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Theory of fermion exchange in massive quantum electrodynamics at high energy. IV. Tenth-order perturbation theory*

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The detailed calculations of the preceding two papers are continued to tenth order. There are 41 Feynman diagrams which must be considered. These diagrams fall into 3 categories: (1) fifteen which contribute to the leading real part, (2) six which individually contribute to the leading imaginary part that cancel in pairs in the sum, and (3) twenty which contribute to the leading imaginary part which do not cancel identically in the sum.

I. INTRODUCTION

In the preceding two papers[1,2] we have presented the details of our calculation of the asymptotic behavior as $s\to\infty$ for t fixed of fermion exchange in massive quantum electrodynamics in sixth- and eighth-order perturbation theory. In this paper we continue this study by computing tenth-order perturbation theory.

In the sixth- and eighth-order calculations our computations have been very explicit and we have laboriously written out many long tedious formulas. Unfortunately, since in tenth-order there are 41 Feynman diagrams which contribute to the leading real or imaginary part, such a detailed algebraic calculation would result in a paper of prohibitive length. However, in these previous papers we were able to achieve some simplification by introducing the momentum-flow diagram. In this paper we will enlarge upon that formalism to such a degree that we will be able to dispense entirely with the need to write out any formulas at all except the final answers. Accordingly, most of this paper will consist of illustrations. The use of these illustrations will be explained in Sec. II.

In eighth order there were seven Feynman diagrams that contributed to the real part of the amplitude and an additional five that were needed to compute the leading imaginary part. In tenth order there are 15 Feynman diagrams which contribute to the leading real part (Fig. 1) and there are an additional 26 Feynman diagrams which contribute to the leading imaginary part. However, these 26 are naturally divided into two subclasses since six of them cancel in pairs (Fig. 2). The remaining 20 diagrams (Fig. 3) are analogous to the five diagrams of eighth order that contribute only to the imaginary part.

Again, as in eighth order, there are many more tenth-order diagrams than those explicitly shown here. We again assure the reader that there are no other diagrams which contribute to the real or imaginary leading order. In particular, it may be verified that the diagram of Fig. 4 does not contribute.

The conventions of the three previous papers[1,2,3] will be used throughout. In particular, when discussing an individual Feynman diagram the integrations over transverse momentum are cut off at $\vec{k}_{\perp\max}$. The limit $\vec{k}_{\perp\max}\to\infty$ is taken only after all diagrams are summed together.

To write out the final answer in compact form we use the notation

$$\alpha(2\vec{r}_\perp)=g^2(2\vec{\not r}_\perp+m)\times\int\frac{d^2\vec{k}_\perp}{(2\pi)^3}\,\frac{2\vec{\not r}_\perp+\vec{\not k}_\perp-m}{[(2\vec{r}_\perp+\vec{k}_\perp)^2+m^2](\vec{k}_\perp{}^2+\lambda^2)}\tag{1.1}$$

and

$$\gamma(\vec{k}_\perp)=4g^2(2\pi)^{-3}(\vec{k}_\perp{}^2+\lambda^2)^{-1}\tag{1.2}$$

and the definition of the convolution operation,

$$f_1(\vec{k}_\perp)*f_2(\vec{k}_\perp)=\int d^2\vec{k}'_\perp f_1(\vec{k}'_\perp)f_2(\vec{k}_\perp-\vec{k}'_\perp).\tag{1.3}$$

Then, defining $\overline{\mathfrak{M}}^{(n)}(s,r_1)$ for pair annihilation by

$$\overline{\mathfrak{M}}^{(n)}_{\mu\nu}(s,r_1)=\bar{v}(r_3+r_1)\gamma_\mu\overline{\mathfrak{M}}^{(n)}(s,r_1)\gamma_\nu u(r_2-r_1)\tag{1.4a}$$

and $\tilde{\mathfrak{M}}^{(n)}(s,r_1)$ for backward Compton scattering by

$$\tilde{\mathfrak{M}}^{(n)}_{\mu\nu}(s,r_1)=\bar{u}(r_3-r_1)\gamma_\mu\tilde{\mathfrak{M}}^{(n)}(s,r_1)\gamma_\nu u(r_2-r_1),\tag{1.4b}$$

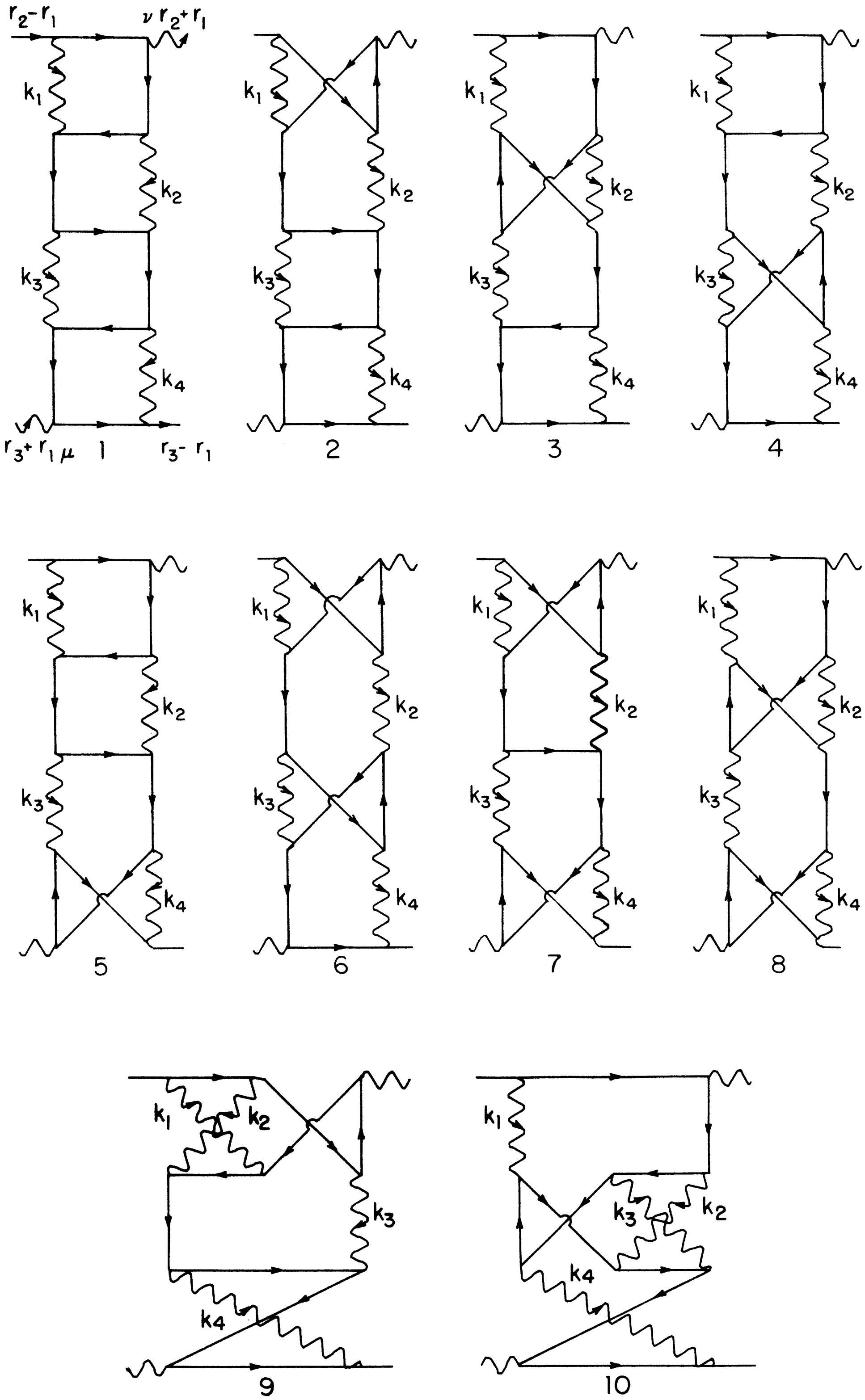


FIG. 1. *(Continued on following page)*

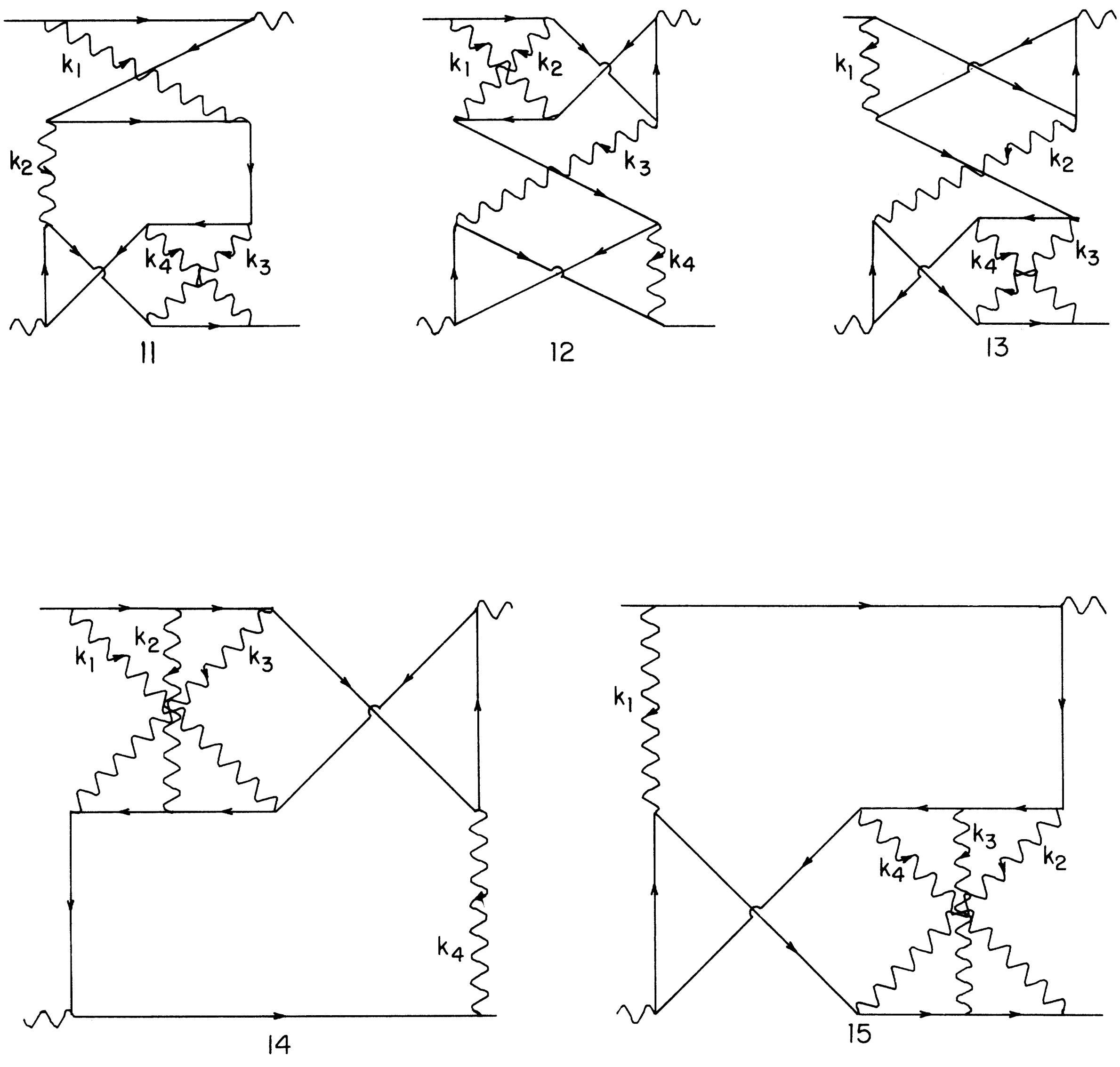


FIG. 1. The 15 Feynman diagrams which contribute to the leading order for $s\to\infty$ of the real part of the tenth-order amplitude for Compton scattering near the backward direction. In all these diagrams the external lines have the momenta indicated on diagram 1. Only the photon momenta are indicated. The other momenta follow from momentum conservation.

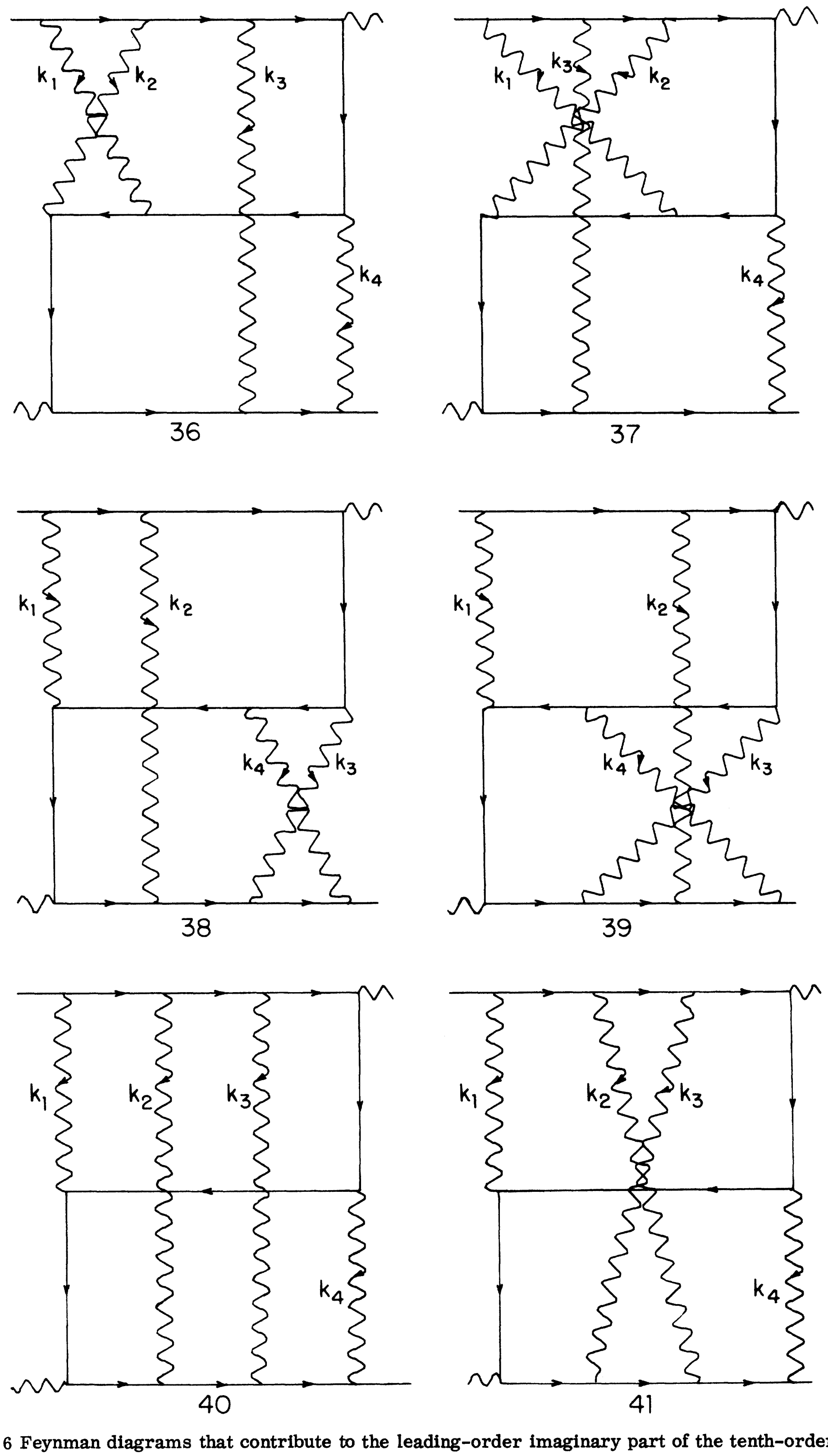


FIG. 2. The 6 Feynman diagrams that contribute to the leading-order imaginary part of the tenth-order backward Compton amplitude which cancel (to leading order) in pairs. The pairs are 36 and 37, 38 and 39, and 40 and 41. The external lines have the momenta indicated on diagram 1.

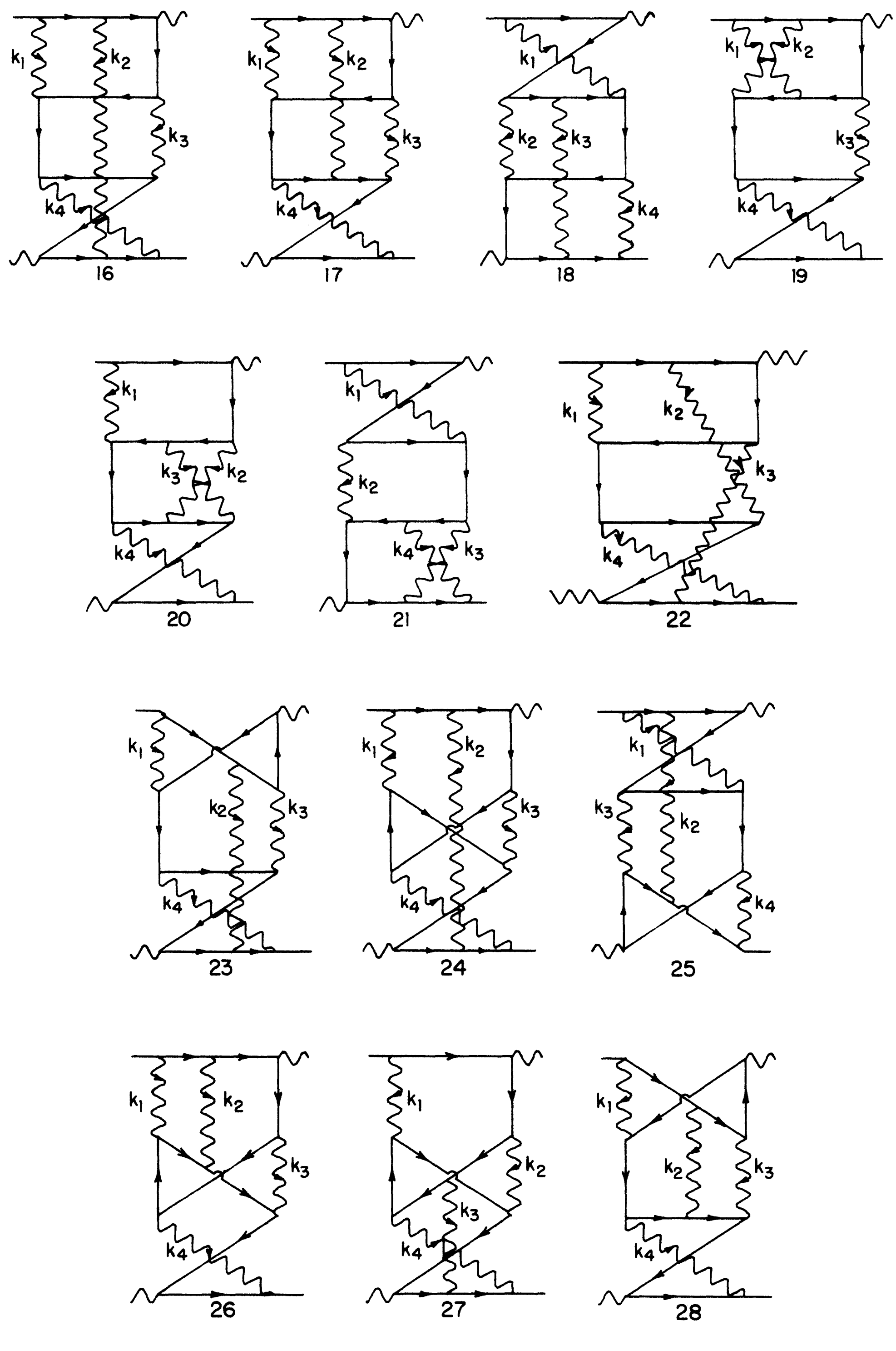


FIG. 3. *(Continued on following page)*

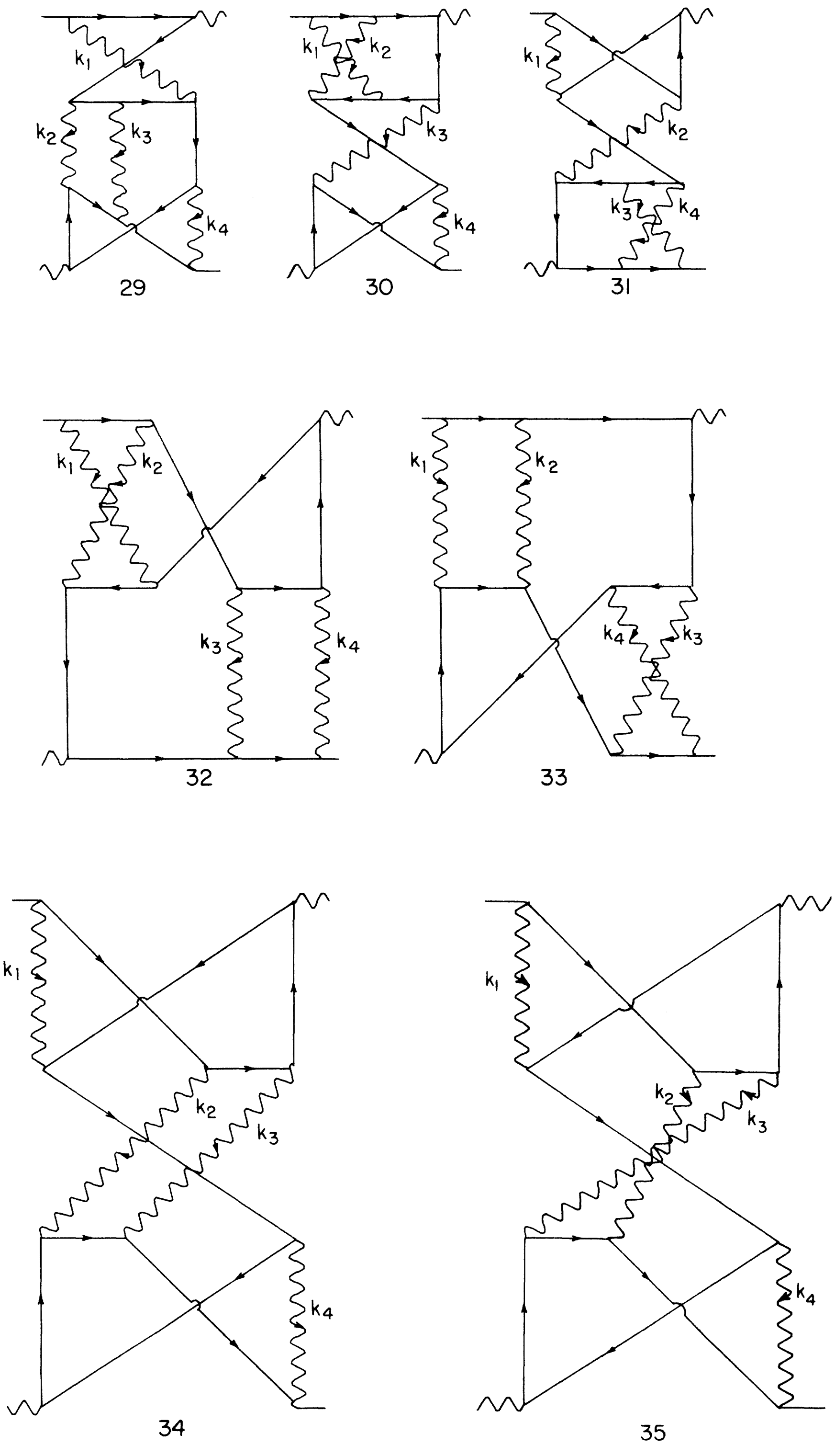


FIG. 3. The remaining 20 Feynman diagrams which contribute to the leading imaginary part of the tenth-order backward Compton amplitudes. The external lines have the momenta indicated on diagram 1.

we have

$$
\begin{aligned}
\mathfrak{M}^{(4)}(s,r_1) \doteq {} & \frac{1}{4!}(\ln^4 s - 4\pi i \ln^3 s)\frac{g^2}{2\not{\vec{r}}_\perp + m}\,\alpha^4(2\vec{r}_\perp) \\
& + \frac{1}{3!}\pi i g^2 \ln^3 s\Bigg\{-\tfrac{1}{2}\gamma(2\vec{r}_\perp) * \frac{\alpha^3(2\vec{r}_\perp)}{2\not{\vec{r}}_\perp + m} + 2\alpha(2\vec{r}_\perp)\tfrac{1}{2}\gamma(2\vec{r}_\perp) * \frac{\alpha^2(2\vec{r}_\perp)}{2\not{\vec{r}}_\perp + m} - 3\alpha^2(2\vec{r}_\perp)\tfrac{1}{4}\gamma(2\vec{r}_\perp) * \frac{\alpha(2\vec{r}_\perp)}{2\not{\vec{r}}_\perp + m} \\
& - 2(2\not{\vec{r}}_\perp + m)\left[\tfrac{1}{4}\gamma(2\vec{r}_\perp) * \frac{\alpha(2\vec{r}_\perp)}{2\not{\vec{r}}_\perp + m}\right]^2 \\
& - 2\int d^2\vec{k}_{2\perp}\int d^2\vec{k}_{3\perp}\,\tfrac{1}{4}\gamma(\vec{k}_{2\perp})\tfrac{1}{4}\gamma(\vec{k}_{3\perp})\,\alpha(\vec{k}_{3\perp} + 2\vec{r}_\perp)\frac{\not{\vec{k}}_{2\perp} + \not{\vec{k}}_{3\perp} + 2\not{\vec{r}}_\perp - m}{(\vec{k}_{2\perp} + \vec{k}_{3\perp} + 2\vec{r}_\perp)^2 + m^2}\,\alpha(\vec{k}_{2\perp} + 2\vec{r}_\perp)\Bigg\}
\end{aligned}
\tag{1.5a}
$$

and

$$
\begin{aligned}
\overline{\mathfrak{M}}^{(4)}(s,r_1) \doteq {} & \frac{1}{4!}\ln^4 s\,\frac{g^2}{2\not{\vec{r}}_\perp + m}\,\alpha^4(2\vec{r}_\perp) \\
& - \frac{1}{3!}\pi i g^2 \ln^3 s\Bigg\{-\tfrac{1}{2}\gamma(2\vec{r}_\perp) * \frac{\alpha^3(2\vec{r}_\perp)}{2\not{\vec{r}}_\perp + m} + 2\alpha(2\vec{r}_\perp)\tfrac{1}{2}\gamma(2\vec{r}_\perp) * \frac{\alpha^2(2\vec{r}_\perp)}{2\not{\vec{r}}_\perp + m} - 3\alpha^2(2\vec{r}_\perp)\tfrac{1}{4}\gamma(2\vec{r}_\perp) * \frac{\alpha(2\vec{r}_\perp)}{2\not{\vec{r}}_\perp + m} \\
& - 2(2\not{\vec{r}}_\perp + m)\left[\tfrac{1}{4}\gamma(2\vec{r}_\perp) * \frac{\alpha(2\vec{r}_\perp)}{2\not{\vec{r}}_\perp + m}\right]^2 \\
& - 2\int d^2\vec{k}_{2\perp}\int d^2\vec{k}_{3\perp}\,\tfrac{1}{4}\gamma(\vec{k}_{2\perp})\tfrac{1}{4}\gamma(\vec{k}_{3\perp})\,\alpha(\vec{k}_{3\perp} + 2\vec{r}_\perp)\frac{\not{\vec{k}}_{2\perp} + \not{\vec{k}}_{3\perp} + 2\not{\vec{r}}_\perp - m}{(\vec{k}_{2\perp} + \vec{k}_{3\perp} + 2\vec{r}_\perp)^2 + m^2}\,\alpha(\vec{k}_{2\perp} + 2\vec{r}_\perp)\Bigg\}.
\end{aligned}
\tag{1.5b}
$$

It is unrealistic to expect that the casual reader will read the whole of this paper. Therefore, we recommend that on first reading one read only Secs. II and V, leaving Secs. III, IV, and VI for reference. Section II is necessary for further extensions of this work to higher order and Sec. V is necessary to illustrate the cancellation in pairs, which first occurs in tenth order. The remainder of the paper is mostly a verification that all the cancellations expected do, in fact, occur. If it is accepted that these cancellations do in fact occur, then the labor of calculation decreases tremendously. This simplification will be exploited in a subsequent paper[4] on higher orders of perturbation theory.

II. DIAGRAMMATIC CALCULATIONS

In this section we expand the device of the momentum-flow diagram as used in the two preceding papers[1,2] to the point that almost all of the computation will be presented graphically in the illustrations. We will discuss the new notational devices in separate subsections.

There is, of course, neither any new physics nor any new mathematics being presented here. We are merely introducing a streamlining of the procedure previously employed. On the other hand, this notation and streamlining will prove to be essential in our treatment of higher orders of perturbation theory to be presented in a following paper.[4]

In writing this section we have assumed a familiarity with the device of the momentum-flow diagram presented in the previous paper on sixth-order perturbation theory.[1] In addition, we will cease to specify the momentum loops used to de-

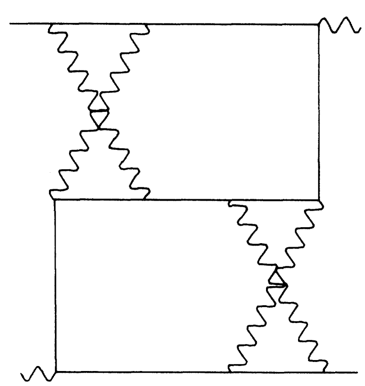


FIG. 4. A Feynman diagram which does not contribute to the leading-order imaginary part in tenth order.

termine which poles to close on and will merely indicate the poles themselves. If in some loop there are more than one pole to close on, this will be indicated.

A. Choice of momenta

In the Feynman diagrams of Figs. 1, 2, and 3 the internal momenta are specified by choosing the momenta of the internal photons as independent variables. This is the same choice of momenta that was made in the previous papers and in those papers these same coordinates were employed for each separate momentum-flow diagram. We then had to specify the region of integration in k space and for the various momentum-flow diagrams there were many different regions, although most of the time the regions were tetrahedrons (of some appropriate dimension) in some orientation.

Some degree of standardization and simplification is obtained if for the plus and minus components of momenta we use a different coordinate specification from what we use for the transverse momenta. We will leave the transverse momenta as specified by Figs. 1, 2, and 3 but, for each momentum-flow diagram, will allow the plus and minus components of momenta (denoted by q instead of k) to be chosen differently for each diagram. We choose these new coordinates q_{+j} and q_{-j} so that q_{-j} always obeys the restriction

$$q_{-j} \geq 0. \tag{2.1}$$

Since q_{+j} are always integrated out we will only specify q_{-j} on the momentum-flow diagram and will usually omit the subscript $-$.

Usually there is only one choice of q_j such that (2.1) when combined with some inequality involving 2ω will guarantee that all the momenta p_{-k} of the diagram will be positive. For example, in Figs. 7, 8, 9, and 10 it is easily seen that the choice of q_j given is (up to permutations) the only possible one. Occasionally, such as in Fig. 31(j), an additional inequality such as

$$q_1 + q_3 - q_2 \geq 0 \tag{2.2}$$

is needed. Such a momentum region is called nontetrahedral (for obvious reasons). For these there are several ways to choose coordinates q_j.

B. Transverse diagrams

The result of expanding any Feynman diagram of $[2(n+1)]$th order for $s\to\infty$ and t fixed is, to leading order, always an expression of the form

$$\frac{1}{n!}\left[m_1 \ln^n s + m_2(\ln^n s - n\pi i \ln^{n-1} s)\right] f(t), \tag{2.3}$$

where $f(t)$ is a $2n$-dimensional integral over the transverse momenta $\vec{k}_{\perp j}$ of the original Feynman diagram and m_1 and m_2 are integers. Indeed, as is seen in fourth, sixth, and eighth order for every momentum-flow diagram associated with a given Feynman diagram, (2.3) holds with the same $f(t)$ for each momentum-flow diagram.

Now it might be thought that the integrand of this integral could be rather arbitrary. In fact, however, inspection of the fourth-, sixth-, and eighth-order results reveals that this integrand is very closely related to the original Feynman diagram and can essentially be written down by inspection without any calculation at all. This relation is illustrated in Fig. 5 where to each contributing Feynman diagram of fourth, sixth, and eighth order we assign a corresponding "transverse diagram" which gives the factor $f(t)$. For the moment we consider only the denominator factors of $f(t)$. The numerator is discussed in Sec. II D.

(i) For each internal photon line in the transverse diagram (indicated by a dashed line) we have the factor

$$\frac{1}{\vec{k}_{\perp j}^{\,2} + \lambda^2}, \tag{2.4}$$

where $\vec{k}_{\perp j}$ is the transverse momentum of the photon.

(ii) For each internal electron line of the transverse diagram (indicated by a solid line) we have the factor

$$\frac{1}{\vec{p}_{\perp j}^{\,2} + m^2}, \tag{2.5}$$

where $\vec{p}_{\perp j}$ is the transverse momentum of the electron.

(iii) There is a factor of $-(i/2)(2\pi)^{-3}$ for each independent variable $\vec{k}_{\perp j}$.

It is not difficult to see that the same process of contraction that leads to the transverse diagrams of Fig. 5 in fourth-, sixth-, and eighth-order perturbation theory will reduce the tenth-order Feynman diagrams of Figs. 1, 2, and 3 to the corresponding transverse diagrams of Fig. 6.

Several remarks are in order.

(1) There is no guess work involved in passing from the Feynman diagram to the transverse diagram. All the reader needs to do is to keep track of all the $\vec{k}_\perp$-dependent factors. This was done in the two previous papers but, since it saves a lot of space by omitting them, we will leave these factors out from now on.

(2) There can be more than one Feynman diagram that leads to the same transverse diagram. For example, in Fig. 6 Feynman diagrams 34 and 35 lead to the same transverse diagram.

(3) Many of the transverse diagrams involve a photon line that leaves one vertex, turns around,

and comes back to the same vertex making a loop that we will call an "ear." Each of these ears contributes a factor

$$\int \frac{d^2\vec{k}_\perp}{(2\pi)^3} \frac{1}{\vec{k}_\perp{}^2 + \lambda^2} . \tag{2.6}$$

This, of course, diverges when the cutoff is removed. These divergences will be canceled by other divergences caused by factors in the numerator of other transverse diagrams.

C. Momentum-flow diagrams

Since the function $f(t)$ is specified by the transverse diagram (and the numerator reduction) it remains only to have a formalism to compute the pair of integers (m_1, m_2) of (2.3) for each momentum-flow diagram.

In the procedure followed previously, after we integrate over q_{+j} we are left with an integral over q_{-j} where each line contributes a factor to the

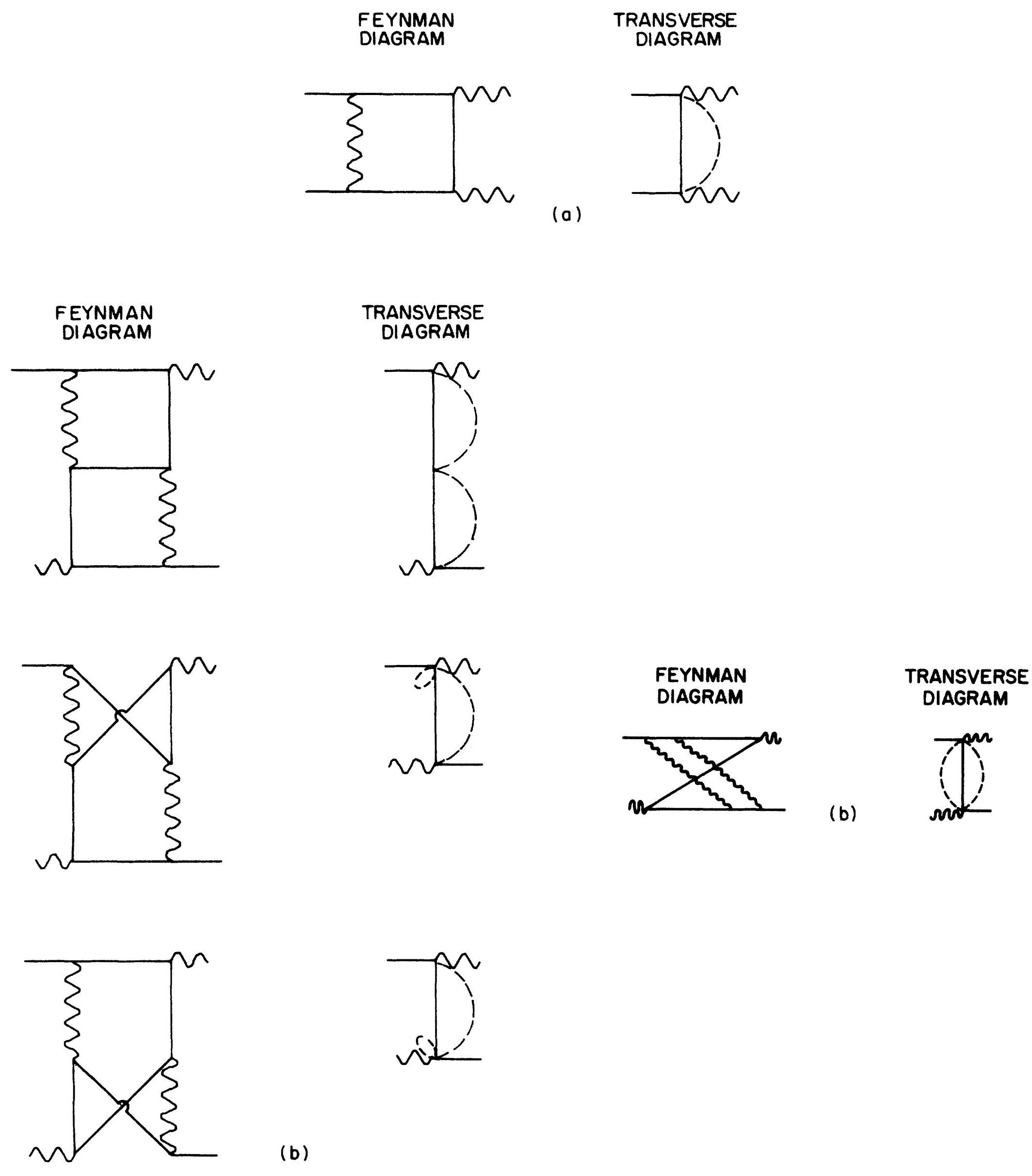


FIG. 5. *(Continued on following page)*

integrand. The factors are the following:

(i) If the propagator of the line is closed on, the factor is

$$\frac{1}{|p_-|}\,. \tag{2.7}$$

(Here p is the momentum of the line under consideration. A momentum p is in general a linear combination of the q_j.)

