Quark self-energy

R. L. Stuller
Physics Department, Imperial College, London SW7 2BZ, England
(Received 5 June 1975)

A model of colored quarks interacting through non-Abelian vector gauge potentials is analyzed. An analogy with the Kondo effect of solid-state physics is exploited to suggest approximations to the coupled integral equations for the quark field two-point function. Conditions are described for which the *self-consistently* determined quark mass equals infinity. A covariant bound-state equation for color singlet mesons is derived which is essentially *three*-dimensional. The internal Minkowski space, parametrized by the relative coordinate four-vector x^μ, is replaced by a hyperplane in that space defined by $x^\mu P_\mu = 0$, P^μ being the four-momentum of the meson. Thus, "quark confinement" to the "inside" of the meson induces a contraction of the four-dimensional, internal Minkowski space to that hypersurface orthogonal to P^μ. Some pecularities of the equation are briefly discussed.

INTRODUCTION

In recent years more and more attention has been focused on theories of weak, electromagnetic, and strong interactions consisting of spinor fields interacting via non-Abelian vector gauge potentials. This interest was rekindled by the demonstration that such theories are renormalizable[1] and further intensified by the discovery that they are asymptotically free.[2] Complementary to this "softness" in the ultraviolet region, it is suggested by renormalization-group arguments that such theories will exhibit a very strongly coupled infrared behavior—that is to say in the region of time-like momenta. This heuristic argument taken together with the practical fact that, in perturbation theory at least, the Green's functions of such theories are beset with uncontrollable infrared sigularities near the mass shells has given rise to the hope that infrared effects could conspire to "confine the quarks." Said another way, despite the persistent use of quark fields in the formal construction of currents and in various phenomenological models for the hadron spectrum, the basic elementary quantum of those fields—the quark itself—have never been observed. The present work analyzes this puzzle within the framework of certain physically motivated approximations to the quark self-energy equations. We shall see that the key to understanding how the basic spinor fields $\chi(x)$ can escape having to create long-lived, colored, single-particle states when applied to the physical vacuum lies in there being specific dynamical processes which *inhibit the localization of those color charges carried by the quark fields*. The quarks, then, as well-defined particles carrying color and having a finite mass are not permitted to exist.

THE KONDO MECHANISM

We assume that the strong gauge group $\mathcal{G} = G \times SU(3)$ (color) with G the spectrum-classifying factor. All we have to say concentrates on the color SU(3) factor. Suppressing indices with respect to G one has typically

$$\mathcal{L}(x) = -\tfrac{1}{4} G_a^{\mu\nu}(x) G_{\mu\nu}^a(x) - \bar{\chi}(x)\gamma^\mu \left[\frac{1}{i}\partial_\mu - g\vec{t}\cdot\vec{A}_\mu(x)\right]\chi(x) + \cdots ,$$

where

$G_a^{\mu\nu}(x) = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + igA_b^\mu (T_a)^{bc} A_c^\nu$ and t and T are the representation matrices appropriate to χ and A^μ, respectively. What we need is an efficient language for discussing at least in a qualitative way the different kinds of physics that can lie hidden in this symmetrical Lagrangian function. To this end, introduce $\chi \rightleftharpoons \chi + A$ as the symbolic expression for the coupling between the two fields χ and A. The double arrow $\rightleftharpoons$ is intended to express that energy, momentum, and color (as well as other elementary properties) flow back and forth between the two fields. Clearly it is only under quite special circumstances that one can speak of an *equilibrium situation*, a stationary state of the so-expressed dynamics. These are the conditions that we seek.

SU(3), being a group of rank two, has its irreducible representations labeled by the eigenvalues of two Casimir operators. For the purpose of illustration and simplicity it will suffice to analyze the case of SU(2), the generalization to SU(3) being straightforward.

Write

$$\mathcal{L}(x) = -\bar{\chi}(x)\gamma^\mu\left(\frac{1}{i}\partial_\mu - f\tfrac{1}{2}\vec{\tau}\cdot\vec{A}_\mu\right)\chi - \tfrac{1}{4}G_a^{\mu\nu}G_{\mu\nu}^a .$$

The complete list of elementary transitions is given by

$$\chi^{\pm 1/2} \rightarrow \chi^{\pm 1/2} + A^0; \quad \chi^{\pm 1/2} \rightarrow \chi^{\mp 1/2} + A^{\pm} \tag{1a}$$

and

$$A^{\pm} \rightarrow A^{\pm} + A^0; \quad A^0 \rightarrow A^0 + A^{\pm} + A^{\mp} . \tag{1b}$$

Such a system may be termed the minimal "completion" of an Abelian theory by the consistent inclusion of *charge-exchange* process.

Let us now assume that the lowest excitation in the spinor spectrum occurs from $m > 0$ and in the vector spectrum for $M^2 = 0$ and furthermore that m and M^2 are proportional to their respective unit matrices. To make clear what we have in mind let us begin by asking the following question: Since the vector spectrum extends to zero mass do we expect the spinor particles (assuming there are spinor particles) to be surrounded by a Coulomb-type field just as in Abelian electrodynamics? Well, just how does the Coulomb field come about? The elementary process $e^+ \rightarrow e^+ + \gamma$ implies that (in the limit of soft, long-wavelength γ's) the states $|e\rangle, |e\gamma\rangle, \cdots, |e\gamma \cdots \gamma\rangle$ are degenerate in energy. The Coulomb field is the dynamical expression of this degeneracy. In the language of chemical equilibrium the relative concentrations of e and γ (see Ref. 3) adjust themselves so as to yield a stationary state. That stationary state is the physical electron. Crucial to this conclusion is that the electron charge is localized in a volume $\sim \lambda^3 = (m^{-1})^3$. The charge's inability to escape from this volume allows establishment of a final equilibrium with the electromagnetic field. How does this picture change upon the inclusion of explicit charge-exchange processes? Or, in other words, how do the reactions $\chi^{\pm 1/2} \rightarrow \chi^{\mp 1/2} + A^{\pm}$ affect the above arguments? We have insisted that the charged as well as the neutral vector spectra extend down to zero mass. Now focus on those transitions in which A^+ (just as γ above) is soft (wavelength $\rightarrow \infty$). One encounters a catastrophe: The charge $+\frac{1}{2}f$ which was assumed initially localized in a small three-dimensional volume $\sim (m)^{-3}$ —that is, after all, the operational definition of a particle—has dissolved into the boson field, becoming thereby delocalized and diffuse and leaving behind a charge $-\frac{1}{2}f$. Obviously our hypothesized single-charged-particle state is not an eigenstate of all dynamically allowed processes. By now it should be clear that what is at issue here is the connection between the *assumed* spectra of the individual fields in an interacting system and the primitive mechanisms which couple those fields—in short, the problem of self-energy. A single-particle state, to be long lived, must be an equilibrium configuration of *all* the elementary mechanisms operating.

