sigma_T^{\,2}(s)}\frac{1}{s^r}, \tag{56}$$

$$r = 2[1-\alpha_{\text{eff}}(\langle t\rangle')], \tag{57}$$

where K contains all the factors independent of s. Now, imposing (54) we get that

$$K \approx \sigma_T^{\,2}(s_{\min})s_{\min}^{r},$$

finally giving

$$h(s,\langle t\rangle') \approx \left(\frac{\sigma_T(s_{\min})}{\sigma_T(s)}\right)^2\left(\frac{s_{\min}}{s}\right)^r. \tag{58}$$

In view of the general smoothness of $\sigma_T(s)$ with s, we also could have absorbed $\sigma_T(s)$ into K. However, keeping $\sigma_T(s)$ explicitly in (58) we will be able to compare $\langle m_{\text{ch}}(s)\rangle$ with data starting from a very low s, where $\sigma_T(s)$ varies significantly with s. Finally, taking into account (51), (53), and (46), we have for the integral average multiplicity of produced secondary charged pions in the lower-energy regime the expression

$$\langle m_{\text{ch}}(s)\rangle = [1-(\sigma_T(s_{\min})/\sigma_T(s))^4(s_{\min}/s)^{2r}]^{1/2}\tfrac{1}{2}\ln\left\{\frac{1+[1-(\sigma_T(s_{\min})/\sigma_T(s))^4(s_{\min}/s)^{2r}]^{1/2}}{1-[1-(\sigma_T(s_{\min})/\sigma_T(s))^4(s_{\min}/s)^{2r}]^{1/2}}\right\}. \tag{59}$$

The integral average multiplicity of all produced charged particles in the final state of pp collisions is

$$\langle n_{\text{ch}}(s)\rangle = 2 + \langle m_{\text{ch}}(s)\rangle. \tag{60}$$

In what follows, the numerical values for $\sigma_T(s)$ will be taken from the graph in Ref. 15. Now, to be able to compare (59) and (60) with experiments, we still have to determine two parameters $s_{\min}$ and r. Looking at experimental data[16] for $\langle n_{\pi^+}\rangle$ and $\langle n_{\pi^-}\rangle$ in Fig. 2, we see that at $s\approx 5.5\ \text{GeV}^2$ they are not observed. Also, at $s\approx 5.5\ \text{GeV}^2$ the elastic pp cross section has a maximum value after which it starts to decrease rather sharply. This, we believe, is connected to the production of pions. Consequently, as far as the production of secondary pions is concerned, we choose

$$s_{\min} = 5.5\ \text{GeV}^2. \tag{61}$$

In order to determine r, we first identify experimental $\langle n_{\pi^+}\rangle + \langle n_{\pi^-}\rangle$ with $\langle m_{\text{ch}}\rangle$ (in our theory for pp collisions $n_{\pi^\pm} = m_\pm$, so $\langle m_{\text{ch}}\rangle = 2\langle n_{\pi^+}\rangle = 2\langle n_{\pi^-}\rangle$). Then, since the lower-energy regime is supposed to