(ii) If the line contains a momentum $p_+ = O(2\omega)$, then the factor is

$$\frac{1}{2\omega p_- - \vec{p}_\perp^{\,2} - m^2 + i\epsilon} \text{ or } \frac{1}{-2\omega p_- - \vec{p}_\perp^{\,2} - m^2 + i\epsilon}\,, \tag{2.8}$$

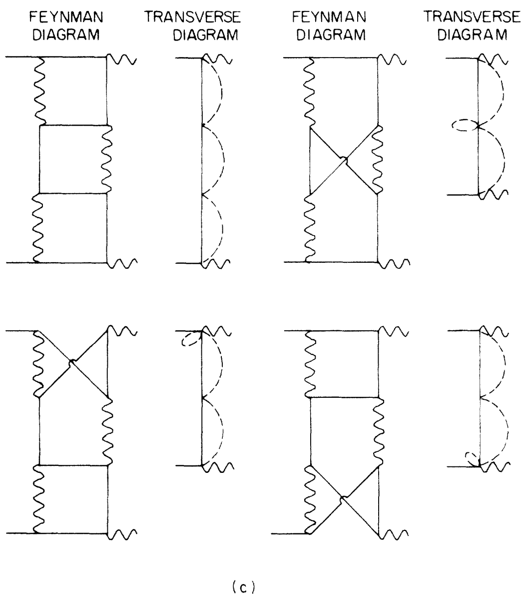
depending on the sign of p_+.

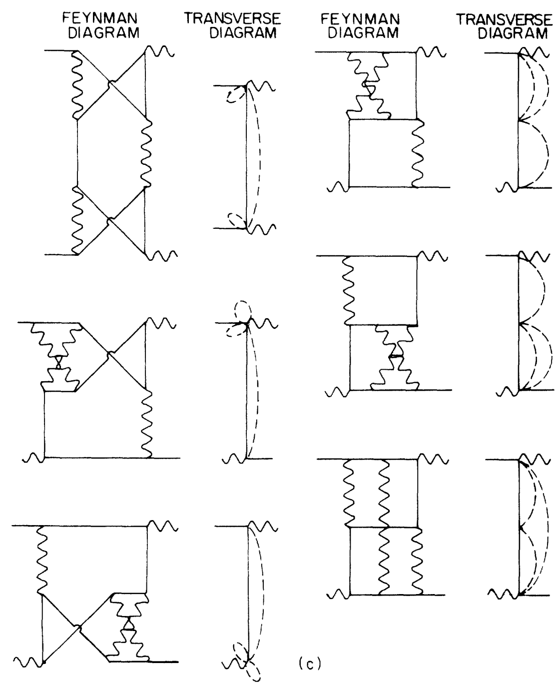
(iii) All other propagators contribute

$$\frac{1}{p_-p_+ - \vec{p}_\perp^{\,2} - m^2 + i\epsilon} \quad \text{(for electrons)} \tag{2.9a}$$

or

$$\frac{1}{p_-p_+ - \vec{p}_\perp^{\,2} - \lambda^2 + i\epsilon} \quad \text{(for photons)}, \tag{2.9b}$$





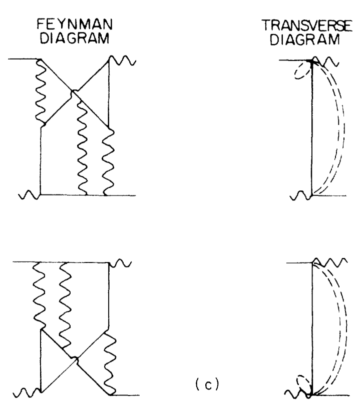


FIG. 5. The Feynman diagrams and their related transverse diagrams for fourth-, sixth-, and eighth-order perturbation theory. We draw either the backward Compton or the annihilation channel as is convenient. (a) is fourth order, (b) is sixth order, and (c) is eighth order.

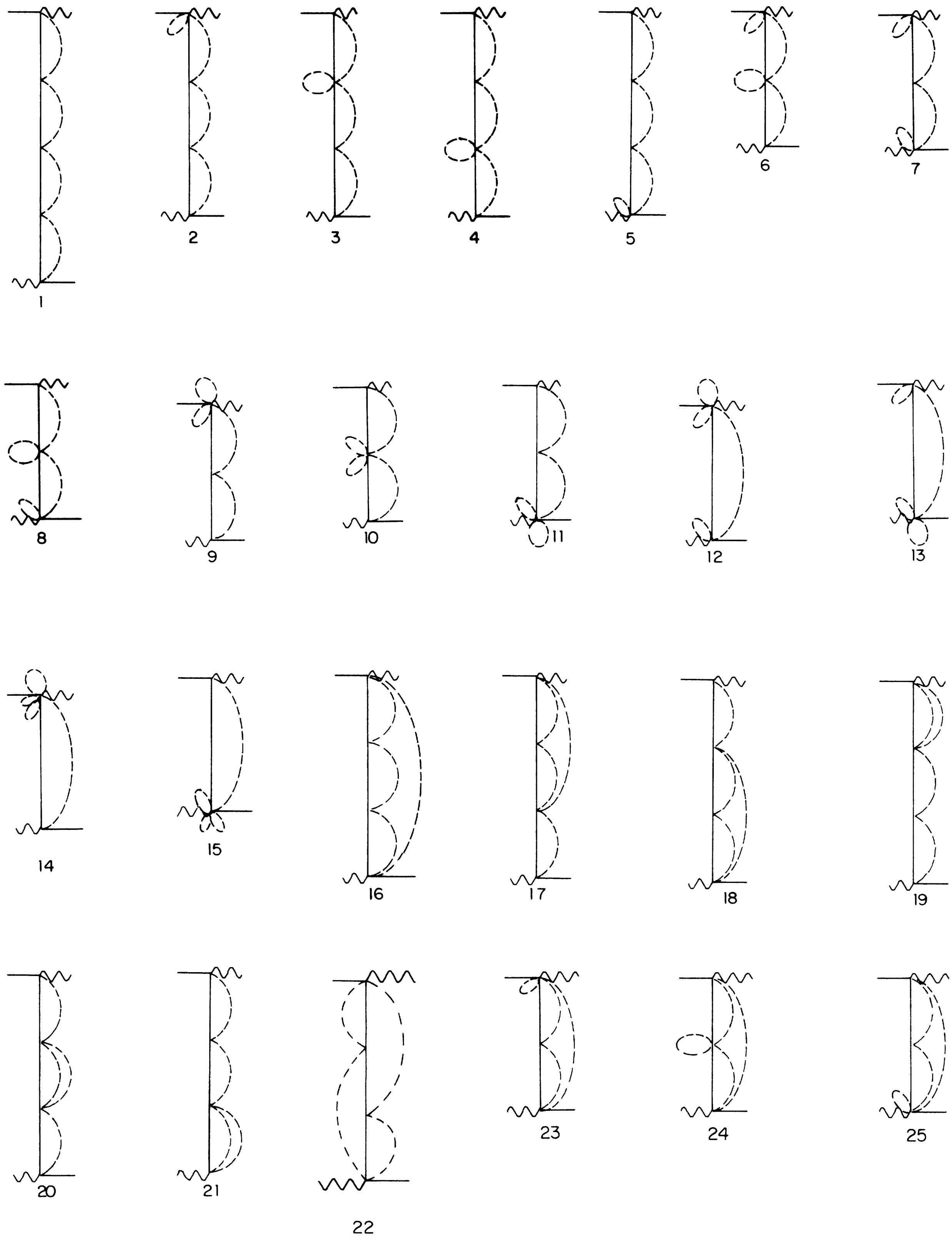


FIG. 6. *(Continued on following page)*

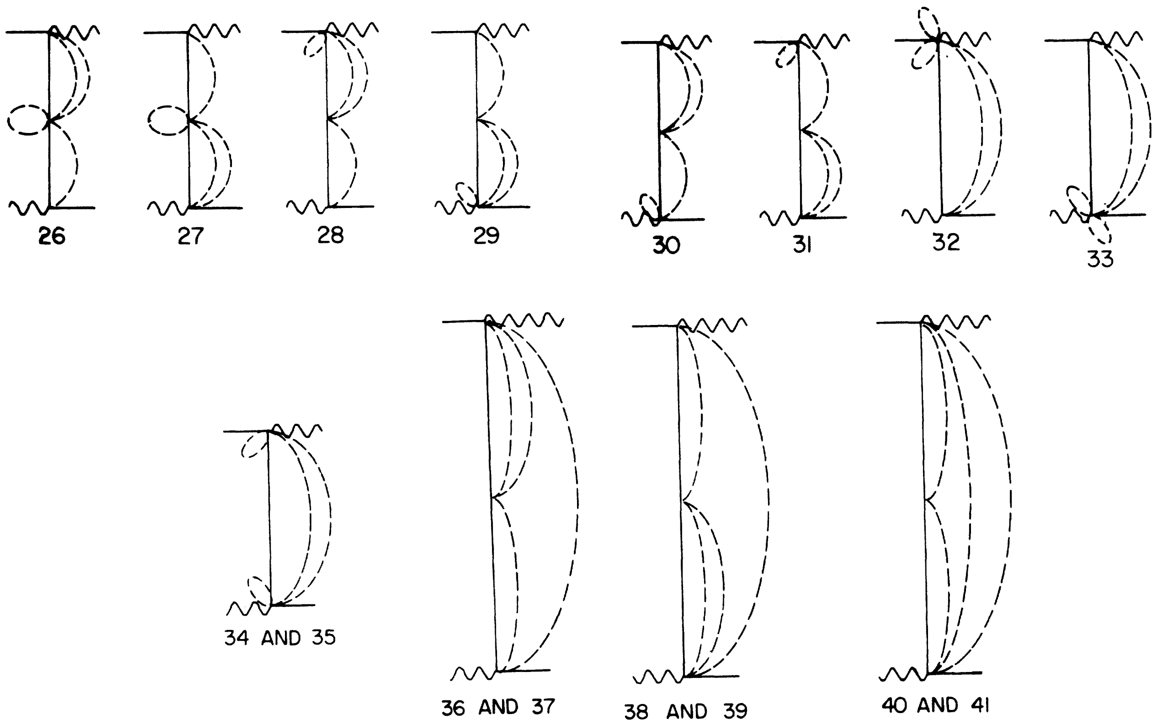


FIG. 6. The transverse diagrams that correspond to the Feynman diagrams of Figs. 1, 2, and 3.

where p_+ must be expressed in terms of the p_{+k} of the poles closed on.

For our expanded momentum-flow diagram we will indicate in a box by each line the factor contributed by that line. We use the following notations corresponding to the 3 types of lines discussed above:

(i) For each propagator closed on,

$$\frac{1}{p_-} \tag{2.10}$$

(we do not need an absolute-value sign since our coordinates are always chosen so that $p_- > 0$). If $p_- = O(2\omega)$ we suppress the 2ω and write the factor as 1.

(ii) For each line containing $p_+ = O(2\omega)$,

$$\frac{1}{p_-}\,. \tag{2.11}$$

(The sign of p_+ will be taken care of later.)

(iii) For all other propagators we proceed as follows: First of all, express p_+ of the line under consideration as

$$p_+ = \sum a_k p_{+k}\,, \tag{2.12}$$

where p_{+k} refers to the poles closed upon, and satisfy

$$p_{+k} = \frac{\vec{p}_{\perp k}^{\,2} + m_k^2}{p_{-k}} \tag{2.13}$$

(where m_k is m if k refers to a fermion and is λ if k refers to a photon). It will turn out that $a_k = \pm 1, 0$. Then, since we are suppressing all such factors as $\vec{p}_{\perp k}^{\,2} + m_k^2$ we write

$$\left[-1, \frac{a_1^{-1}p_{-1}, a_2^{-1}p_{-2}, \ldots, a_n^{-1}p_{-n}}{p_-}\right]. \tag{2.14}$$

[If in (2.12) some $a_k = 0$, these k are omitted from (2.14).] The meaning of this symbol is that for any ordering of momenta under consideration we are to take the *smallest* factor inside the brackets. Note: If $p_- = O(2\omega)$ we will omit the term -1 (since -1 is always larger than the remaining factors) and we will omit p_- as well.

From now on we will use the term "momentum-flow diagram" to mean a diagram with all of these notations included.

There will at times be more than one set of poles which must be closed on to compute a given momentum-flow diagram. Usually, for the cases considered here the choice of poles will occur only in one of the momentum loops. When this is the case we will label these poles as (a) and (b). Whenever the factors are different for the two choices of poles, the box will be split into two parts. The factor in the upper half refers to pole (a) while the factor in the lower half refers to pole (b).

Once in a while there will be more sets of poles than can conveniently be handled on one diagram. Fortunately, in the cases considered here most of these poles do not contribute. In cases like this the poles which can be shown not to contribute will be marked by a dotted cross instead of a solid one.

D. Numerator

The numerator of each Feynman diagram must be approximated just as was done for fourth, sixth, and eighth orders. However, we will dispense with the demonstration that the omitted terms actually lead to a smaller contribution than the terms retained.

There are several points to consider:

(i) If the electron has a momentum p such that $p_+ = O(2\omega)$, then the factor p_+ appears in the numerator. This is usually approximated by 2ω. However, the factor of p_+ suppresses any pole closed on that electron line. Therefore, poles on these fast electron lines will be omitted from the diagrams unless there is some loop momentum p_+ for which the only poles are poles on the fast electron line. Such momentum-flow diagrams can never contribute and hence the calculation is stopped at this point.

(ii) The numerator reduction is carried out in terms of the coordinate system of the original Feynman diagram. As a result of this reduction some of the momenta may have their plus or minus component as a factor. This will be indicated on the momentum-flow diagram by (+) or (−) by the appropriate line. The direction of the momentum q may not be the same as the direction of the momentum of the electron in the original Feynman diagram. To compute the correct relative sign for the numerator one traces the electron line through the diagram from top to bottom. All (+) and (−) lines whose momentum arrows point in the direction of the trace get a plus sign; all (+) and (−) lines whose momentum arrows lie opposite to the trace get a minus sign. This signed combination of + and − factors is indicated on the diagram as "NUM =."

(iii) All the rest of the numerator factors depend only on transverse momenta and are recorded in the text.

E. Computation of (m_1, m_2)

To compute the pair of numbers (m_1, m_2) for each momentum-flow diagram consider the product formed by the factors in the boxes on the diagram and the numerator factors given as "NUM =." This product is to be examined for all allowed orderings of the momenta q_i such as $0 < q_{P_1} \ll q_{P_2} \ll q_{P_3} \cdots \ll q_{P_n} \ll 2\omega$. For each ordering that leads to an expression of the form

$$\frac{1}{q_1}\frac{1}{q_2}\cdots\frac{1}{q_n} \tag{2.15}$$

there will be a contribution of either +1 or −1 to m_1 or m_2. If the smallest q points in the direction of the electron line (as given in the original Feynman diagram), then the contribution has an imaginary part and hence contributes to m_2. If the smallest q points opposite to the direction of the electron line, there is no imaginary part and the contribution is to m_1.

To determine whether the contribution is +1 or −1 we compute the product of the signs of:

(i) all contributing factors in boxes,

(ii) the sign of the numerator factor,

(iii) +1 for each q on a fast line (with $|q_+| = 2\omega$) which points in the direction of the electron, and

(iv) −1 for each q on a fast line (with $|q_+| = 2\omega$) which points opposite to the direction of the electron.

III. DIAGRAMS CONTRIBUTING TO THE REAL PART

The 15 Feynman diagrams which contribute to the leading real part are displayed in Fig. 1. We will treat each Feynman diagram in a separate subsection. The diagrams will be added together in Sec. VI.

To save writing we use the notation

$$(m_1, m_2) = \frac{1}{4!}\left[m_1 \ln^4 s + m_2(\ln^4 s - 4\pi i \ln^3 s)\right]. \tag{3.1}$$

A. Feynman diagram 1

The numerator for Feynman diagram 1 is

$$\begin{aligned} N_1 = {} & - g^{10}\bar{u}(r_3 - r_1)\gamma_{\alpha_1}(\not{r}_3 - \not{r}_1 - \not{k}_4 + m)\gamma_\mu(-\not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_2}(-\not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_1} \\ & \times(-\not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_3}(-\not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_2}(-\not{k}_2 - 2\not{r}_1 + m)\gamma_{\alpha_4} \\ & \times(-\not{k}_1 - \not{k}_2 - 2\not{r}_1 + m)\gamma_{\alpha_3}(-\not{k}_1 - 2\not{r}_1 + m)\gamma_\nu(\not{r}_2 - \not{k}_1 - \not{r}_1 + m)\gamma_{\alpha_4}u(r_2 - r_1). \end{aligned} \tag{3.2}$$

This is approximated by

$$16(2\omega - k_{1+})(2\omega - k_{4-})\bar{u}(r_3 - r_1)\gamma_\mu \bar{N}_1 \gamma_\nu u(r_2 - r_1), \tag{3.3a}$$

where

$$\bar{N}_1 = -g^{10}(-\not{\vec{k}}_{4\perp} - 2\not{\vec{r}}_\perp + m)(\not{\vec{k}}_{3\perp} + \not{\vec{k}}_{4\perp} + 2\not{\vec{r}}_\perp + m)(-\not{\vec{k}}_{3\perp} - 2\not{\vec{r}}_\perp + m)$$
$$\times(\not{\vec{k}}_{2\perp} + \not{\vec{k}}_{3\perp} + 2\not{\vec{r}}_\perp + m)(-\not{\vec{k}}_{2\perp} - 2\not{\vec{r}}_\perp + m)(\not{\vec{k}}_{1\perp} + \not{\vec{k}}_{2\perp} + 2\not{\vec{r}}_\perp + m)(-\not{\vec{k}}_{1\perp} - 2\not{\vec{r}}_\perp + m). \tag{3.3b}$$

Using this numerator with the momentum-flow diagram of Fig. 7 we find

$$\mathfrak{M}_1^{(4)} \doteq (0, 1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_1 D_1^{-1}, \tag{3.4a}$$

where

$$D_1 = (\vec{k}_{1\perp}^2 + \lambda^2)(\vec{k}_{2\perp}^2 + \lambda^2)(\vec{k}_{3\perp}^2 + \lambda^2)(\vec{k}_{4\perp}^2 + \lambda^2)[(\vec{k}_{1\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{2\perp} + 2\vec{r}_\perp)^2 + m^2]$$
$$\times[(\vec{k}_{3\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{4\perp} + 2\vec{r}_\perp)^2 + m^2]. \tag{3.4b}$$

B. Feynman diagram 2

The numerator for Feynman diagram 2 is

$$N_2 = -g^{10}\bar{u}(r_3 - r_1)\gamma_{\alpha_1}(\not{r}_3 - \not{r}_1 - \not{k}_4 + m)\gamma_\mu(-\not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_2}(-\not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_1}$$
$$\times(-\not{k}_2 - 2\not{r}_1 + m)\gamma_{\alpha_3}(-\not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_2}(-\not{k}_2 - 2\not{r}_1 + m)\gamma_{\alpha_4}$$
$$\times(-\not{k}_1 - \not{k}_2 - 2\not{r}_1 + m)\gamma_\nu(\not{r}_2 - \not{k}_1 - \not{k}_2 - \not{r}_1 + m)\gamma_{\alpha_3}(\not{r}_2 - \not{k}_1 - \not{r}_1 + m)\gamma_{\alpha_4}u(r_2 - r_1). \tag{3.5}$$

This is approximated by

$$16(2\omega - k_{1+})(2\omega - k_{1+} - k_{2+})(2\omega - k_{4-})(-k_{1-} - k_{2-})\bar{u}(r_3 - r_1)\gamma_\mu \bar{N}_2\gamma_\nu u(r_2 - r_1), \tag{3.6a}$$

where

$$\bar{N}_2 = -g^{10}(-\not{\vec{k}}_{4\perp} - 2\not{\vec{r}}_\perp + m)(\not{\vec{k}}_{3\perp} + \not{\vec{k}}_{4\perp} + 2\not{\vec{r}}_\perp + m)(-\not{\vec{k}}_{3\perp} - 2\not{\vec{r}}_\perp + m)(\not{\vec{k}}_{2\perp} + \not{\vec{k}}_{3\perp} + 2\not{\vec{r}}_\perp + m)(-\not{\vec{k}}_{2\perp} - 2\not{\vec{r}}_\perp + m). \tag{3.6b}$$

Using this numerator with the momentum-flow diagram of Fig. 8 we find

$$\mathfrak{M}_2^{(4)} \doteq (0, -1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_2 D_2^{-1}, \tag{3.7}$$

where

$$D_2 = (\vec{k}_{1\perp}^2 + \lambda^2)(\vec{k}_{2\perp}^2 + \lambda^2)(\vec{k}_{3\perp}^2 + \lambda^2)(\vec{k}_{4\perp}^2 + \lambda^2)[(\vec{k}_{2\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{3\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{4\perp} + 2\vec{r}_\perp)^2 + m^2]. \tag{3.8}$$

C. Feynman diagram 3

The numerator for Feynman diagram 3 is

$$N_3 = -g^{10}\bar{u}(r_3 - r_1)\gamma_{\alpha_1}(\not{r}_3 - \not{r}_1 - \not{k}_4 + m)\gamma_\mu(-\not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_2}(-\not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_1}$$
$$\times(-\not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_3}(-\not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_4}(-\not{k}_1 - \not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_2}$$
$$\times(-\not{k}_1 - \not{k}_2 - 2\not{r}_1 + m)\gamma_{\alpha_3}(-\not{k}_1 - 2\not{r}_1 + m)\gamma_\nu(\not{r}_2 - \not{k}_1 - \not{r}_1 + m)\gamma_{\alpha_4}u(r_2 - r_1). \tag{3.9}$$

This is approximated by

$$16(2\omega - k_{1-})(2\omega - k_{4+})(-k_{2-} - k_{3-})(-k_{1+} - k_{2+})\bar{u}(r_3 - r_1)\gamma_\mu \bar{N}_3\gamma_\nu u(r_2 - r_1), \tag{3.9$'$}$$

where

$$\bar{N}_3 = -g^{10}(-\not{\vec{k}}_{4\perp} - 2\not{\vec{r}}_\perp + m)(\not{\vec{k}}_{3\perp} + \not{\vec{k}}_{4\perp} + 2\not{\vec{r}}_\perp + m)(-\not{\vec{k}}_{3\perp} - 2\not{\vec{r}}_\perp + m)(\not{\vec{k}}_{1\perp} + \not{\vec{k}}_{2\perp} + \not{\vec{k}}_{3\perp} + 2\not{\vec{r}}_\perp + m)(-\not{\vec{k}}_{1\perp} - 2\not{\vec{r}}_\perp + m). \tag{3.9$''$}$$

Using this numerator with the momentum-flow diagram of Fig. 9 we find

$$\mathfrak{M}_3^{(4)} \doteq (0, -1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_3 D_3^{-1}, \tag{3.10}$$

where

$$D_3 = (\vec{k}_{1\perp}{}^2 + \lambda^2)(\vec{k}_{2\perp}{}^2 + \lambda^2)(\vec{k}_{3\perp}{}^2 + \lambda^2)(\vec{k}_{4\perp}{}^2 + \lambda^2)[(\vec{k}_{1\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{3\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{4\perp} + 2\vec{r}_\perp)^2 + m^2]. \qquad (3.11)$$

D. Feynman diagram 4

Feynman diagram 4 is essentially diagram 3 turned upside down. Therefore, we immediately have from Sec. III C

$$\mathfrak{M}_4^{(4)} \doteq (0, -1) \int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3} \frac{d^2\vec{k}_{2\perp}}{(2\pi)^3} \frac{d^2\vec{k}_{3\perp}}{(2\pi)^3} \frac{d^2\vec{k}_{4\perp}}{(2\pi)^3} \bar{N}_4 D_4{}^{-1}, \qquad (3.12)$$

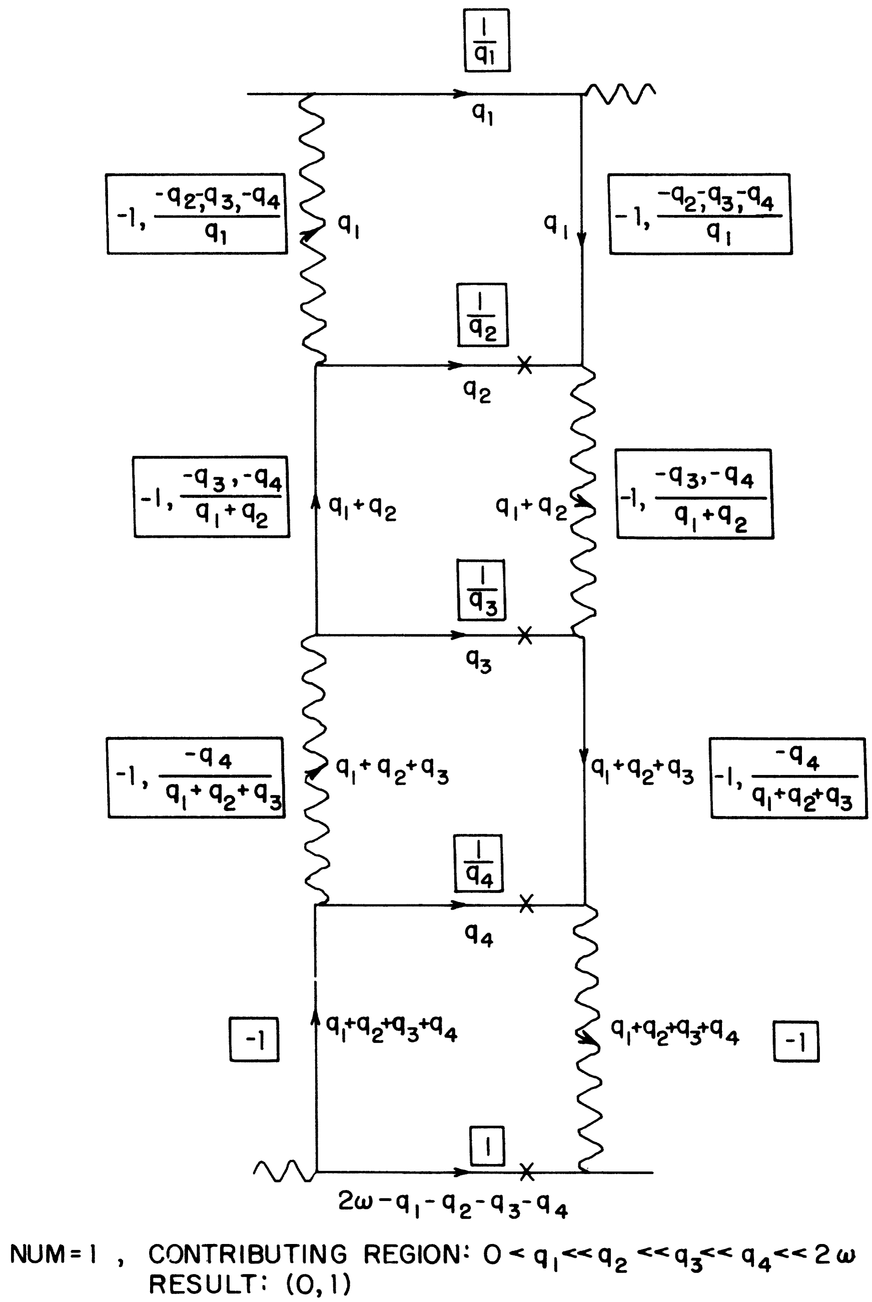


FIG. 7. The momentum-flow diagrams for Feynman diagram 1.

where

$$\bar{N}_4 = -g^{10}(-\not{K}_{4\perp} - 2\not{r}_\perp + m)(\not{K}_{2\perp} + \not{K}_{3\perp} + \not{K}_{4\perp} + 2\not{r}_\perp + m)(-\not{K}_{2\perp} - 2\not{r}_\perp + m)(\not{K}_{1\perp} + \not{K}_{2\perp} + 2\not{r}_\perp + m)(-\not{K}_{1\perp} - 2\not{r}_\perp + m) \tag{3.13}$$

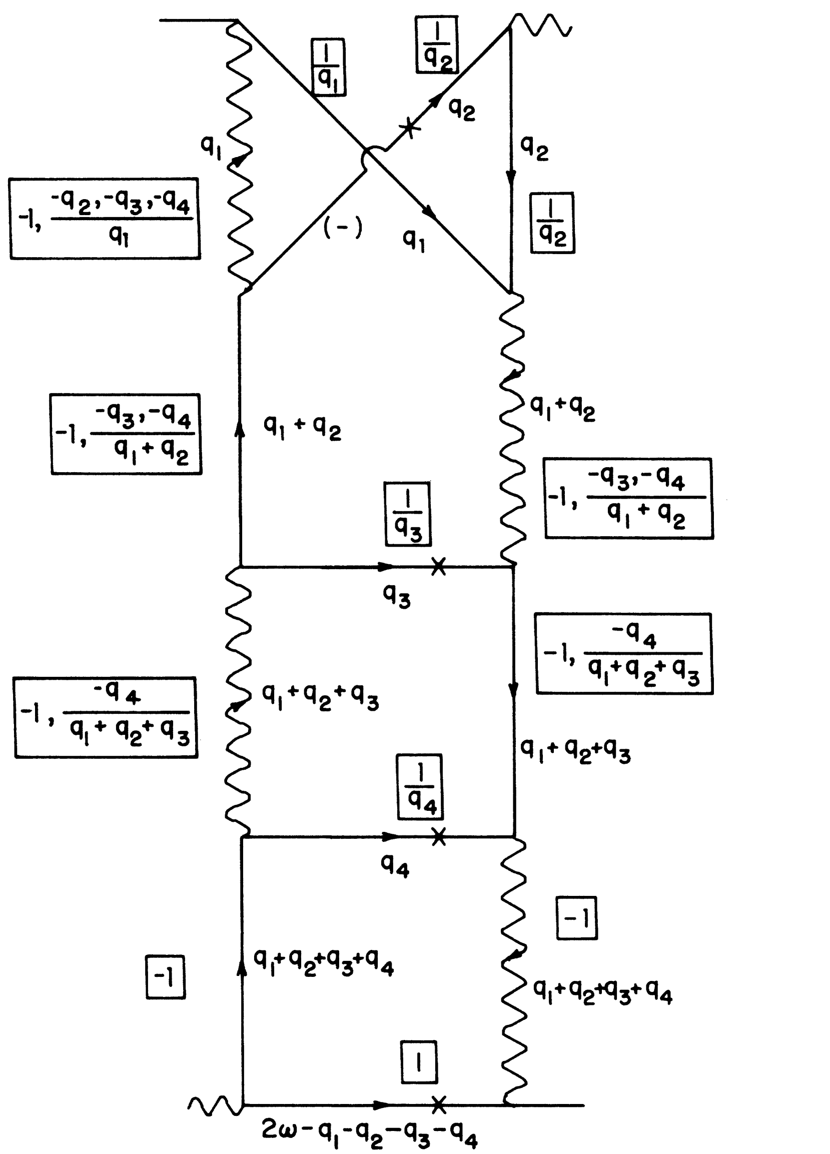
and

$$D_4 = (\vec{k}_{1\perp}{}^2 + \lambda^2)(\vec{k}_{2\perp}{}^2 + \lambda^2)(\vec{k}_{3\perp}{}^2 + \lambda^2)(\vec{k}_{4\perp}{}^2 + \lambda^2)[(\vec{k}_{1\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{2\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{4\perp} + 2\vec{r}_\perp)^2 + m^2]\,. \tag{3.14}$$

E. Feynman diagram 5

Feynman diagram 5 is essentially diagram 2 turned upside down. Therefore, we immediately have from Sec. III B

$$\bar{\mathfrak{M}}_5^{(4)} \doteq (0, -1) \int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3} \frac{d^2\vec{k}_{2\perp}}{(2\pi)^3} \frac{d^2\vec{k}_{3\perp}}{(2\pi)^3} \frac{d^2\vec{k}_{4\perp}}{(2\pi)^3} \bar{N}_5 D_5{}^{-1}\,, \tag{3.15}$$



NUM $= -q_2$, CONTRIBUTING REGION: $0 \ll q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
RESULT: (0,-1)

FIG. 8. The momentum-flow diagram for Feynman diagram 2.

where

$$\bar{N}_5 = -g^{10}(-\not{\vec{k}}_{3\perp} - 2\not{\vec{r}}_\perp + m)(\not{\vec{k}}_{2\perp} + \not{\vec{k}}_{3\perp} + 2\not{\vec{r}}_\perp + m)(-\not{\vec{k}}_{2\perp} - 2\not{\vec{r}}_\perp + m)(\not{\vec{k}}_{1\perp} + \not{\vec{k}}_{2\perp} + 2\not{\vec{r}}_\perp + m)(-\not{\vec{k}}_{1\perp} - 2\not{\vec{r}}_\perp + m) \tag{3.16}$$

and

$$D_5 = (\vec{k}_{1\perp}{}^2 + \lambda^2)(\vec{k}_{2\perp}{}^2 + \lambda^2)(\vec{k}_{3\perp}{}^2 + \lambda^2)(\vec{k}_{4\perp}{}^2 + \lambda^2)[(\vec{k}_{1\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{2\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{3\perp} + 2\vec{r}_\perp)^2 + m^2]. \tag{3.17}$$

F. Feynman diagram 6

The numerator for Feynman diagram 6 is

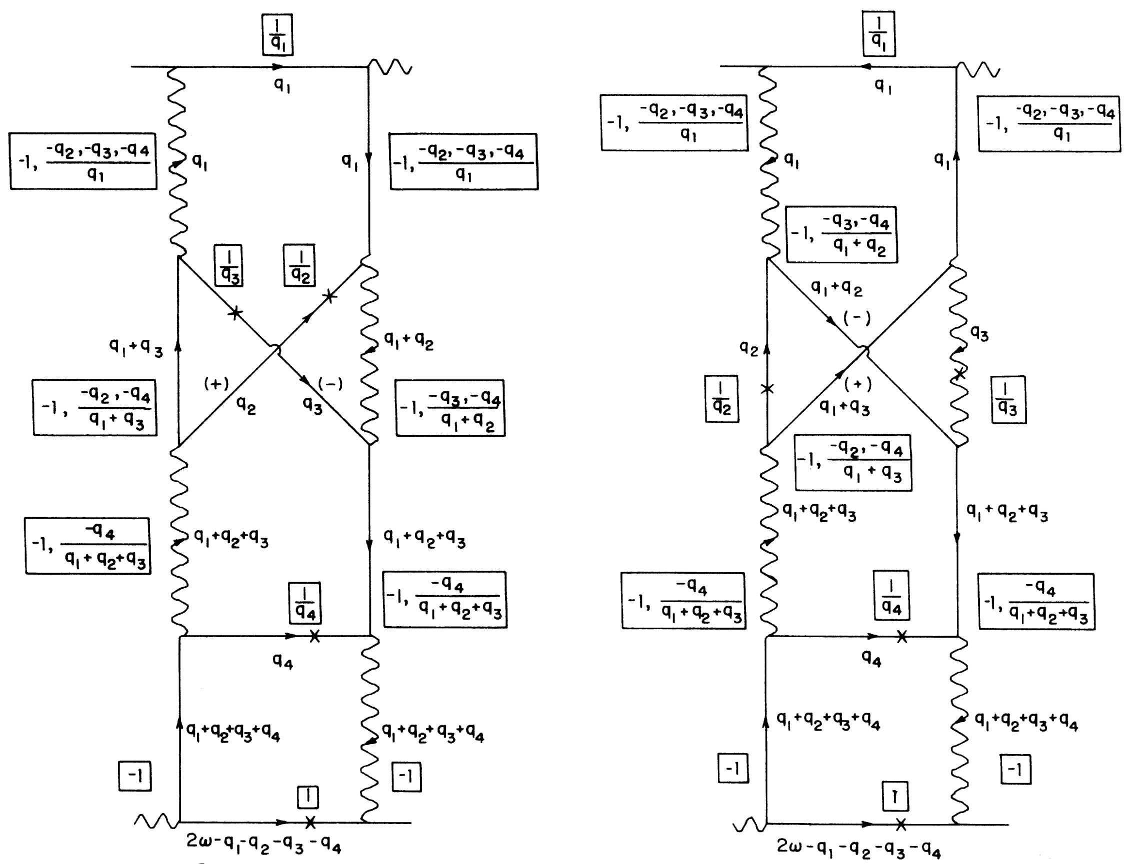
$$\begin{aligned} N_6 = -g^{10}\bar{u}(r_3 - r_1)\gamma_{\alpha_1}(\not{r}_3 - \not{r}_1 - \not{k}_4 + m)\gamma_\mu(-\not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_2}(-\not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_3} \\ \times(-\not{k}_2 - \not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_1}(-\not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_2}(-\not{k}_2 - 2\not{r}_1 + m)\gamma_{\alpha_4} \\ \times(-\not{k}_1 - \not{k}_2 - 2\not{r}_1 + m)\gamma_\nu(\not{r}_2 - \not{k}_1 - \not{k}_2 - \not{r}_1 + m)\gamma_{\alpha_3}(\not{r}_2 - \not{k}_1 + m)\gamma_{\alpha_4}u(r_2 - r_1). \end{aligned} \tag{3.18}$$

This is approximated by

$$16(2\omega - k_{1-} - k_{2-})(2\omega - k_{4+})(2\omega - k_{1-})(-k_{1+} - k_{2+})(-k_{3-} - k_{4-})(-k_{2+} - k_{3+})\bar{u}(r_3 - r_1)\gamma_\mu \bar{N}_6 \gamma_\nu u(r_2 - r_1), \tag{3.19}$$

where

$$\bar{N}_6 = -g^{10}(-\not{\vec{k}}_{4\perp} - 2\not{\vec{r}}_\perp + m)(\not{\vec{k}}_{2\perp} + \not{\vec{k}}_{3\perp} + \not{\vec{k}}_{4\perp} + 2\not{\vec{r}}_\perp + m)(-\not{\vec{k}}_{2\perp} - 2\not{\vec{r}}_\perp + m). \tag{3.20}$$



(a) NUM $= -q_{3-}\,q_{2+} = -\dfrac{q_3}{q_2}$,

CONTRIBUTING REGION: $0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

RESULT: (0,-1)

(b) NUM $= -(q_{1-} + q_{2-})(q_{1+} + q_{3+}) = (q_1 + q_2)\max\left[\dfrac{1}{q_2}, \dfrac{1}{q_4}\right]$,

NO REGION CONTRIBUTES

RESULT: (0, 0)

FIG. 9. The 2 momentum-flow diagrams for Feynman diagram 3.