What then do we conclude from the above observation? If

(i) $m > 0$ and the vector spectrum begins at $M^2 = 0$,

(ii) the matrices m, M^2 are proportional to their respective unit matrices, i.e., $m = 1m$, $M^2 = 1M^2$,

(iii) the charge-exchange $\chi^{\pm 1/2} \rightarrow \chi^{\mp 1/2} + A^{\pm}$ transitions are treated *symmetrically* with the neutral process $\chi^{\pm} \rightarrow \chi^{\pm} + A^0$, and

(iv) the strength of these transitions is sufficiently strong in the limit of soft (long-wavelength) A^0, $A^{\pm}$,

then long-lived, charged, single-particle-like states $\chi^{\pm 1/2}$ cannot exist.

Phenomena of this genre are familiar from solid-state physics. The Kondo effect[4] arises due to the coupling of a spin-$\frac{1}{2}$ impurity to soft collective spin waves in the solid. An initially localized impurity is not an eigenstate of the total Hamiltonian as the boson spin waves are capable of transporting spin out of the initial region of localization. Thus the spin-$\frac{1}{2}$ impurity quickly "disappears." Another viewpoint is that the soft, spin-carrying transitions induce an uncontrollable averaging over the up (down) configuration of the spin, resulting in an observable value of zero. This reasoning applied to our Yang-Mills example suggests that insisting on the gluon's spectrum extending to zero mass will, under favorable dynamical conditions (still to be discovered, of course), induce an uncontrollable, energetically favorable averaging over the charge states available to the fermion, resulting in there being no fermion of definite charge at all. In honor of the solid-state example which inspired this observation, we term this effect the Yang-Mills-Kondo mechanism. The above argument generalizes trivially to SU(3) and forms the basis for our conviction that soft Yang-Mills excitations are the agency ultimately responsible for the nonexistence of color quarks, and explains how quark fields can play such an important role while the quanta of those fields simply do not exist.

Let us now begin translating this qualitative picture into the quantitative language of Green's functions. The mathematical statement of the quark's not being permitted to exist will be that the only *self-consistent solutions to the self-energy equations* for the renormalized, physical quark mass m, will be $m = \infty$. Let us see how this comes about.

THE QUARK SELF-ENERGY

Any nonperturbative question about the quark single-particle spectrum must be addressed to the Schwinger-Dyson equation for the quark propagator $G(p)$, namely

$$G^{-1}(p)=\gamma\cdot p+m_0+ig^2\int\frac{(dk)}{(2\pi)^4}\gamma^\mu t_a G(p-k)\Gamma_b^\nu(p-k,p)\mathcal{G}_{\mu\nu}^{ab}(k)\,, \tag{2}$$

where we have included a bare mass for reference purposes. The $t_a=\frac{1}{2}\lambda_a$ [$a=1,\cdots,8$, the fundamental representation of SU(3) color]. All reference to other quantum numbers (ordinary electric charge, hypercharge, and perhaps charm) is suppressed. We are explicitly assuming that as far as the determination of the single-quark spectrum is concerned, any dynamics that may be coupled to these quantum numbers can be ignored for the present, to be treated later as a perturbation.

Clearly, without additional information about Γ_a^ν and $\mathcal{G}_{ab}^{\mu\nu}$ Eq. (2) is useless as it stands, being simply the first in an infinite hierarchy of coupled equations. Using the physics of the Kondo mechanism as a guide, however, we shall guess reasonable small-momentum-transfer behavior for Γ^ν and small-k^2 behavior for $\mathcal{G}^{\mu\nu}$ in terms of which we shall be able to conclude something about the quark spectrum.

The first Slavnov[5] identity guarantees that $\mathcal{G}_{ab}^{\mu\nu}(k)$ may be decomposed as

$$\mathcal{G}_{ab}^{\mu\nu}(k)=\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\right)\frac{d_{ab}(k^2)}{k^2}+\delta_{ab}\xi\frac{k^\mu k^\nu}{(k^2)^2}, \tag{3}$$

with ξ the gauge parameter ($\xi=0$ the Landau gauge, $\xi=1$ the Feynman gauge). All dynamical information is contained in the invariant, *dimensionless* function $d_{ab}(k^2)$. Complying with point (ii) above that we are seeking *symmetric* solutions, we put $d_{ab}(k^2)=\delta_{ab}d(k^2)$. Our fundamental assumption—that the vector spectrum extends down to zero mass—we parametrize as the behavior of $d(k^2)$ near $k^2=0$, namely

$$\lim_{k^2\to 0}d(k^2)=\left(\frac{\Omega^2}{k^2-i\epsilon}\right)^\lambda. \tag{4}$$

The case $\lambda=0$ would correspond to a pure single-particle pole as in quantum electrodynamics. One expects in general, however, $\lambda\neq 0$, for certainly the fact that the openings of all multiboson thresholds collapse on top of the one-particle pole will be enough to transform what would have been a pole into a cut. See Fig. 1. Equation (4) is, of course, not the most general imaginable behavior (there could be logarithmic dependences superposed on top of the power as well as essentially singular behavior) but will be general enough for our purposes. The quantity λ (in general a function of g^2) is at this stage to be regarded as a parameter of the *Ansatz*. It is left to a more complete future theory to determine $\lambda(g^2)$ self-consistently by solving the integral equations for the gluon propagators. The present work makes no further comment about this difficult problem.

As $d(k^2)$ is dimensionless the *Ansatz* in Eq. (4) introduces a further parameter Ω^2 having dimension (mass)^2, which has no counterpart in the original Lagrangian. This mass parameter will ultimately be relatable to the normalization point chosen to define the mass-zero Yang-Mills theory and as we shall see is the quantity which sets the scale for the physical hadron masses.

It is, of course, not enough to know $d(k^2)$ for small k^2 only as the $\int(dk)$ in Eq. (2) instructs us to integrate over the entire Minkowski space. To this end we assume $d(k^2)\to 1$ as $k^2\to\infty$. This assumption may oversimplify the ultraviolet behavior, but as it is an assessment of the consequences of specific *infrared* behavior that we are after, a more sophisticated treatment of ultraviolet asymptotics will be deferred to a later date.[6] Thus, the full *Ansatz* for $d(k^2)$ becomes

$$d(k^2)=\left(\frac{\Omega^2}{k^2-i\epsilon}\right)^{\lambda(g^2)}+1\,. \tag{5}$$

For large k^2 the infrared term dies out (for $\lambda>0$ as we assume) leaving the canonical "1" behind. If in the process of solving Eq. (2) an ultraviolet divergence is encountered (as it will be) it will be cut off at some maximum momentum Λ.