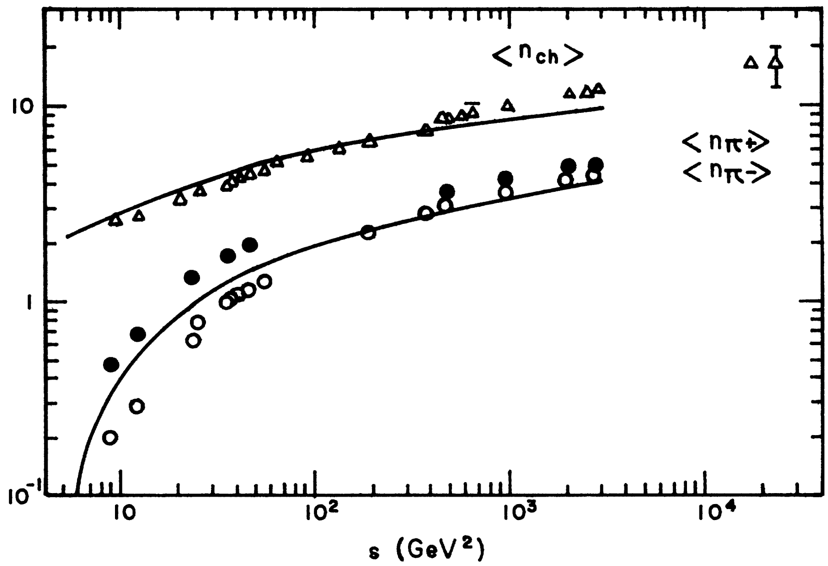

FIG. 2. Integral average multiplicities through the CERN-ISR range of energies in inelastic pp collisions. The lower curve is the prediction for $[\langle n_{\pi^+}(s)\rangle + \langle n_{\pi^-}(s)\rangle]/2$ while the upper curve is the prediction for $\langle n_{ch}(s)\rangle$ ignoring, however, the production of kaons and antinucleons. The curves are formally valid only in the lower-energy regime (see the text).

extend up to $s \approx 60$ GeV^2, we choose r so that at $s = 60$ GeV^2, $\langle n_{ch}\rangle$ from (60) equals experimental $\langle n_{ch}\rangle \approx 5$. This gives

$$2r \approx 2.2, \tag{62}$$

which, when combined with (57) and (49), gives

$$\langle |t| \rangle' \approx 1.4\ (\mathrm{GeV}/c)^2. \tag{63}$$

In Fig. 2 we compare Eqs. (59) and (60) with the data.[15,16] The lower curve represents $\langle m_{ch}(s)\rangle/2$, which is to be compared with the experimental $[\langle n_{\pi^+}(s)\rangle + \langle n_{\pi^-}(s)\rangle]/2$ while the upper curve represents $\langle n_{ch}(s)\rangle$. As we can see, the agreement with experiments is not only very good at energies belonging to the lower-energy regime ($s \lesssim 60$ GeV^2), but also amazingly good at ISR energies. Even at $s \approx 3000$ GeV^2 the computed $\langle m_{ch}(s)\rangle/2$ fits the data quite well, while the computed $\langle n_{ch}(s)\rangle$ is somewhat below the data. This we attribute mainly to the fact that the production of secondary kaons and antinucleons becomes non-negligible at this point and, since $\langle n_{ch}(s)\rangle$ from (60) does not take them into account, it must predict smaller values than those from experiments.

Finally, we would like to see why the lower-energy regime expression for $\langle n_{ch}(s)\rangle$ is describing the trends of experimental data so well also at very high energies. Looking at relation (46) we see that $\langle m_{ch}(s,t)\rangle$ is a function of s and t through $h(s,t)$ only. From relation (45) it is easy to deduce that as s increases, $h(s,t)$ will change from values comparable to unity to values comparable to zero. Consequently, at rather high energies $\langle m_{ch}(s,t)\rangle$ will behave basically as $\ln[1/h(s,t)]$, since in (46) the factor $[1-h^2(s,t)]^{1/2}$ will mostly have values comparable to 1 and since the argument of the logarithm behaves asymptotically as $1/h(s,t)$ for small $h(s,t)$. Thus it is entirely possible that this approximate logarithmic dependence is responsible for a fairly smooth behavior in t of $\langle m_{ch}(s,t)\rangle$. Furthermore, with the assumption that $\langle t\rangle$ does not change drastically with s, we expect $\langle |t| \rangle$ at higher energies to be not too different from $\langle |t| \rangle' \approx 1.4$ (GeV/c)^2, which we deduced to be an average of $\langle |t| \rangle$ in the lower-energy regime. With this it is then reasonable to expect $\langle m_{ch}(s,\langle t\rangle')\rangle$ to describe quite well the experimental trends of $\langle m_{ch}(s)\rangle$ also at higher energies.