Using this numerator with the momentum-flow diagrams of Fig. 10, we find

$$\mathfrak{M}_6^{(4)} \doteq (0,1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\,\bar{N}_6 D_6^{-1}\,, \tag{3.21}$$

where

$$D_6 = (\vec{k}_{1\perp}^{\,2}+\lambda^2)(\vec{k}_{2\perp}^{\,2}+\lambda^2)(\vec{k}_{3\perp}^{\,2}+\lambda^2)(\vec{k}_{4\perp}^{\,2}+\lambda^2)[(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\,. \tag{3.22}$$

G. Feynman diagram 7

The numerator for Feynman diagram 7 is

$$\begin{aligned} N_7 = -g^{10}\bar{u}(r_3-r_1)\gamma_{\alpha_1}&(\not{r}_3-\not{r}_1-\not{k}_4+m)\gamma_{\alpha_2}(\not{r}_3-\not{r}_1-\not{k}_3-\not{k}_4+m)\gamma_\mu(-\not{k}_3-\not{k}_4-2\not{r}_1+m)\gamma_{\alpha_1}(-\not{k}_3-2\not{r}_1+m)\gamma_{\alpha_3}\\ &\times(-\not{k}_2-\not{k}_3-2\not{r}_1+m)\gamma_{\alpha_2}(-\not{k}_2-2\not{r}_1+m)\gamma_{\alpha_4}(-\not{k}_1-\not{k}_2-2\not{r}_1+m)\gamma_\nu(\not{r}_2-\not{k}_1-\not{k}_2-\not{r}_1+m)\gamma_{\alpha_3}\\ &\times(\not{r}_2-\not{k}_1-\not{r}_1+m)\gamma_{\alpha_4}u(r_2-r_1)\,. \end{aligned} \tag{3.23}$$

This is approximated by

$$16(2\omega-k_{4-})(2\omega-k_{1+}-k_{2+})(2\omega-k_{3-}-k_{4-})(2\omega-k_{1-})(-k_{3+}-k_{4+})(-k_{1-}-k_{2-})\bar{u}(r_3-r_1)\gamma_\mu\bar{N}_7\gamma_\nu u(r_2-r_1)\,, \tag{3.24}$$

where

$$\bar{N}_7 = -g^{10}(-\not{\vec{k}}_{3\perp}-2\not{\vec{r}}_\perp+m)(\not{\vec{k}}_{2\perp}+\not{\vec{k}}_{3\perp}+2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{2\perp}-2\not{\vec{r}}_\perp+m)\,. \tag{3.25}$$

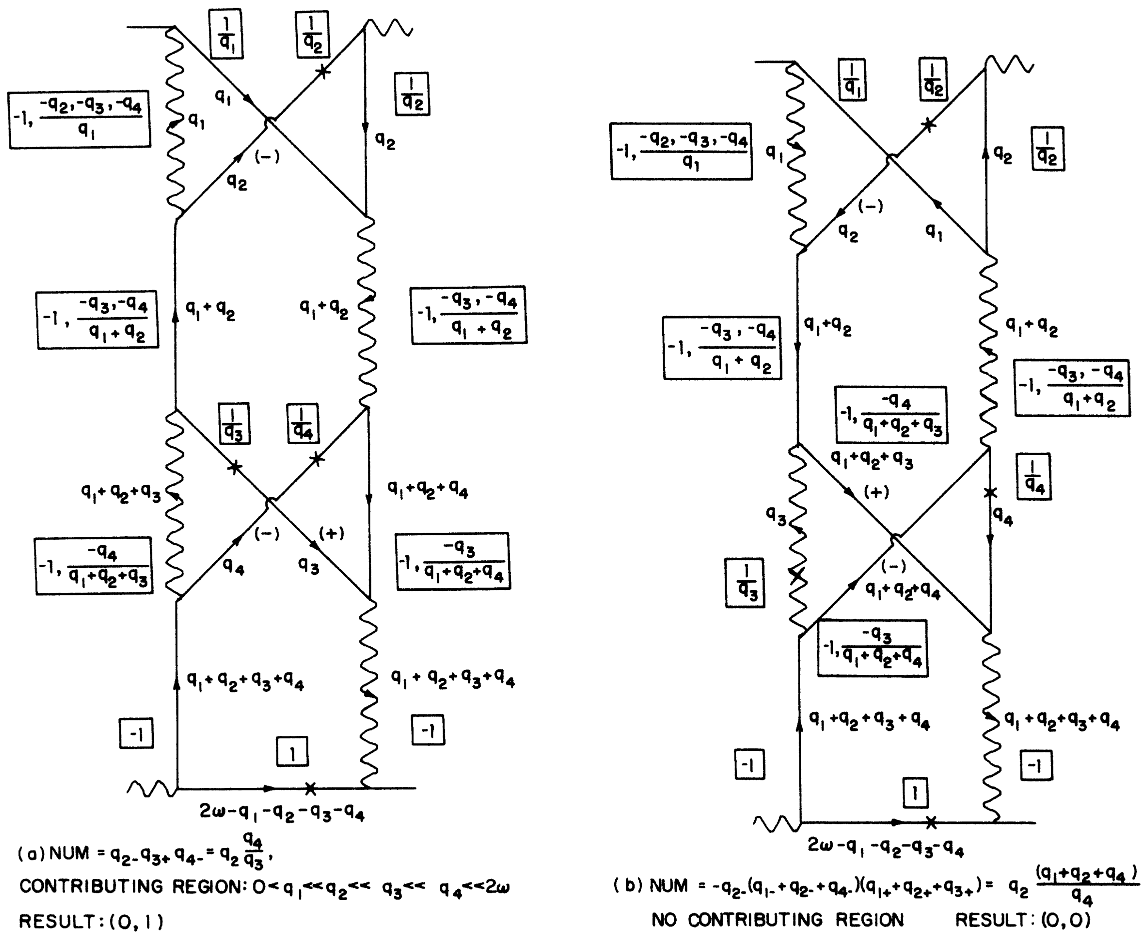


FIG. 10. The 2 momentum-flow diagrams for Feynman diagram 6.

Using this numerator with the momentum-flow diagram of Fig. 11, we find

$$\mathfrak{M}_7^{(4)} \doteq (0,1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\,\overline{N}_7 D_7^{-1}\,, \tag{3.26}$$

where

$$D_7 = (\vec{k}_{1\perp}^{\,2}+\lambda^2)(\vec{k}_{2\perp}^{\,2}+\lambda^2)(\vec{k}_{3\perp}^{\,2}+\lambda^2)(\vec{k}_{4\perp}^{\,2}+\lambda^2)[(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m][(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]\,. \tag{3.27}$$

H. Feynman diagram 8

Feynman diagram 8 is essentially diagram 6 turned upside down. Therefore, we immediately have from Sec. III F

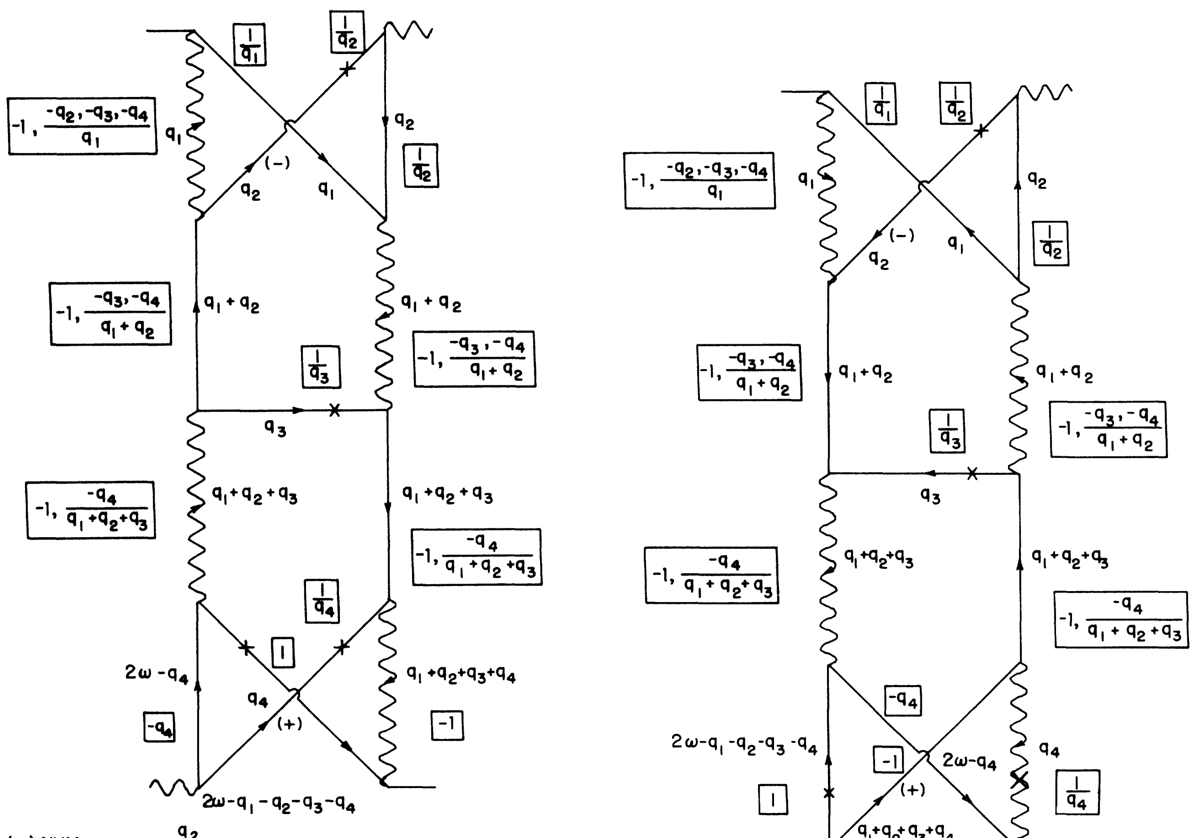
$$\mathfrak{M}_8^{(4)} \doteq (0,1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\,\overline{N}_8 D_8^{-1}\,, \tag{3.28}$$

where

$$\overline{N}_8 = -g^{10}(-\not{k}_{3\perp}-2\not{r}_\perp+m)(\not{k}_{1\perp}+\not{k}_{2\perp}+\not{k}_{3\perp}+2\not{r}_\perp+m)(-\not{k}_{1\perp}-2\not{r}_\perp+m) \tag{3.29}$$

and

$$D_8 = (\vec{k}_{1\perp}^{\,2}+\lambda^2)(\vec{k}_{2\perp}^{\,2}+\lambda^2)(\vec{k}_{3\perp}^{\,2}+\lambda^2)(\vec{k}_{4\perp}^{\,2}+\lambda^2)[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]\,. \tag{3.30}$$



(a) NUM = $q_{2-}q_{4+} = \frac{q_2}{q_4}$,
CONTRIBUTING REGION $0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
RESULT: (0, 1)

(b) NUM = $q_{2-}(q_{1+}+q_{2+}+q_{3+}+q_{4+}) = \frac{q_{2-}}{2\omega}$, NO REGION CONTRIBUTES
RESULT: (0,0)

FIG. 11. The 2 momentum-flow diagrams for Feynman diagram 7.

I. Feynman diagram 9

The numerator for Feynman diagram 9 is

$$N_9 = -g^{10}\bar{u}(r_3 - r_1)\gamma_{\alpha_1}(\not{r}_3 - \not{r}_1 - \not{k}_4 + m)\gamma_\mu(-\not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_2}(-\not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_1}$$
$$\times(-\not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_3}(-\not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_4}(-\not{k}_1 - \not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_\nu$$
$$\times(\not{r}_2 - \not{k}_1 - \not{k}_2 - \not{k}_3 - \not{r}_1 + m)\gamma_{\alpha_2}(\not{r}_2 - \not{k}_1 - \not{k}_2 - \not{r}_1 + m)\gamma_{\alpha_3}(\not{r}_2 - \not{k}_1 - \not{r}_1 + m)\gamma_{\alpha_4}u(r_2 - r_1)\,. \qquad (3.31)$$

This is approximated as

$$16(2\omega - k_{1+} - k_{2+} - k_{3+})(2\omega - k_{4-})(2\omega - k_{1+} - k_{2+})(2\omega - k_{1+})(-k_{2-} - k_{3-})$$
$$\times(-k_{1-} - k_{2-} - k_{3-})\bar{u}(r_3 - r_1)\gamma_\mu \bar{N}_9 \gamma_\nu u(r_2 - r_1)\,, \qquad (3.32)$$

where

$$\bar{N}_9 = -g^{10}(-\vec{\not{k}}_{4\perp} - 2\vec{\not{r}}_\perp + m)(\vec{\not{k}}_{3\perp} + \vec{\not{k}}_{4\perp} + 2\vec{\not{r}}_\perp + m)(-\vec{\not{k}}_{3\perp} - 2\vec{\not{r}}_\perp + m)\,. \qquad (3.33)$$

Using this numerator with the momentum-flow diagram of Fig. 12, we find

$$\mathfrak{M}_9^{(4)} \doteq (1,0)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_9 D_9^{-1}\,, \qquad (3.34)$$

where

$$D_9 = (\vec{k}_{1\perp}^{\,2} + \lambda^2)(\vec{k}_{2\perp}^{\,2} + \lambda^2)(\vec{k}_{3\perp}^{\,2} + \lambda^2)(\vec{k}_{4\perp}^{\,2} + \lambda^2)[(\vec{k}_{3\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{4\perp} + 2\vec{r}_\perp)^2 + m^2]\,. \qquad (3.35)$$

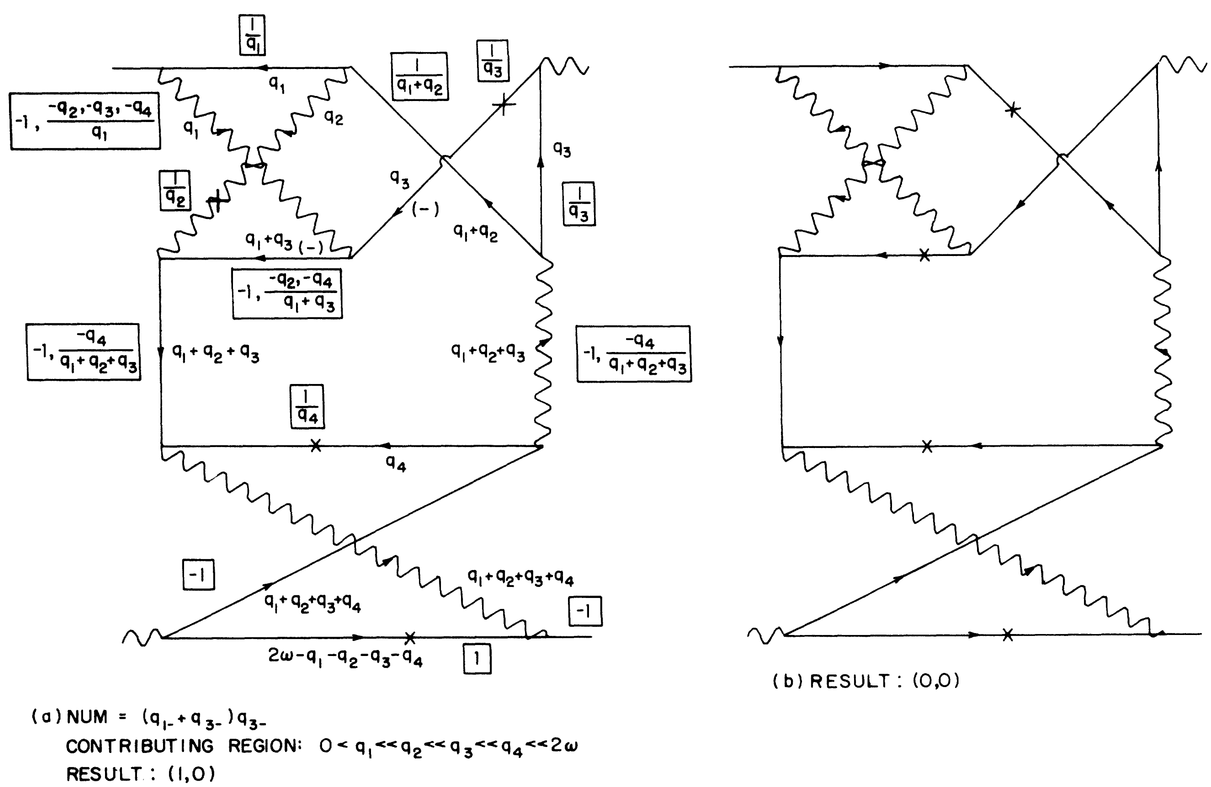


FIG. 12. The 2 momentum-flow diagrams for Feynman diagram 9.

J. Feynman diagram 10

The numerator for Feynman diagram 10 is

$$N_{10} = -g^{10}\bar{u}(r_3 - r_1)\gamma_{\alpha_1}(\not{r}_3 - \not{r}_1 - \not{k}_4 + m)\gamma_\mu(-\not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_2}(-\not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_3}(-\not{k}_2 - \not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_4}$$
$$\times(-\not{k}_1 - \not{k}_2 - \not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_1}(-\not{k}_1 - \not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_2}(-\not{k}_1 - \not{k}_2 - 2\not{r}_1 + m)\gamma_{\alpha_3}(-\not{k}_1 - 2\not{r}_1 + m)\gamma_\nu$$
$$\times(\not{r}_2 - \not{r}_1 - \not{k}_1 + m)\gamma_{\alpha_4}u(r_2 - r_1). \tag{3.36}$$

This is approximated as

$$16(2\omega - k_{1+})(2\omega - k_{4-})(-k_{1+} - k_{2+} - k_{3+})(-k_{3-} - k_{4-})(-k_{1+} - k_{2+})(-k_{2-} - k_{3-} - k_{4-})\bar{u}(r_3 - r_1)\gamma_\mu \bar{N}_{10}\gamma_\nu u(r_2 - r_1), \tag{3.37}$$

where

$$\bar{N}_{10} = -g^{10}(-\vec{\not{k}}_{4\perp} - 2\vec{\not{r}}_\perp + m)(\vec{\not{k}}_{1\perp} + \vec{\not{k}}_{2\perp} + \vec{\not{k}}_{3\perp} + \vec{\not{k}}_{4\perp} + 2\vec{\not{r}}_\perp + m)(-\vec{\not{k}}_{1\perp} - 2\vec{\not{r}}_\perp + m). \tag{3.38}$$

Using this numerator with the momentum-flow diagram of Fig. 13, we find

$$\mathfrak{M}_{10}^{(4)} \doteq (1,0)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{10}D_{10}{}^{-1}, \tag{3.39}$$

where

$$D_{10} = (\vec{k}_{1\perp}{}^2 + \lambda^2)(\vec{k}_{2\perp}{}^2 + \lambda^2)(\vec{k}_{3\perp}{}^2 + \lambda^2)(\vec{k}_{4\perp}{}^2 + \lambda^2)[(\vec{k}_{1\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{4\perp} + 2\vec{r}_\perp)^2 + m^2]. \tag{3.40}$$

K. Feynman diagram 11

Feynman diagram 11 is essentially diagram 9 turned upside down. Therefore, we immediately have from Sec. III I

$$\mathfrak{M}_{11}^{(4)} \doteq (1,0)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{11}D_{11}{}^{-1}, \tag{3.41}$$

where

$$\bar{N}_{11} = -g^{10}(-\vec{\not{k}}_{2\perp} - 2\vec{\not{r}}_\perp + m)(\vec{\not{k}}_{1\perp} + \vec{\not{k}}_{2\perp} + 2\vec{\not{r}}_\perp + m)(-\vec{\not{k}}_{1\perp} - 2\vec{\not{r}}_\perp + m) \tag{3.42}$$

and

$$D_{11} = (\vec{k}_{1\perp}{}^2 + \lambda^2)(\vec{k}_{2\perp}{}^2 + \lambda^2)(\vec{k}_{3\perp}{}^2 + \lambda^2)(\vec{k}_{4\perp}{}^2 + \lambda^2)[(\vec{k}_{1\perp}{}^2 + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{2\perp} + 2\vec{r}_\perp)^2 + m^2]. \tag{3.43}$$

L. Feynman diagram 12

The numerator for Feynman diagram 12 is

$$N_{12} = -g^{10}\bar{u}(r_3 - r_1)\gamma_{\alpha_1}(\not{r}_3 - \not{r}_1 - \not{k}_4 + m)\gamma_{\alpha_2}(\not{r}_3 - \not{r}_1 - \not{k}_3 - \not{k}_4 + m)\gamma_\mu(-\not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_1}(-\not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_3}$$
$$\times(-\not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_4}(-\not{k}_1 - \not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_\nu(\not{r}_2 - \not{k}_1 - \not{k}_2 - \not{k}_3 - \not{r}_1 + m)\gamma_{\alpha_2}$$
$$\times(\not{r}_2 - \not{k}_1 - \not{k}_2 - \not{r}_1 + m)\gamma_{\alpha_3}(\not{r}_2 - \not{k}_1 - \not{r}_1 + m)\gamma_{\alpha_4}u(r_2 - r_1). \tag{3.44}$$

This is approximated as

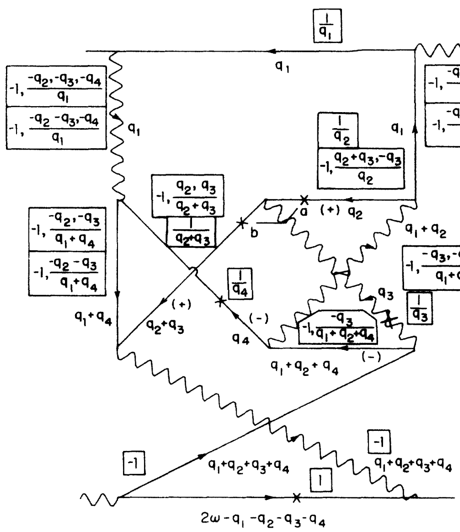
$$16(2\omega - k_{1+} - k_{2+} - k_{3+})(2\omega - k_{3-} - k_{4-})(2\omega - k_{4-})(2\omega - k_{1+} - k_{2+})(2\omega - k_{1+})$$
$$\times(-k_{3+} - k_{4+})(-k_{2-} - k_{3-})(-k_{1-} - k_{2-} - k_{3-})\bar{u}(r_3 - r_1)\gamma_\mu \bar{N}_{12}\gamma_\nu u(r_2 - r_1), \tag{3.45}$$

where

$$\bar{N}_{12} = -g^{10}(-\vec{\not{k}}_{3\perp} - 2\vec{\not{r}}_\perp + m) \tag{3.46}$$

Using this numerator with the momentum-flow diagram of Fig. 14, we find

$$\mathfrak{M}_{12}^{(4)} \doteq (-1,0)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{12}D_{12}{}^{-1}, \tag{3.47}$$



(a) NUM $= q_{2+}(q_{2+}+q_{3+})q_{4-}(q_{1-}+q_{2-}+q_{4-})$

Pole a: NUM $= \frac{q_4}{q_2}(q_1+q_2+q_4)\max\left(\frac{1}{q_2},\frac{1}{q_3}\right)$

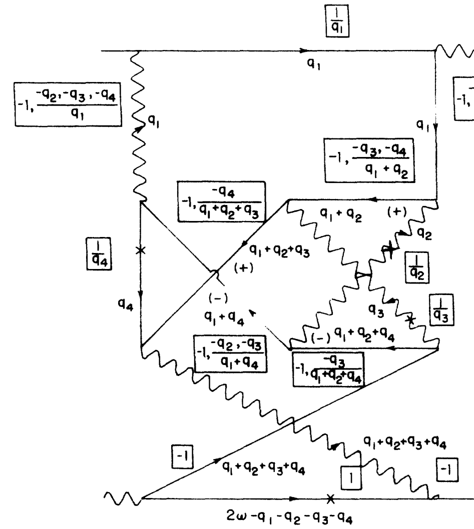
CONTRIBUTING REGION: $0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

RESULT: (1,0)

Pole b: NUM $= \frac{q_4(q_1+q_2+q_4)}{q_2+q_3}\max\left(\frac{1}{q_2+q_3}, -\frac{1}{q_3}\right)$

NO REGION CONTRIBUTES, RESULT: (0,0)

TOTAL RESULT: (1,0)

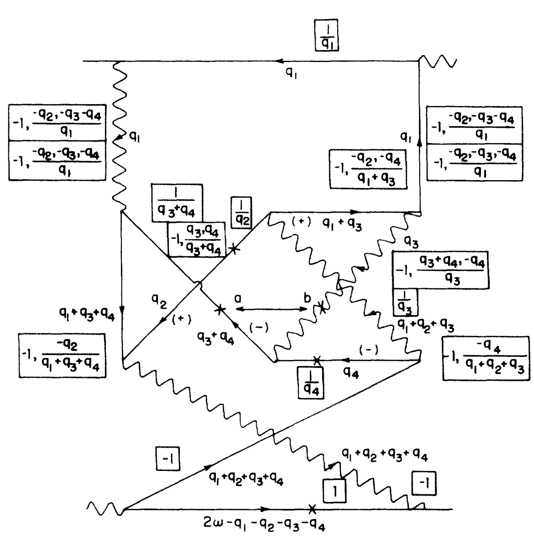


(b) NUM $= (q_{1+}+q_{2+})(q_{1+}+q_{2+}+q_{3+})(q_{1-}+q_{4-})(q_{1-}+q_{2-}+q_{4-})$

$= -\frac{(q_1+q_4)}{q_4}(q_1+q_2+q_4)\max\left(\frac{1}{q_3},\frac{1}{q_4}\right)$

NO REGION CONTRIBUTES

RESULT: (0,0)



(c) NUM $= -q_{2+}(q_{3-}+q_{4-})(q_{1+}+q_{3+})q_{4-} = \frac{q_4(q_3+q_4)}{q_2}\max\left(\frac{1}{q_2},\frac{1}{q_4}\right)$

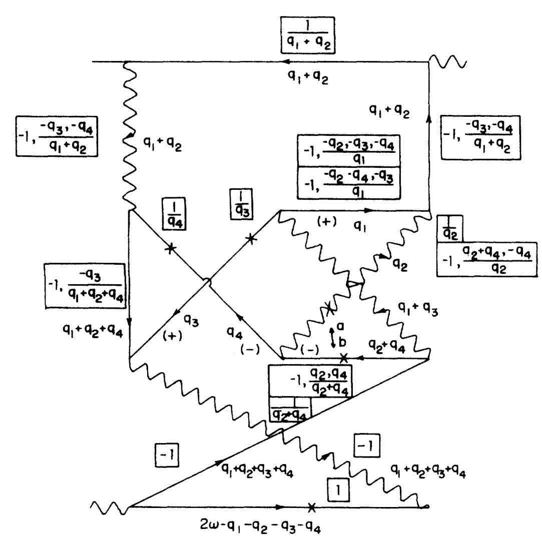
Pole a: CONTRIBUTING REGION: $0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

RESULT: (1,0)

Pole b: CONTRIBUTING REGION: $0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

RESULT: (-1,0)

TOTAL RESULT: (0,0)

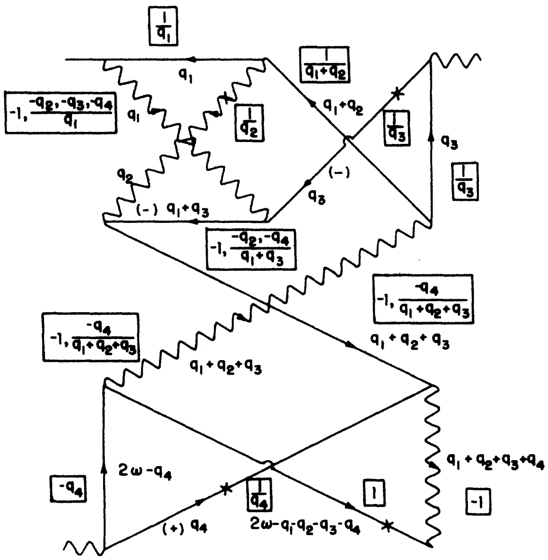


(d) NUM $= -q_{1+}q_{3+}q_{4-}(q_{2-}+q_{4-})$

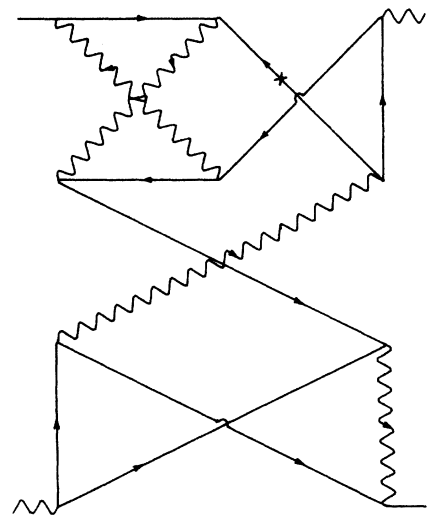
NO REGION CONTRIBUTES

FOR EITHER POLE

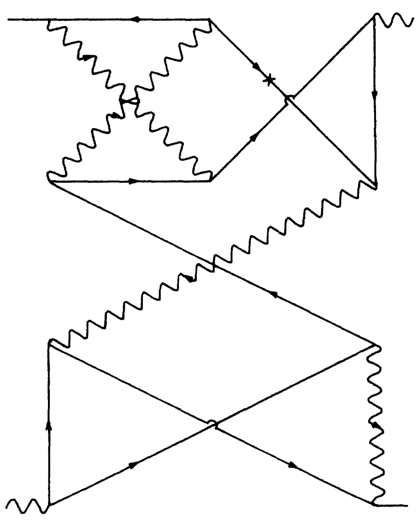
FIG. 13. The 4 momentum-flow diagrams for Feynman diagram 10.



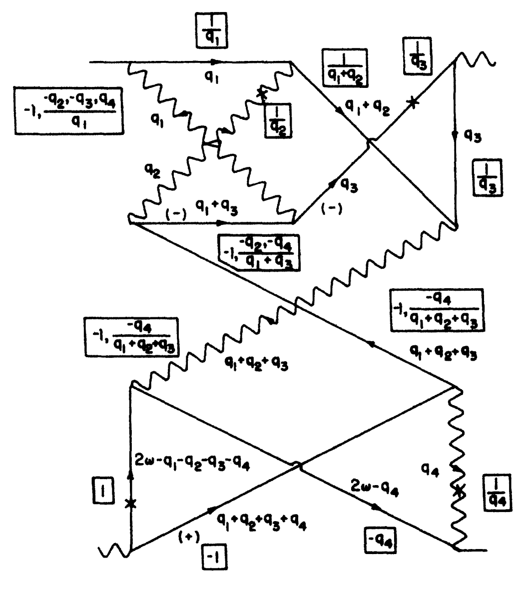
(a) NUM = $-q_{3-}(q_{1-}+q_{3-})q_{4+} = \frac{q_3}{q_4}(q_1+q_3)$
CONTRIBUTING REGION: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
RESULT: (-1,0)



(b) NO CONTRIBUTION



(c) NO CONTRIBUTION



(d) NUM = $q_{3-}(q_{1-}+q_{3-})(q_{1+}+q_{2+}+q_{3+}+q_{4+})$
NO REGION CONTRIBUTES

FIG. 14. The 4 momentum-flow diagrams for Feynman diagram 12.

where

$$D_{12} = (\vec{k}_{1\perp}^{\,2} + \lambda^2)(\vec{k}_{2\perp}^{\,2} + \lambda^2)(\vec{k}_{3\perp}^{\,2} + \lambda^2)(\vec{k}_{4\perp}^{\,2} + \lambda^2)\,[(\vec{k}_{3\perp} + 2\vec{r}_\perp)^2 + m^2]. \quad (3.48)$$

M. Feynman diagram 13

Feynman diagram 13 is essentially diagram 12 turned upside down. Therefore, we immediately have from Sec. III L

$$\mathfrak{M}_{13}^{(4)} \doteq (-1, 0)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\,\overline{N}_{13} D_{13}^{-1}, \quad (3.49)$$

where

$$\overline{N}_{13} = -g^{10}(-\not{k}_{2\perp} - 2\not{r}_\perp + m) \quad (3.50)$$

and

$$D_{13} = (\vec{k}_{1\perp}^{\,2} + \lambda^2)(\vec{k}_{2\perp}^{\,2} + \lambda^2)(\vec{k}_{3\perp} + \lambda^2)(\vec{k}_{4\perp}^{\,2} + \lambda^2)\,[(\vec{k}_{2\perp} + 2\vec{r}_\perp)^2 + m^2]. \quad (3.51)$$

N. Feynman diagram 14

The numerator for Feynman diagram 14 is

$$\begin{aligned} N_{14} = -g^{10}\bar{u}(r_3 - r_1)\gamma_{\alpha_1}(\not{r}_3 - \not{r}_1 - \not{k}_4 + m)\gamma_\mu(-\not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_2}(-\not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_3} \\ \times(-\not{k}_2 - \not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_4}(-\not{k}_1 - \not{k}_2 - \not{k}_4 - 2\not{r}_1 + m)\gamma_\nu \\ \times(\not{r}_2 - \not{k}_1 - \not{k}_2 - \not{k}_3 - \not{k}_4 - \not{r}_1 + m)\gamma_{\alpha_1}(\not{r}_2 - \not{k}_1 - \not{k}_2 - \not{k}_3 - \not{r}_1 + m)\gamma_{\alpha_2} \\ \times(\not{r}_2 - \not{k}_1 - \not{k}_2 - \not{r}_1 + m)\gamma_{\alpha_3}(\not{r}_2 - \not{k}_1 - \not{r}_1 + m)\gamma_{\alpha_4} u(r_2 - r_1). \end{aligned} \quad (3.52)$$

This is approximated as

$$\begin{aligned} 16(2\omega - k_{4-})(2\omega - k_{1+} - k_{2+} - k_{3+} - k_{4+}) \\ \times(2\omega - k_{1+} - k_{2+} - k_{3+})(2\omega - k_{1+} - k_{2+}) \\ \times(2\omega - k_{1+})\bar{u}(r_3 - r_1)\gamma_\mu \overline{N}_{14}\gamma_\nu u(r_2 - r_1), \end{aligned} \quad (3.53)$$

where

$$\overline{N}_{14} = -g^{10}(-\not{k}_{4\perp} - 2\not{r}_\perp + m). \quad (3.54)$$

Using this numerator with the momentum-flow diagrams of Fig. 15, we find

$$\mathfrak{M}_{14}^{(4)} \doteq (0, 1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\,\overline{N}_{14} D_{14}^{-1}, \quad (3.55)$$

where

$$\begin{aligned} D_{14} = (\vec{k}_{1\perp}^{\,2} + \lambda^2)(\vec{k}_{2\perp}^{\,2} + \lambda^2)(\vec{k}_{3\perp}^{\,2} + \lambda^2)(\vec{k}_{4\perp}^{\,2} + \lambda^2) \\ \times[(\vec{k}_{4\perp} + 2\vec{r}_\perp)^2 + m^2]. \end{aligned} \quad (3.56)$$

O. Feynman diagram 15

Feynman diagram 15 is essentially diagram 14 turned upside down. Therefore, from Sec. III N we find

$$\mathfrak{M}_{15}^{(4)} \doteq (0, -1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\,\overline{N}_{15} D_{15}^{-1}, \quad (3.57)$$

where

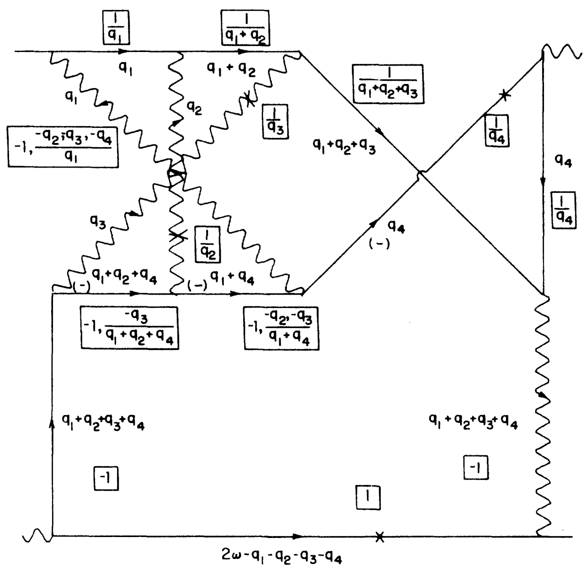
$$\overline{N}_{15} = -g^{10}(-\not{k}_{1\perp} - 2\not{r}_\perp + m) \quad (3.58)$$

and

$$\begin{aligned} D_{15} = (\vec{k}_{1\perp}^{\,2} + \lambda^2)(\vec{k}_{2\perp}^{\,2} + \lambda^2)(\vec{k}_{3\perp}^{\,2} + \lambda^2)(\vec{k}_{4\perp}^{\,2} + \lambda^2) \\ \times[(\vec{k}_{1\perp} + 2\vec{r}_\perp)^2 + m^2]. \end{aligned} \quad (3.59)$$

IV. DIAGRAMS THAT CONTRIBUTE TO THE IMAGINARY PART

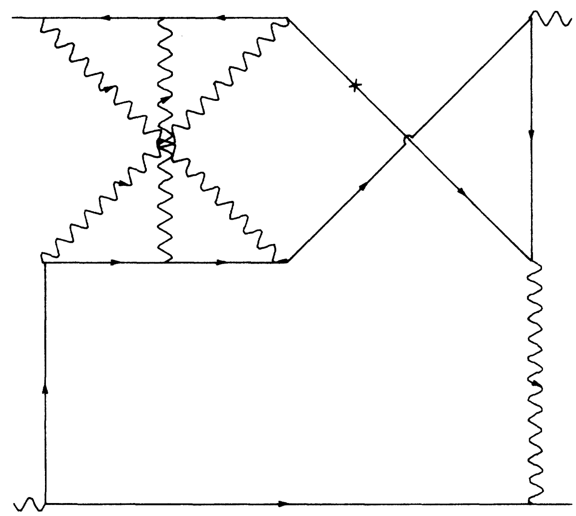
In this section we will evaluate the 20 Feynman diagrams of Fig. 3 which contribute to the leading imaginary part of $\mathfrak{M}^{(4)}$ and which do not cancel in pairs. The 6 diagrams of Fig. 2 which cancel in pairs will be studied in Sec. V. As in Sec. III, each diagram will be treated in a separate subsection. The diagrams will be summed in Sec. VI.



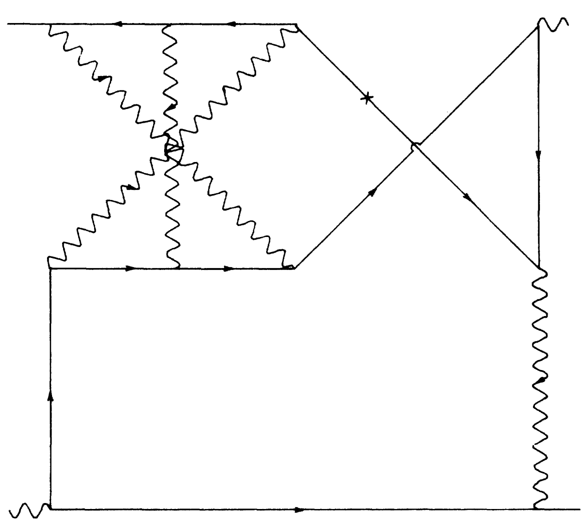
(a) NUM $=-q_4(q_1+q_4)(q_1+q_2+q_4)$

CONTRIBUTING REGION: $0<q_1\ll q_2\ll q_3<q_4\ll 2\omega$

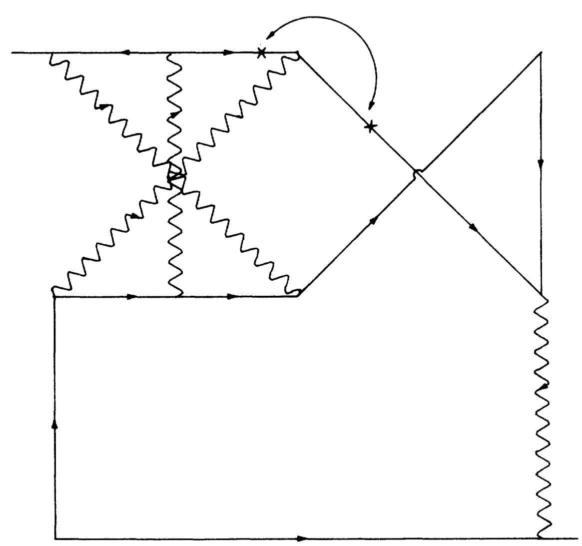
RESULT : (0,-1)



(b) NO CONTRIBUTION



(c) NO CONTRIBUTION



(d) NO CONTRIBUTION

FIG. 15. The 4 momentum-flow diagrams for Feynman diagram 14.