Our final input is the assumption that

$$\Gamma_a^\mu(p-k,p)=\gamma^\mu t_a \tag{6}$$

for small momentum transfers and for $-p^2\approx m^2$ (m being the quark mass). This structure reflects points (iii) and (iv) above, namely that the

FIG. 1. Cut structure of the gluon propagator. Anticipated structure of $d(k^2)$ has all cuts telescoping onto zero.

transitions $\chi \leftrightarrow \chi + A$ continue to occur symmetrically even in the limit of very soft A's. From the point of view of our four-dimensional equation this is a drastic assumption. It does, however, have the virtue of simplicity and emphasizes the nonsuppression of low-frequency, long-wavelength transitions.

Now we can make a bit of progress. Defining $G^{-1}(p) \equiv A(p^2)[\gamma\cdot p + m(p^2)]$, Eq. (2) can be written as two coupled equations for $A(p^2)$ and $m(p^2)$,

$$A(p^2)m(p^2) = m_0 - ic_0 g^2 \int \frac{(dk) m((p-k)^2)}{(2\pi)^4 m^2((p-k)^2) + (p-k)^2} g^{\mu\nu} \mathcal{G}_{\mu\nu}(k) \frac{1}{A((p-k^2))} \tag{7}$$

and

$$[A(p^2) - 1](-p^2) = \frac{ic_0 g^2}{4} \mathrm{Tr} \int \frac{(dk)}{(2\pi)^4} \frac{\gamma\cdot p \gamma^\mu \gamma\cdot(k-p)\gamma^\nu}{m^2((p-k)^2) + (p-k)^2} \mathcal{G}_{\mu\nu}(k) \frac{1}{A((p-k)^2)} \tag{8}$$

with

$$\mathcal{G}_{\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \frac{1}{k^2} \left[\left(\frac{\Omega^2}{k^2}\right)^{\lambda(g^2)} + 1\right] + \xi \frac{k_\mu k_\nu}{(k^2)^2} \tag{9}$$

and c_0 is the Casimir invariant defined by $\sum_{a=1}^{8}(t^a)^2 = c_0 1$. Two features prevent the straightforward extraction of physical information from Eqs. (7) and (8)—the nonlinearity of this highly coupled system and, because $G(p)$ is a Green's function of a colored field, its gauge dependence. This gauge dependence enters implicitly via the explicit dependence of $\mathcal{G}^{\mu\nu}$ on ξ. Now, structure which depends on the gauge is useless for drawing physical conclusions. However, there is one feature of $G(p)$ which must be independent of gauge if the theory is to be consistent, namely the postion of its pole and cut discontinuities corresponding to one quark and one quark plus multi $q\bar{q}$ pair thresholds. The residues of these singularities will, in general, be gauge dependent and not directly connectable to properties of physical states. An equation for the quark mass itself, however, is physically reliable information and in fact precisely that information we are after. We now propose to use the gauge dependence to achieve a partial decoupling of Eqs. (7) and (8). Assume that $\lambda(g^2)$ is sufficiently large and positive so that the dominant region of integration is from small k^μ (what is meant by sufficiently large will be determined later). Since the eigenvalue equation for the quark mass reads $m(p^2)|_{-p^2=m^2} = m$ we ignore the k dependence of $m((p-k)^2)$ and $A((p-k)^2)$ under the integral and choose the gauge so that $A(p^2)|_{-p^2=m^2} = 1$, i.e., we fix the gauge so that in the vicinity of the assumed quark mass shell we have

$$\mathrm{Tr} \int \frac{(dk)}{(2\pi)^4} \frac{\gamma\cdot p \gamma^\mu \gamma\cdot(k-p)\gamma^\nu}{m^2 + (p-k)^2} \mathcal{G}_{\mu\nu}(k)\bigg|_{-p^2=m^2} = 0 . \tag{10}$$

This will be a gauge resembling the Yennie gauge in quantum electrodynamics where all soft-photon corrections to the residue of the one-electron singularity (in the two-point function) are removed by the choice $\xi = 3$. We are now left with an eigenvalue equation for the quark mass m

$$m = m_0 - ic_0 g^2 \int \frac{(dk)}{(2\pi)^4} \frac{m}{m^2 + (p-k)^2} g^{\mu\nu} \mathcal{G}_{\mu\nu}(k)\bigg|_{-p^2=m^2}, \tag{11}$$

which will be the equation of concern in the rest of this paper. Arguments such as the above have many predecessors in the literature.[7,8] All previous cases, however, were concerned with approximations (and/or *Ansätze*) tailored to the "expected exact ultraviolet behavior." We are reversing the priorities here by anticipating that the greater portion of the self-consistently determined quark mass will be controlled by the low-momentum region of integration in Eq. (11).

Let us now define

$$g^{\mu\nu} \mathcal{G}_{\mu\nu}(k) = \frac{3}{k^2}\left(\frac{\Omega^2}{k^2}\right)^\lambda + (3+\xi)\frac{1}{k^2} \equiv \mathcal{G}^{\mathrm{IR}} + \mathcal{G}^{\mathrm{UV}} \tag{12}$$

with the λ-dependent term emphasizing the infrared contribution and the $1/k^2$ term the ultraviolet contribution. Furthermore, as many of our integrals are potentially infrared divergent we include a control parameter μ in Eq. (12) so that

$$\mathcal{G}^{\mathrm{IR}} = \frac{3}{k^2+\mu^2}\left(\frac{\Omega^2}{k^2+\mu^2}\right)^\lambda$$

with the understanding that μ is set equal to zero at the end. This trick enables us to assess the role played by formal infrared divergences in our self-energy equations. It will turn out, when we come to constructing bound-state equations, that a criterion for which states may or may not exist (having finite energy and standard parity properties, etc.) is that the equation generating those states as eigenstates be independent of μ, that is to say, free of infrared divergences.

THE SELF-CONSISTENTLY DETERMINED QUARK MASS

To illustrate the general case with a specific example let us temporarily set $\lambda = 1$ and perform the (dk) integration. The result can be written in the spectral form

$$m(p^2) = m_0 + \frac{3g^2c_0\Omega^2}{(4\pi)^2} m(p^2) \int_{(m+\mu)^2}^{\infty} \frac{dM^2}{M^2} \frac{1}{M^2+p^2-i\epsilon} \frac{M^2+m^2-\mu^2}{[M^2-(m+\mu)^2]^{1/2}[M^2-(m-\mu)^2]^{1/2}}$$

$$+\frac{(3+\xi)}{(4\pi)^2} c_0 g^2 m(p^2) \int_{m^2}^{\Lambda^2} \frac{dM^2}{M^2} \frac{M^2-m^2}{M^2+p^2-i\epsilon}, \tag{13}$$

where we have restored the p^2 dependence of $m(p^2)$ momentarily and Λ^2 is the ultraviolet cutoff. Now let $-p^2 \to m^2$ and obtain the eigenvalue equation

$$m = m_0 + \frac{3g^2c_0\Omega^2}{(4\pi)^2} m \int_{(m+\mu)^2}^{\infty} \frac{dM^2}{M^2} \frac{1}{M^2-m^2} \frac{M^2+m^2-\mu^2}{[M^2-(m+\mu)^2]^{1/2}[M^2-(m-\mu)^2]^{1/2}} + \frac{(3+\xi)g^2c_0}{(4\pi)^2} m \int_{m^2}^{\Lambda^2} \frac{dM^2}{M^2}. \tag{14}$$