IV. DISCUSSION AND CONCLUSION

As we have seen, despite the fact that we have used the lower-energy-regime approximation ($s \lesssim 60$ GeV^2) in evaluation of the integral mean multiplicities, their excellent agreements with experiments extends well into the ISR region: $s \approx 300$ GeV^2. Beyond $s \approx 300$ GeV^2 we still have a rather good agreement, although discrepancies between the theoretical curves and the data increase as s increases. In any case, the trend of the data is definitely described well by expressions (59) and (60). Consequently, we must conclude that, unlike the integral correlation parameters, the integral average multiplicities are relatively immune to the existence of the lower- and higher-energy regimes.

It is not difficult to see that $\langle m_{ch}(s)\rangle$ from (59) can be written as a power series

$$\langle m_{ch}(s)\rangle = \sum_{k=1}^{\infty}\frac{1}{2k-1}X^k(s), \tag{64}$$

where

$$X(s) = 1 - \left(\frac{\sigma_T(s_{\min})}{\sigma_T(s)}\right)^4\left(\frac{s_{\min}}{s}\right)^{2r}. \tag{65}$$

Now, if s is not too far away from $s_{\min}$, $X(s)$ can be considered small [$X(s)$ can never exceed 1], so we can stop in the summation of (64) at, say, $k=3$. This gives

$$\langle m_{ch}(s)\rangle \approx \frac{4}{3} - \frac{5}{3}\left(\frac{\sigma_T(s_{\min})}{\sigma_T(s)}\right)^4\left(\frac{s_{\min}}{s}\right)^{2r} + \frac{1}{3}\left(\frac{\sigma_T(s_{\min})}{\sigma_T(s)}\right)^8\left(\frac{s_{\min}}{s}\right)^{4r}. \tag{66}$$

In view of the slow variation of $\sigma_T(s)$ with s, relation (66) gives $\langle m_{ch}(s)\rangle$ [and consequently $\langle n_{ch}(s)\rangle$] in a low-s region essentially as a combination of terms, each of which has a different power dependence on s. This agrees generally with claims from Berger and Krzywicki,[2] except that the region in which $\langle m_{ch}(s)\rangle$ can be "economically" written in such a way does not extend all the way up to $p_{lab}\approx 100$ GeV/c but more likely to $p_{lab}\approx 30$ GeV/c [having perhaps a few more terms in (66)].

In view of the fact that the computed $\langle m_{ch}(s)\rangle$ and $\langle n_{ch}(s)\rangle$ [relations (59) and (60)] are in rather good agreement with experiments in the ISR energy region, it makes sense to look at $\langle m_{ch}(s)\rangle$ for very large s. With little work, we get from (59) that at very large s

$$\langle m_{ch}(s)\rangle \approx \ln\left(2\frac{\sigma_T(s)}{\sigma_T(s_{\min})}\right) + r\ln\left(\frac{s}{s_{\min}}\right), \tag{67}$$

If one neglects the variation of $\sigma_T(s)$ with s, this expression is very similar to the one for the average multiplicity that can be obtained within the so-called Feynman's bremsstrahlung analogy.[10,17]

The value of 1.4 (GeV/c)^2, which we obtained for $\langle|t|\rangle'$ [an "average" of $\langle|t|\rangle$ over the whole lower-energy regime, see (63)], at first glance seems too high. However, if we compare it with $s\approx 60$ GeV^2, which is the upper value of s in the lower-energy regime, then we see that this value for $\langle|t|\rangle'$ is actually quite reasonable. From the physical point of view one should indeed expect $\langle|t|\rangle' \neq 0$, for, as previously noted, no secondary particle production is possible if $\langle t\rangle = 0$.

The value for $s_{\min}$ of about 5.5 GeV^2 [see relation (55)] refers only to the production of secondary charged pions. The meaning is clear—5.5 GeV^2 is the cutoff value for s below which practically no secondary pions are produced. We believe that each kind of produced secondary particle should have its own value for $s_{\min}$. For example, the produced secondary kaons in pp collisions should have $s_{\min}$ distinctly larger than 5.5 GeV^2.