A. Feynman diagram 16

The numerator for Feynman diagram 16 is

$$N_{16} = -g^{10}\bar{u}(r_3 - r_1)\gamma_{\alpha_1}(\not{r}_3 - \not{r}_1 - \not{k}_4 + m)\gamma_{\alpha_2}(\not{r}_3 - \not{r}_1 - \not{k}_2 - \not{k}_4 + m)\gamma_\mu$$
$$\times(-\not{k}_2 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_3}(-\not{k}_2 - \not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_1}(-\not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_4}$$
$$\times(-\not{k}_1 - \not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_3}(-\not{k}_1 - \not{k}_2 - 2\not{r}_1 + m)\gamma_\nu(\not{r}_2 - \not{k}_1 - \not{k}_2 - \not{r}_1 + m)\gamma_{\alpha_2}(\not{r}_2 - \not{k}_1 - \not{r}_1 + m)\gamma_{\alpha_4}u(r_2 - r_1). \quad (4.1)$$

This is approximated by

$$16(2\omega - k_{1+})(2\omega - k_{1+} - k_{2+})(2\omega - k_{4+})(2\omega - k_{2+} - k_{4+})\bar{u}(r_3 - r_1)\gamma_\mu \bar{N}_{16}\gamma_\nu u(r_2 - r_1), \quad (4.2)$$

where

$$\bar{N}_{16} = -g^{10}(-\not{k}_{2\perp} - \not{k}_{4\perp} - 2\not{r}_\perp + m)(\not{k}_{2\perp} + \not{k}_{3\perp} + \not{k}_{4\perp} + 2\not{r}_\perp + m)(-\not{k}_{2\perp} - \not{k}_{3\perp} - 2\not{r}_\perp + m)$$
$$\times(\not{k}_{1\perp} + \not{k}_{2\perp} + \not{k}_{3\perp} + 2\not{r}_\perp + m)(-\not{k}_{1\perp} - \not{k}_{2\perp} - 2\not{r}_\perp + m). \quad (4.3)$$

We now take advantage of a "symmetry" of this diagram to reduce the number of momentum-flow diagrams we need to evaluate by a factor of two. If we reverse any momentum-flow diagram of diagram 16 left for right and reverse the direction of all arrows, then we obtain another momentum-flow diagram. The denominators of these two distinct momentum-flow diagrams will be the same, but their numerators will be different. However, the leading order (4.2) shows that these two numerators are also the same. Therefore, in Fig. 16 we have not drawn diagrams obtainable from the ones shown by reversing right for left and reversing all arrows and we have used two for the numerator factor instead of one. Then, using Fig. 16 and (4.2) we find

$$\mathfrak{M}_{16}^{(4)} \doteq (-2, 2)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{16}D_{16}^{-1}, \quad (4.4)$$

where

$$D_{16} = (\vec{k}_{1\perp}^2 + \lambda^2)(\vec{k}_{2\perp}^2 + \lambda^2)(\vec{k}_{3\perp}^2 + \lambda^2)(\vec{k}_{4\perp}^2 + \lambda^2)[(\vec{k}_{1\perp} + \vec{k}_{2\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{3\perp} + \vec{k}_{2\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{4\perp} + \vec{k}_{2\perp} + 2\vec{r}_\perp)^2 + m^2]. \quad (4.5)$$

B. Feynman diagram 17

The numerator for Feynman diagram 17 is

$$N_{17} = -g^{10}\bar{u}(r_3 - r_1)\gamma_{\alpha_1}(\not{r}_3 - \not{r}_1 - \not{k}_4 + m)\gamma_\mu(-\not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_2}(-\not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_3}(-\not{k}_2 - \not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_1}$$
$$\times(-\not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_4}(-\not{k}_1 - \not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_2}(-\not{k}_1 - \not{k}_2 - 2\not{r}_1 + m)\gamma_\nu$$
$$\times(\not{r}_2 - \not{r}_1 - \not{k}_1 - \not{k}_2 + m)\gamma_{\alpha_3}(\not{r}_2 - \not{r}_1 - \not{k}_1 + m)\gamma_{\alpha_4}u(r_2 - r_1). \quad (4.6)$$

This is approximated as

$$16(2\omega - k_{4-})(2\omega - k_{1+} - k_{2+})(2\omega - k_{1+})(-k_{3-} - k_{4-})$$
$$\times\bar{u}(r_3 - r_1)\gamma_\mu \bar{N}_{17}\gamma_\nu u(r_2 - r_1), \quad (4.7)$$

where

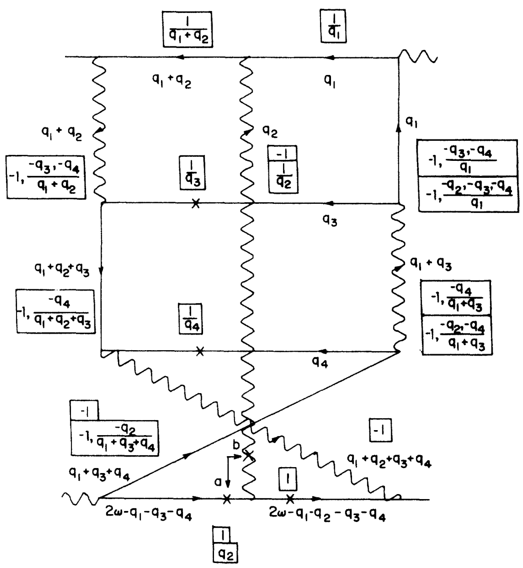
$$\bar{N}_{17} = -g^{10}(-\not{k}_{4\perp} - 2\not{r}_\perp + m)(\not{k}_{2\perp} + \not{k}_{3\perp} + \not{k}_{4\perp} + 2\not{r}_\perp + m)$$
$$\times(-\not{k}_{2\perp} - \not{k}_{3\perp} - 2\not{r}_\perp + m)(\not{k}_{1\perp} + \not{k}_{2\perp} + \not{k}_{3\perp} + 2\not{r}_\perp + m)$$
$$\times(-\not{k}_{1\perp} - \not{k}_{2\perp} - 2\not{r}_\perp + m). \quad (4.8)$$

We again reduce the number of momentum-flow diagrams by considering the pairs of diagrams related by reversing right for left and reversing the direction of all arrows. This time the − components in the approximate numerator (4.8) are not invariant under the operation and hence the two terms on the diagram for NUM = are slightly different. Then, using (4.8) with the momentum-flow diagrams of Fig. 17 we find

$$\mathfrak{M}_{17}^{(4)} \doteq (2, -2)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{17}D_{17}^{-1}, \quad (4.9)$$

where

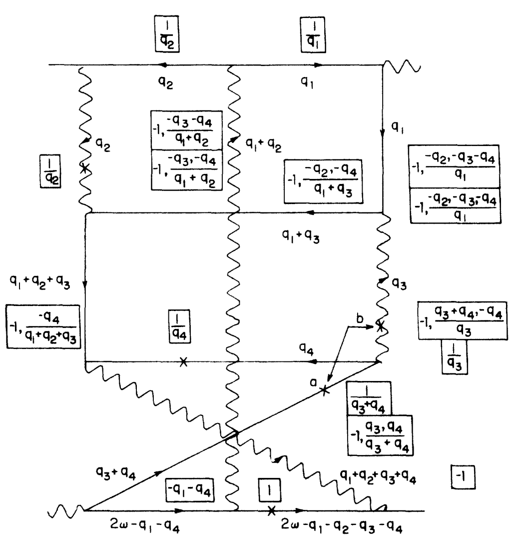
$$D_{17} = (\vec{k}_{1\perp}^2 + \lambda^2)(\vec{k}_{2\perp}^2 + \lambda^2)(\vec{k}_{3\perp}^2 + \lambda^2)(\vec{k}_{4\perp}^2 + \lambda^2)$$
$$\times[(\vec{k}_{1\perp} + \vec{k}_{2\perp} + 2\vec{r}_\perp)^2 + m^2]$$
$$\times[(\vec{k}_{2\perp} + \vec{k}_{3\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{4\perp} + 2\vec{r}_\perp)^2 + m^2]. \quad (4.10)$$



(a) NUM = 2

POLE a: CONTRIBUTING REGION: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
RESULT: (-2,0)

POLE b: NO CONTRIBUTION

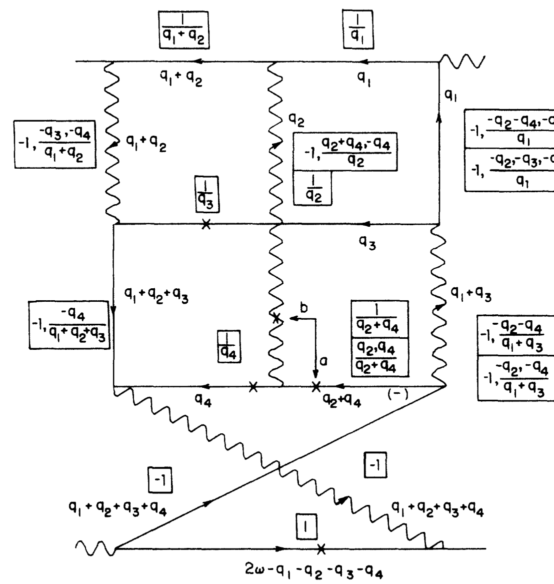


(b) NUM = 2

POLE a: CONTRIBUTING REGION: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
RESULT: (0,2)

POLE b: NO CONTRIBUTION

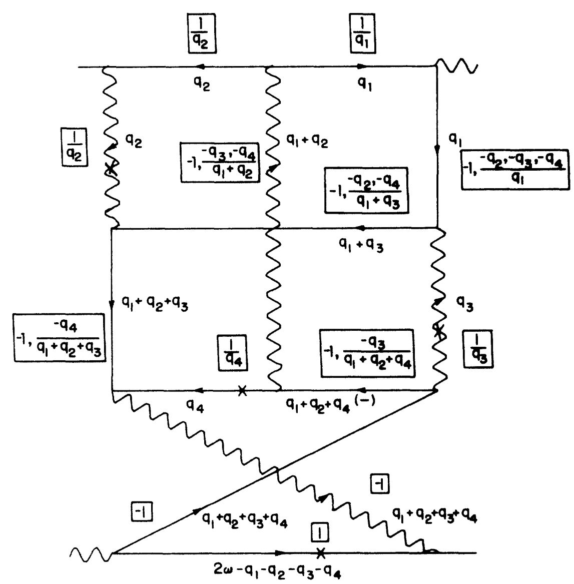
FIG. 16. Two of the 4 momentum-flow diagrams for Feynman diagram 16. The other 2 are obtained from these by turning the arrows on the diagram right for left and then reversing the direction of all the arrows.



(a) NUM = $-(q_{2-}+q_{4-})-q_{4-}$

POLE a: CONTRIBUTING REGION: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
RESULT: (2,0)

POLE b: NO CONTRIBUTION



(b) NUM = $-(q_1+q_2+q_4)-q_4$

CONTRIBUTING REGION: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
RESULT: (0,-2)

FIG. 17. Two of the 4 momentum-flow diagrams for Feynman diagram 17. The other two are obtained from them by the right-left symmetry operation.

C. Feynman diagram 18

Feynman diagram 18 is essentially diagram 17 turned upside down. Therefore, from Sec. IV B we find

$$\mathfrak{M}_{18}^{(4)} \doteq (2,-2)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\overline{N}_{18}D_{18}^{-1}, \tag{4.11}$$

where

$$\overline{N}_{18}=-g^{10}(-\vec{k}\!\!/_{4\perp}-\vec{k}\!\!/_{3\perp}-2\vec{r}\!\!/_{\perp}+m)(\vec{k}\!\!/_{2\perp}+\vec{k}\!\!/_{3\perp}+\vec{k}\!\!/_{4\perp}+2\vec{r}\!\!/_{\perp}+m)\times(-\vec{k}\!\!/_{2\perp}-\vec{k}\!\!/_{3\perp}-2\vec{r}\!\!/_{\perp}+m)\times(\vec{k}\!\!/_{1\perp}+\vec{k}\!\!/_{2\perp}+\vec{k}\!\!/_{3\perp}+2\vec{r}\!\!/_{\perp}+m)(-\vec{k}\!\!/_{1\perp}-2\vec{r}\!\!/_{\perp}+m) \tag{4.12}$$

and

$$D_{18}=(\vec{k}_{1\perp}{}^2+\lambda^2)(\vec{k}_{2\perp}{}^2+\lambda^2)(\vec{k}_{3\perp}{}^2+\lambda^2)(\vec{k}_{4\perp}{}^2+\lambda^2)\times[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]\times[(\vec{k}_{3\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]. \tag{4.13}$$

D. Feynman diagram 19

The numerator for Feynman diagram 19 is

$$N_{19}=-g^{10}\bar{u}(r_3-r_1)\gamma_{\alpha_1}(\not{r}_3-\not{r}_1-\not{k}_4+m)\gamma_\mu(-\not{k}_4-2\not{r}_1+m)\gamma_{\alpha_2}(-\not{k}_3-\not{k}_4-2\not{r}_1+m)\gamma_{\alpha_1}\times(-\not{k}_3-2\not{r}_1+m)\gamma_{\alpha_3}(-\not{k}_2-\not{k}_3-2\not{r}_1+m)\gamma_{\alpha_4}(-\not{k}_1-\not{k}_2-\not{k}_3-2\not{r}_1+m)\gamma_{\alpha_2}\times(-\not{k}_1-\not{k}_2-2\not{r}_1+m)\gamma_\nu(\not{r}_2-\not{r}_1-\not{k}_1-\not{k}_2+m)\gamma_{\alpha_3}(\not{r}_2-\not{r}_1-\not{k}_1+m)\gamma_{\alpha_4}u(r_2-r_1). \tag{4.14}$$

This is approximated as

$$16(2\omega-k_{1+})(2\omega-k_{1+}-k_{2+})(2\omega-k_{4+})(-k_{2-}-k_{3-})\bar{u}(r_3-r_1)\gamma_\mu\overline{N}_{19}\gamma_\nu u(r_2-r_1), \tag{4.15}$$

where

$$\overline{N}_{19}=-g^{10}(-\vec{k}\!\!/_{4\perp}-2\vec{r}\!\!/_{\perp}+m)(\vec{k}\!\!/_{3\perp}+\vec{k}\!\!/_{4\perp}+2\vec{r}\!\!/_{\perp}+m)(-\vec{k}\!\!/_{3\perp}+2\vec{r}\!\!/_{\perp}+m)(\vec{k}\!\!/_{1\perp}+\vec{k}\!\!/_{2\perp}+\vec{k}\!\!/_{3\perp}+2\vec{r}\!\!/_{\perp}+m)(-\vec{k}\!\!/_{1\perp}-\vec{k}\!\!/_{2\perp}-2\vec{r}\!\!/_{\perp}+m). \tag{4.16}$$

Using this numerator with the momentum-flow diagram of Fig. 18, we find

$$\mathfrak{M}_{19}^{(4)} \doteq (-1,1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\overline{N}_{19}D_{19}^{-1}, \tag{4.17}$$

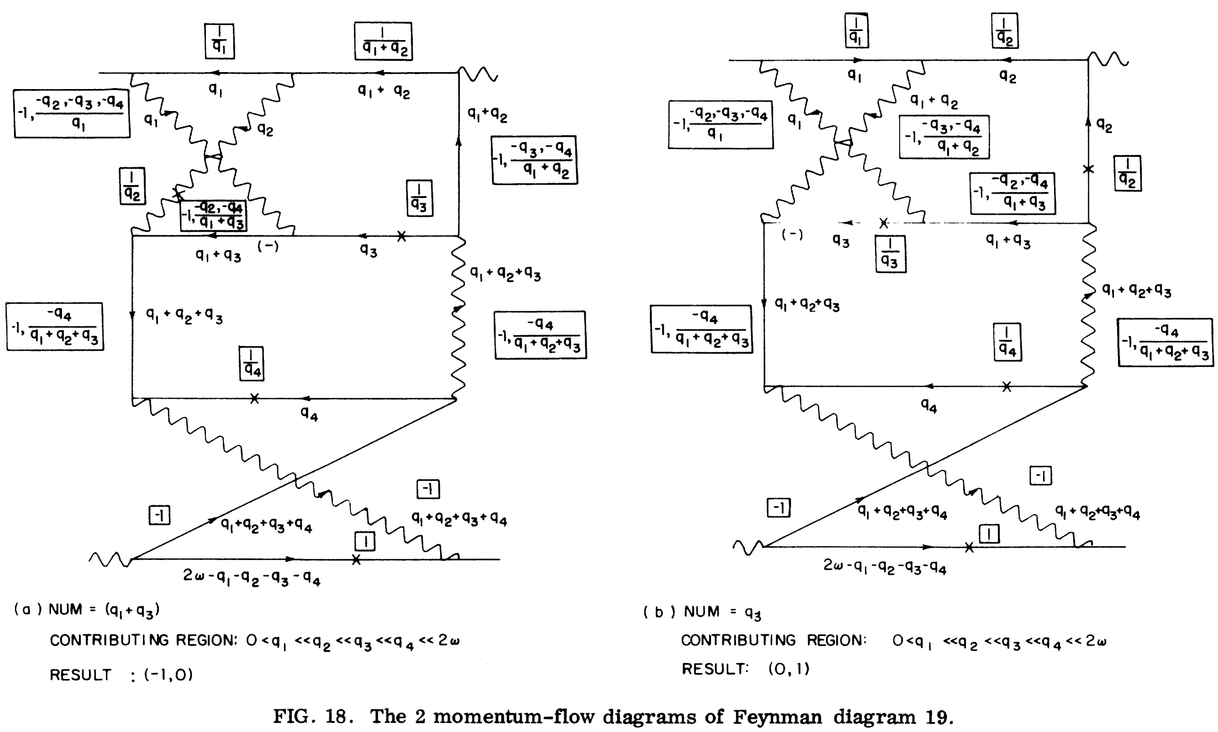


FIG. 18. The 2 momentum-flow diagrams of Feynman diagram 19.

where

$$D_{19} = (\vec{k}_{1\perp}^{\,2} + \lambda^2)(\vec{k}_{2\perp}^{\,2} + \lambda^2)(\vec{k}_{3\perp}^{\,2} + \lambda^2)(\vec{k}_{4\perp}^{\,2} + \lambda^2)[(\vec{k}_{1\perp} + \vec{k}_{2\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{3\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{4\perp} + 2\vec{r}_\perp)^2 + m^2]. \tag{4.18}$$

E. Feynman diagram 20

The numerator for Feynman diagram 20 is

$$\begin{aligned} N_{20} = -g^{10}\bar{u}(r_3 - r_1)\gamma_{\alpha_1}(\not{r}_3 - \not{r}_1 - \not{k}_4 + m)\gamma_\mu(-\not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_2}(-\not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_3} \\ \times(-\not{k}_2 - \not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_1}(-\not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_4}(-\not{k}_1 - \not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_2} \\ \times(-\not{k}_1 - \not{k}_2 - 2\not{r}_1 + m)\gamma_{\alpha_3}(-\not{k}_1 - 2\not{r}_1 + m)\gamma_\nu(\not{r}_2 - \not{k}_1 + m)\gamma_{\alpha_4}u(r_2 - r_1). \end{aligned} \tag{4.19}$$

This is approximated by

$$16(2\omega - k_{1+})(2\omega - k_{4-})(-k_{1+} - k_{2+})(-k_{3-} - k_{4-})\bar{u}(r_3 - r_1)\gamma_\mu \bar{N}_{20}\gamma_\nu u(r_2 - r_1), \tag{4.20}$$

where

$$\bar{N}_{20} = -g^{10}(-\vec{\not{k}}_{4\perp} - 2\vec{\not{r}}_\perp + m)(\vec{\not{k}}_{2\perp} + \vec{\not{k}}_{3\perp} + \vec{\not{k}}_{4\perp} + 2\vec{\not{r}}_\perp + m)(-\vec{\not{k}}_{2\perp} - \vec{\not{k}}_{3\perp} - 2\vec{\not{r}}_\perp + m)(\vec{\not{k}}_{1\perp} + \vec{\not{k}}_{2\perp} + \vec{\not{k}}_{3\perp} + 2\vec{\not{r}}_\perp + m)(-\vec{\not{k}}_{1\perp} - 2\vec{\not{r}}_\perp + m). \tag{4.21}$$

Using this numerator with the momentum-flow diagrams of Fig. 19, we find

$$\tilde{\mathfrak{M}}_{20}^{(4)} \doteq (-1,1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{20}D_{20}^{-1}, \tag{4.22}$$

where

$$D_{20} = (\vec{k}_{1\perp}^{\,2} + \lambda^2)(\vec{k}_{2\perp}^{\,2} + \lambda^2)(\vec{k}_{3\perp}^{\,2} + \lambda^2)(\vec{k}_{4\perp}^{\,2} + \lambda^2)[(\vec{k}_{1\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{2\perp} + \vec{k}_{3\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{4\perp} + 2\vec{r}_\perp)^2 + m^2]. \tag{4.23}$$

F. Feynman diagram 21

Feynman diagram 21 is essentially diagram 19 turned upside down. Therefore, we find from Sec. IV D that

$$\tilde{\mathfrak{M}}_{21}^{(4)} \doteq (-1,1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{21}D_{21}^{-1}, \tag{4.24}$$

where

$$\bar{N}_{21} = -g^{10}(-\vec{\not{k}}_{3\perp} - \vec{\not{k}}_{4\perp} - 2\vec{\not{r}}_\perp + m)(\vec{\not{k}}_{2\perp} + \vec{\not{k}}_{3\perp} + \vec{\not{k}}_{4\perp} + 2\vec{\not{r}}_\perp + m)(-\vec{\not{k}}_{2\perp} - 2\vec{\not{r}}_\perp + m)(\vec{\not{k}}_{1\perp} + \vec{\not{k}}_{2\perp} + 2\vec{\not{r}}_\perp + m)(-\vec{\not{k}}_{1\perp} - 2\vec{\not{r}}_\perp + m) \tag{4.25}$$

and

$$D_{21} = (\vec{k}_{1\perp}^{\,2} + \lambda^2)(\vec{k}_{2\perp}^{\,2} + \lambda^2)(\vec{k}_{3\perp}^{\,2} + \lambda^2)(\vec{k}_{4\perp}^{\,2} + \lambda^2)[(\vec{k}_{1\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{2\perp} + 2\vec{r}_\perp)^2 + m^2][(\vec{k}_{3\perp} + \vec{k}_{4\perp} + 2\vec{r}_\perp)^2 + m^2]. \tag{4.26}$$

G. Feynman diagram 22

The numerator for Feynman diagram 22 is

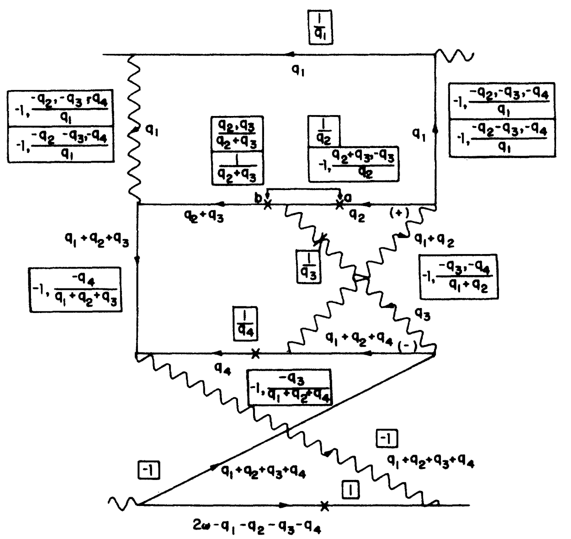
$$\begin{aligned} N_{22} = -g^{10}\bar{u}(r_3 - r_1)\gamma_{\alpha_1}(\not{r}_3 - \not{r}_1 - \not{k}_4 + m)\gamma_{\alpha_2}(\not{r}_3 - \not{r}_1 - \not{k}_3 - \not{k}_4 + m)\gamma_\mu \\ \times(-\not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_3}(-\not{k}_2 - \not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_1}(-\not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_4} \\ \times(-\not{k}_1 - \not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_2}(-\not{k}_1 - \not{k}_2 - 2\not{r}_1 + m)\gamma_\nu(\not{r}_2 - \not{r}_1 - \not{k}_1 - \not{k}_2 + m)\gamma_{\alpha_3} \\ \times(\not{r}_2 - \not{r}_1 - \not{k}_1 + m)\gamma_{\alpha_4}u(r_2 - r_1). \end{aligned} \tag{4.27}$$

This is approximated by

$$16(2\omega - k_{1+})(2\omega - k_{1+} - k_{2+})(2\omega - k_{3-} - k_{4-})(2\omega - k_{4-})\bar{u}(r_3 - r_1)\gamma_\mu\bar{N}_{22}\gamma_\nu u(r_2 - r_1), \tag{4.28}$$

where

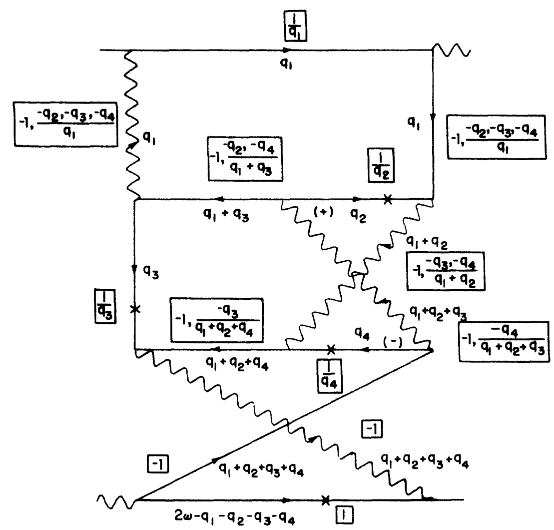
$$\begin{aligned} \bar{N}_{22} = -g^{10}(-\vec{\not{k}}_{3\perp} - \vec{\not{k}}_{4\perp} - 2\vec{\not{r}}_\perp + m)(\vec{\not{k}}_{2\perp} + \vec{\not{k}}_{3\perp} + \vec{\not{k}}_{4\perp} + 2\vec{\not{r}}_\perp + m)(-\vec{\not{k}}_{2\perp} - \vec{\not{k}}_{3\perp} - 2\vec{\not{r}}_\perp + m) \\ \times(\vec{\not{k}}_{1\perp} + \vec{\not{k}}_{2\perp} + \vec{\not{k}}_{3\perp} + 2\vec{\not{r}}_\perp + m)(-\vec{\not{k}}_{1\perp} - \vec{\not{k}}_{2\perp} - 2\vec{\not{r}}_\perp + m). \end{aligned} \tag{4.29}$$



(a) $\text{NUM} = -q_{2+}(q_{1-}+q_{2-}+q_{4-}) = \begin{cases} -\dfrac{q_1+q_2+q_4}{q_2} & \text{POLE a} \\ -(q_1+q_2+q_4)\max\left[\dfrac{1}{q_2+q_3}, -\dfrac{1}{q_3}\right] & \text{POLE b} \end{cases}$

POLE a: CONTRIBUTING REGION: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
RESULT : (-1,0)

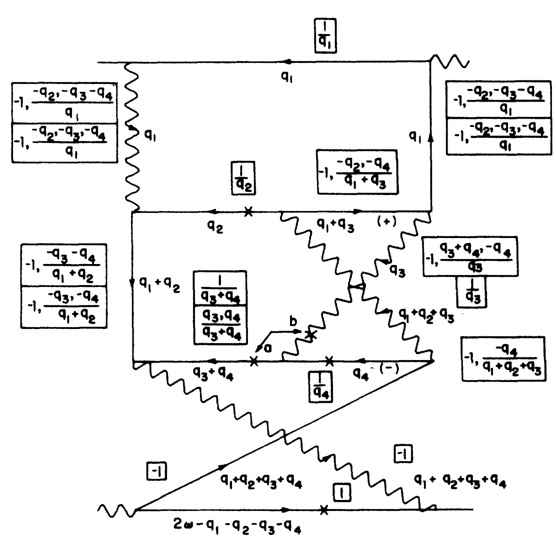
POLE b: NO CONTRIBUTION



(b) $\text{NUM} = q_{2+}q_{4-} = \dfrac{q_4}{q_2}$

CONTRIBUTING REGION: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

RESULT : (0,1)

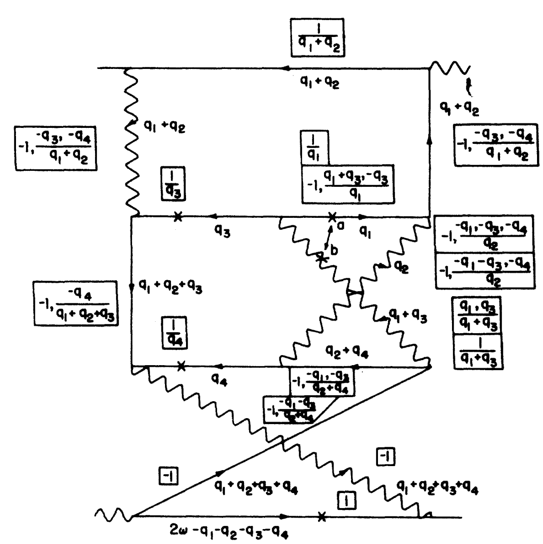


(c) $\text{NUM} = (q_{1+}+q_{3+})q_{4-} = -q_4 \max\left[\dfrac{1}{q_2}, \dfrac{1}{q_4}\right]$

POLE a: CONTRIBUTING REGION: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
RESULT: (-1,0)

POLE b: CONTRIBUTING REGION: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
RESULT: (1,0)

TOTAL RESULT: (0,0)



(d) $\text{NUM} = q_{1+}(q_{2-}+q_{4-})$

NO REGION CONTRIBUTES FOR EITHER POLE

FIG. 19. The 4 momentum-flow diagrams of Feynman diagram 20.

Using this numerator with the momentum-flow diagrams of Fig. 20, we find

$$\mathfrak{M}_{22}^{(4)} \doteq (-2,2)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{22}D_{22}^{-1}, \tag{4.30}$$

where

$$D_{22}=(\vec{k}_{1\perp}^{\,2}+\lambda^2)(\vec{k}_{2\perp}^{\,2}+\lambda^2)(\vec{k}_{3\perp}^{\,2}+\lambda^2)(\vec{k}_{4\perp}^{\,2}+\lambda^2)[(\vec{k}_{1\perp}+\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{3\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]. \tag{4.31}$$

H. Feynman diagram 23

The numerator for Feynman diagram 23 is

$$\begin{aligned}N_{23}=-g^{10}\bar{u}(r_3-r_1)\gamma_{\alpha_1}(\not{r}_3-\not{r}_1-\not{k}_4+m)\gamma_{\alpha_2}(\not{r}_3-\not{r}_1-\not{k}_2-\not{k}_4+m)\gamma_\mu(-\not{k}_2-\not{k}_4-2\not{r}_1+m)\gamma_{\alpha_3}(-\not{k}_2-\not{k}_3-\not{k}_4-2\not{r}_1+m)\gamma_{\alpha_1}\\ \times(-\not{k}_2-\not{k}_3-2\not{r}_1+m)\gamma_{\alpha_4}(-\not{k}_1-\not{k}_2-\not{k}_3-2\not{r}_1+m)\gamma_\nu(\not{r}_2-\not{r}_1-\not{k}_1-\not{k}_2-\not{k}_3+m)\gamma_{\alpha_3}\\ \times(\not{r}_2-\not{r}_1-\not{k}_1-\not{k}_2+m)\gamma_{\alpha_2}(\not{r}_2-\not{r}_1-\not{k}_1+m)\gamma_{\alpha_4}u(r_2-r_1).\end{aligned} \tag{4.32}$$

This is approximated by

$$\begin{aligned}16(2\omega-k_{1+})(2\omega-k_{1+}-k_{2+})(2\omega-k_{1+}-k_{2+}-k_{3+})(2\omega-k_{4-})(2\omega-k_{2-}-k_{4-})(-k_{1-}-k_{2-}-k_{3-})\\ \times\bar{u}(r_3-r_1)\gamma_\mu\bar{N}_{23}\gamma_\nu u(r_2-r_1),\end{aligned} \tag{4.33}$$

where

$$\bar{N}_{23}=-g^{10}(-\not{k}_{2\perp}-\not{k}_{4\perp}-2\not{r}_\perp+m)(\not{k}_{2\perp}+\not{k}_{3\perp}+\not{k}_{4\perp}+2\not{r}_\perp+m)(-\not{k}_{2\perp}-\not{k}_{3\perp}-2\not{r}_\perp+m). \tag{4.34}$$

Using the numerator with the momentum-flow diagrams of Fig. 21, we find

$$\mathfrak{M}_{23}^{(4)} \doteq (2,-2)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{23}D_{23}^{-1}, \tag{4.35}$$

where

$$D_{23}=(\vec{k}_{1\perp}^{\,2}+\lambda^2)(\vec{k}_{2\perp}^{\,2}+\lambda^2)(\vec{k}_{3\perp}^{\,2}+\lambda^2)(\vec{k}_{4\perp}^{\,2}+\lambda^2)[(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{2\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]. \tag{4.36}$$

I. Feynman diagram 24

The numerator for Feynman diagram 24 is

$$\begin{aligned}N_{24}=-g^{10}\bar{u}(r_3-r_1)\gamma_{\alpha_1}(\not{r}_3-\not{r}_1-\not{k}_4+m)\gamma_{\alpha_2}(\not{r}_3-\not{r}_1-\not{k}_2-\not{k}_4+m)\gamma_\mu\\ \times(-\not{k}_2-\not{k}_4-2\not{r}_1+m)\gamma_{\alpha_3}(-\not{k}_2-\not{k}_3-\not{k}_4-2\not{r}_1+m)\gamma_{\alpha_4}(-\not{k}_1-\not{k}_2-\not{k}_3-\not{k}_4-2\not{r}_1+m)\gamma_{\alpha_1}\\ \times(-\not{k}_1-\not{k}_2-\not{k}_3-2\not{r}_1+m)\gamma_{\alpha_3}(-\not{k}_1-\not{k}_2-2\not{r}_1+m)\gamma_\nu(\not{r}_2-\not{r}_1-\not{k}_1-\not{k}_2+m)\gamma_{\alpha_2}(\not{r}_2-\not{r}_1-\not{k}_1+m)\gamma_{\alpha_4}u(r_2-r_1).\end{aligned} \tag{4.37}$$

This is approximated as

$$\begin{aligned}16(2\omega-k_{1+})(2\omega-k_{1+}-k_{2+})(2\omega-k_{4-})(2\omega-k_{2-}-k_{4-})(-k_{2-}-k_{3-}-k_{4-})(-k_{1+}-k_{2+}-k_{3+})\\ \times\bar{u}(r_3-r_1)\gamma_\mu\bar{N}_{24}\gamma_\nu u(r_2-r_1),\end{aligned} \tag{4.38}$$

where

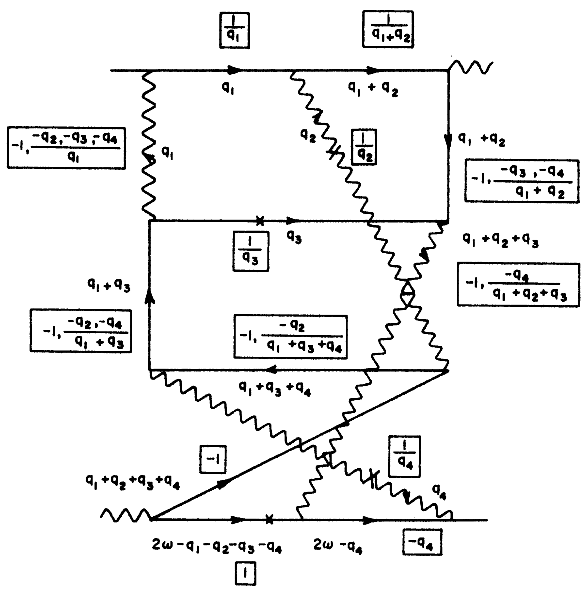
$$\bar{N}_{24}=-g^{10}(-\not{k}_{2\perp}-\not{k}_{4\perp}-2\not{r}_\perp+m)(\not{k}_{1\perp}+\not{k}_{2\perp}+\not{k}_{3\perp}+\not{k}_{4\perp}+2\not{r}_\perp+m)(-\not{k}_{1\perp}-\not{k}_{2\perp}-2\not{r}_\perp+m). \tag{4.39}$$

Using this numerator with the momentum-flow diagrams of Fig. 22, we find

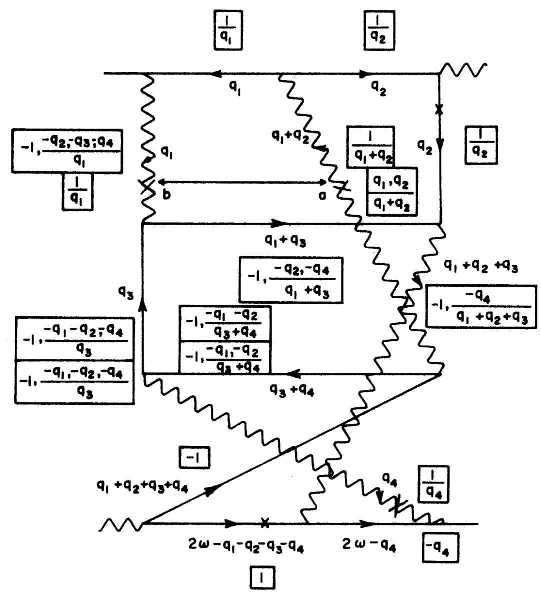
$$\mathfrak{M}_{24}^{(4)} \doteq (2,-2)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{24}D_{24}^{-1}, \tag{4.40}$$

where

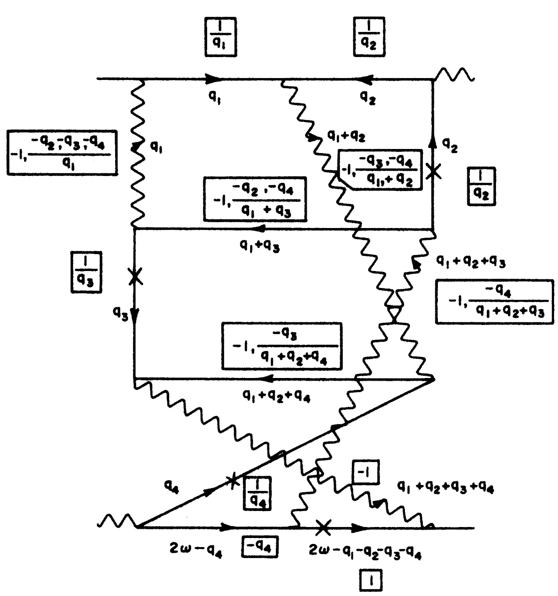
$$D_{24}=(\vec{k}_{1\perp}^{\,2}+\lambda^2)(\vec{k}_{2\perp}^{\,2}+\lambda^2)(\vec{k}_{3\perp}^{\,2}+\lambda^2)(\vec{k}_{4\perp}^{\,2}+\lambda^2)[(\vec{k}_{1\perp}+\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{2\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]. \tag{4.41}$$