The reader may have been wondering why we have bothered to make specific reference to a bare mass m_0 in the above. Here we see that its sole role is to resolve an ambiguity inherent to Eq. (14). If m_0 were zero Eq. (14) clearly has as a solution $m = 0$ which we want to reject. This solution, incidentally, would be the one favored by ordinary perturbation theory. In any event, once m_0 has ensured that the system chooses the *nonperturbative* mode, we set $m_0 = 0$ and find

$$1 = \frac{3g^2c_0\Omega^2}{(4\pi)^2} \int_{(m+\mu)^2}^{\infty} \frac{dM^2}{M^2} \frac{1}{M^2-m^2} \frac{M^2+m^2-\mu^2}{[M^2-(m+\mu)^2]^{1/2}[M^2-(m-\mu)^2]^{1/2}} + \frac{(3+\xi)g^2c_0}{(4\pi)^2} \int_{m^2}^{\Lambda^2} \frac{dM^2}{M^2}. \tag{15}$$

Suppose for the moment that $\Omega = 0$; then the solution to (15) is

$$m^2 = \Lambda^2 \exp\left[-\left(\frac{4\pi}{g}\right)^2 \frac{1}{c_0(3+\xi)}\right], \tag{16}$$

which could have been anticipated as Λ, the ultraviolet cutoff, is the only dimensional parameter left upon which m could depend. This result is typical of those solutions found in Ref. 7. With $\Omega \neq 0$ an entirely new possibility presents itself. Ignoring the ultraviolet term in Eq. (14) for the moment, recall that the physical situation is *defined* by the limit $\mu \to 0$. But the first integral in Eq. (15) is infrared divergent if we blindly set $\mu = 0$, that is to say there is a divergence coming from the vicinity of the lower limit of integration, $M \approx m^2$. How then is it possible that this integral can satisfy the sum rule, namely Eq. (15)? This will only be possible if as $\mu \to 0$ the region of integration shrinks accordingly so that this *shrinking of the integration region* and the *divergence of the integrand* succeed in compensating one another. This is what happens. The integration in Eq. (15) may be performed, yielding

$$1 = \frac{3g^2c_0\Omega^2}{(4\pi)^2} \left\{ \left(2 - \frac{\mu^2}{m^2}\right) \frac{1}{(4m^2\mu^2-\mu^4)^{1/2}} \left[\frac{\pi}{2} - \sin^{-1}\frac{\mu}{m}\right] - \frac{1}{m^2}\ln\frac{m}{\mu} \right\} + \frac{g^2c_0(3+\xi)}{(4\pi)^2} \ln\frac{\Lambda^2}{m^2}. \tag{17}$$

Still ignoring the $\ln(\Lambda^2/m^2)$ term we see that Eq. (17) defines a function $m(\Omega, \mu)$ which in the limit $\mu/m \ll 1$ becomes $m = [g^2c_0/(4\pi)^2](2\Omega^2/\mu)$, i.e., the only value of $m(\mu)$ compatible with $\mu \to 0$ is $m \to \infty$. No finite value of the quark mass is possible.

It is a delicate question whether the presence of the ultraviolet cutoff Λ can alter this conclusion. As our treatment of the large-momentum region is provisional anyway a completely conclusive argument can not be given but one can say something. It is clearly meaningless to speak of a quark mass $m \gg \Lambda$ as Λ represents a *maximum* momentum, but we can imagine m being driven large until it becomes of the order m. Write Eq. (17) as

$$1 = a_1 \frac{\Omega^2}{m\mu} + a_2(\xi) \ln\frac{\Lambda}{m} \tag{17'}$$

with $a_1 = 3g^2c_0/(4\pi)^2$ and $a_2(\xi) = 2g^2c_0(3+\xi)/(4\pi)^2$. As $m \to \Lambda$ the ultraviolet term vanishes (in a gauge-independent manner) and we obtain the previous solution: $m \to \infty$ in the limit $\mu \to 0$. The implied interdependence of the *ultraviolet* and *infrared* cutoffs was to have been expected since as $m \to \infty$ there ceases to be a "region of momenta where all mass parameters may be ignored." Consequences of this intriguing intertwining of the ultraviolet and infrared will be pursued elsewhere.

Let us now analyze Eq. (11) for the case of arbitrary λ. Dropping the ultraviolet term which is the same as above, we find

$$1 = \frac{3g^2 c_0 (\Omega^2)^\lambda \Gamma(\lambda)}{(4\pi)^2 \Gamma(1+\lambda)} \int_0^1 \frac{dz(1-z)^\lambda}{[-p^2z^2 + (m^2+p^2)z + \mu^2(1-z) - i\epsilon]^\lambda} \Bigg|_{-p^2=m^2} . \tag{18}$$

Evaluating the right-hand side in the limit $\mu/m \ll 1$ gives

$$1 = \frac{3g^2 c_0}{(4\pi)^2} \frac{\Gamma(\lambda)}{\Gamma(1+\lambda)} \times \begin{cases} \left(\dfrac{\Omega^2}{m^2}\right)^\lambda B(1+\lambda,\ 1-2\lambda), & 0<\lambda<\frac{1}{2} \\ \left(\dfrac{\Omega^2}{\mu^2}\right)^\lambda \left(\dfrac{\mu}{m}\right) C(\lambda), & \frac{1}{2}<\lambda \end{cases} \tag{19}$$

where $B(1+\lambda, 1-2\lambda)$ is the usual B function, and

$$C(\lambda) \equiv \lim_{x\to\infty} \int_0^x dz\, \frac{(1-z/x)^\lambda}{(1-z/x+z^2)^\lambda} \quad \text{with } C(1) = \pi/2 . \tag{20}$$

Clearly, for $\lambda > \frac{1}{2}$, $m(\mu) \to \infty$ as $\mu \to 0$ just as in our test case $\lambda = 1$. What is interesting is that for $\lambda < \frac{1}{2}$ the infrared singularity is insufficiently strong to overcome phase space and m is not forced to infinity. Under these circumstances the ultraviolet region becomes at least as important in determining the physical mass, and an entirely different situation ensues. For the rest of this paper we assume that $\lambda(g^2) > \frac{1}{2}$.[9]