As we have seen, as the energy increases, the experimental $f_2^-(s)$ for pp collisions changes from negative to positive values [already at 50 GeV/c of proton laboratory momentum $f_2^-(s)>0$]. As a consequence, $P_{m_-}(s)$ cannot be described with $W_{m_-}(s,\langle t\rangle)$ alone [which can give only $f_2^-(s)<0$], but it also needs the second term at higher energies, as indicated in (43):

$$P_{m_-}(s) = W_{m_-}(s,\langle t\rangle) + \tilde{W}_{m_-}(s), \tag{68}$$

where we have denoted

$$\tilde{W}_{m_-}(s) = \frac{\langle t^2\rangle - \langle t\rangle^2}{2}\left(\frac{d^2W_{m_-}(s,t)}{dt^2}\right)_{t=\langle t\rangle}.$$

Since $W_{m_-}(s,\langle t\rangle)$ and $\tilde{W}_{m_-}(s)$ cannot in general be proportional to each other [$\sum_{m_-}W_{m_-}(s,\langle t\rangle)=1$, $\sum_{m_-}\tilde{W}_{m_-}(s)=0$], they in a way constitute two independent "components" of $P_{m_-}(s)$. In this sense our formalism shows a formal similarity with Wilson's idea of a two-component description of many-particle production amplitudes.[18] The two components in the Wilson approach refer to diffractive and nondiffractive mechanisms.[18] Such a two-component description seems to be particularly applicable[8] for $p_{lab}\gtrsim 50$ GeV/c, which is actually the region where relation (68) for $P_{m_-}(s)$ holds.

Let us now briefly review the main ingredients of the usual two-component model of multiparticle production. Since the two mechanisms populate different regions of the many-particle phase space (the diffractive mechanism contributes to the fragmentation region, while the nondiffractive contributes to the entire pionization region), Harari and Rabinovici[8] assume that there are no interference terms between them in their contribution to the total many-particle cross section. The general conclusion which they reach from the experimental fits is that while the nondiffractive mechanism is dominant, the contributions from the diffractive mechanism, although small, are essential.

In addition to the assumption that the diffractive contribution corresponds to a low-multiplicity cross section, they also assume that the diffractive parts of σ_{m_-}, $\langle m_-\rangle$, and f_2^- are constant in energy. For the nondiffractive parts of σ_{m_-} and f_2^- they assume $\ln(s)$ dependence, as well as that $f_k^-=0$ $(k\geq 3)$ for the nondiffractive part. Denoting now with D_{m_-} and $M_{m_-}(s)$ the diffractive and nondiffractive parts of $P_{m_-}(s)$, we have

$$P_{m_-}(s)=D_{m_-}+M_{m_-}(s), \tag{69}$$

where D_{m_-} is assumed to be s independent. Exactly how $W_{m_-}(s,\langle t\rangle)$ and $\tilde{W}_{m_-}(s)$ are related to D_{m_-} and $M_{m_-}(s)$ is difficult to tell. However, Harari and Rabinovici[8] assume that $D_{m_-}=0$ for $m_-\geq 3$, which gives the formal connection

$$\begin{aligned} &W_{m_-}(s,\langle t\rangle)+\tilde{W}_{m_-}(s)=D_{m_-}+M_{m_-}(s), \quad m_-<3\\ &W_{m_-}(s,\langle t\rangle)+\tilde{W}_{m_-}(s)=M_{m_-}(s), \quad m_-\geq 3, \end{aligned} \tag{70}$$