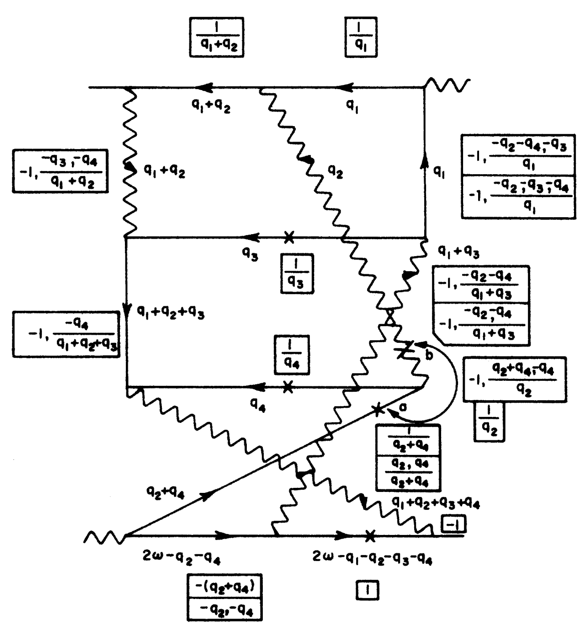
(a) NUM = 1
NO CONTRIBUTION



(b) NUM = 1
NO CONTRIBUTION

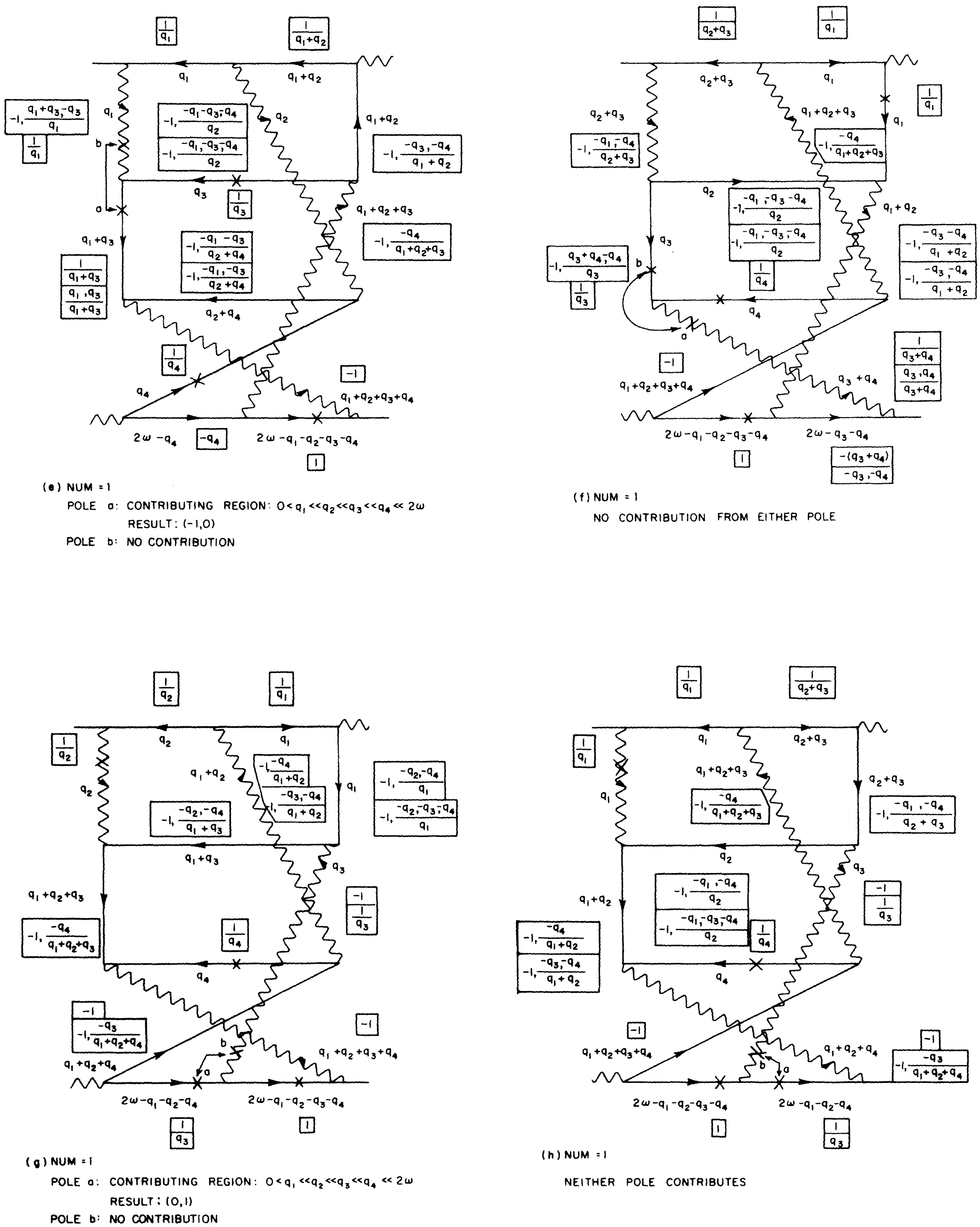


(c) NUM = 1
CONTRIBUTING REGION: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
RESULT: (0,1)



(d) NUM = 1
POLE a: CONTRIBUTING REGION: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
RESULT: (-1,0)
POLE b: NO CONTRIBUTION

FIG. 20. *(Continued on following page)*



(e) NUM = 1

POLE a: CONTRIBUTING REGION: $0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

RESULT: (−1,0)

POLE b: NO CONTRIBUTION

(f) NUM = 1

NO CONTRIBUTION FROM EITHER POLE

(g) NUM = 1

POLE a: CONTRIBUTING REGION: $0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

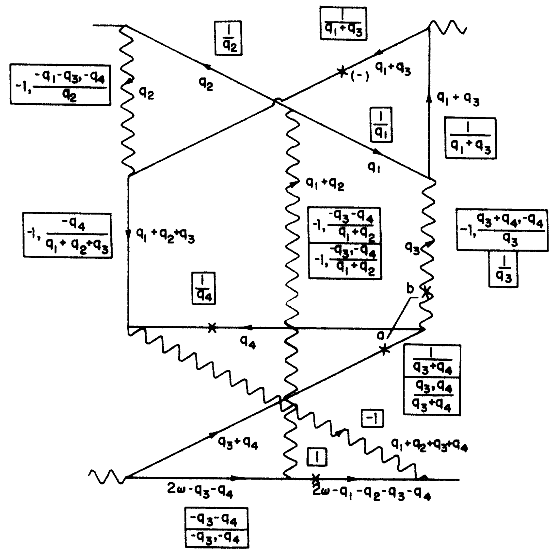
RESULT: (0,1)

POLE b: NO CONTRIBUTION

(h) NUM = 1

NEITHER POLE CONTRIBUTES

FIG. 20. The 8 momentum-flow diagrams of Feynman diagram 22.



(a) NUM = q_1+q_3

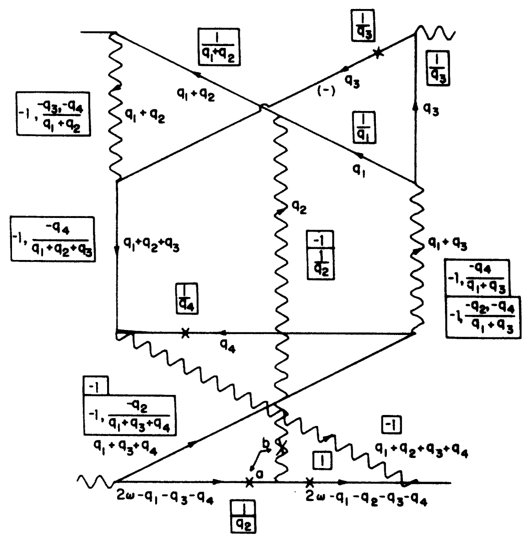
POLE a: CONTRIBUTING REGIONS:

$0<q_1\ll q_2\ll q_3\ll q_4\ll 2\omega$

$0<q_2\ll q_1\ll q_3\ll q_4\ll 2\omega$

RESULT: (-1,-1)

POLE b: NO CONTRIBUTION

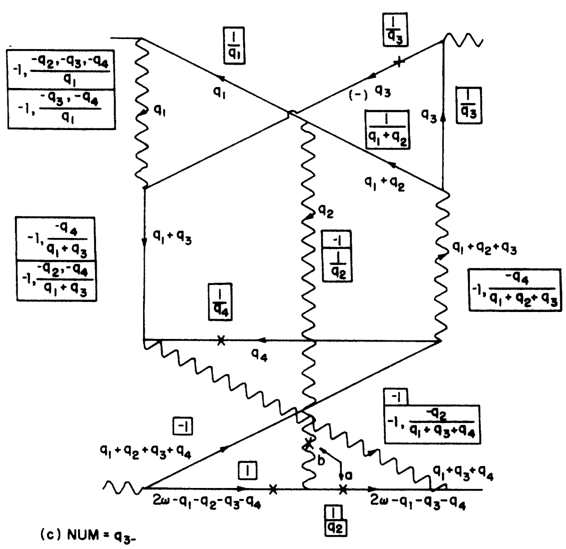


(b) NUM = q_3

POLE a: CONTRIBUTING REGION: $0<q_1\ll q_2\ll q_3\ll q_4\ll 2\omega$

RESULT: (1,0)

POLE b: NO CONTRIBUTION



(c) NUM = q_3-

POLE a: CONTRIBUTING REGIONS:

$0<q_1\ll q_2\ll q_3\ll q_4\ll 2\omega$

$0<q_1\ll q_3\ll q_2\ll q_4\ll 2\omega$

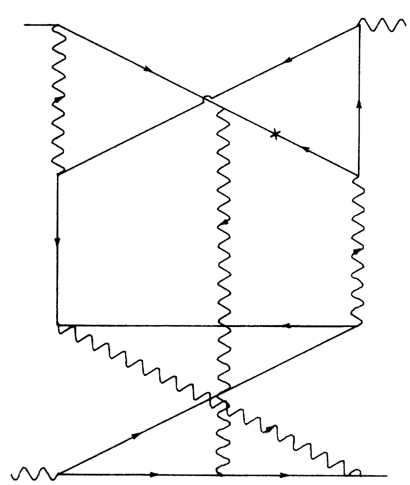
RESULT: (2,0)

POLE b: CONTRIBUTING REGION:

$0<q_1\ll q_3\ll q_2\ll q_4\ll 2\omega$

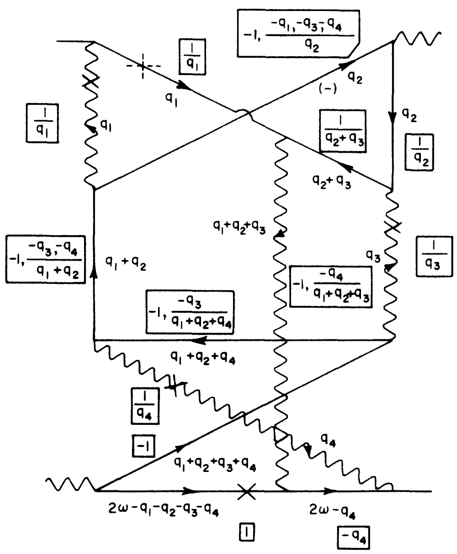
RESULT: (-1,0)

TOTAL RESULT: (1,0)



(d) NO CONTRIBUTION

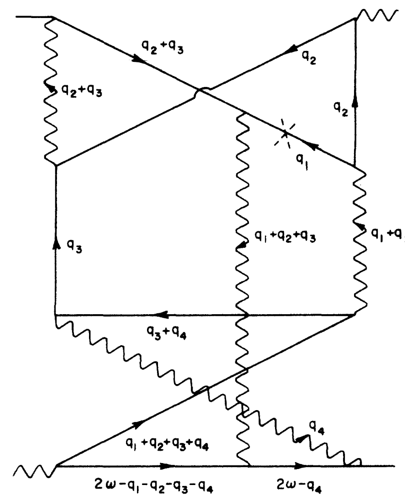
FIG. 21. *(Continued on following page)*



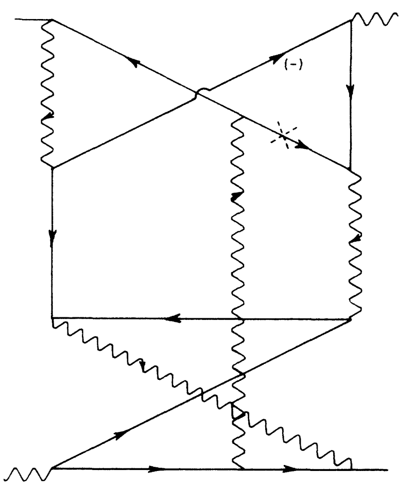
(e) NUM = $-q_2$

CONTRIBUTING REGION: $0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

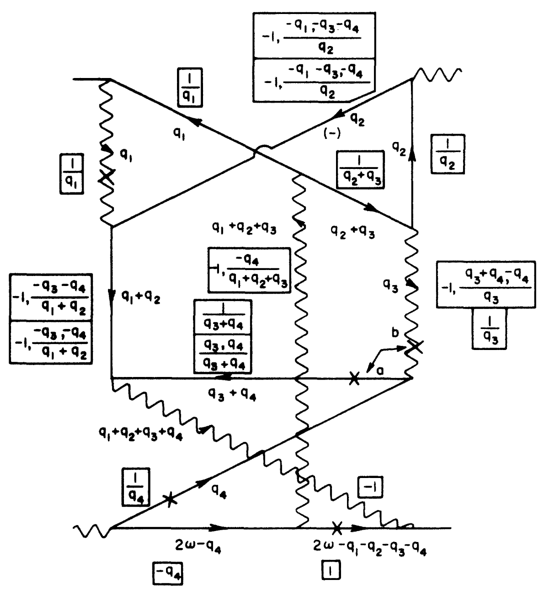
RESULT: (0,-1)



(f) NO CONTRIBUTION



(g) NO CONTRIBUTION



(h) NUM $= q_2$

POLE a: CONTRIBUTING REGION: $0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

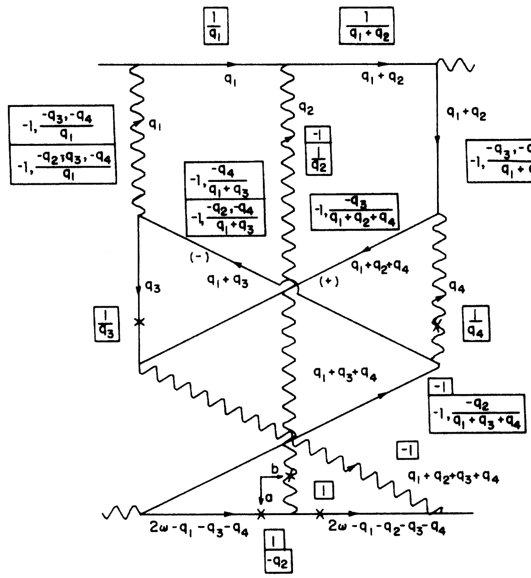
RESULT: (-1,0)

POLE b: CONTRIBUTING REGION: $0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

RESULT: (1,0)

TOTAL RESULT: (0,0)

FIG. 21. The 8 momentum-flow diagrams of Feynman diagram 23.



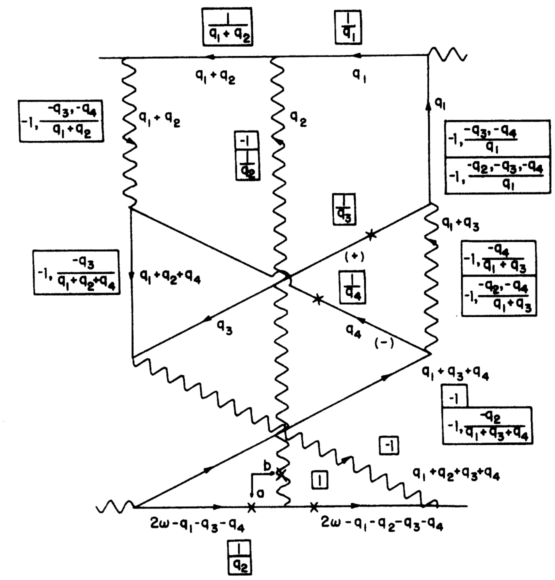
(a) NUM $= -(q_{1+}+q_{2+}+q_{4+})(q_{1-}+q_{3-}) - (q_{1+}+q_{3+})(q_{1-}+q_{2-}+q_{4-})$

POLE a: NUM $= \frac{q_1+q_3}{q_3} + \frac{q_1+q_2+q_4}{q_4}$

NO CONTRIBUTION

POLE b: NUM $= \frac{q_1+q_3}{q_3} + (q_1+q_2+q_4)\max\left[\frac{1}{q_2}, \frac{1}{q_4}\right]$

NO CONTRIBUTION



(b) NUM $= -q_{3+}q_{4-} - q_{4+}q_{3-} = -\frac{q_4}{q_3} - \frac{q_3}{q_4}$

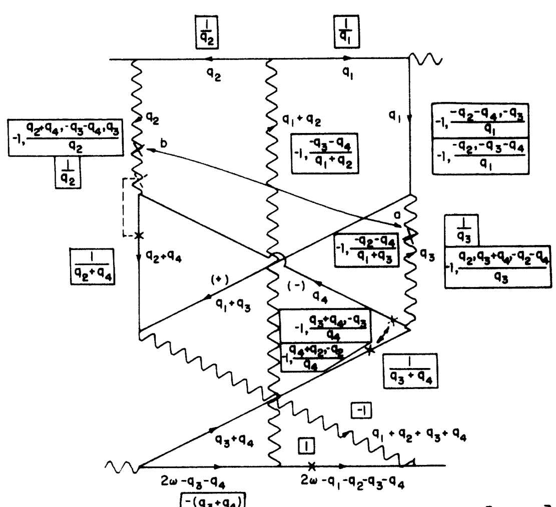
POLE a: CONTRIBUTING REGIONS:

$0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

$0 < q_1 \ll q_2 \ll q_4 \ll q_3 \ll 2\omega$

RESULT: (2,0)

POLE b: NO CONTRIBUTION



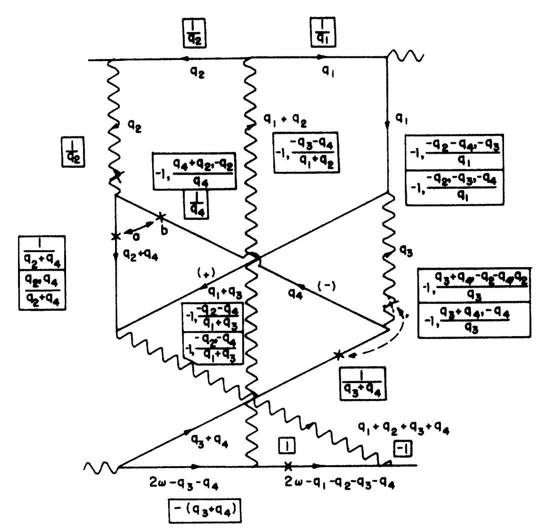
(c) POLE a: NUM $= -(q_{1+}+q_{3+})q_{4-} - q_{4+}(q_{1-}+q_{3-}) = \frac{q_4}{q_2+q_4} + (q_1+q_3)\max\left[\frac{1}{q_3}, \frac{-1}{q_3+q_4}\right]$

NO CONTRIBUTION

POLE b: NUM $= \frac{q_4}{q_2+q_4} + (q_1+q_3)\max\left(\frac{1}{q_2}, -\frac{1}{q_4+q_2}\right)$

CONTRIBUTING REGION: $0 < q_1 \ll q_2 \ll q_4 \ll q_3 \ll 2\omega$

RESULT: (0, 1)



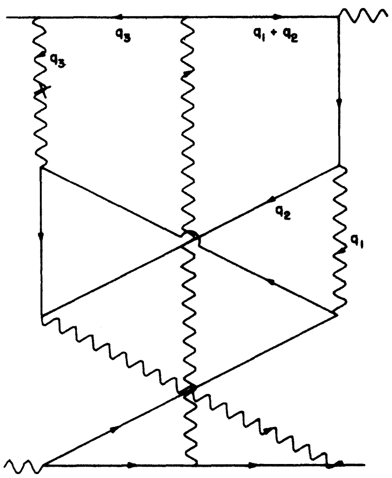
(c′) POLE a: NUM $= \frac{q_4}{q_2+q_4} + (q_1+q_3)\max\left(\frac{1}{q_2}, -\frac{1}{q_2+q_4}\right)$

CONTRIBUTING REGION: $0 < q_1 \ll q_2 \ll q_4 \ll q_3 \ll 2\omega$

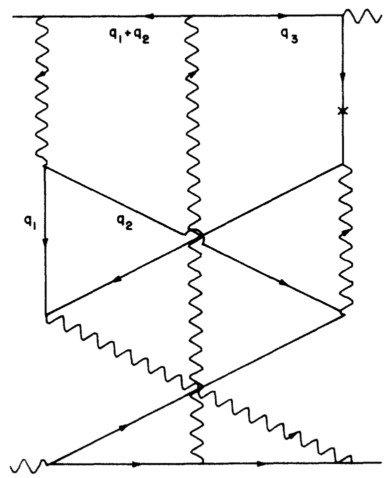
RESULT: (0,−1)

POLE b: NO CONTRIBUTION

FIG. 22. *(Continued on following page)*



(d) NO CONTRIBUTION



(e) NO CONTRIBUTION

FIG. 22. The 5 momentum-flow diagrams of Feynman diagram 24. Momentum-flow diagram (c) has been drawn twice in order to accommodate all the poles. In (c) the pole on q_2+q_4 has been used even though there is a second pole in that momentum circuit on q_2. In (c′) the pole on q_2 is treated.

J. Feynman diagram 25

Feynman diagram 25 is essentially diagram 23 turned upside down. Therefore, from Sec. IV H we find

$$\tilde{\mathfrak{M}}_{25}^{(4)} \doteq (2,-2)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{25}D_{25}{}^{-1}, \tag{4.42}$$

where

$$\bar{N}_{25} = -g^{10}(-\vec{\not{k}}_{2\perp}-\vec{\not{k}}_{3\perp}-2\vec{\not{r}}_{\perp}+m)(\vec{\not{k}}_{1\perp}+\vec{\not{k}}_{2\perp}+\vec{\not{k}}_{3\perp}+2\vec{\not{r}}_{\perp}+m)(-\vec{\not{k}}_{1\perp}-\vec{\not{k}}_{2\perp}-2\vec{\not{r}}_{\perp}+m) \tag{4.43}$$

and

$$D_{25} = (\vec{k}_{1\perp}{}^2+\lambda^2)(\vec{k}_{2\perp}{}^2+\lambda^2)(\vec{k}_{3\perp}+\lambda^2)(\vec{k}_{4\perp}{}^2+\lambda^2)[(\vec{k}_{1\perp}+\vec{k}_{2\perp}+2\vec{r}_{\perp})^2+m^2][(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_{\perp})^2+m^2]. \tag{4.44}$$

K. Feynman diagram 26

The numerator for Feynman diagram 26 is

$$\begin{aligned} N_{26} = &-g^{10}\bar{u}(r_3-r_1)\gamma_{\alpha_1}(\not{r}_3-\not{r}_1-\not{k}_4+m)\gamma_\mu(-\not{k}_4-2\not{r}_1+m)\gamma_{\alpha_2}(-\not{k}_3-\not{k}_4-2\not{r}_1+m)\gamma_{\alpha_3}(-\not{k}_2-\not{k}_3-\not{k}_4-2\not{r}_1+m)\gamma_{\alpha_4} \\ &\times(-\not{k}_1-\not{k}_2-\not{k}_3-\not{k}_4-2\not{r}_1+m)\gamma_{\alpha_1}(-\not{k}_1-\not{k}_2-\not{k}_3-2\not{r}_1+m)\gamma_{\alpha_2}(-\not{k}_1-\not{k}_2-2\not{r}_1+m)\gamma_\nu(\not{r}_2-\not{r}_1-\not{k}_1-\not{k}_2+m)\gamma_{\alpha_3} \\ &\times(\not{r}_2-\not{r}_1-\not{k}_1+m)\gamma_{\alpha_4}u(r_2-r_1). \end{aligned} \tag{4.45}$$

This is approximated by

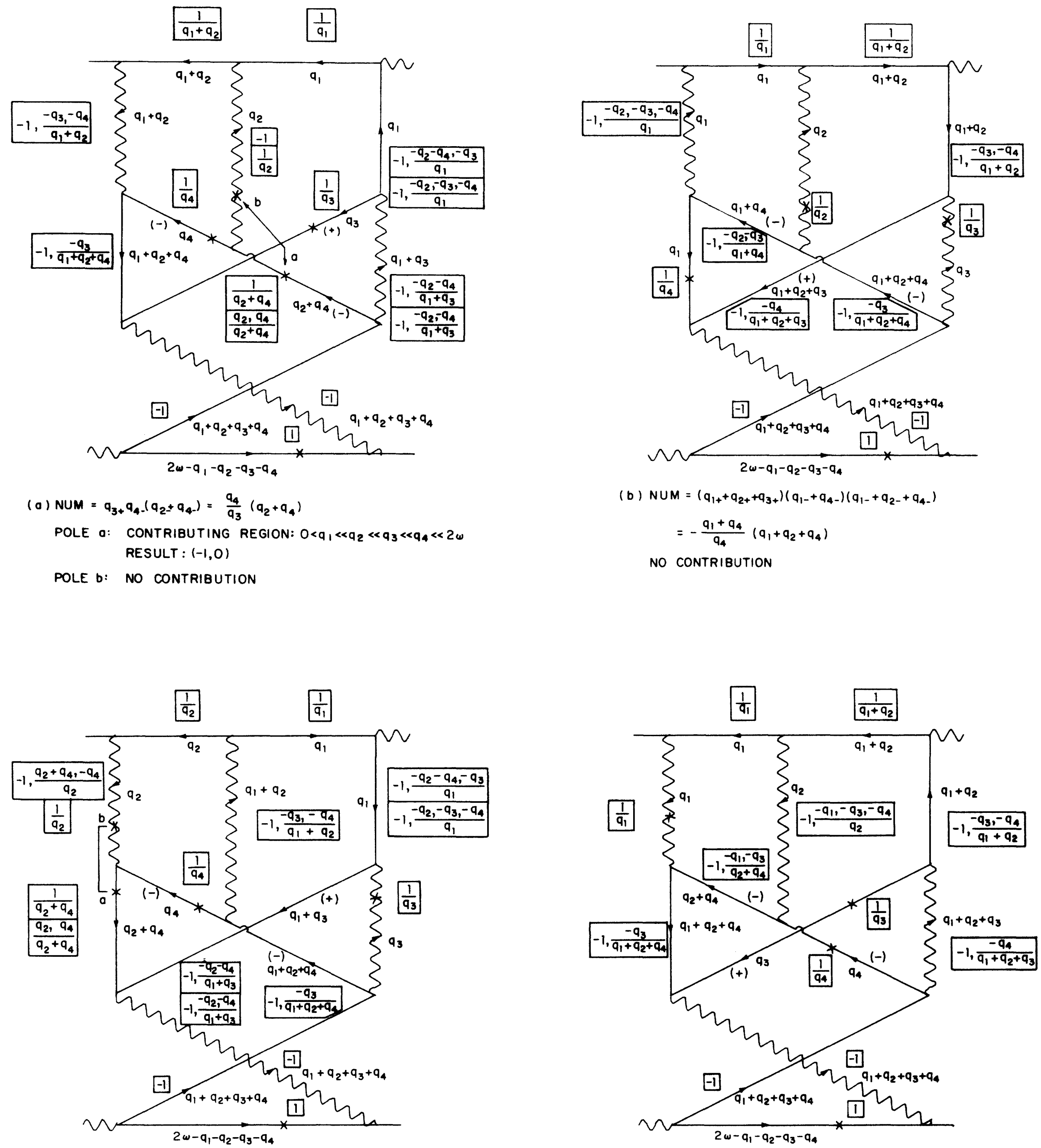
$$16(2\omega-k_{1+})(2\omega-k_{1+}-k_{2+})(2\omega-k_{4-})(-k_{1+}-k_{2+}-k_{3+})(-k_{2-}-k_{3-}-k_{4-})(-k_{3-}-k_{4-})\bar{u}(r_3-r_1)\gamma_\mu\bar{N}_{26}\gamma_\nu u(r_2-r_1), \tag{4.46}$$

where

$$\bar{N}_{26} = -g^{10}(-\vec{\not{k}}_{4\perp}-2\vec{\not{r}}_{\perp}+m)(\vec{\not{k}}_{1\perp}+\vec{\not{k}}_{2\perp}+\vec{\not{k}}_{3\perp}+\vec{\not{k}}_{4\perp}+2\vec{\not{r}}_{\perp}+m)(-\vec{\not{k}}_{1\perp}-\vec{\not{k}}_{2\perp}-2\vec{\not{r}}_{\perp}+m). \tag{4.47}$$

Using this numerator with the momentum-flow diagrams of Fig. 23, we find

$$\tilde{\mathfrak{M}}_{26}^{(4)} \doteq (-1,1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{26}D_{26}{}^{-1}, \tag{4.48}$$



(a) NUM $= q_{3+}q_{4-}(q_{2+}q_{4-}) = \frac{q_4}{q_3}(q_2+q_4)$

POLE a: CONTRIBUTING REGION: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

RESULT: $(-1,0)$

POLE b: NO CONTRIBUTION

(b) NUM $= (q_{1+}+q_{2+}+q_{3+})(q_{1-}+q_{4-})(q_{1-}+q_{2-}+q_{4-})$

$= -\frac{q_1+q_4}{q_4}(q_1+q_2+q_4)$

NO CONTRIBUTION

(c) NUM $= (q_{1+}+q_{3+})q_{4-}(q_{1-}+q_{2-}+q_{4-})$

POLE a: NO CONTRIBUTION

POLE b: NUM $= -q_4(q_1+q_2+q_4)\max\left(\frac{1}{q_2}, \frac{1}{q_4}\right)$

CONTRIBUTING REGION: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

RESULT: $(0,1)$

(d) NUM $= q_{3+}q_{4-}(q_{2-}+q_{4-}) = \frac{q_4}{q_3}(q_2+q_4)$

NO CONTRIBUTION

FIG. 23. The 4 momentum-flow diagrams for Feynman diagram 26.

where

$$D_{26}=(\vec{k}_{1\perp}^{\,2}+\lambda^2)(\vec{k}_{2\perp}^{\,2}+\lambda^2)(\vec{k}_{3\perp}^{\,2}+\lambda^2)(\vec{k}_{4\perp}^{\,2}+\lambda^2)\,[(\vec{k}_{1\perp}+\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2]\,[(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]. \tag{4.49}$$

L. Feynman diagram 27

Feynman diagram 27 is essentially diagram 26 turned upside down. Therefore, from Sec. IV J we find

$$\tilde{\mathfrak{M}}_{27}^{(4)} \doteq (-1,1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\,\overline{N}_{27}D_{27}^{-1}, \tag{4.50}$$

where

$$\overline{N}_{27}=-g^{10}(-\vec{\not k}_{4\perp}-\vec{\not k}_{3\perp}-2\vec{\not r}_\perp+m)(\vec{\not k}_{1\perp}+\vec{\not k}_{2\perp}+\vec{\not k}_{3\perp}+\vec{\not k}_{4\perp}+2\vec{\not r}_\perp+m)(-\vec{\not k}_{1\perp}-2\vec{\not r}_\perp+m) \tag{4.51}$$

and

$$D_{27}=(\vec{k}_{1\perp}^{\,2}+\lambda^2)(\vec{k}_{2\perp}^{\,2}+\lambda^2)(\vec{k}_{3\perp}^{\,2}+\lambda^2)(\vec{k}_{4\perp}^{\,2}+\lambda^2)\,[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2]\,[(\vec{k}_{3\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]. \tag{4.52}$$

M. Feynman diagram 28

The numerator for diagram 28 is

$$\begin{aligned}N_{28}=&-g^{10}\bar{u}(r_3-r_1)\gamma_{\alpha_1}(\not r_3-\not r_1-\not k_4+m)\gamma_\mu(-\not k_4-2\not r_1+m)\gamma_{\alpha_2}(-\not k_3-\not k_4-2\not r_1+m)\gamma_{\alpha_3}(-\not k_2-\not k_3-\not k_4-2\not r_1+m)\gamma_{\alpha_1}\\&\times(-\not k_2-\not k_3-2\not r_1+m)\gamma_{\alpha_4}(-\not k_1-\not k_2-\not k_3-2\not r_1+m)\gamma_\nu(\not r_2-\not k_1-\not k_2-\not k_3-\not r_1+m)\gamma_{\alpha_2}(\not r_2-\not k_1-\not k_2-\not r_1+m)\gamma_{\alpha_3}\\&\times(\not r_2-\not k_1-\not r_1+m)\gamma_{\alpha_4}u(r_2-r_1).\end{aligned} \tag{4.53}$$

This is approximated by

$$\begin{aligned}&16(2\omega-k_{1+})(2\omega-k_{1+}-k_{2+})(2\omega-k_{1+}-k_{2+}-k_{3+})\\&\times(2\omega-k_{4-})(-k_{1-}-k_{2-}-k_{3-})(-k_{3-}-k_{4-})\\&\times\bar{u}(r_3-r_1)\gamma_\mu\overline{N}_{28}\gamma_\nu u(r_2-r_1),\end{aligned} \tag{4.54}$$

where

$$\begin{aligned}\overline{N}_{28}=&-g^{10}(-\vec{\not k}_{4\perp}-2\vec{\not r}_\perp+m)(\vec{\not k}_{2\perp}+\vec{\not k}_{3\perp}+\vec{\not k}_{4\perp}+2\vec{\not r}_\perp+m)\\&\times(-\vec{\not k}_{2\perp}-\vec{\not k}_{3\perp}-2\vec{\not r}_\perp+m).\end{aligned} \tag{4.55}$$

Using this numerator with the momentum-flow diagrams of Fig. 24, we obtain

$$\tilde{\mathfrak{M}}_{28}^{(4)} \doteq (-1,1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\,\overline{N}_{28}D_{28}^{-1}, \tag{4.56}$$

where

$$\begin{aligned}D_{28}=&(\vec{k}_{1\perp}^{\,2}+\lambda^2)(\vec{k}_{2\perp}^{\,2}+\lambda^2)(\vec{k}_{3\perp}^{\,2}+\lambda^2)(\vec{k}_{4\perp}^{\,2}+\lambda^2)\\&\times[(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]\\&\times[(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2].\end{aligned} \tag{4.57}$$

N. Feynman diagram 29

Feynman diagram 29 is essentially diagram 28 turned upside down. Therefore, from Sec. IV L we find

$$\tilde{\mathfrak{M}}_{29}^{(4)} \doteq (-1,1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\,\overline{N}_{29}D_{29}^{-1} \tag{4.58}$$

where

$$\begin{aligned}\overline{N}_{29}=&-g^{10}(-\vec{\not k}_{2\perp}-\vec{\not k}_{3\perp}-2\vec{\not r}_\perp+m)\\&\times(\vec{\not k}_{1\perp}+\vec{\not k}_{2\perp}+\vec{\not k}_{3\perp}+2\vec{\not r}_\perp+m)(-\vec{\not k}_{1\perp}-2\vec{\not r}_\perp+m)\end{aligned} \tag{4.59}$$

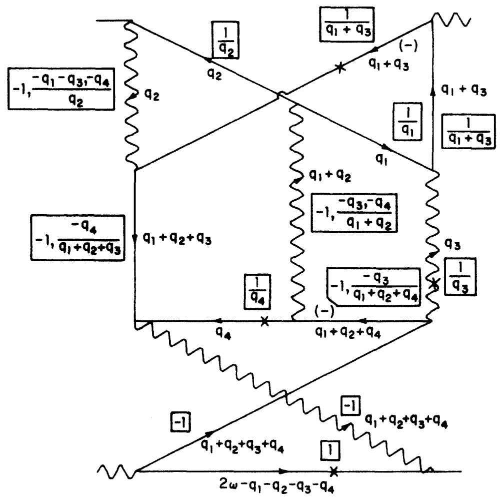
and

$$\begin{aligned}D_{29}=&(\vec{k}_{1\perp}^{\,2}+\lambda^2)(\vec{k}_{2\perp}^{\,2}+\lambda^2)(\vec{k}_{3\perp}^{\,2}+\lambda^2)(\vec{k}_{4\perp}^{\,2}+\lambda^2)\\&\times[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2]\\&\times[(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2].\end{aligned} \tag{4.60}$$

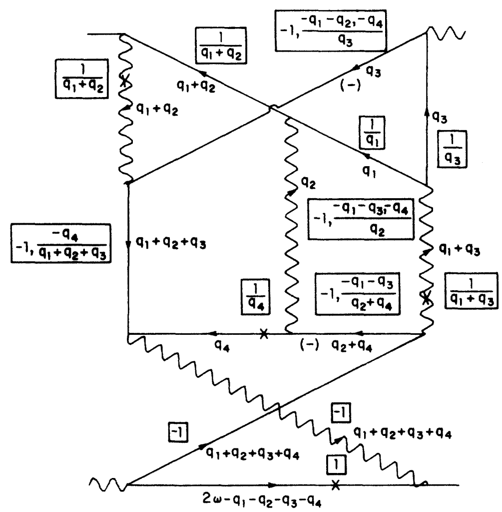
O. Feynman diagram 30

The numerator for diagram 30 is

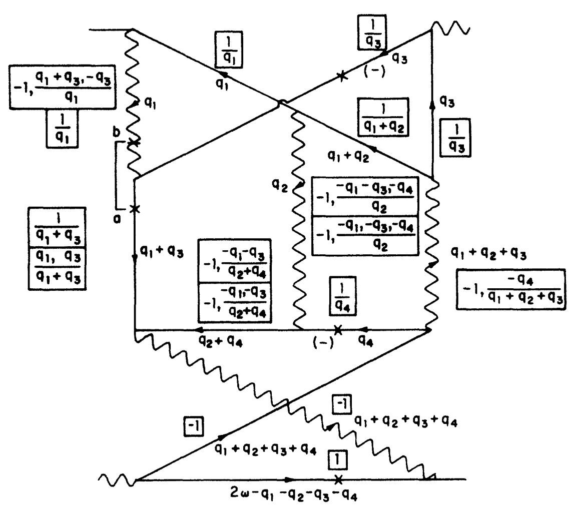
$$\begin{aligned}N_{30}=&-g^{10}\bar{u}(r_3-r_1)\gamma_{\alpha_1}(\not r_3-\not r_1-\not k_4+m)\gamma_{\alpha_2}(\not r_3-\not r_1-\not k_3-\not k_4+m)\gamma_\mu(-\not k_3-\not k_4-2\not r_1+m)\gamma_{\alpha_1}(-\not k_3-2\not r_1+m)\gamma_{\alpha_3}\\&\times(-\not k_2-\not k_3-2\not r_1+m)\gamma_{\alpha_4}(-\not k_1-\not k_2-\not k_3-2\not r_1+m)\gamma_{\alpha_2}(-\not k_1-\not k_2-2\not r_1+m)\gamma_\nu(\not r_2-\not r_1-\not k_1-\not k_2+m)\gamma_{\alpha_3}\\&\times(\not r_2-\not r_1-\not k_1+m)\gamma_{\alpha_4}u(r_2-r_1).\end{aligned} \tag{4.61}$$



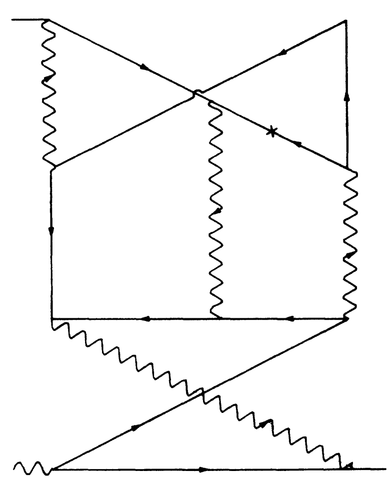
(a) NUM = $-(q_1+q_3)(q_1+q_2+q_4)$
CONTRIBUTING REGIONS:
$0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
$0 < q_2 \ll q_1 \ll q_3 \ll q_4 \ll 2\omega$
RESULT: (1,1)



(b) NUM = $-q_3(q_2+q_4)$
CONTRIBUTING REGION: $0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
RESULT: (-1,0)



(c) NUM = $-q_3 q_4$
POLE a: CONTRIBUTING REGION: $0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
RESULT: (-1,0)
POLE b: NO CONTRIBUTION



(d) NO CONTRIBUTION

FIG. 24. The 4 momentum-flow diagrams for Feynman diagram 28.