COMMENTS ON THE RESULT

Let us now ask for the probable stability of the result ($m \to \infty$ provided $\lambda > \frac{1}{2}$) if we were to have more detailed information about the behavior of the functions $d(k^2)$, $\Gamma^\mu(p-k,p)$, and $A((p-k)^2)$ in the region of small k^μ near $-p^2 = m^2$. A glance at Eq. (7) reveals that if only the ratio $\Gamma^\mu(p-k,p)/A((p-k)^2)$ remains finite as $k^\mu \to 0$, then for $\lambda > \frac{1}{2}$ the infrared divergence will obtain and the self-consistent mass m will be driven to infinity. Consider the contribution to the integral in Eq. (7) from the origin in k space. It will be $(-p^2 \approx m^2)$

$$\int \frac{(dk)}{(2\pi)^4} \frac{m}{m^2+p^2-2p\cdot k+k^2} \frac{(\Omega^2)^\lambda}{(k^2+\mu^2)^{1+\lambda}} \sim (\Omega^2)^\lambda \int \frac{(dk)}{(2\pi)^4} \frac{m}{-2p\cdot k} \frac{1}{(k^2+\mu^2)^{1+\lambda}} , \tag{21}$$

which diverges as $(1/\mu)^{2\lambda-1}$ as $\mu \to 0$. Seen in this light our result has a very simple, almost trivial interpretation. The Yang-Mills propagator is made sufficiently singular so that the contribution of soft radiation of the self-mass is infinitely large.

That $m = \infty$ is caused by an infrared effect will be of importance when we begin discussing in the next section which operators are expected to produce finite-mass states when acting on the vacuum as opposed to the action of a single-quark operator $\chi(x)$ which only produces states of infinite energy. Operators carrying color quantum numbers we expect to suffer the same fate as the quarks themselves as the Yang-Mills fields couple universally to color. If, however, an operator is *neutral* it can be expected to decouple from the infinite-mass-producing *soft* radiation and has a chance at least of exciting only *finite*-mass states out of the vacuum. This argument, by the way, would clearly be false if $m = \infty$ had been caused by an ultraviolet effect.

The time has come to face up to two serious problems our approach engenders.

(i) Having demonstrated that $m \to \infty$ the propagation of single quarks is described by a two-point function $G = 0$. How then are we to ever build up higher Green's function which are not trivially also equal to zero?

(ii) At the heart of our proposal is the requirement that the colored Yang-Mills gluon spectrum extends down to zero mass and is sufficiently concentrated at $M^2 = 0$ so that $\lambda(g^2) > \frac{1}{2}$. But does this not require the existence of massless, strongly interacting particles for which there is no experimental evidence? If that were the case our scheme would have to be rejected. One would have solved the quark puzzle forcing $m \to \infty$ only to be plagued by massless gluons which could not be confined. We feel that it would be being premature to conclude that this model predicts "zero-mass effects" acting between *physical* hadrons until we have clarified point (i) above as to the detailed nature and inner structure of those states. If, for example, we could show that during the collision of colorless hadrons the radiation of zero-mass colored bosons is dynamically suppressed then we will have trouble neither with unitarity nor with observation. We will come back to this difficult point later and proceed now to the construction of bound states.

BOUND STATES

We begin our discussion of bound states with the fermion-antifermion Bethe-Salpeter[10] equation

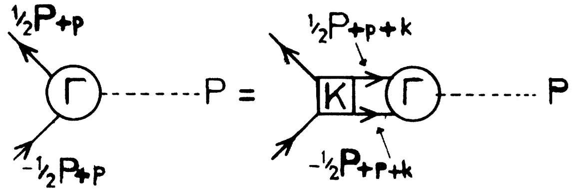

FIG. 2. Bound-state Bethe-Salpeter equation.

where $-P^2=M^2$ are the eigenvalues we seek and K is the Bethe-Salpeter kernel. In the approximation that $K=ig^2\gamma^\mu t_a \mathcal{G}^{ab}_{\mu\nu}\gamma^\nu t_b$ we can write (see Fig. 2)

$$\Gamma_P(p)=-ig^2\int\frac{(dk)}{(2\pi)^4}\,t_a\gamma^\mu G(\tfrac{1}{2}P+p-k)\Gamma_P(p-k)$$
$$\times G(-\tfrac{1}{2}P+p-k)t_b\gamma^\nu\mathcal{G}^{ab}_{\mu\nu}(k)\,, \quad (22)$$

with a similar equation for the fermion-fermion case. It is our purpose to describe how $\Gamma_P(p)$ can exist (and be nontrivial) for *finite* eigenvalues $-P^2$ even though the individual factors $G(\pm\frac{1}{2}P+p-k)$ when taken alone are, in accordance with $m\to\infty$, equal to zero. This vanishing of individual factors requires all Bethe-Salpeter equations for composite operators to be *homogeneous*. Thus $m\to\infty$ has caused these equations to all become "of bound-state type" as is to be expected if the physical constituent thresholds recede to infinity.

The form of Eq. (22) provokes two questions:

(i) Upon writing in color indices $\Gamma_P^{ab}(p)=\Gamma^{(1)}\delta^{ab}+\sum_{A=1}^{8}\Gamma_A^{(8)a}(t_A)^b$ can one see any qualitative distinction between the singlet and octet vertices directly from the equation?

(ii) How reliable does one expect the ladder approximation for the kernel to be? Is there a degree of trustworthiness left in any information obtained from such an equation?

These two questions are intimately related. To see this we make explicit the dependence of $\mathcal{G}^{\mu\nu}$ on the color indices, i.e., $\mathcal{G}^{\mu\nu}_{ab}=\delta_{ab}\mathcal{G}^{\mu\nu}(k)$, and define the group-theoretic factors $\sum_{a=1}^{8}t^a t^a=c_0 1$; $\sum_{a=1}^{8}t^a t^b t^a=c_1 t^b$. One obtains

$$\Gamma_P^{(1)}(p)=-ig^2c_0\int\frac{(dk)}{(2\pi)^4}\gamma^\mu G(Q_+-k)\Gamma_P^{(1)}(p-k)$$
$$\times G(Q_--k)\gamma^\nu\mathcal{G}_{\mu\nu}(k) \quad (23a)$$

and

$$\Gamma_P^{(8)}(p)=-ig^2c_1\int\frac{(dk)}{(2\pi)^4}\gamma^\mu G(Q_+-k)\Gamma_P^{(8)}(p-k)$$
$$\times G(Q_--k)\gamma^\nu\mathcal{G}_{\mu\nu}(k)\,, \quad (23b)$$