showing that at higher multiplicities the contribution of $\tilde{W}_{m_-}(s)$ is quite important to the nondiffractive component of $P_{m_-}(s)$. One may think that since $W_{m_-}(s,\langle t\rangle)$ dominates $P_{m_-}(s)$ at lower energies, perhaps as the energy increases its contribution to $P_{m_-}(s)$ becomes less significant. However, this is not so. Namely, because of $\sum_{m_-}\tilde{W}_{m_-}(s)=0$, some of the $\tilde{W}_{m_-}(s)$ must be negative, and since $P_{m_-}(s)\geq 0$ these negative $\tilde{W}_{m_-}(s)$ must be balanced out by $W_{m_-}(s,\langle t\rangle)$ [incidentally, $W_{m_-}(s,\langle t\rangle)\geq 0$]. Consequently, neither $W_{m_-}(s,\langle t\rangle)$ nor $\tilde{W}_{m_-}(\bar{s})$ can be directly associated with either $D_{m_-}(s)$ or $M_{m_-}(s)$, both of which are positive-definite.

Thus, it appears that from the "dynamical" point of view we cannot directly relate the charge-conserving hadronic-bremsstrahlung model with the usual two-component model, although at higher energies ($p_{\text{lab}}\gtrsim 50$ GeV/c for pp collisions) they both use two components in the integral multiplicity distribution function. This actually should not be surprising if we note that the emphasis in the charge-conserving hadronic-bremsstrahlung model is on the charge conservation itself, while the emphasis in the usual two-component model is on the detailed "dynamical" picture, which as we saw is not really simple but consists of two dynamical mechanisms, diffractive and nondiffractive.

However, looking at relation (70) one can easily see that at higher energies the charge conservation [entering on the left-hand side of relation (70)] will influence what the mixture of diffractive and nondiffractive mechanisms [entering on the right-hand side of relation (70)] should be. Of course, this will greatly depend on how "strong" the constraints of the charge conservation are. We believe that the contribution to $P_{m_-}(s)$ of D_{m_-} can be chosen to be much smaller than that of $M_{m_-}(s)$ [in Ref. 8, D_{m_-} by itself gives $f_{2_-}=-0.26$, which clearly has to be outweighed by the contribution from $M_{m_-}(s)$ as making the total $f_{2_-}(s)$ positive] because the constraints of charge conservation on $f_2^-(s)$ are relaxed for pp collisions in the higher-energy regime [$f_2^-(s)>0$ which is reflected in the fact that $\tilde{W}_{m_-}(s)\neq 0$ in relation (70)]. As the energy decreases and we get to the lower-energy regime ($p_{\text{lab}}\lesssim 30$ GeV/c), the usual two-component picture is less applicable, for now the constraints of charge conservation require $f_2^-(s)$ to be negative and s-dependent. This, however, could only be achieved by altering the usual assumptions of the two-component model (see Ref. 8) as, for example, by assuming that at lower energies the contribution of $M_{m_-}(s)$ to $P_{m_-}(s)$ is negligible, by allowing D_{m_-} to be energy-dependent in such a way as to dominate $P_{m_-}(s)$, and by giving the energy-dependent (negative) $f_2^-(s)$. This, however, no longer has anything to do with either the usual two-component model[8] or the diffractive model alone, which is supposed to have energy-independent $\langle n\rangle$ and f_2.

Another reason why the usual two-component model seems to be defective at lower energies is that it ignores for $\langle n_{\pi^-}(s)\rangle$ the nonleading terms of the form $\langle n_{\pi^-}\rangle\approx s^{-k}$, which seem to be needed for $p_{\text{lab}}\lesssim 30$ GeV/c (see, for example, Ref. 19 and 20). However, in this energy region the charge-conserving hadronic-bremsstrahlung model describes $P_{m_{\text{ch}}}(s)$ well with just one term (see Ref. 3, 5, and 7) and at rather low energies gives for $\langle n_{\text{ch}}(s)\rangle$ an expression in which the "nonleading" terms are approximately of the form s^{-k} [see relation (66)].

Despite the fact that the charge-conserving hadronic-bremsstrahlung model can describe only gross features of multiparticle production processes, so far it has been quite successful in describing the experimental data.[3,6,7] We attribute this to the fact that this model takes into account the charge conservation in particle production that makes it distinctly different from the ordinary hadronic-bremsstrahlung model.

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