This is approximated as

$$16(2\omega - k_{1+})(2\omega - k_{1+} - k_{2+})(2\omega - k_{3-} - k_{4-}) \times (2\omega - k_{4-})(-k_{2-} - k_{3-})(-k_{3+} - k_{4+}) \times \bar{u}(r_3 - r_1)\gamma_\mu \bar{N}_{30}\gamma_\nu u(r_2 - r_1), \quad (4.62)$$

where

$$\bar{N}_{30} = -g^{10}(-\not{\vec{k}}_{3\perp} - 2\not{\vec{r}}_\perp + m)(\not{\vec{k}}_{1\perp} + \not{\vec{k}}_{2\perp} + \not{\vec{k}}_{3\perp} + 2\not{\vec{r}}_\perp + m) \times (-\not{\vec{k}}_{1\perp} - \not{\vec{k}}_{2\perp} - 2\not{\vec{r}}_\perp + m). \quad (4.63)$$

Using this numerator with the momentum-flow diagrams of Fig. 25, we find

$$\tilde{\mathfrak{M}}_{30}^{(4)} \doteq (1, -1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{30}D_{30}^{-1}, \quad (4.64)$$

where

$$D_{30} = (\vec{k}_{1\perp}^2 + \lambda^2)(\vec{k}_{2\perp}^2 + \lambda^2)(\vec{k}_{3\perp}^2 + \lambda^2)(\vec{k}_{4\perp}^2 + \lambda^2) \times [(\vec{k}_{1\perp} + \vec{k}_{2\perp} + 2\vec{r}_\perp)^2 + m^2]\,[(\vec{k}_{3\perp} + 2\vec{r}_\perp)^2 + m^2]. \quad (4.65)$$

P. Feynman diagram 31

Feynman diagram 31 is essentially diagram 30 turned upside down. Therefore, we find from Sec. IV N that

$$\tilde{\mathfrak{M}}_{31}^{(4)} \doteq (1, -1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{31}D_{31}^{-1}, \quad (4.66)$$

where

$$\bar{N}_{31} = -g^{10}(-\not{\vec{k}}_{3\perp} - \not{\vec{k}}_{4\perp} - 2\not{\vec{r}}_\perp + m) \times (\not{\vec{k}}_{2\perp} + \not{\vec{k}}_{3\perp} + \not{\vec{k}}_{4\perp} + 2\not{\vec{r}}_\perp + m)\,(-\not{\vec{k}}_{2\perp} - 2\not{\vec{r}}_\perp + m) \quad (4.67)$$

and

$$D_{31} = (\vec{k}_{1\perp}^2 + \lambda^2)(\vec{k}_{2\perp}^2 + \lambda^2)(\vec{k}_{3\perp}^2 + \lambda^2)(\vec{k}_{4\perp}^2 + \lambda^2) \times [(\vec{k}_{2\perp} + 2\vec{r}_\perp)^2 + m^2]\,[(\vec{k}_{3\perp} + \vec{k}_{4\perp} + 2\vec{r}_\perp)^2 + m^2]. \quad (4.68)$$

Q. Feynman diagram 32

The numerator for Feynman diagram 32 is

$$N_{32} = -g^{10}\bar{u}(r_3 - r_1)\gamma_{\alpha_1}(\not{r}_3 - \not{r}_1 - \not{k}_4 + m)\gamma_{\alpha_2}(\not{r}_3 - \not{r}_1 - \not{k}_3 - \not{k}_4 + m)\gamma_\mu(-\not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_3}(-\not{k}_2 - \not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_4} \times (-\not{k}_1 - \not{k}_2 - \not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_\nu(\not{r}_2 - \not{r}_1 - \not{k}_1 - \not{k}_2 - \not{k}_3 - \not{k}_4 + m)\gamma_{\alpha_1}(\not{r}_2 - \not{r}_1 - \not{k}_1 - \not{k}_3 + m)\gamma_{\alpha_2} \times (\not{r}_2 - \not{r}_1 - \not{k}_1 - \not{k}_2 + m)\gamma_{\alpha_3}(\not{r}_2 - \not{r}_1 - \not{k}_1 + m)\gamma_{\alpha_4}u(r_2 - r_1). \quad (4.69)$$

This is approximated as

$$16(2\omega - k_{1+})(2\omega - k_{1+} - k_{2+})(2\omega - k_{1+} - k_{2+} - k_{3+})(2\omega - k_{1+} - k_{2+} - k_{3+} - k_{4+}) \times (2\omega - k_{3-} - k_{4-})(2\omega - k_{4-})(-k_{1-} - k_{2-} - k_{3-} - k_{4-})(-k_{2-} - k_{3-} - k_{4-})(-k_{1-} - k_{2-} - k_{3-})\bar{u}(r_3 - r_1)\gamma_\mu\bar{N}_{32}\gamma_\nu u(r_2 - r_1), \quad (4.70)$$

where

$$\bar{N}_{32} = -g^{10}(-\not{\vec{k}}_{3\perp} - \not{\vec{k}}_{4\perp} - 2\not{\vec{r}}_\perp + m). \quad (4.71)$$

Using this numerator with the momentum-flow diagrams of Fig. 26, we find

$$\tilde{\mathfrak{M}}_{32}^{(4)} \doteq (-1, 1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{32}D_{32}^{-1}, \quad (4.72)$$

where

$$D_{32} = (\vec{k}_{1\perp}^2 + \lambda^2)(\vec{k}_{2\perp}^2 + \lambda^2)(\vec{k}_{3\perp}^2 + \lambda^2)(\vec{k}_{4\perp}^2 + \lambda^2) \times [(\vec{k}_{3\perp} + \vec{k}_{4\perp} + 2\vec{r}_\perp)^2 + m^2]. \quad (4.73)$$

R. Feynman diagram 33

Feynman diagram 33 is essentially diagram 32 turned upside down. Therefore, we find from Sec. IV P that

$$\tilde{\mathfrak{M}}_{33}^{(4)} \doteq (-1, 1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{33}D_{33}^{-1}, \quad (4.74)$$

where

$$\bar{N}_{33} = -g^{10}(-\not{\vec{k}}_{1\perp} - \not{\vec{k}}_{2\perp} - 2\not{\vec{r}}_\perp + m) \quad (4.75)$$

and

$$D_{33} = (\vec{k}_{1\perp}^2 + \lambda^2)(\vec{k}_{2\perp}^2 + \lambda^2)(\vec{k}_{3\perp}^2 + \lambda^2)(\vec{k}_{4\perp}^2 + \lambda^2) \times [(\vec{k}_{1\perp} + \vec{k}_{2\perp} + 2\vec{r}_\perp)^2 + m^2]. \quad (4.76)$$

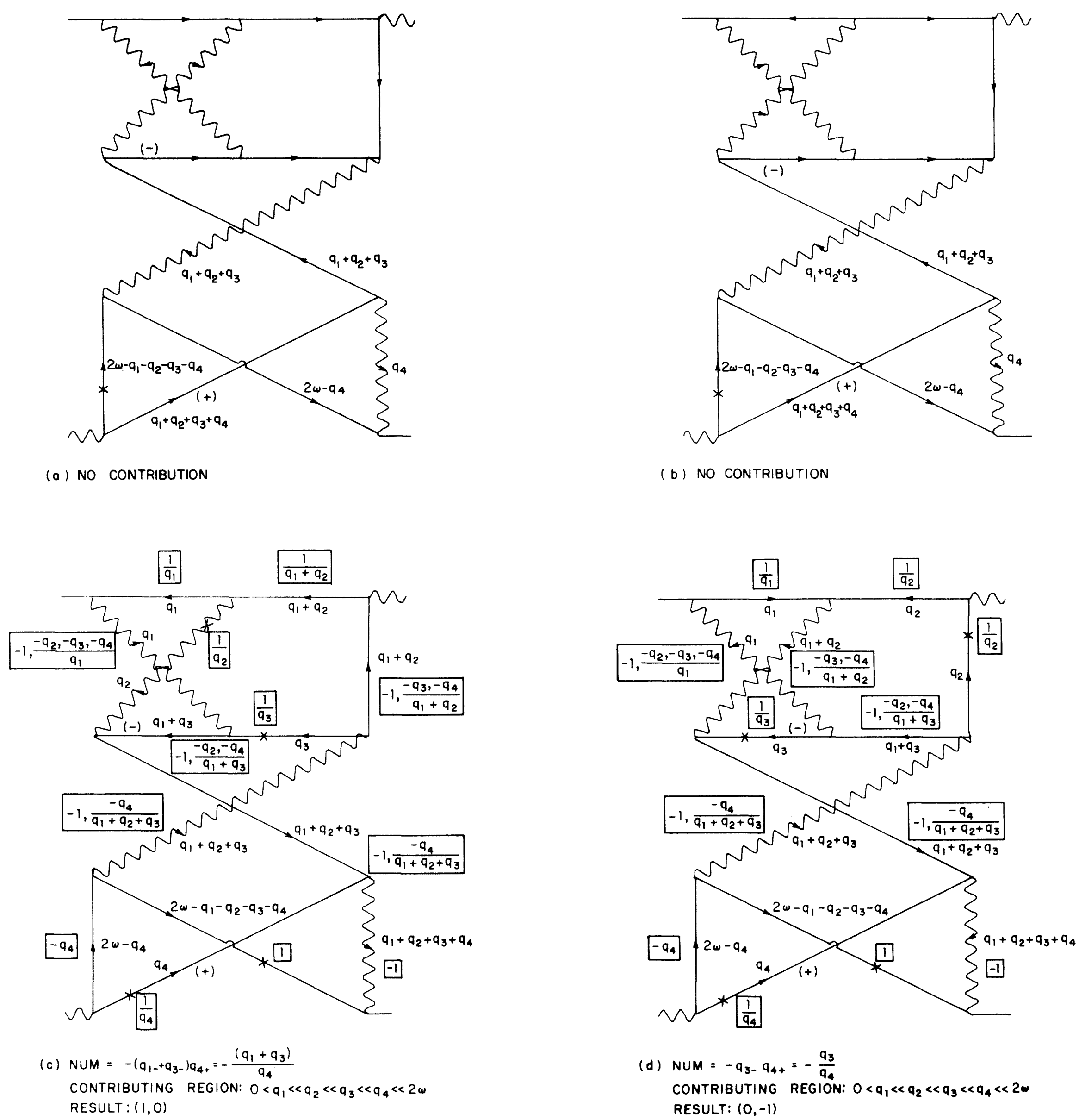


FIG. 25. The 4 momentum-flow diagrams for Feynman diagram 30.

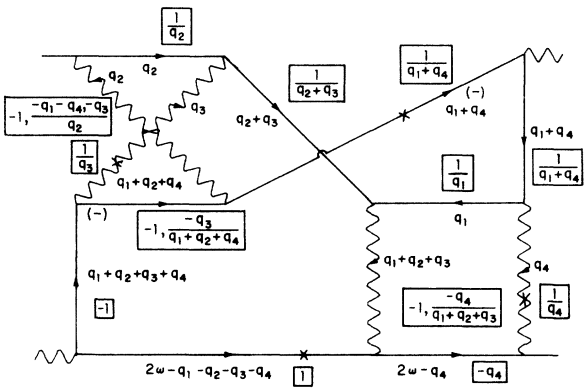
S. Feynman diagram 34

The numerator for Feynman diagram 34 is

$$N_{34} = -g^{10}\bar{u}(r_3 - r_1)\gamma_{\alpha_1}(\not{r}_3 - \not{r}_1 - \not{k}_4 + m)\gamma_{\alpha_2}(\not{r}_3 - \not{r}_1 - \not{k}_3 - \not{k}_4 + m)\gamma_{\alpha_3}(\not{r}_3 - \not{r}_1 - \not{k}_2 - \not{k}_3 - \not{k}_4 + m)\gamma_\mu$$
$$\times(-\not{k}_2 - \not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_1}(-\not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_4}(-\not{k}_1 - \not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_\nu(\not{r}_2 - \not{r}_1 - \not{k}_1 - \not{k}_2 - \not{k}_3 + m)\gamma_{\alpha_2}$$
$$\times(\not{r}_2 - \not{r}_1 - \not{k}_1 - \not{k}_2 + m)\gamma_{\alpha_3}(\not{r}_2 - \not{r}_1 - \not{k}_1 + m)\gamma_{\alpha_4}u(r_2 - r_1)\ . \qquad (4.77)$$

This is approximated as

$$16(2\omega - k_{1+})(2\omega - k_{1+} - k_{2+})(2\omega - k_{1+} - k_{2+} - k_{3+})(2\omega - k_{4-})(2\omega - k_{3-} - k_{4-})$$
$$\times(2\omega - k_{2-} - k_{3-} - k_{4-})(-k_{1-} - k_{2-} - k_{3-})(-k_{2+} - k_{3+} - k_{4+})\bar{u}(r_3 - r_1)\gamma_\mu \bar{N}_{34}\gamma_\nu u(r_2 - r_1)\ , \qquad (4.78)$$



(a) NUM = $(q_1+q_4)(q_1+q_2+q_4)$

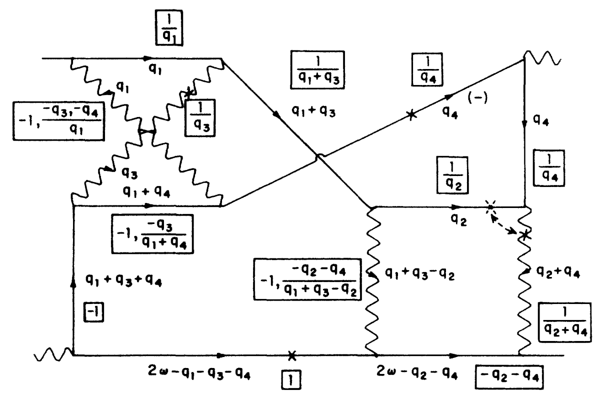
CONTRIBUTING REGIONS:

$0<q_1\ll q_2\ll q_3\ll q_4\ll 2\omega$

$0<q_2\ll q_1\ll q_3\ll q_4\ll 2\omega$

$0<q_2\ll q_3\ll q_1\ll q_4\ll 2\omega$

RESULT: (-1,-2)



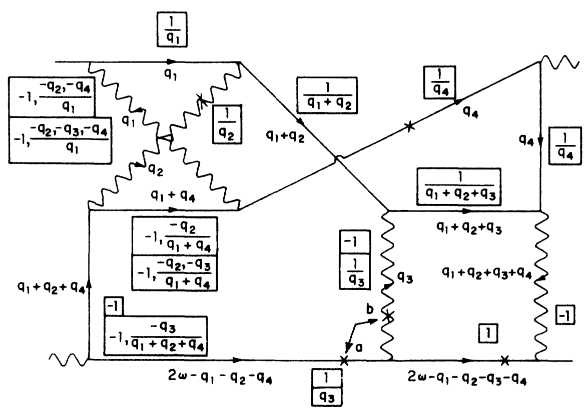
(b) NUM = $q_4(q_1+q_4)$

CONTRIBUTING REGIONS:

$0<q_1\ll q_2\ll q_3\ll q_4\ll 2\omega$

$0<q_2\ll q_1\ll q_3\ll q_4\ll 2\omega$

RESULT: (0,2)



(c) NUM = $q_4(q_1+q_4)$

POLE a : CONTRIBUTING REGIONS:

$0<q_1\ll q_2\ll q_3\ll q_4\ll 2\omega$

$0<q_1\ll q_2\ll q_4\ll q_3\ll 2\omega$

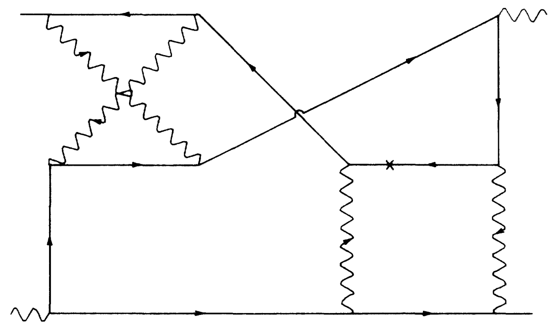
RESULT: (0,0)

POLE b : CONTRIBUTING REGION:

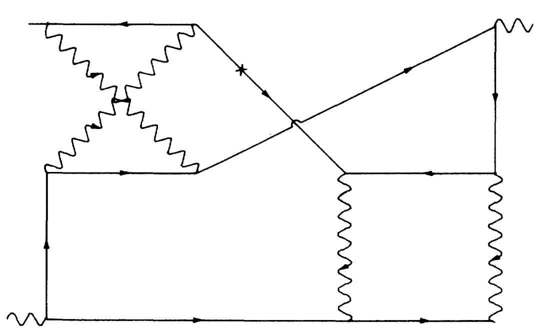
$0<q_1\ll q_2\ll q_4\ll q_3\ll 2\omega$

RESULT: (0,1)

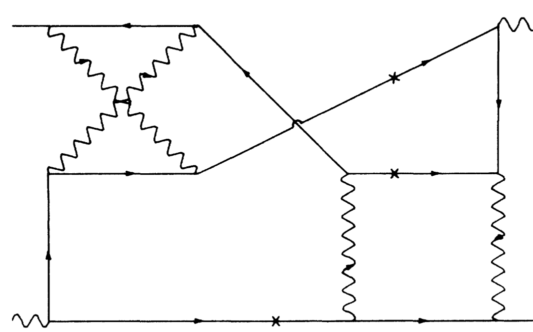
TOTAL RESULT: (0,1)



(d) NO CONTRIBUTION

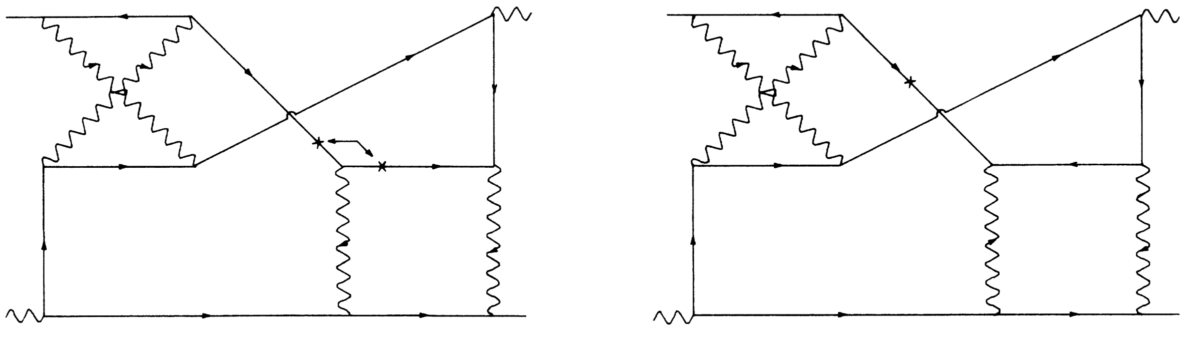


(e) NO CONTRIBUTION



(f) NO CONTRIBUTION

FIG. 26. *(Continued on following page)*



(g) NO CONTRIBUTION (h) NO CONTRIBUTION

FIG. 26. The 8 momentum-flow diagrams for Feynman diagram 32.

where

$$\bar{N}_{34} = -g^{10}(-\not{\vec{k}}_{2\perp} - \not{\vec{k}}_{3\perp} - 2\not{\vec{r}}_{\perp} + m). \tag{4.79}$$

Using this numerator with the momentum-flow diagrams of Fig. 27, we find

$$\mathfrak{M}_{34}^{(4)} \doteq (1, -1)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{34}D_{34}^{-1}, \tag{4.80}$$

where

$$D_{34} = (\vec{k}_{1\perp}^{\,2} + \lambda^2)(\vec{k}_{2\perp}^{\,2} + \lambda^2)(\vec{k}_{3\perp}^{\,2} + \lambda^2)(\vec{k}_{4\perp}^{\,2} + \lambda^2)[(\vec{k}_{2\perp} + \vec{k}_{3\perp} + 2\vec{r}_\perp)^2 + m^2]. \tag{4.81}$$

T. Feynman diagram 35

The numerator for Feynman diagram 35 is

$$\begin{aligned} N_{35} = -g^{10}\bar{u}(r_3 - r_1)\gamma_{\alpha_1}(\not{r}_3 - \not{r}_1 - \not{k}_4 + m)\gamma_{\alpha_2}(\not{r}_3 - \not{r}_1 - \not{k}_2 - \not{k}_4 + m)\gamma_{\alpha_3}(\not{r}_3 - \not{r}_1 - \not{k}_2 - \not{k}_3 - \not{k}_4 + m)\gamma_\mu \\ \times(-\not{k}_2 - \not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_1}(-\not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_{\alpha_4}(-\not{k}_1 - \not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_\nu(\not{r}_2 - \not{k}_1 - \not{k}_2 - \not{k}_3 - \not{r}_1 + m)\gamma_{\alpha_3} \\ \times(\not{r}_2 - \not{k}_1 - \not{k}_2 - \not{r}_1 + m)\gamma_{\alpha_2}(\not{r}_2 - \not{k}_1 - \not{r}_1 + m)\gamma_{\alpha_4}u(r_2 - r_1). \end{aligned} \tag{4.82}$$

This is approximated by

$$\begin{aligned} 16(2\omega - k_{1+})(2\omega - k_{1+} - k_{2+})(2\omega - k_{1+} - k_{2+} - k_{3+})(2\omega - k_{4-})(2\omega - k_{2-} - k_{4-}) \\ \times(2\omega - k_{2-} - k_{3-} - k_{4-})(-k_{1-} - k_{2-} - k_{3-})(-k_{2+} - k_{3+} - k_{4+})\bar{u}(r_3 - r_1)\gamma_\mu\bar{N}_{34}\gamma_\nu u(r_2 - r_1), \end{aligned} \tag{4.83}$$

where $\bar{N}_{34}$ is given by (4.79). Using this numerator with the momentum-flow diagrams of Fig. 28 we find

$$\mathfrak{M}_{35}^{(4)} \doteq -2\mathfrak{M}_{34}^{(4)}. \tag{4.84}$$

V. DIAGRAMS WHICH CANCEL IN PAIRS

In Fig. 2 we display the six Feynman diagrams which, to leading order, cancel in pairs. The pair 38 and 39 is essentially 36 and 37 upside down. Therefore, we need only consider the pairs 36, 37 and 40, 41. These will be discussed in separate subsections.

A. Feynman diagrams 36 and 37

The numerator for Feynman diagram 36 is

$$\begin{aligned} N_{36} = -g^{10}\bar{u}(r_3 - r_1)\gamma_{\alpha_1}(\not{r}_3 - \not{r}_1 - \not{k}_4 + m)\gamma_{\alpha_2}(\not{r}_3 - \not{r}_1 - \not{k}_3 - \not{k}_4 + m)\gamma_\mu(-\not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_3}(-\not{k}_2 - \not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_4} \\ \times(-\not{k}_1 - \not{k}_2 - \not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_1}(-\not{k}_1 - \not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_\nu(\not{r}_2 - \not{k}_1 - \not{k}_2 - \not{r}_1 + m)\gamma_{\alpha_3} \\ \times(\not{r}_2 - \not{k}_1 - \not{r}_1 + m)\gamma_{\alpha_4}u(r_2 - r_1). \end{aligned} \tag{5.1}$$

This is approximated by

$$16(2\omega-k_{1+})(2\omega-k_{1+}-k_{2+})(2\omega-k_{1+}-k_{2+}-k_{3+})(2\omega-k_{4-})(2\omega-k_{3-}-k_{4-})(-k_{2-}-k_{3-}-k_{4-})$$
$$\times\bar{u}(r_3-r_1)\gamma_\mu\bar{N}_{36}\gamma_\nu u(r_2-r_1), \quad (5.2)$$

where

$$\bar{N}_{36}=-g^{10}(-\vec{k}_{3\perp}-\vec{k}_{4\perp}-2\vec{r}_\perp+m)(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp+m)(-\vec{k}_{1\perp}-\vec{k}_{2\perp}-\vec{k}_{3\perp}-2\vec{r}_\perp+m). \quad (5.3)$$

Using this numerator with the momentum-flow diagrams of Fig. 29, we find that

$$\tilde{\mathfrak{M}}_{36}^{(4)} \doteq (2,-2)\int\frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{36}D_{36}^{-1}, \quad (5.4)$$

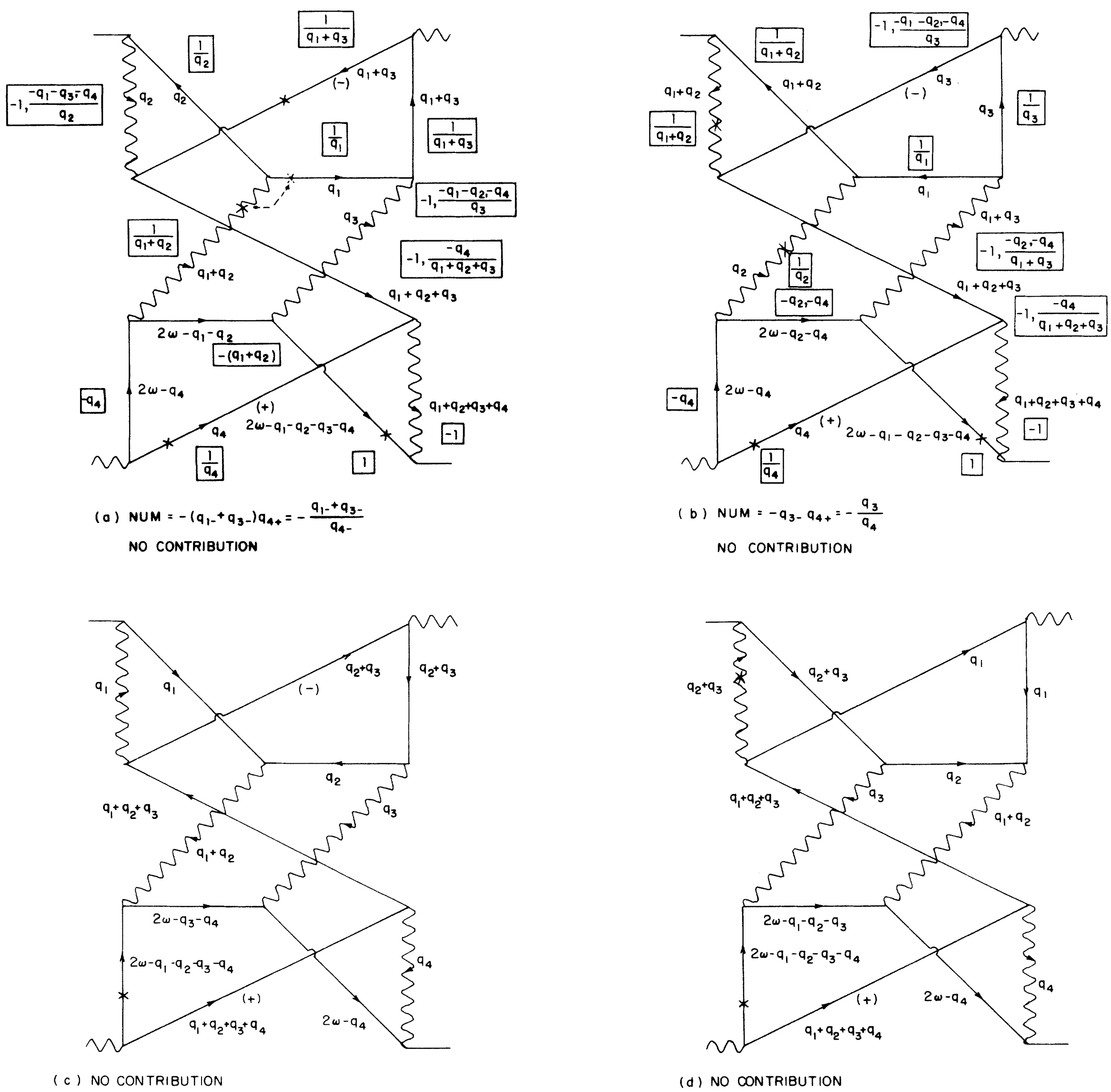


FIG. 27. *(Continued on following page)*

where

$$D_{36}=(\vec{k}_{1\perp}{}^2+\lambda^2)(\vec{k}_{2\perp}{}^2+\lambda^2)(\vec{k}_{3\perp}{}^2+\lambda^2)(\vec{k}_{4\perp}{}^2+\lambda^2)[(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{3\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]. \quad (5.5)$$

The numerator for Feynman diagram 37 is

$$N_{37}=-g^{10}\bar{u}(r_3-r_1)\gamma_{\alpha_1}(\not{r}_3-\not{r}_1-\not{k}_4+m)\gamma_{\alpha_2}(\not{r}_3-\not{r}_1-\not{k}_3-\not{k}_4+m)\gamma_\mu(-\not{k}_3-\not{k}_4-2\not{r}_1+m)\gamma_{\alpha_3}(-\not{k}_2-\not{k}_3-\not{k}_4-2\not{r}_1+m)\gamma_{\alpha_4}$$
$$\times(-\not{k}_1-\not{k}_2-\not{k}_3-\not{k}_4-2\not{r}_1+m)\gamma_{\alpha_1}(-\not{k}_1-\not{k}_2-\not{k}_3-2\not{r}_1+m)\gamma_\nu$$
$$\times(\not{r}_2-\not{r}_1-\not{k}_1-\not{k}_2-\not{k}_3+m)\gamma_{\alpha_3}(\not{r}_2-\not{r}_1-\not{k}_1-\not{k}_3+m)\gamma_{\alpha_2}(\not{r}_2-\not{r}_1-\not{k}_1+m)\gamma_{\alpha_4}u(r_2-r_1), \quad (5.6)$$

which is approximated by

$$16(2\omega-k_{1+})(2\omega-k_{1+}-k_{3+})(2\omega-k_{1+}-k_{2+}-k_{3+})(2\omega-k_{4-})(2\omega-k_{3-}-k_{4-})(-k_{2-}-k_{3-}-k_{4-})$$
$$\times\bar{u}(r_3-r_1)\gamma_\mu\bar{N}_{36}\gamma_\nu u(r_2-r_1). \quad (5.7)$$

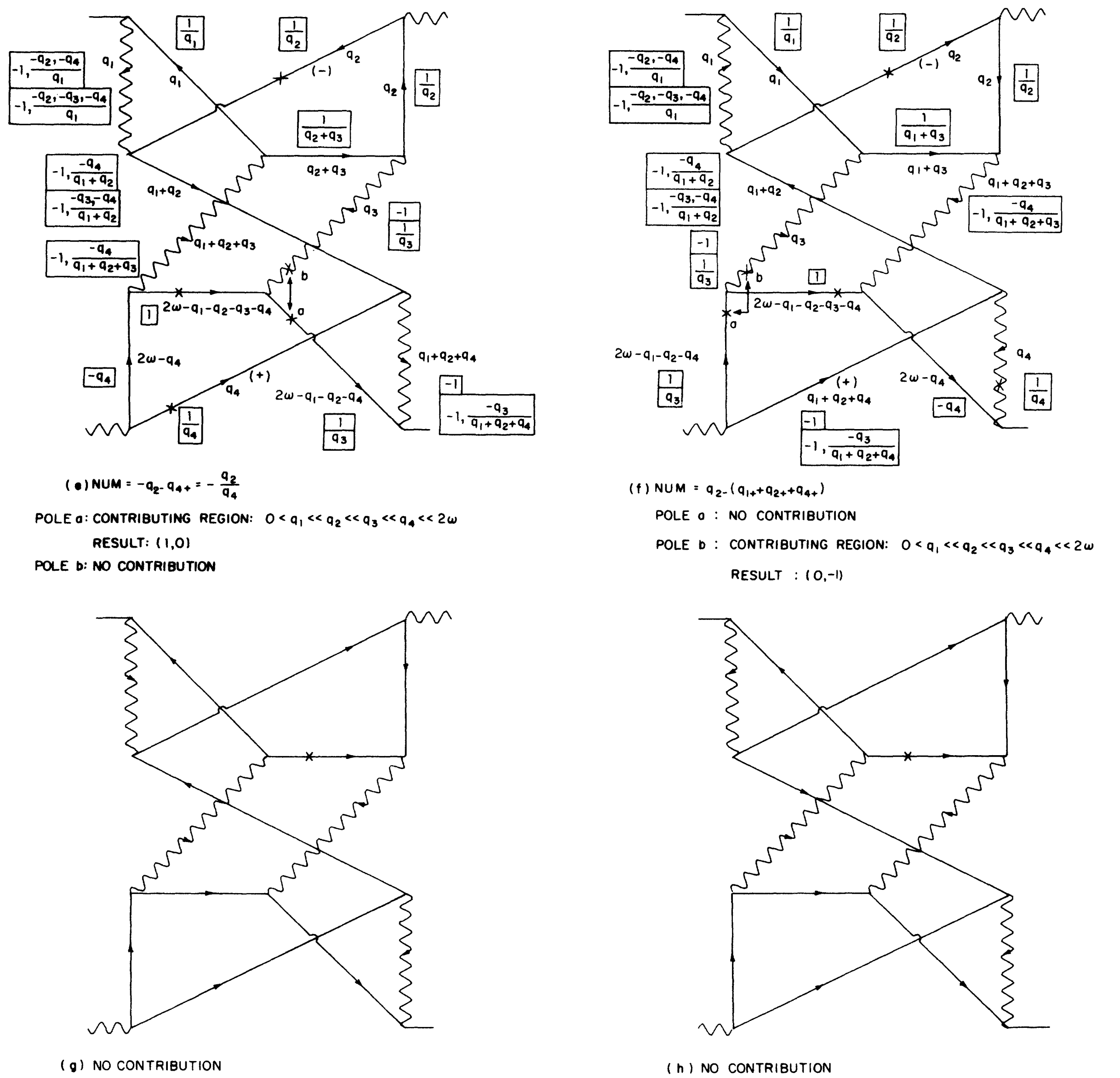
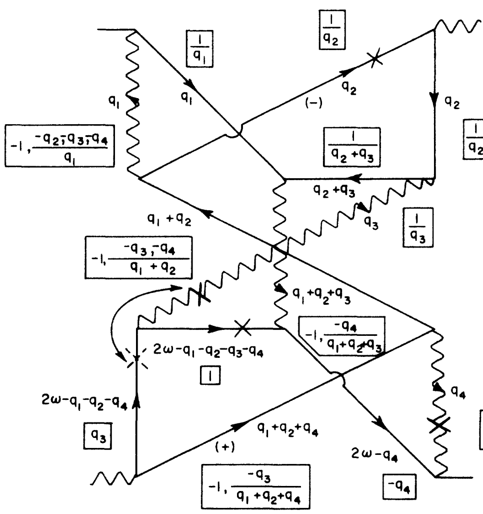


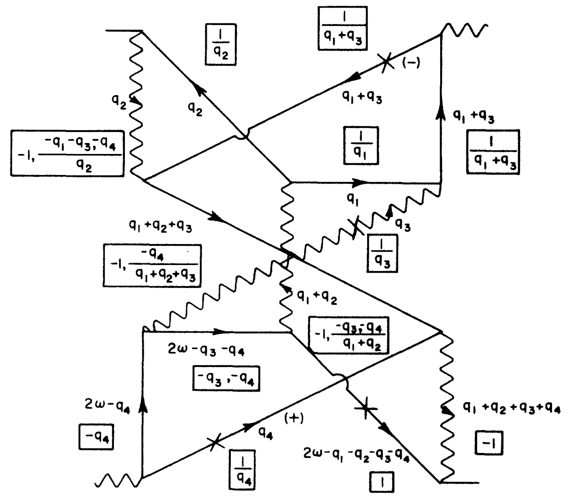
FIG. 27. The 8 momentum-flow diagrams for Feynman diagram 34.