where $Q_\pm=\pm\frac{1}{2}P+p$. The only visible difference between Eqs. (23a) and (23b) is the appearance of different effective coupling constants g^2c_0 and g^2c_1 in the singlet and octet channels, respectively, and hence we must conclude that, in the ladder approximation at least, octet solutions will appear if singlet solutions do and vice versa. But color-carrying representations were supposed to have had infinite energy. What has gone wrong? The culprit may be seen to be the ladder approximation itself. For imagine we could solve (23b) exactly for a set of eigenvalues $\{-P^2=M^2\}$ and let $\psi_M^{(8)}(X)$ be an effective wave function describing the center-of-mass motion of one of these "particles." Our guiding principle has been that a state can be long-lived only if it is an eigenstate of *all* dynamical mechanisms at work in the system. But since $\psi_M^{(8)}$ describes a *localized* color carrier (degree of localizability $\sim M^{-1}$) there *must* exist an additional, uncontrollable, minimal (for wavelengths $\gg M^{-1}$) coupling of $\psi_M^{(8)}(X)$ to the colored gauge field. This is so because it is the defining role of such fields to couple *universally* to color wherever it be found (and in whatever form). In the language of an effective Lagrangian one would have

$$\mathcal{L}_{\text{eff}}=-\bar\psi_M^{(8)}H^\mu\left(\frac{1}{i}\partial_\mu-T^aA_{\mu a}\right)\psi_M^{(8)}(X)+\cdots\,, \quad (24)$$

where H^μ are a set of matrices appropriate to the spin of $\psi_M^{(8)}(X)$ and one expects this Lagrangian to be accurate for wavelengths (of A_a^μ) $\gg M^{-1}$. But now the same scenario which forces the quark mass $m\to\infty$ causes an infinite infrared "renormalization" of the octet eigenvalues (calculated in the ladder approximation). To see this remember that our quark equation was as in Fig. 3. Calling ---○--- the propagator for $\psi_M^{(8)}$ one anticipates a similar self-energy equation (Fig. 4) whose solution will inevitably require a shift of the eigenvalues ${M_{(8)}}^2\to\infty$. The set of numbers $\{{M_8}^2\}$ play the same role in the $\psi_M^{(8)}$ self-energy equation as the bare quark mass m_0 in Eq. (2). Thus, once the infinite-mass-producing mechanism is established for the quarks themselves, it becomes universal in the sense that any localized color state suffers the same fate.

This is not so for the singlet eigenvalues $\{{M_{(1)}}^2\}$. Precisely because the corresponding states do not carry color, they decouple from the long-wavelength Yang-Mills degrees of freedom and escape being infinitely renormalized. In this sense the singlet eigenvalues $\{{M_{(1)}}^2\}$ computed in the ladder approximation can be expected to be roughly correct (apart from finite shifts which may come from "short-distance" improvements of the kernel, quark form factors, etc.) while the octet eigenvalues $\{{M_{(8)}}^2\}$ so computed are totally unreliable.

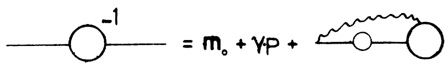

FIG. 3. Quark self-energy equation.

In graphical language this means that corrections to the kernel coming from graphs of the form shown in Fig. 5 are expected to be crucial in the octet channel as they form the basis for the self-energy process in Fig. 4. In the singlet channel the above arguments suggest (but of course do not prove) that as far as the gross features of the spectrum $\{M_{(1)}{}^2\}$ are concerned, the process schematized in Fig. 5 may be ignored. From now on we concentrate on the singlet equation (23a).

It is convenient to define a wave function $\Phi_P(p)$ by $\Phi_P(p) = G^{-1}(\frac{1}{2}P + p)\Gamma_P(p)G^{-1}(-\frac{1}{2}P + p)$ so that Eq. (23a) becomes

$$G^{-1}(\tfrac{1}{2}P + p)\Phi_P(p)G^{-1}(-\tfrac{1}{2}P + p) = -ic_0 g^2 \int \frac{(dk)}{(2\pi)^4}\gamma^\mu \Phi_P(p-k)\gamma^\nu \mathcal{G}_{\mu\nu}(k)\,. \quad (25)$$

Introducing the relative internal coordinate x^μ (conjugate to p^μ) by

$$\Phi_P(x) = \int \frac{(dp)}{(2\pi)^4} e^{ip\cdot x}\,\Phi_P(p) \quad (26)$$

we can write Eq. (25) in internal coordinate space as

$$[m + \gamma\cdot(\tfrac{1}{2}P + p_x)]\,\Phi_P(x)[m + \gamma\cdot(-\tfrac{1}{2}P + p_x)] = -ic_0 g^2 \mathcal{G}_{\mu\nu}(x)\gamma^\mu \Phi_P(x)\gamma^\nu\,, \quad (27)$$

where we have replaced $G^{-1}(p)$ by $m + \gamma\cdot p$ and $p_x^\mu = -i(\partial/\partial x_\mu)$. In terms of quark field operators it can be shown[10] that

$$\Phi_P(x) = \left\langle P \left| \left(\chi\left(\frac{x}{2}\right)\bar{\chi}\left(-\frac{x}{2}\right)\right)_+ \right| 0 \right\rangle. \quad (28)$$

Now we are searching for a meson wave equation that is implied by the specific quark-quenching mechanism proposed here. As written, Eq. (27) still retains explicit dependence on m (or what is the same thing on the infrared cutoff μ) and is, therefore, unsatisfactory. We now *define* acceptable hadronic states to be those that survive the limit $\mu \to 0$ for it is precisely these states for which the dependence on the quark mass drops out of the equation and whose scale of length will be set by Ω^{-1} and not by m^{-1}. Since

$$\mathcal{G}^{\mu\nu}(x) = \int \frac{(dk)}{(2\pi)^4} e^{ik\cdot x}\frac{1}{k^2+\mu^2}\left[\left(\frac{\Omega^2}{k^2+\mu^2}\right)^\lambda + 1\right] \times (g^{\mu\nu} + \text{gauge terms})\,, \quad (29)$$

the right-hand side of Eq. (27) also depends on μ, but only in an innocuous way as the limit $\mu \to 0$ there is smooth. This is not so on the left-hand side, of course. To understand physically what must happen in the limit $m = \infty$ recall the meaning[11] of the relative time variable (in the rest frame of P^μ) introduced in Eq. (28). At $t = -x^0/2$ the operator $\bar{\chi}$ acts on the vacuum and produces a "particle." *After the elapse of x^0 seconds* the operator χ acts to produce the companion "antiparticle"; the two objects proceed then to interact and form a bound state. The point we wish to stress is that x^0 measures the length of time one particle can exist by itself *before* its bound-state partner is even created. There is no logical objection to this possibility when the constituent mass is finite. However, as $m \to \infty$ it becomes impossible to create one of the particles alone, thus leading us to expect that in some sense $\Phi_P(x) = 0$ unless $x^0 = 0$ or, said another way, that the internal space becomes essentially three-dimensional.

To see how this can come about in detail, introduce for an arbitrary vector V^μ its projection orthogonal to p^μ by

$$\bar{V}^\mu = \left(g^{\mu\nu} - \frac{P^\mu P^\nu}{P^2}\right)V_\nu\,,\quad \bar{V}P = 0 \quad (30)$$

and consider the left-hand side of Eq. (27) written as

$$\left(m + \gamma\cdot P\,\frac{(p\cdot P)}{P^2} + \gamma\cdot\bar{p} + \tfrac{1}{2}\gamma\cdot P\right) \times \Phi_P(p)\left(m + \gamma\cdot P\,\frac{(p\cdot P)}{P^2} + \gamma\cdot\bar{p} - \tfrac{1}{2}\gamma\cdot P\right). \quad (31)$$

As the eigenvalues of the matrix $\gamma\cdot P/p^2$ are $\pm M^{-1}$ we see that if only it is required that $-p\cdot P \to Mm$ we have a good chance of eliminating all explicit m dependence. For values of p^μ not satisfying the relation, $\Phi_P(p) = 0$.[12] This hunch is corroborated by inspecting

$$\Phi_P(x) = \int \frac{(dp)}{(2\pi)^4}\,\Phi_P(p)\exp(i\bar{p}\cdot x)\exp(ix\cdot P)\,\frac{(p\cdot P)}{P^2}\,. \quad (32)$$

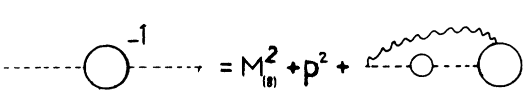

FIG. 4. Bound-state self-energy equation.

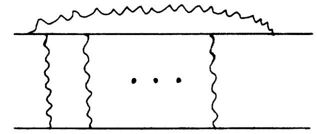

FIG. 5. Typical process contributing to the bound-state self-energy of Fig. 4.

If the integration is made to extend over only those p^μ for which $-p\cdot P \to Mm \to \infty$ ($-P^2 = M^2$ fixed) we must have $\Phi_P(x) = 0$ unless $x\cdot P = 0$. This is the covariant generalization of $x^0 = 0$ in the rest frame of P^μ. The matrix character of Eq. (31) unfortunately makes this passage to the limit a bit more delicate. Not only do we anticipate a support constraint on $\Phi_P(p)$ but also restrictions on the various spins and parities as well. If we expand

$$\Phi = \Phi^S 1 + \Phi^P \gamma_5 + \Phi^V_\mu \gamma^\mu + \Phi^A_\mu \gamma^\mu i\gamma_5 + \tfrac{1}{2}\Phi^T_{\mu\nu}\sigma^{\mu\nu}$$

it can be shown that in the limit $m \to \infty$ $\Phi^S = \Phi^A_\mu = \Phi^T_{\mu\nu} = 0$, the only surviving amplitudes being pseudoscalar and vector. The tedious details of the derivation will be published elsewhere.[13] Suffice it to say that the solutions to Eq. (27) take the form

$$\Phi^i_P(x) = \exp(-im\tau)\Phi^i_P(\bar{x}), \quad i = P, V \tag{33}$$

where $\bar{x}\cdot P = 0$ by definition and the covariant proper time τ is given by $\tau = -(x\cdot P)/M$. $\phi^P(\bar{x})$ and $\phi^V_\mu(\bar{x})$ satisfy

$$(\tfrac{1}{4}M^2 + \bar{p}^2\ )\phi^P(\bar{x}) = \frac{c_0 g^2}{i}\ \mathcal{G}_{\alpha\beta}(\bar{x})[g^{\alpha\beta}\phi^P(\bar{x})], \tag{34a}$$

$$(\tfrac{1}{4}M^2 + \bar{p}^2)\phi^V_\mu(\bar{x}) = \frac{c_0 g^2}{i}\ \mathcal{G}_{\alpha\beta}(\bar{x})[g^{\alpha\beta}\phi^V_\mu(\bar{x}) - g^\alpha_\mu \phi^{V\,\beta}(\bar{x}) - g^\beta_\mu \phi^{V\,\alpha}(\bar{x})] \tag{34b}$$

supplemented by $P^\mu \phi^V_\mu = \bar{p}^\mu \phi^V_\mu = 0$. It is easy to show that if we reassemble ϕ^P and ϕ^V_μ into the matrix $\phi = \phi^P\gamma_5 + \phi^V_\mu\gamma^\mu$ that (34a) and (34b) are derived as appropriate projections of the single matrix equation

$$(\tfrac{1}{2}M + \gamma\cdot\bar{p})\phi(\bar{x})(\tfrac{1}{2}M + \gamma\cdot\bar{p}) = \frac{c_0 g^2}{i}\ \mathcal{G}_{\alpha\beta}(\bar{x})\gamma^\alpha\phi(\bar{x})\gamma^\beta\ . \tag{35}$$

Since $\bar{p}\cdot x = \bar{p}\cdot\bar{x} = p\cdot\bar{x}$ and $\bar{\gamma}\cdot P = 0$ one has

$$\frac{1}{i}\bar{\gamma}^\mu \frac{\partial}{\partial x^\mu}\phi_P(x) = \frac{1}{i}\bar{\gamma}^\mu \frac{\partial}{\partial \bar{x}^\mu}\phi_P(\bar{x})\ . \tag{36}$$

Equation (35) is our proposed covariant wave equation for the mesons.

It is apart from our main purpose in this paper to begin constructing solutions to Eqs. (34a) and (34b). This will be done in a sequel to the present article.[13] We feel, however, that several attractive features warrant specific mention:

(i) The meson wave equation is essentially three-dimensional. The limit $m \to \infty$ has caused what was an internal *four*-dimensional Minkowski space with indefinite metric to shrink to a *three*-dimensional Euclidean space with positive-definite metric. The vector space $\{x^\mu\}$ is replaced by a hypersurface in that space characterized by $x\cdot P = 0$. That the internal configuration space has become three-dimensional will be important for eliminating the "timelike ghost" problems usually associated with Bethe-Salpeter[10] equations.