(a) NUM = $q_{2-}(q_{1+}+q_{2+}+q_{4+}) = -\frac{q_2}{q_3}$

CONTRIBUTING REGION: $0 \ll q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

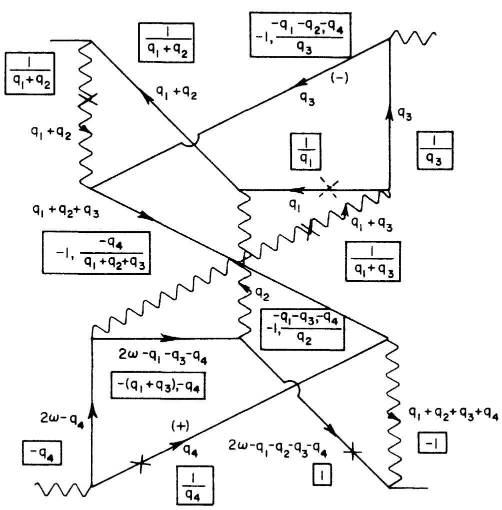
RESULT: (0,1)



(b) NUM = $-(q_{1-}+q_{3-})q_{4+} = -\frac{(q_1+q_3)}{q_4}$

CONTRIBUTING REGIONS: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
$0<q_2 \ll q_1 \ll q_3 \ll q_4 \ll 2\omega$

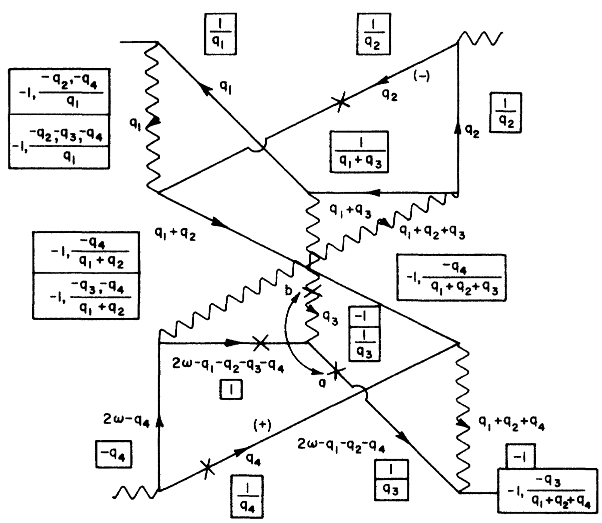
RESULT: (1,1)



(c) NUM = $-q_{3-}\, q_{4+} = -\frac{q_3}{q_4}$

CONTRIBUTING REGION: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

RESULT: (−1,0)



(d) NUM = $-q_{2-}q_{4+} = -\frac{q_2}{q_4}$

POLE a: CONTRIBUTING REGIONS: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
$0<q_1 \ll q_3 \ll q_2 \ll q_4 \ll 2\omega$

RESULT: (−2,0)

POLE b: NO CONTRIBUTION

FIG. 28. *(Continued on following page)*

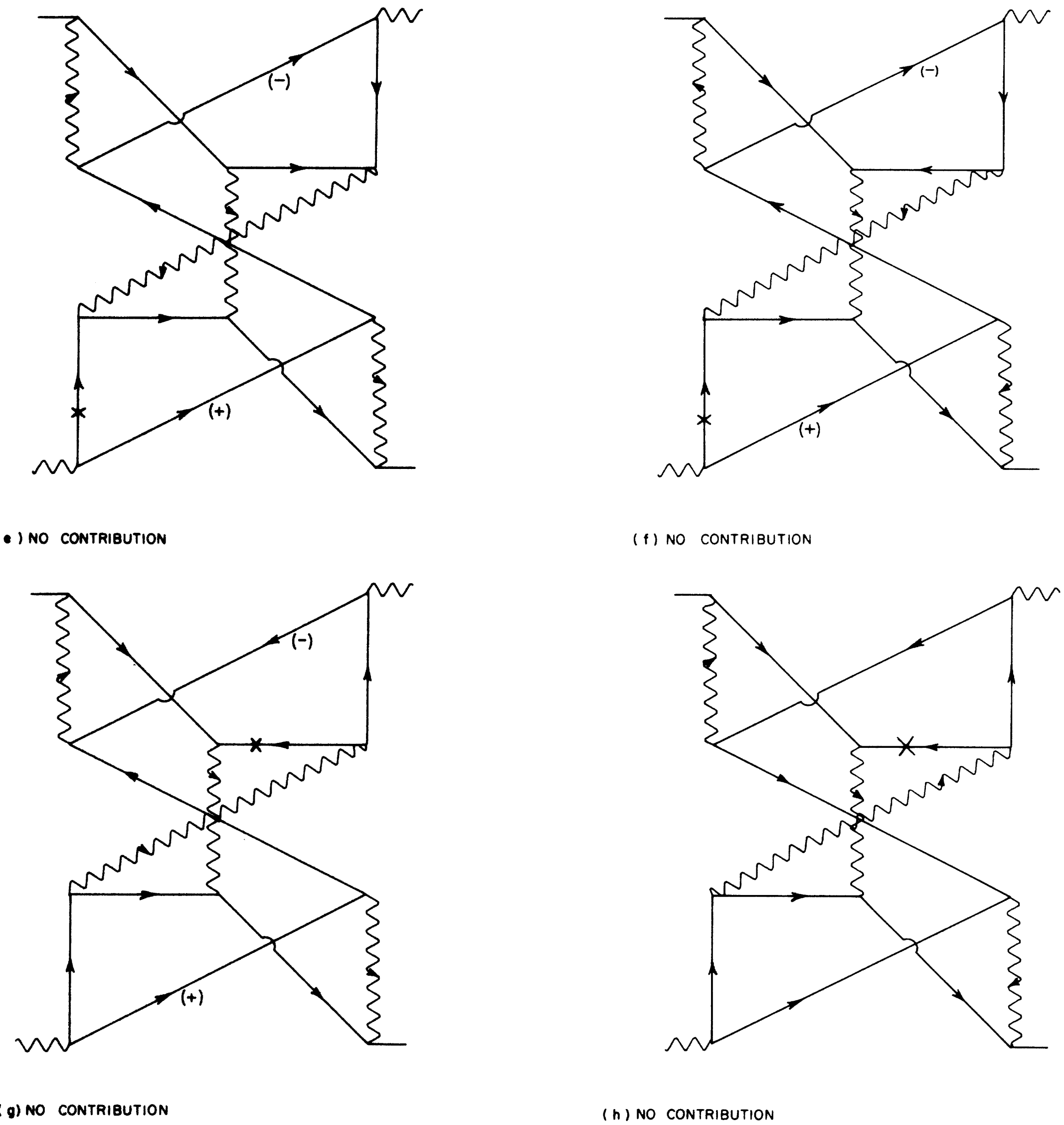


FIG. 28. The 8 momentum-flow diagrams for Feynman diagram 35.

Then using the momentum-flow diagrams of Fig. 30, we have

$$\bar{\mathfrak{M}}_{37}^{(4)} \doteq (-2,2)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{36}D_{36}^{-1} \tag{5.8}$$

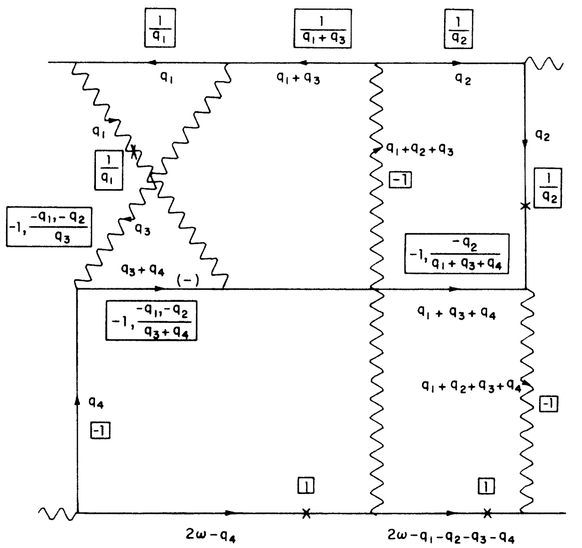
and therefore to leading order

$$\bar{\mathfrak{M}}_{36}^{(4)} + \bar{\mathfrak{M}}_{37}^{(4)} \doteq 0\,. \tag{5.9}$$

B. Feynman diagrams 40 and 41

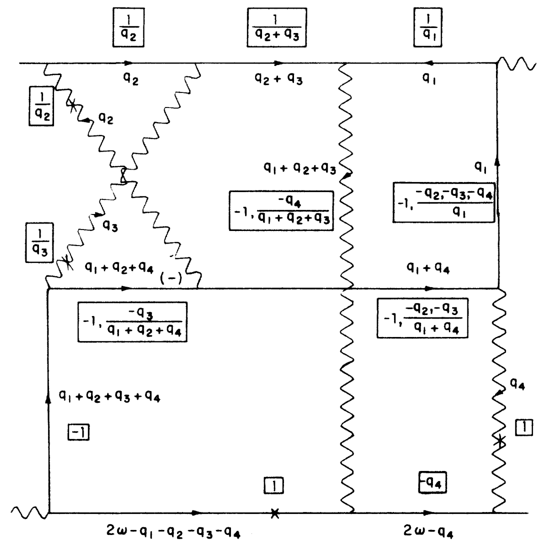
The numerator for Feynman diagram 40 is

$$\begin{aligned} N_{40} = -g^{10}\bar{u}(r_3 - r_1)\gamma_{\alpha_1}&(\not{r}_3 - \not{r}_1 - \not{k}_4 + m)\gamma_{\alpha_2}(\not{r}_3 - \not{r}_1 - \not{k}_3 - \not{k}_4 + m)\gamma_{\alpha_3}(\not{r}_3 - \not{r}_1 - \not{k}_1 - \not{k}_2 - \not{k}_3 + m)\gamma_\mu \\ &\times(-\not{k}_2 - \not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_4}(-\not{k}_1 - \not{k}_2 - \not{k}_3 - \not{k}_4 - 2\not{r}_1 + m)\gamma_{\alpha_1}(-\not{k}_1 - \not{k}_2 - \not{k}_3 - 2\not{r}_1 + m)\gamma_\nu \\ &\times(\not{r}_2 - \not{r}_1 - \not{k}_1 - \not{k}_2 - \not{k}_3 + m)\gamma_{\alpha_2}(\not{r}_2 - \not{r}_1 - \not{k}_1 - \not{k}_2 + m)\gamma_{\alpha_3}(\not{r}_2 - \not{r}_1 - \not{k}_1 + m)\gamma_{\alpha_4}u(r_2 - r_1)\,. \end{aligned} \tag{5.10}$$



(a) NUM = $-(q_3+q_4)$

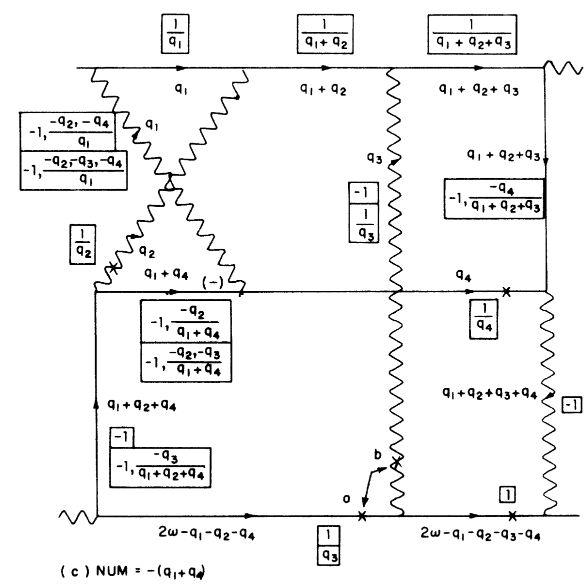
NO CONTRIBUTION



(b) NUM = $-(q_1+q_2+q_4)$

CONTRIBUTING REGION $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

RESULT: (1,0)

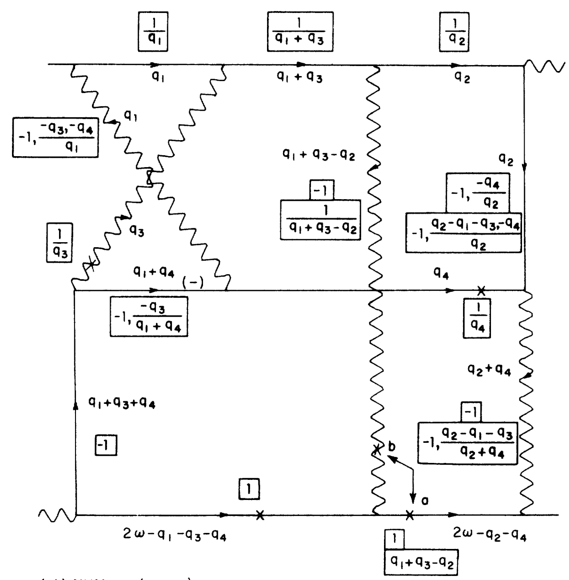


(c) NUM = $-(q_1+q_4)$

POLE a : CONTRIBUTING REGION: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

RESULT: (0,−1)

POLE b : NO CONTRIBUTION



(d) NUM = $-(q_1+q_4)$

POLE a : CONTRIBUTING REGIONS:

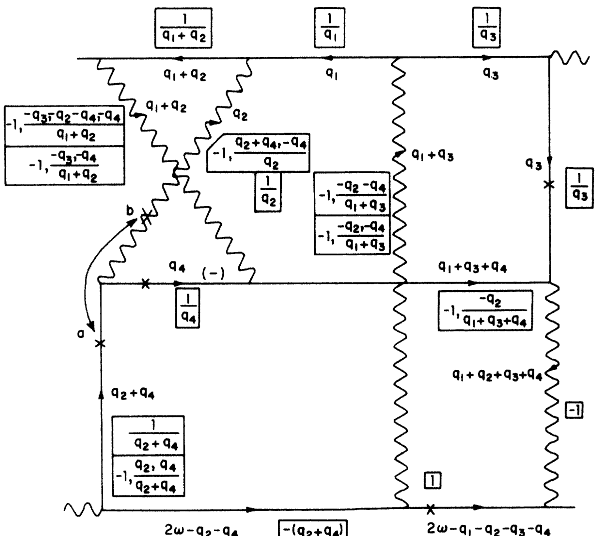
$0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

$0<q_2 \ll q_1 \ll q_3 \ll q_4 \ll 2\omega$

RESULT : (0,−2)

POLE b : NO CONTRIBUTION

FIG. 29. *(Continued on following page)*

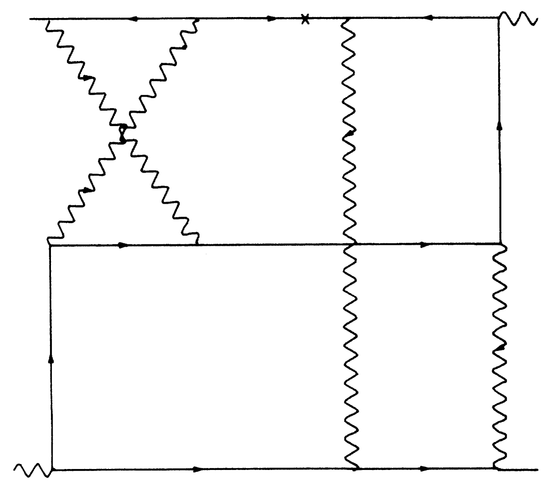


(e) NUM = $-q_4$

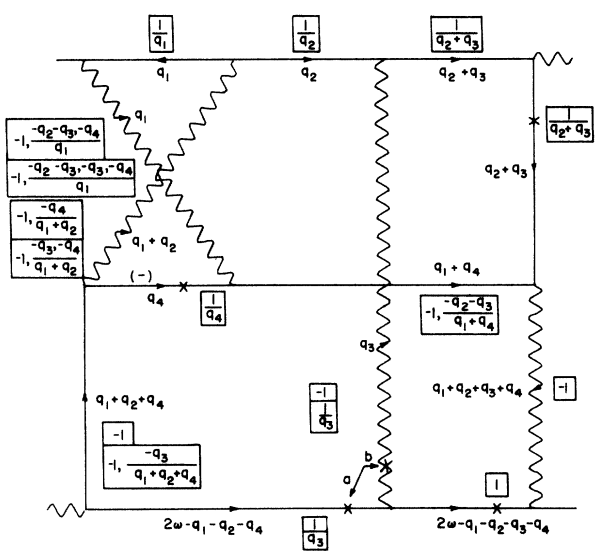
POLE a : CONTRIBUTING REGION: $0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

RESULT : (−1, 0)

POLE b : NO CONTRIBUTION



(f) NO CONTRIBUTION



(g) NUM = $-q_4$

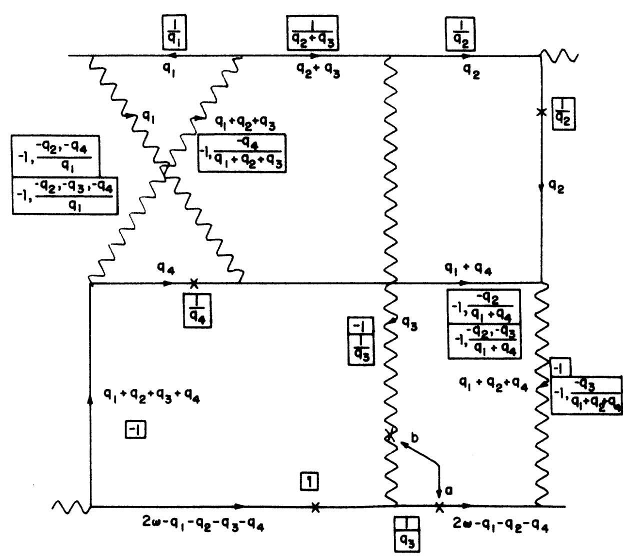
POLE a: CONTRIBUTING REGIONS:

$0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

$0 < q_2 \ll q_1 \ll q_3 \ll q_4 \ll 2\omega$

RESULT : (1, 1)

POLE b: NO CONTRIBUTION



(h) NUM = $-q_4$

POLE a: CONTRIBUTING REGION: $0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

RESULT : (1, 0)

POLE b: NO CONTRIBUTION

FIG. 29. The 8 momentum-flow diagrams for Feynman diagram 36.

This is approximated by

$$16(2\omega-k_{1+})(2\omega-k_{1+}-k_{2+})(2\omega-k_{1+}-k_{2+}-k_{3+})(2\omega-k_{4-})(2\omega-k_{3-}-k_{4-})(2\omega-k_{2-}-k_{3-}-k_{4-}) \times \bar{u}(r_3-r_1)\gamma_\mu \bar{N}_{40}\gamma_\nu u(r_2-r_1), \quad (5.11)$$

where

$$\bar{N}_{40}=-g^{10}(-\not{k}_{2\perp}-\not{k}_{3\perp}-\not{k}_{4\perp}-2\not{r}_\perp+m)(\not{k}_{1\perp}+\not{k}_{2\perp}+\not{k}_{3\perp}+\not{k}_{4\perp}+2\not{r}_\perp+m)(-\not{k}_{1\perp}-\not{k}_{2\perp}-\not{k}_{3\perp}-2\not{r}_\perp+m). \quad (5.12)$$

This diagram is symmetric under right-left reversal plus reversing the direction of all arrows. Under this operation momentum-flow diagrams (i) and (j) of Fig. 31 go into themselves while the other momentum-flow diagrams are transformed into a different momentum-flow diagram. For these latter we treat explicitly only one member of the pair and use 2 for the numerator instead of 1. Therefore, we find

$$\mathfrak{M}_{40}^{(4)} \doteq (-2,2)\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{40}D_{40}^{-1}, \quad (5.13)$$

where

$$D_{40}=(\vec{k}_{1\perp}^{\,2}+\lambda^2)(\vec{k}_{2\perp}^{\,2}+\lambda^2)(\vec{k}_{3\perp}^{\,2}+\lambda^2)(\vec{k}_{4\perp}^{\,2}+\lambda^2)[(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{2\perp}+\vec{k}_{3\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]. \quad (5.14)$$

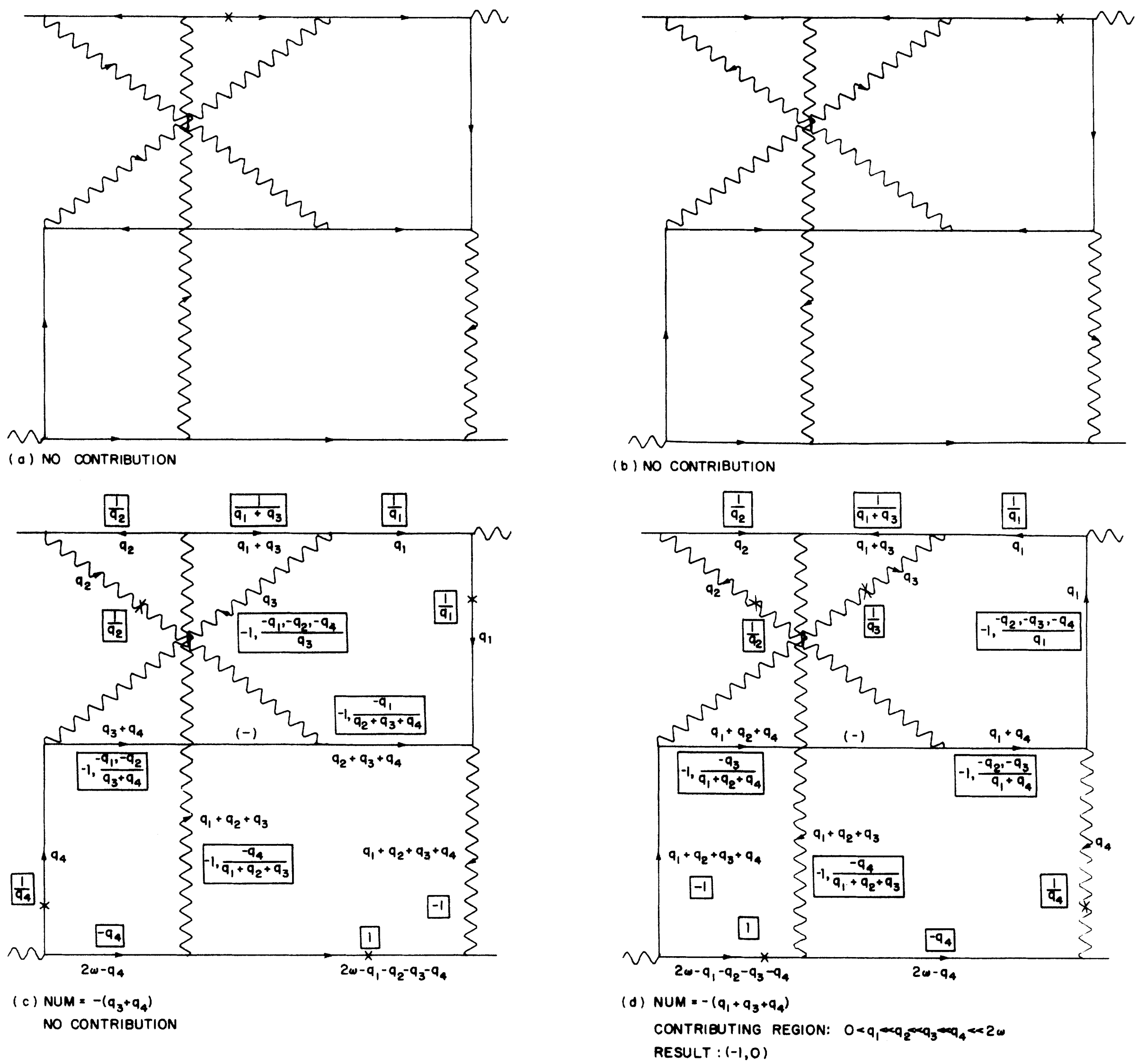
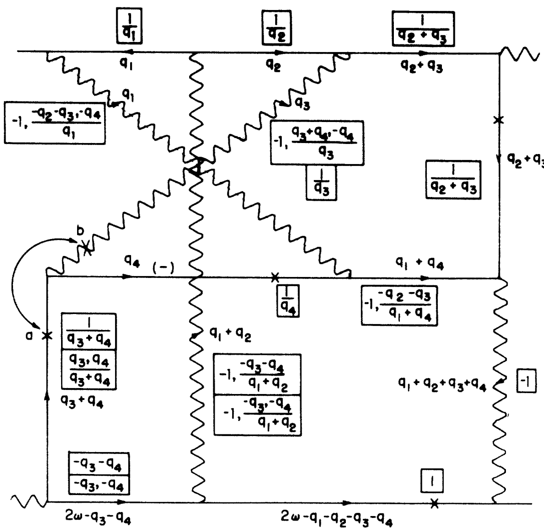


FIG. 30. *(Continued on following page)*



(e) NUM = $-q_4$

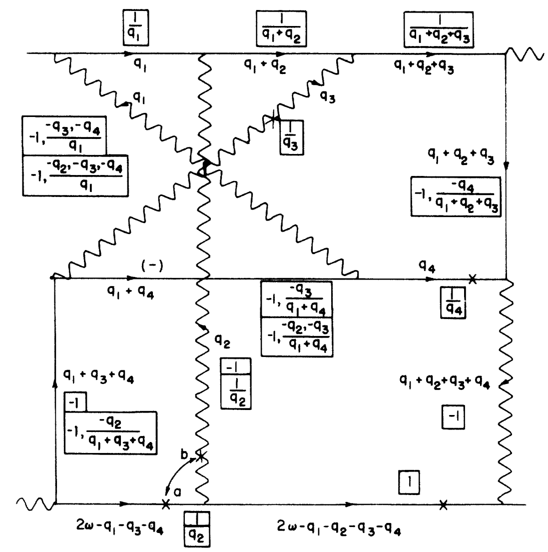
POLE a: CONTRIBUTING REGIONS:

$0<q_1\ll q_2\ll q_3\ll q_4\ll 2\omega$

$0<q_2\ll q_1\ll q_3\ll q_4\ll 2\omega$

RESULT: (1,1)

POLE b: NO CONTRIBUTION

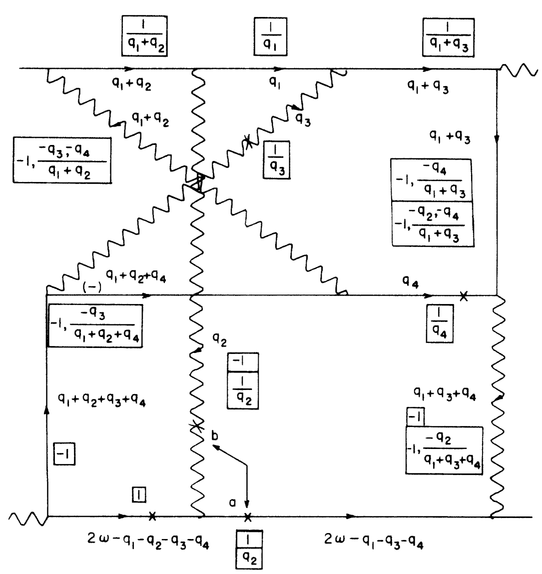


(f) NUM = $-(q_1+q_4)$

POLE a: CONTRIBUTING REGION: $0<q_1\ll q_2\ll q_3\ll q_4\ll 2\omega$

RESULT: (0,-1)

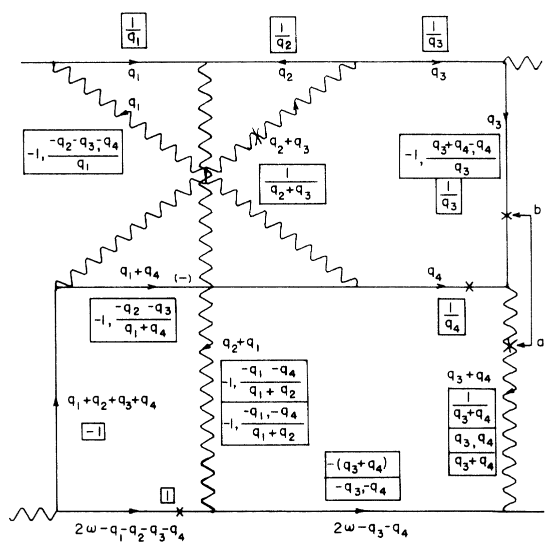
POLE b: NO CONTRIBUTION



(g) NUM = $-(q_1+q_2+q_4)$

POLE a: CONTRIBUTING REGION: $0<q_1\ll q_2\ll q_3\ll q_4\ll 2\omega$

RESULT: (0,-1)



(h) NUM = $-(q_3+q_4)$

POLE a: CONTRIBUTING REGIONS:

$0<q_1\ll q_2\ll q_3\ll q_4\ll 2\omega$ $\quad 0\ll q_1\ll q_3\ll q_2\ll q_4\ll 2\omega$

$0<q_2\ll q_1\ll q_3\ll q_4\ll 2\omega$ $\quad 0\ll q_3\ll q_1\ll q_2\ll q_4\ll 2\omega$

RESULT: (1,3)

POLE b: NO CONTRIBUTION

FIG. 30. *(Continued on following page)*

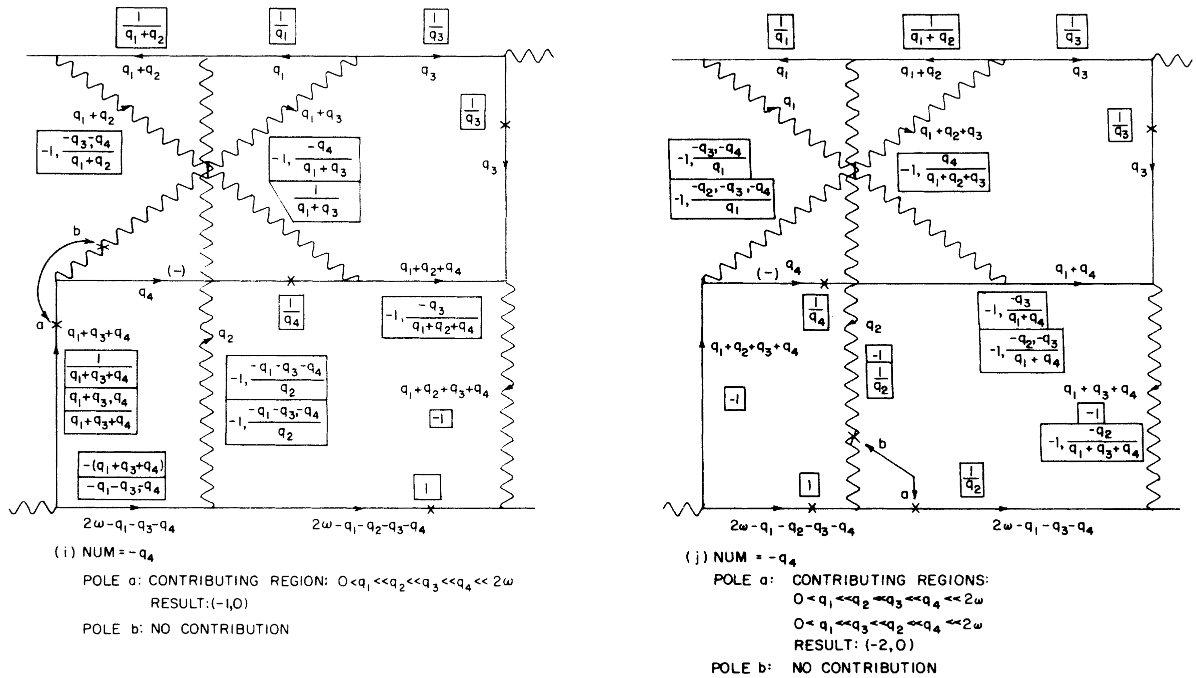


FIG. 30. The 10 momentum-flow diagrams for Feynman diagram 37.

The numerator of Feynman diagram 41 is

$$\begin{aligned} N_{41} = -g^{10}\bar{u}(r_3-r_1)\gamma_{\alpha_1}(\not{r}_3-\not{r}_1-\not{k}_4+m)\gamma_{\alpha_2}(\not{r}_3-\not{r}_1-\not{k}_2-\not{k}_4+m)\gamma_{\alpha_3}(\not{r}_3-\not{r}_1-\not{k}_2-\not{k}_3-\not{k}_4+m)\gamma_\mu \\ \times(-\not{k}_2-\not{k}_3-\not{k}_4-2\not{r}_1+m)\gamma_{\alpha_4}(\not{k}_1+\not{k}_2+\not{k}_3+\not{k}_4+2\not{r}_1+m)\gamma_{\alpha_1}(-\not{k}_1-\not{k}_2-\not{k}_3-2\not{r}_1+m)\gamma_\nu \\ \times(\not{r}_2-\not{r}_1-\not{k}_1-\not{k}_2-\not{k}_3)\gamma_{\alpha_3}(\not{r}_2-\not{r}_1-\not{k}_1-\not{k}_2+m)\gamma_{\alpha_2}(\not{r}_2-\not{r}_1-\not{k}_1+m)\gamma_{\alpha_4}u(r_2-r_1)\,, \end{aligned} \tag{5.15}$$

which is approximated by

$$\begin{aligned} 16(2\omega-k_{1+})(2\omega-k_{1+}-k_{2+})(2\omega-k_{1+}-k_{2+}-k_{3+})(2\omega-k_{4-})(2\omega-k_{2-}-k_{4-})(2\omega-k_{2-}-k_{3-}-k_{4-}) \\ \times\bar{u}(r_3-r_1)\gamma_\mu\bar{N}_{40}\gamma_\nu u(r_2-r_1)\,. \end{aligned} \tag{5.16}$$

Using this numerator with the momentum-flow diagrams of Fig. 32, (where the right-left symmetry has been used) we obtain

$$\bar{\mathfrak{M}}_{41}^{(4)} \doteq (2,-2)\int\frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\bar{N}_{40}D_{40}^{-1}\,. \tag{5.17}$$

Therefore, to leading order

$$\bar{\mathfrak{M}}_{40}^{(4)}+\bar{\mathfrak{M}}_{41}^{(4)}\doteq 0\,. \tag{5.18}$$

VI. SUMMATION OF DIAGRAMS

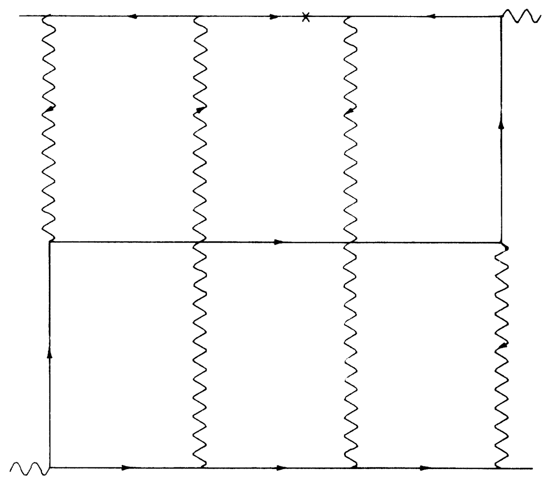
It remains to sum together all the terms previously computed. This can, of course, be done by brute force, but an appropriate grouping of terms can simplify the labor somewhat. We first consider the 15 terms which contribute to the real part and note that Feynman diagrams 1–8 and 14–15 are proportional to (0, 1) while diagrams 9–13 are proportional to (1, 0). Define $\hat{\mathfrak{M}}_i^{(4)}$ by

$$\hat{\mathfrak{M}}_i^{(4)} = (1,-1)\int\frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}N_iD_i^{-1}\,, \tag{6.1}$$

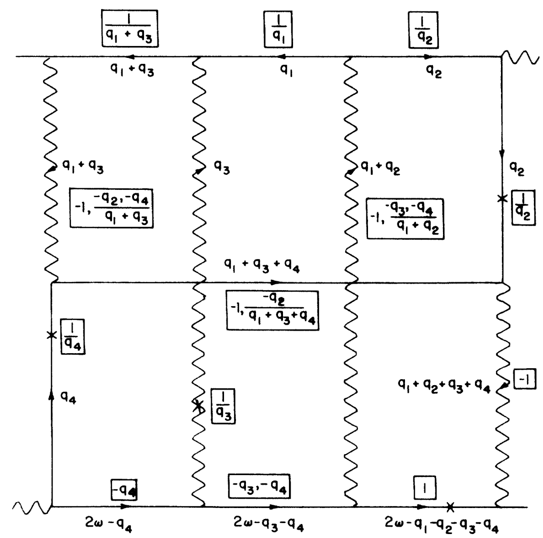
for $i=9$, 10, 11, 12, and 13. Then $\bar{\mathfrak{M}}_i^{(4)}-\hat{\mathfrak{M}}_i^{(4)}$ is proportional to (0, 1) for $i=9$–13.

Define $K^{(4)}$ to be the coefficient of (0, 1) in the sum of all contributing tenth-order diagrams. Then

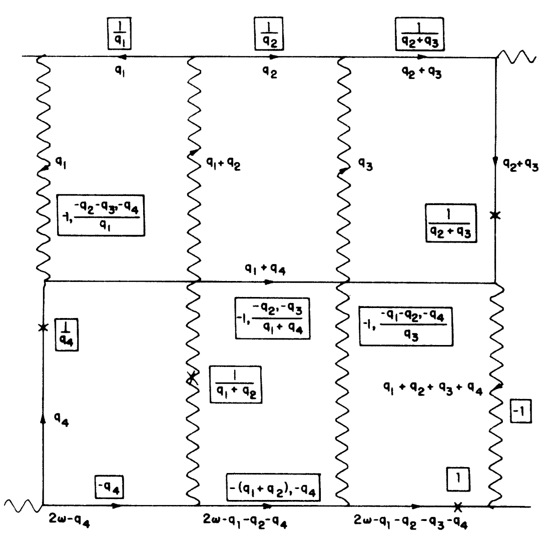
$$\sum_{i=1}^{15}\bar{\mathfrak{M}}_i^{(4)} = (0,1)K^{(4)}+\sum_{i=9}^{13}\hat{\mathfrak{M}}_i^{(4)} \tag{6.2}$$



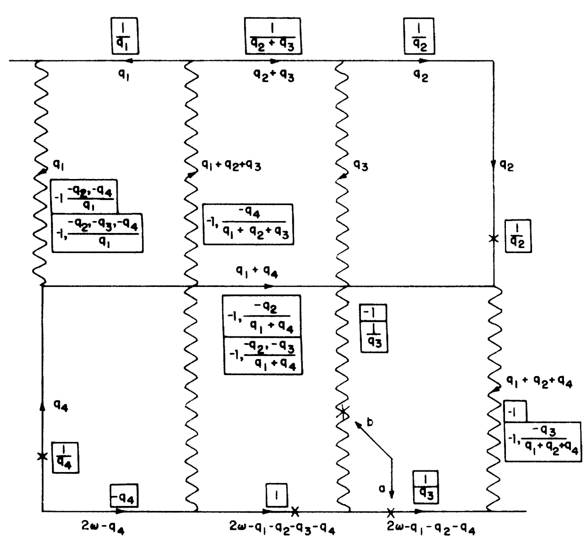
(a) NUM = 1
NO CONTRIBUTION



(b) NUM = 2
NO CONTRIBUTION

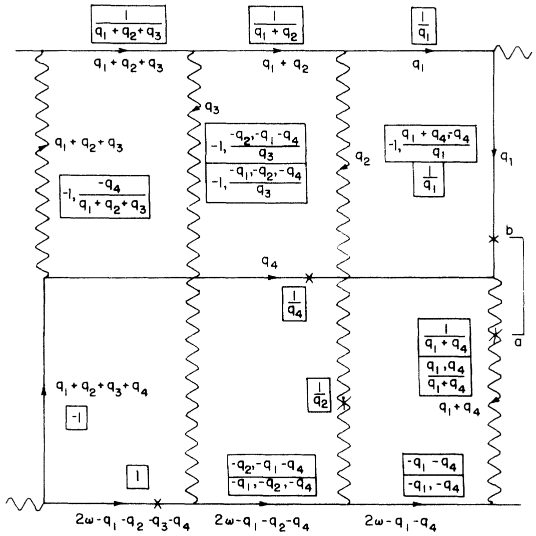


(c) NUM = 2
NO CONTRIBUTION



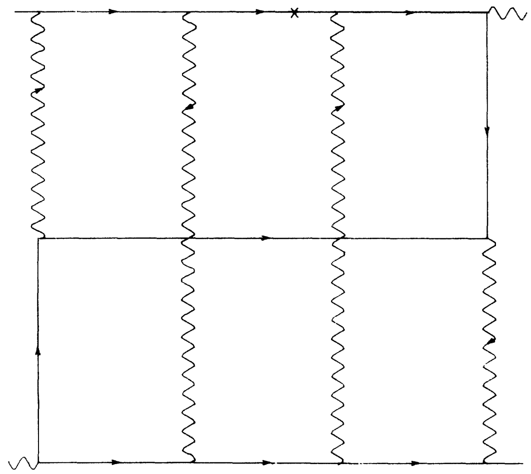
(d) NUM = 2
POLE a: CONTRIBUTING REGION: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
RESULT: (-2,0)
POLE b: NO CONTRIBUTION

FIG. 31. *(Continued on following page)*



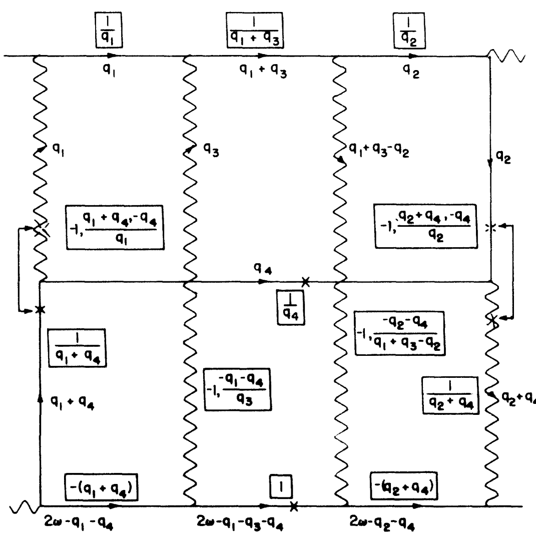
(e) NUM = 2

NO CONTRIBUTION



(f) NUM = 1

NO CONTRIBUTION



(g) NUM = 1

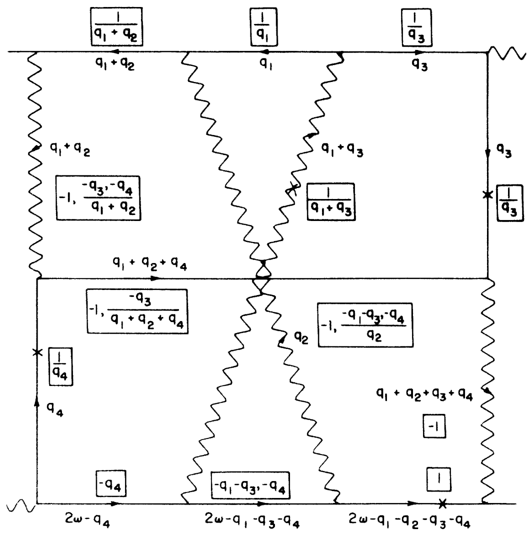
CONTRIBUTING REGIONS:

$0 < q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$

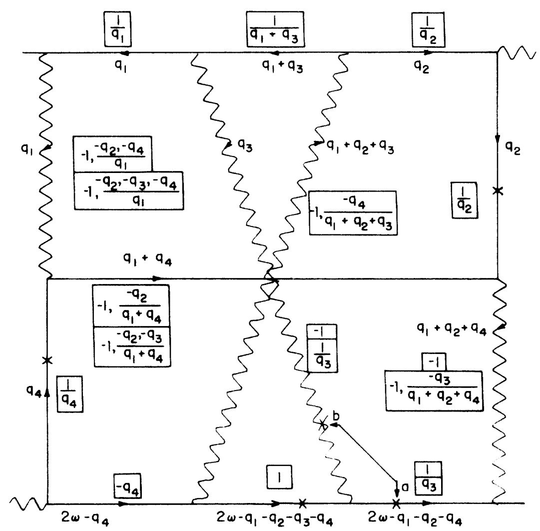
$0 < q_2 \ll q_1 \ll q_3 \ll q_4 \ll 2\omega$

RESULT: (0,2)

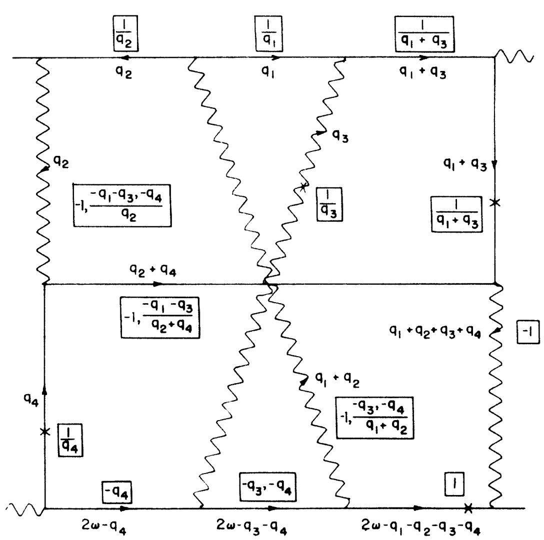
FIG. 31. Seven of the 11 momentum-flow diagrams for Feynman diagram 40. The additional diagrams are obtained from diagrams (b), (c), (d), and (e) by the right-left symmetry operation discussed in the text.