(ii) The factors $\gamma\cdot\bar{p} + \frac{1}{2}M$ may be interpreted as defining what is meant *kinematically* by a quark inside a hadron. The eigenvalue M refers to the state in which the quark is propagating. This "inverse propagator" has, of course, no meaning except when applied to a color-singlet meson wave function. Nevertheless, it is instructive to ask whether the factor $\gamma\cdot\bar{p} + \frac{1}{2}M$ can have any zeros, as the zeros of inverse propagators specify energy and momentum relationships for physical asymptotic states. For definiteness ask whether

$$\tfrac{1}{2}M + \gamma\cdot\bar{p} = 0 \tag{37}$$

has any solutions. If so, then

$$\tfrac{1}{4}M^2 + p^2 - \frac{(p\cdot P)^2}{P^2} = 0\ . \tag{38}$$

By assumption $-P^2 = M^2 > 0$ as lightlike solutions would not be relevant for hadrons; but the combination $p^2 - (p\cdot P)^2/P^2 = \bar{p}^2$ is positive definite. Thus there can be no solution to Eq. (38) for real mementa. Physically this means that our quark propagators describe excitations which can never be forced onto their mass shell no matter how much momentum one expends although under certain circumstances they may act as if they "had a mass equal to $\frac{1}{2}$ of the meson mass." This fact is consistent with $m \to \infty$.

(iii) The absence[14] of constituent thresholds promises a purely discrete spectrum of eigenvalues $\{-P^2\}$ which should be welcome for identification with Regge (?) families of states.

(iv) Equations (34a) and (34b) reduce to equations of Schrödinger type in the rest frame of P^μ

$$\left[-\nabla^2 + \left(-\frac{c_0 g^2}{i} g_{\alpha\beta}\mathcal{G}^{\alpha\beta}(\vec{x})\right)\right]\phi^P(\vec{x}) = -\tfrac{1}{4}M^2\phi^P(\vec{x})\ . \tag{39}$$

This essentially nonrelativistic-looking dynamics taken together with only ϕ^V_μ and ϕ^P being nonzero dovetails nicely with the prediction of the naive quark model that the expected families of states are to be $\{J^{PC} = 0^{-+}, 1^{+-}, 2^{-+}, \ldots\}$ and $\{J^{PC} = 1^{--}, 2^{++}, 3^{--}, \ldots\}$. Note also that the eigenvalues of this equation are the *squares* of the meson masses.

COMMENTS AND PROBLEMS

Before we can assess the implications of this model for deep-inelastic scattering, form factors, e^+e^- annihilation into hadrons, etc., it is necessary to study the solutions to Eq. (35) in detail. This will presumably also shed light on the difficult question of whether there are long-range forces, due to the underlying mass-zero spectrum of the gauge fields, capable of acting between *colorless* hadrons. We emphasize that this question cannot be settled by group-theoretic arguments alone, but rather only by detailed examination of the dynamics, i.e., Eq. (35). Re-expressed in the language of "unitarity" the question devolves on whether or not the space of color-singlet states is complete. At this stage we can only hope that it is.

A related question concerns possible ways of determining the function $\lambda(g^2)$. At our present stage of development we have been content to regard λ as a phenomenological parameter. But being that parameter which fixes the strength of the singularity in the gauge field two-point functions at $k^2=0$ it must be determinable by self-consistently solving the Schwinger-Dyson equation for the gluon propagator. As any such program necessarily must come to grips with the fierce self-coupling of the pure mass-zero Yang-Mills system, this poses quite a technical challenge. It seems unlikely, however, that $\lambda(g^2)$ considered as an analytic function of g^2 would have a behavior as to make $\mathrm{Re}\lambda(g^2)>\frac{1}{2}$ impossible. Thus it is quite natural to assume that for *some* g^2 we can arrange $\lambda(g^2)>\frac{1}{2}$. Whether that g^2 is "small" or "large" is a question only to be answered after solving the gluon self-energy integral equations.

Equation (35) bears much in common with the phenomenological equation employed by Feynman, Kislinger, and Ravndal[15] to analyze low-energy baryon and meson data. We see now that the *ad hoc* "throwing away of the timelike excitations" advocated by these authors is intertwined with the very problem of quark confinement itself. This suggests that—taken from a purely phenomenological point of view—one should investigate classes of equations where the confined-quark differential operators are given by $\gamma\cdot\bar{p}_{\bar{x}}$ instead of $\gamma\cdot p_x$ and the meson and baryon wave functions become $\phi(\bar{x}_{12})$ and $\psi(\bar{x}_{12},\bar{x}_{23},\bar{x}_{31})$, respectively. Such a program will occupy us for some time to come.

ACKNOWLEDGMENTS

The author gratefully acknowledges numerous useful conversations with Charles Nash.

[1]G. 't Hooft, Nucl. Phys. B35, 167 (1971).

[2]D. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973); H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973); D. Gross and F. Wilczek, *ibid.* 30, 1343 (1973).

[3]In the case of the Coulomb field, it is the timelike photons, those that do not carry any intrinsic angular momentum, that are involved.

[4]J. Kondo, Adv. Solid State Phys. 23, 183 (1969). The Kondo mechanism has received attention in a Yang-Mills context from other authors. See P. W. Anderson, Cambridge University Report No. TCM/7/1975 (unpublished) and K. Wilson, Cornell Report No. CLNS 271, 1974 (unpublished). There is a considerable difference in detail between what these authors suggest and our point of view. We are merely using the physics of the Kondo mechanism to suggest approximations to integral equations. The above authors want to use solid-state techniques directly to define and solve the four-dimensional field theory.

[5]A. A. Slavnov, Teor. Mat. Fiz. 10, 153 (1972) [Theor. Math. Phys. 10, 99 (1972)].

[6]Since we will discover that the renormalized quark mass $m\to\infty$, the usual renormalization-group type of analysis will have to be reworked.

[7]Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); M. Baker and S. L. Glashow, *ibid.* 128, 2462 (1962).

[8]M. Baker, K. Johnson, and R. Willey, Phys. Rev. 136, B1111 (1964).

[9]It must be remarked that the solid-state Kondo effect more closely resembles the case $\lambda<\frac{1}{2}$. The long-wavelength spin waves are real, physical excitations, the effects of which can be measured in the laboratory. The case $\lambda>\frac{1}{2}$ represents a generalization where $m=\infty$ and we expect any direct effect of the mass-zero gluons to be "hidden."

[10]H. Bethe and E. E. Salpeter, Phys. Rev. 84, 1232 (1951); see also N. Nakanishi, Prog. Theor. Phys. Suppl. 43, 1 (1969).

[11]S. Mandelstam, Proc. R. Soc. (London) A233, 248 (1955); J. Schwinger, lecture notes Harvard, 1968 (unpublished).

[12]We are assuming, of course, that the right-hand side of Eq. (27) remains well defined in the limit $\mu\to 0$. The uncancelled large factors m must then be compensated by the vanishing of Φ itself. It is conceivable that the diverging m's on the left could be compensated by a divergence of the *integral* on the right. We reject this possibility (for the moment) as being "less natural" in that it does not correspond to our motivating observation that $\Phi_P(x)$ should be zero unless $x^0=0$.

[13]R. L. Stuller (unpublished).

[14]The properties also obtain in a two-dimensional model studied by G. 't Hooft, CERN Report No. TH-1820, 1974 (unpublished).

[15]R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D 3, 2706 (1971).