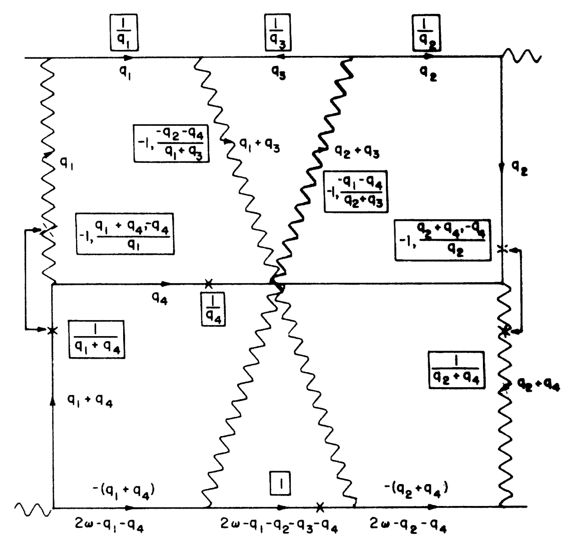
(a) NUM = 2
CONTRIBUTING REGION: $0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
RESULT : (2,0)



(b) NUM = 2
POLE a: CONTRIBUTING REGIONS:
$0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
$0<q_1 \ll q_3 \ll q_2 \ll q_4 \ll 2\omega$
RESULT : (4,0)
POLE b: NO CONTRIBUTION

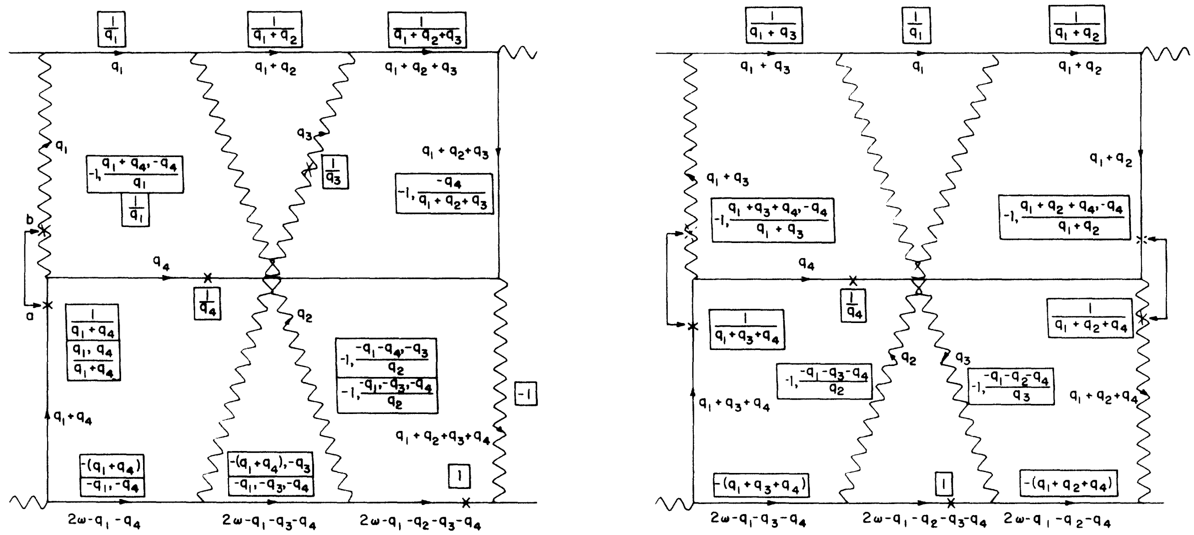


(c) NUM = 2
CONTRIBUTING REGIONS:
$0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$
$0<q_2 \ll q_1 \ll q_3 \ll q_4 \ll 2\omega$
RESULT: (-2,-2)



(d) NUM = 1
CONTRIBUTING REGIONS:
$0<q_1 \ll q_2 \ll q_3 \ll q_4 \ll 2\omega$ $0<q_1 \ll q_3 \ll q_2 \ll q_4 \ll 2\omega$
$0<q_2 \ll q_1 \ll q_3 \ll q_4 \ll 2\omega$ $0<q_2 \ll q_3 \ll q_1 \ll q_4 \ll 2\omega$
$0<q_3 \ll q_1 \ll q_2 \ll q_4 \ll 2\omega$ $0<q_3 \ll q_2 \ll q_1 \ll q_4 \ll 2\omega$
RESULT : (-2,-4)

FIG. 32. *(Continued on following page)*



(e) NUM = 2

POLE a: CONTRIBUTING REGION: $0<q_1\ll q_2\ll q_3\ll q_4\ll 2\omega$

RESULT: (0,2)

POLE b: NO CONTRIBUTION

(f) NUM = 1

CONTRIBUTING REGIONS:

$0<q_1\ll q_2\ll q_3\ll q_4\ll 2\omega$

$0<q_1\ll q_3\ll q_2\ll q_4\ll 2\omega$

RESULT: (0,2)

FIG. 32. Six of the 10 momentum-flow diagrams for Feynman diagram 41. The additional diagrams are obtained from diagrams (a), (b), (c), and (e) by the right-left symmetry operation discussed in the text.

and we find

$$
\begin{aligned}
K^{(4)}=-g^{10}\int &\frac{d^2\vec k_{1\perp}}{(2\pi)^3}\frac{d^2\vec k_{2\perp}}{(2\pi)^3}\frac{d^2\vec k_{3\perp}}{(2\pi)^3}\frac{d^2\vec k_{4\perp}}{(2\pi)^3}\{(\vec k_{1\perp}{}^2+\lambda^2)(\vec k_{2\perp}{}^2+\lambda^2)(\vec k_{3\perp}{}^2+\lambda^2)(\vec k_{4\perp}{}^2+\lambda^2)[(\vec k_{1\perp}+2\vec r_\perp)^2+m^2][(\vec k_{2\perp}+2\vec r_\perp)^2+m^2]\\
&\times[(\vec k_{3\perp}+2\vec r_\perp)^2+m^2][(\vec k_{4\perp}+2\vec r_\perp)^2+m^2]\}^{-1}\\
&\times\{(-\not{k}_{4\perp}-2\not{r}_\perp+m)(\not{k}_{3\perp}+\not{k}_{4\perp}+2\not{r}_\perp+m)(-\not{k}_{3\perp}-2\not{r}_\perp+m)(\not{k}_{2\perp}+\not{k}_{3\perp}+2\not{r}_\perp+m)\\
&\quad\times(-\not{k}_{2\perp}-2\not{r}_\perp+m)(\not{k}_{1\perp}+\not{k}_{2\perp}+2\not{r}_\perp+m)(-\not{k}_{1\perp}-2\not{r}_\perp+m)\\
&\quad-[(\vec k_{1\perp}+2\vec r_\perp)^2+m^2](-\not{k}_{4\perp}-2\not{r}_\perp+m)(\not{k}_{3\perp}+\not{k}_{4\perp}+2\not{r}_\perp+m)(-\not{k}_{3\perp}-2\not{r}_\perp+m)(\not{k}_{2\perp}+\not{k}_{3\perp}+2\not{r}_\perp+m)(-\not{k}_{2\perp}-2\not{r}_\perp+m)\\
&\quad-[(\vec k_{2\perp}+2\vec r_\perp)^2+m^2](-\not{k}_{4\perp}-2\not{r}_\perp+m)(\not{k}_{3\perp}+\not{k}_{4\perp}+2\not{r}_\perp+m)(-\not{k}_{3\perp}-2\not{r}_\perp+m)\\
&\quad\times(\not{k}_{1\perp}+\not{k}_{2\perp}+\not{k}_{3\perp}+2\not{r}_\perp+m)(-\not{k}_{1\perp}-2\not{r}_\perp+m)\\
&\quad-[(\vec k_{3\perp}+2\vec r_\perp)^2+m^2](-\not{k}_{4\perp}-2\not{r}_\perp+m)(\not{k}_{2\perp}+\not{k}_{3\perp}+\not{k}_{4\perp}+2\not{r}_\perp+m)(-\not{k}_{2\perp}-2\not{r}_\perp+m)\\
&\quad\times(\not{k}_{1\perp}+\not{k}_{2\perp}+2\not{r}_\perp+m)(-\not{k}_{1\perp}-2\not{r}_\perp+m)\\
&\quad-[(\vec k_{4\perp}+2\vec r_\perp)^2+m^2](-\not{k}_{3\perp}-2\not{r}_\perp+m)(\not{k}_{2\perp}+\not{k}_{3\perp}+2\not{r}_\perp+m)(-\not{k}_{2\perp}-2\not{r}_\perp+m)(\not{k}_{1\perp}+\not{k}_{2\perp}+2\not{r}_\perp+m)(-\not{k}_{1\perp}-2\not{r}_\perp+m)\\
&\quad+[(\vec k_{3\perp}+2\vec r_\perp)^2+m^2][(\vec k_{4\perp}+2\vec r_\perp)^2+m^2](-\not{k}_{2\perp}-2\not{r}_\perp+m)(\not{k}_{1\perp}+\not{k}_{2\perp}+2\not{r}_\perp+m)(-\not{k}_{1\perp}-2\not{r}_\perp+m)\\
&\quad+[(\vec k_{2\perp}+2\vec r_\perp)^2+m^2][(\vec k_{4\perp}+2\vec r_\perp)^2+m^2](-\not{k}_{3\perp}-2\not{r}_\perp+m)(\not{k}_{1\perp}+\not{k}_{2\perp}+\not{k}_{3\perp}+2\not{r}_\perp+m)(-\not{k}_{1\perp}-2\not{r}_\perp+m)\\
&\quad+[(\vec k_{2\perp}+2\vec r_\perp)^2+m^2][(\vec k_{3\perp}+2\vec r_\perp)^2+m^2](-\not{k}_{4\perp}-2\not{r}_\perp+m)(\not{k}_{1\perp}+\not{k}_{2\perp}+\not{k}_{3\perp}+\not{k}_{4\perp}+2\not{r}_\perp+m)(-\not{k}_{1\perp}-2\not{r}_\perp+m)\\
&\quad+[(\vec k_{1\perp}+2\vec r_\perp)^2+m^2][(\vec k_{4\perp}+2\vec r_\perp)^2+m^2](-\not{k}_{3\perp}-2\not{r}_\perp+m)(\not{k}_{2\perp}+\not{k}_{3\perp}+2\not{r}_\perp+m)(-\not{k}_{2\perp}-2\not{r}_\perp+m)\\
&\quad+[(\vec k_{1\perp}+2\vec r_\perp)^2+m^2][(\vec k_{3\perp}+2\vec r_\perp)^2+m^2](-\not{k}_{4\perp}-2\not{r}_\perp+m)(\not{k}_{2\perp}+\not{k}_{3\perp}+\not{k}_{4\perp}+2\not{r}_\perp+m)(-\not{k}_{2\perp}-2\not{r}_\perp+m)\\
&\quad+[(\vec k_{1\perp}+2\vec r_\perp)^2+m^2][(\vec k_{2\perp}+2\vec r_\perp)^2+m^2](-\not{k}_{4\perp}-2\not{r}_\perp+m)(\not{k}_{3\perp}+\not{k}_{4\perp}+2\not{r}_\perp+m)(-\not{k}_{3\perp}-2\not{r}_\perp+m)\\
&\quad-[(\vec k_{2\perp}+2\vec r_\perp)^2+m^2][(\vec k_{3\perp}+2\vec r_\perp)^2+m^2][(\vec k_{4\perp}+2\vec r_\perp)^2+m^2](-\not{k}_{1\perp}-2\not{r}_\perp+m)\\
&\quad-[(\vec k_{1\perp}+2\vec r_\perp)^2+m^2][(\vec k_{3\perp}+2\vec r_\perp)^2+m^2][(\vec k_{4\perp}+2\vec r_\perp)^2+m^2](-\not{k}_{2\perp}-2\not{r}_\perp+m)\\
&\quad-[(\vec k_{1\perp}+2\vec r_\perp)^2+m^2][(\vec k_{2\perp}+2\vec r_\perp)^2+m^2][(\vec k_{4\perp}+2\vec r_\perp)^2+m^2](-\not{k}_{3\perp}-2\not{r}_\perp+m)\\
&\quad-[(\vec k_{1\perp}+2\vec r_\perp)^2+m^2][(\vec k_{2\perp}+2\vec r_\perp)^2+m^2][(\vec k_{3\perp}+2\vec r_\perp)^2+m^2](-\not{k}_{4\perp}-2\not{r}_\perp+m)\}\,. \qquad (6.3)
\end{aligned}
$$

We will reduce $K^{(4)}$ by comparing it with the eighth-order coefficient of (1, 0) (for backward Compton scattering), $K^{(3)}$, where

$$\begin{aligned}
K^{(3)} = -g^8 \int & \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\{(\vec{k}_{1\perp}{}^2+\lambda^2)(\vec{k}_{2\perp}{}^2+\lambda^2)(\vec{k}_{3\perp}{}^2+\lambda^2)[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]\}^{-1} \\
& \times\{(-\not{\vec{k}}_{3\perp}-2\not{\vec{r}}_\perp+m)(\not{\vec{k}}_{2\perp}+\not{\vec{k}}_{3\perp}+2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{2\perp}-2\not{\vec{r}}_\perp+m)(\not{\vec{k}}_{1\perp}+\not{\vec{k}}_{2\perp}+2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{1\perp}-2\not{\vec{r}}_\perp+m) \\
& \quad -[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{3\perp}-2\not{\vec{r}}_\perp+m)(\not{\vec{k}}_{2\perp}+\not{\vec{k}}_{3\perp}+2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{2\perp}-2\not{\vec{r}}_\perp+m) \\
& \quad -[(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{3\perp}-2\not{\vec{r}}_\perp+m)(\not{\vec{k}}_{1\perp}+\not{\vec{k}}_{2\perp}+\not{\vec{k}}_{3\perp}+2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{1\perp}-2\not{\vec{r}}_\perp+m) \\
& \quad -[(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{2\perp}-2\not{\vec{r}}_\perp+m)(\not{\vec{k}}_{1\perp}+\not{\vec{k}}_{2\perp}+2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{1\perp}-2\not{\vec{r}}_\perp+m) \\
& \quad +[(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{1\perp}-2\not{\vec{r}}_\perp+m)+[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{2\perp}-2\not{\vec{r}}_\perp+m) \\
& \quad +[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{3\perp}-2\not{\vec{r}}_\perp+m)\} \qquad (6.4)
\end{aligned}$$

and we know from the paper on eighth-order perturbation theory that

$$\begin{aligned}
K^{(3)} &= -g^8 \int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\{(\vec{k}_{1\perp}{}^2+\lambda^2)(\vec{k}_{2\perp}{}^2+\lambda^2)(\vec{k}_{3\perp}{}^2+\lambda^2)[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]\}^{-1} \\
&\qquad \times(-\not{\vec{k}}_{3\perp}-2\not{\vec{r}}_\perp+m)(2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{2\perp}-2\not{\vec{r}}_\perp+m)(2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{1\perp}-2\not{\vec{r}}_\perp+m) \\
&= \frac{g^2}{2\not{\vec{r}}_\perp+m}[\alpha(2\vec{r}_\perp)]^3 . \qquad (6.5)
\end{aligned}$$

We use this previous reduction of $K^{(3)}$ by computing

$$\begin{aligned}
& K^{(4)}+g^2\int\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\frac{(-\not{\vec{k}}_{4\perp}-2\not{\vec{r}}_\perp+m)(2\not{\vec{r}}_\perp+m)K^{(3)}}{[(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2](\vec{k}_{4\perp}{}^2+\lambda^2)} \\
&= g^{10}\int\frac{d^2\vec{k}_{1\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\{(\vec{k}_{1\perp}{}^2+\lambda^2)(\vec{k}_{2\perp}{}^2+\lambda^2)(\vec{k}_{3\perp}{}^2+\lambda^2)(\vec{k}_{4\perp}{}^2+\lambda^2)[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2] \\
&\qquad \times[(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\}^{-1} \\
&\quad \times(\{[(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{4\perp}-2\not{\vec{r}}_\perp+m)+[(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{3\perp}-2\not{\vec{r}}_\perp+m)\} \\
&\qquad \times(\not{\vec{k}}_{2\perp}+\not{\vec{k}}_{3\perp}+2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{2\perp}-2\not{\vec{r}}_\perp+m)(\not{\vec{k}}_{1\perp}+\not{\vec{k}}_{2\perp}+2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{1\perp}-2\not{\vec{r}}_\perp+m) \\
&\quad -[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2]\{[(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]^2(-\not{\vec{k}}_{4\perp}-2\not{\vec{r}}_\perp+m)+[(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{3\perp}-2\not{\vec{r}}_\perp+m)\} \\
&\qquad \times(\not{\vec{k}}_{2\perp}+\not{\vec{k}}_{3\perp}+2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{2\perp}-2\not{\vec{r}}_\perp+m) \\
&\quad -[(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2]\{[(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{4\perp}-2\not{\vec{r}}_\perp+m)+[(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{3\perp}-2\not{\vec{r}}_\perp+m)\} \\
&\qquad \times(\not{\vec{k}}_{1\perp}+\not{\vec{k}}_{2\perp}+\not{\vec{k}}_{3\perp}+2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{1\perp}-2\not{\vec{r}}_\perp+m) \\
&\quad -[(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]\{[(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]+(-\not{\vec{k}}_{4\perp}-2\not{\vec{r}}_\perp+m)(\not{\vec{k}}_{2\perp}+\not{\vec{k}}_{3\perp}+2\not{\vec{r}}_\perp+m)\} \\
&\qquad \times(-\not{\vec{k}}_{2\perp}-2\not{\vec{r}}_\perp+m)(\not{\vec{k}}_{1\perp}+\not{\vec{k}}_{2\perp}+2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{1\perp}-2\not{\vec{r}}_\perp+m) \\
&\quad -[(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{3\perp}-2\not{\vec{r}}_\perp+m)(\not{\vec{k}}_{2\perp}+\not{\vec{k}}_{3\perp}+2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{2\perp}-2\not{\vec{r}}_\perp+m)(\not{\vec{k}}_{1\perp}+\not{\vec{k}}_{2\perp}+2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{1\perp}-2\not{\vec{r}}_\perp+m) \\
&\quad +[(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{2\perp}-2\not{\vec{r}}_\perp+m)(\not{\vec{k}}_{1\perp}+\not{\vec{k}}_{2\perp}+2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{1\perp}-2\not{\vec{r}}_\perp+m) \\
&\quad +[(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{3\perp}-2\not{\vec{r}}_\perp+m)(\not{\vec{k}}_{1\perp}+\not{\vec{k}}_{2\perp}+\not{\vec{k}}_{3\perp}+2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{1\perp}-2\not{\vec{r}}_\perp+m) \\
&\quad +[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{3\perp}-2\not{\vec{r}}_\perp+m)(\not{\vec{k}}_{2\perp}+\not{\vec{k}}_{3\perp}+2\not{\vec{r}}_\perp+m)(-\not{\vec{k}}_{2\perp}-2\not{\vec{r}}_\perp+m) \\
&\quad +[(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]\{[(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]+(-\not{\vec{k}}_{4\perp}-2\not{\vec{r}}_\perp+m)(\not{\vec{k}}_{1\perp}+\not{\vec{k}}_{2\perp}+\not{\vec{k}}_{3\perp}+2\not{\vec{r}}_\perp+m)\}(-\not{\vec{k}}_{1\perp}-2\not{\vec{r}}_\perp+m) \\
&\quad +[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]\{[(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]+(-\not{\vec{k}}_{4\perp}-2\not{\vec{r}}_\perp+m)(\not{\vec{k}}_{2\perp}+\not{\vec{k}}_{3\perp}+2\not{\vec{r}}_\perp+m)\}(-\not{\vec{k}}_{2\perp}-2\not{\vec{r}}_\perp+m) \\
&\quad +[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2]\{[(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{3\perp}-2\not{\vec{r}}_\perp+m)+[(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{4\perp}-2\not{\vec{r}}_\perp+m)\} \\
&\quad -[(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{1\perp}-2\not{\vec{r}}_\perp+m) \\
&\quad -[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{2\perp}-2\not{\vec{r}}_\perp+m) \\
&\quad -[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{3\perp}-2\not{\vec{r}}_\perp+m) \\
&\quad -[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2](-\not{\vec{k}}_{4\perp}-2\not{\vec{r}}_\perp+m)) \\
&= 0 . \qquad (6.6)
\end{aligned}$$

To obtain (6.6) we have combined the terms of $K^{(3)}$ as follows [where the first entry is the number of the term in (6.3) and the second entry is the number of the term in (6.4) and a "—" in the second entry means that no term of (6.4) is used]:

(1,1), (2,2), (3,3), (4,4), (5,—), (6,—), (7,—), (9,—), (8,5),

(10,0), (11,7), (12,—), (13,—), (14,—), (15,—).

Therefore,

$$K^{(4)}=\frac{g^2}{2\not{r}_\perp+m}\,[\alpha(2\vec{r}_\perp)]^4. \tag{6.7}$$

In (6.7) the cutoff $k_{\max}$ may be removed. However, this does not allow the cutoff to be removed in (6.2) because $\sum_{i=1}^{13}\hat{\mathfrak{M}}_i^{(4)}$ will diverge in that limit. These extra 5 terms on the right-hand side of (6.2) are purely imaginary and must be combined with the sum $\sum_{i=16}^{34}\tilde{\mathfrak{M}}_i^{(4)}$ before a convergent result may be obtained.

To compute explicitly the leading imaginary part it is convenient to group the terms together as

$$\begin{aligned}\sum_{i=16}^{35}\tilde{\mathfrak{M}}_i^{(4)}+\sum_{i=9}^{13}\hat{\mathfrak{M}}_i^{(4)}={}&(\tilde{\mathfrak{M}}_{16}^{(4)}+\tilde{\mathfrak{M}}_{23}^{(4)}+\tilde{\mathfrak{M}}_{24}^{(4)}+\tilde{\mathfrak{M}}_{25}^{(4)}+2\tilde{\mathfrak{M}}_{32}^{(4)}+2\tilde{\mathfrak{M}}_{33}^{(4)}+2\tilde{\mathfrak{M}}_{34}^{(4)}+2\tilde{\mathfrak{M}}_{35}^{(4)})\\&+(\tilde{\mathfrak{M}}_{17}^{(4)}-\tilde{\mathfrak{M}}_{25}^{(4)}+2\tilde{\mathfrak{M}}_{26}^{(4)}+2\tilde{\mathfrak{M}}_{28}^{(4)}-2\tilde{\mathfrak{M}}_{33}^{(4)}-2\tilde{\mathfrak{M}}_{34}^{(4)}-2\tilde{\mathfrak{M}}_{35}^{(4)})\\&+(\tilde{\mathfrak{M}}_{18}^{(4)}-\tilde{\mathfrak{M}}_{23}^{(4)}+2\tilde{\mathfrak{M}}_{27}^{(4)}+2\tilde{\mathfrak{M}}_{29}^{(4)}-2\tilde{\mathfrak{M}}_{32}^{(4)}-2\tilde{\mathfrak{M}}_{34}^{(4)}-2\tilde{\mathfrak{M}}_{35}^{(4)})\\&+(\tilde{\mathfrak{M}}_{19}^{(4)}-\tilde{\mathfrak{M}}_{26}^{(4)}+\tilde{\mathfrak{M}}_{30}^{(4)}+\tilde{\mathfrak{M}}_{33}^{(4)}+\hat{\mathfrak{M}}_{9}^{(4)}+\hat{\mathfrak{M}}_{12}^{(4)})+(\tilde{\mathfrak{M}}_{20}^{(4)}-\tilde{\mathfrak{M}}_{28}^{(4)}-\tilde{\mathfrak{M}}_{29}^{(4)}+\tilde{\mathfrak{M}}_{34}^{(4)}+\tilde{\mathfrak{M}}_{35}^{(4)}+\hat{\mathfrak{M}}_{10}^{(4)})\\&+(\tilde{\mathfrak{M}}_{21}^{(4)}-\tilde{\mathfrak{M}}_{27}^{(4)}+\tilde{\mathfrak{M}}_{31}^{(4)}+\tilde{\mathfrak{M}}_{32}^{(4)}+\hat{\mathfrak{M}}_{11}^{(4)}+\hat{\mathfrak{M}}_{13}^{(4)})+(\tilde{\mathfrak{M}}_{22}^{(4)}+\tilde{\mathfrak{M}}_{23}^{(4)}+\tilde{\mathfrak{M}}_{25}^{(4)}+2\tilde{\mathfrak{M}}_{34}^{(4)}+2\tilde{\mathfrak{M}}_{35}^{(4)}).\end{aligned} \tag{6.8}$$

Now use the identities

$$\begin{aligned}&(-2,2)\bar{N}_{16}+(2,-2)[(\vec{k}_{1\perp}+\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{23}+(2,-2)[(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{24}\\&\quad+(2,-2)[(\vec{k}_{2\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{25}+2(-1,1)[(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{1\perp}+\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{32}\\&\quad+2(-1,1)[(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{2\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{33}\\&\quad+2(-1,1)[(\vec{k}_{1\perp}+\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{2\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{34}\\&\qquad=-g^{10}(-2,2)(-\not{k}_{2\perp}-\not{k}_{4\perp}-2\not{r}_\perp+m)(-\not{k}_{2\perp}-2\not{r}_\perp-m)(-\not{k}_{2\perp}-\not{k}_{3\perp}-2\not{r}_\perp+m)(-\not{k}_{2\perp}-2\not{r}_\perp-m)(-\not{k}_{1\perp}-\not{k}_{2\perp}-2\not{r}_1+m),\end{aligned} \tag{6.9}$$

$$\begin{aligned}&(2,-2)\bar{N}_{17}-(2,-2)[(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{25}+2(-1,1)[(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{26}\\&\quad+2(-1,1)[(\vec{k}_{1\perp}+\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{28}-2(-1,1)[(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{33}\\&\quad-2(-1,1)[(\vec{k}_{1\perp}+\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{34}\\&\qquad=-g^{10}(2,-2)(-\not{k}_{4\perp}-2\not{r}_\perp+m)(-2\not{r}_\perp-m)(-\not{k}_{2\perp}-\not{k}_{3\perp}-2\not{r}_\perp+m)(-\not{k}_{2\perp}-2\not{r}_\perp-m)(-\not{k}_{1\perp}-\not{k}_{2\perp}-2\not{r}_\perp+m)\\&\qquad\quad-g^{10}(-2,2)[(\vec{k}_{1\perp}+\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2](-\not{k}_{4\perp}-2\not{r}_\perp+m),\end{aligned} \tag{6.10}$$

$$\begin{aligned}&(2,-2)\bar{N}_{18}-(2,-2)[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{23}+2(-1,1)[(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{27}\\&\quad+2(-1,1)[(\vec{k}_{3\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{24}-2(-1,1)[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{32}\\&\quad-2(-1,1)[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{3\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{34}\\&\qquad=-g^{10}(2,-2)(-\not{k}_{4\perp}-\not{k}_{3\perp}-2\not{r}_\perp+m)(-\not{k}_{3\perp}-2\not{r}_\perp+m)(-\not{k}_{2\perp}-\not{k}_{3\perp}-2\not{r}_\perp+m)(-2\not{r}_\perp-m)(-\not{k}_{1\perp}-2\not{r}_\perp+m)\\&\qquad\quad-g^{10}(-2,2)[(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{3\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2](-\not{k}_{1\perp}-2\not{r}_\perp-m),\end{aligned} \tag{6.11}$$

where in $\bar{N}_{23}$ we have made the substitution of variables $k_2\to k_3$,

$$\begin{aligned}&(-1,1)\bar{N}_{19}-(-1,1)[(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{26}+(1,-1)[(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{30}\\&\quad+(-1,1)[(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{33}+(1,-1)[(\vec{k}_{1\perp}+\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{9}\\&\quad+(-1,1)[(\vec{k}_{1\perp}+\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{12}\\&\qquad=-g^{10}(-1,1)(-\not{k}_{4\perp}-2\not{r}_\perp+m)(-2\not{r}_\perp-m)(-\not{k}_{3\perp}-2\not{r}_\perp+m)(-2\not{r}_\perp-m)(-\not{k}_{1\perp}-\not{k}_{2\perp}-2\not{r}_\perp+m)\\&\qquad\quad-g^{10}(1,-1)[(\vec{k}_{1\perp}+\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2](-\not{k}_{4\perp}-2\not{r}_\perp+m),\end{aligned} \tag{6.12}$$

$$(-1,1)\bar{N}_{20}-(-1,1)[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{28}-(-1,1)[(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{29}$$
$$+(-1,1)[(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{34}+(1,-1)[(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{10}$$
$$=-g^{10}(-1,1)(-\not{k}_{4\perp}-2\not{r}_\perp+m)(-2\not{r}_\perp-m)(-\not{k}_{2\perp}-\not{k}_{3\perp}-2\not{r}_\perp+m)(-2\not{r}_\perp-m)(-\not{k}_{1\perp}-2\not{r}_\perp+m)$$
$$-g^{10}(1,-1)[(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]\{[(\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2](-\not{k}_{1\perp}-2\not{r}_\perp+m)+[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2](-\not{k}_{4\perp}-2\not{r}_\perp+m)\},\qquad(6.13)$$

$$(-1,1)\bar{N}_{21}-(-1,1)[(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{27}+(1,-1)[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{31}$$
$$+(-1,1)[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{32}+(1,-1)[(\vec{k}_{3\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{11}$$
$$+(-1,1)[(\vec{k}_{1\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{3\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{13}$$
$$=g^{10}(-1,1)(-\not{k}_{3\perp}-\not{k}_{4\perp}-2\not{r}_\perp+m)(-2\not{r}_\perp-m)(-\not{k}_{2\perp}-2\not{r}_\perp+m)(-2\not{r}_\perp-m)(-\not{k}_{1\perp}-2\not{r}_\perp+m)$$
$$-g^{10}(1,-1)[(\vec{k}_{3\perp}+\vec{k}_{4\perp}+2r_\perp)^2+m^2][(\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2](-\not{k}_{1\perp}-2\not{r}_\perp+m),\qquad(6.14)$$

and

$$(-2,2)\bar{N}_{22}+(2,-2)[(\vec{k}_{1\perp}+\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{23}+(2,-2)[(\vec{k}_{3\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{25}$$
$$+2(-1,1)[(\vec{k}_{1\perp}+\vec{k}_{2\perp}+2\vec{r}_\perp)^2+m^2][(\vec{k}_{3\perp}+\vec{k}_{4\perp}+2\vec{r}_\perp)^2+m^2]\bar{N}_{34}$$
$$=g^{10}(-2,2)(-\not{k}_{3\perp}-\not{k}_{4\perp}-2\not{r}_\perp+m)(-\not{k}_{3\perp}-2\not{r}_\perp-m)(-\not{k}_{2\perp}-\not{k}_{3\perp}-2\not{r}_\perp+m)(-\not{k}_{2\perp}-2\not{r}_\perp-m)(-\not{k}_{1\perp}-\not{k}_{2\perp}-2\not{r}_\perp+m)$$
$$-g^{10}(-2,2)[(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2](-\not{k}_{3\perp}-\not{k}_{4\perp}-2\not{r}_\perp+m)(-2\not{r}_\perp-m)(-\not{k}_{1\perp}-\not{k}_{2\perp}-2\not{r}_\perp+m),\qquad(6.15)$$

and obtain

$$\sum_{i=16}^{35}\mathfrak{M}_i^{(4)}+\sum_{i=9}^{13}\mathfrak{M}_i^{(4)}=g^4(-2,2)\int\frac{d^3\vec{k}_{2\perp}}{(2\pi)^3}\,\frac{1}{\vec{k}_{2\perp}{}^2+\lambda^2}\,\frac{\alpha^3(2\vec{r}_\perp+\vec{k}_{2\perp})}{2\not{r}_\perp+\not{k}_{2\perp}+m}$$
$$+g^4(2,-2)\alpha(2\vec{r}_\perp)\int\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\,\frac{1}{\vec{k}_{2\perp}{}^2+\lambda^2}\,\frac{\alpha^2(2\vec{r}_\perp+\vec{k}_{2\perp})}{2\not{r}_\perp+\not{k}_{2\perp}+m}$$
$$+g^4(2,-2)\alpha(2\vec{r}_\perp)\int\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\,\frac{1}{\vec{k}_{3\perp}{}^2+\lambda^2}\,\frac{\alpha^2(2\vec{r}_\perp+\vec{k}_{3\perp})}{2\not{r}_\perp+\not{k}_{3\perp}+m}$$
$$+g^4(-1,1)\alpha^2(2\vec{r}_\perp)\int\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\,\frac{1}{\vec{k}_{2\perp}{}^2+\lambda^2}\,\frac{\alpha(2\vec{r}_\perp+\vec{k}_{2\perp})}{2\not{r}_\perp+\not{k}_{2\perp}+m}$$
$$+g^4(-1,1)\alpha^2(2\vec{r}_\perp)\int\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\,\frac{1}{\vec{k}_{3\perp}{}^2+\lambda^2}\,\frac{\alpha(2\vec{r}_\perp+\vec{k}_{3\perp})}{2\not{r}_\perp+\not{k}_{3\perp}+m}$$
$$+g^4(-1,1)\alpha^2(2\vec{r}_\perp)\int\frac{d^2\vec{k}_{4\perp}}{(2\pi)^3}\,\frac{1}{\vec{k}_{4\perp}{}^2+\lambda^2}\,\frac{\alpha(2\vec{r}_\perp+\vec{k}_{4\perp})}{2\not{r}_\perp+\not{k}_{4\perp}+m}$$
$$+g^6(-2,2)\int\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\,\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\,\frac{1}{\vec{k}_{2\perp}{}^2+\lambda^2}\,\frac{1}{\vec{k}_{3\perp}{}^2+\lambda^2}\,\alpha(\vec{k}_{3\perp}+2\vec{r}_\perp)\frac{(\not{k}_{2\perp}+\not{k}_{3\perp}+2\not{r}_\perp-m)}{[(\vec{k}_{2\perp}+\vec{k}_{3\perp}+2\vec{r}_\perp)^2+m^2]}\,\alpha(\vec{k}_{2\perp}+2\vec{r}_\perp)$$
$$+g^6(-2,2)(2\not{r}_\perp+m)\int\frac{d^2\vec{k}_{3\perp}}{(2\pi)^3}\,\frac{1}{\vec{k}_{3\perp}{}^2+\lambda^2}\,\frac{\alpha(\vec{k}_{3\perp}+2\vec{r}_\perp)}{\not{k}_{3\perp}+2\not{r}_\perp+m}\int\frac{d^2\vec{k}_{2\perp}}{(2\pi)^3}\,\frac{1}{\vec{k}_{2\perp}{}^2+\lambda^2}\,\frac{\alpha(\vec{k}_{2\perp}+2\vec{r}_\perp)}{\not{k}_{2\perp}+2\not{r}_\perp+m},\qquad(6.16)$$

where $\alpha(2\vec{r}_\perp)$ is given by (1.1).

Finally we combine (6.2), (6.7), and (6.16) and use the definitions (1.2), (1.3), and (3.1) and obtain the desired result (1.5).

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