Hamiltonian formulation of monopole theories with strings*

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We consider the Hamiltonian formulation of the theory resulting from Dirac's monopole action supplemented by a mass term for the gauge field. The original (zero-mass gauge field) theory is also discussed and its Hamiltonian is shown to be essentially the same as that of the two-potential formalism. In this case, the coordinates of the string are absorbed into what turn out to be the physically meaningful variables for the particles and the field. In the massive case, the string does play a significant role and gives rise to a static linear potential and a Yukawa potential between the monopoles. Such a potential has also been found by Nambu and others and may lead to an acceptable model for interactions of quarks.

I. INTRODUCTION

The reasons for a serious study of Dirac's hypothesis of the existence of isolated magnetic charges or magnetic monopoles have been discussed by several authors.[1] In addition to the well-known reasons that such an hypothesis leads to (i) symmetic electrodynamics, (ii) quantization of electric charge, and (iii) a possible natural explanation of T violation, in recent years it has become apparent[2] that Dirac's theory of monopoles[3] suitably modified may provide a theory of the hadronic spectrum as well as hadronic interactions. This idea stems from the fact that the "singularity line" or the so-called Dirac string assumes dynamical significance if the gauge field in the theory has a nonvanishing mass. One can then establish a connection between the Dirac string and the dual model string.[4] One can also show, in a certain approximation scheme, that the interaction between the monopoles is like that of Yukawa at short distances and varies directly as the distance for large distances. Further, it has become plausible recently that the form of the hadronic interaction should have this character if we wish to understand scaling and quark confinement.

Our main emphasis in this paper is on the Hamiltonian formulation based on the canonical procedure set up by Dirac.[5] In an earlier paper, Balachandran, Rupertsberger, and Schechter[6] have constructed the Hamiltonian corresponding to the Zwanziger Lagrangian[7] and discussed the quantization procedure using Dirac's method.[5] Their study shows that while Zwanziger's formulation has the advantage of being a local field theory, it suffers in general from a lack of rotational invariance when the gauge field is massive. In this paper, we consider Dirac's theory[3] and discuss its Hamiltonian formulation and the problems involved in its quantization.

In the next section, we discuss the equations of motion, the canonical momenta, and the constraints that follow from Dirac's action supplemented by a mass term for the vector field. In Sec. III, we find an appropriate set of physically significant dynamical variables, both when the vector field is massless and massive. These variables are unaffected by the presence of constraints in the sense that they have vanishing Poisson brackets with all of them on the constraint surface. In this way, for the zero-mass case Dirac's action leads to a particularly simple form for the Hamiltonian, which reflects the symmetry between the charges and monopoles explicitly. We indicate how this Hamiltonian might be quantized when the vector field is massless. The static limit in the massive case is also briefly considered. The final section is devoted to concluding remarks.

II. EQUATIONS OF MOTION AND CONSTRAINTS

A. Equations of motion

We begin with the classical Dirac action[3] for a system consisting of a point electric charge and a point magnetic monopole (the generalization to a system of several electric charges and magnetic monopoles is straightforward) interacting with a vector field which we may allow to have a mass μ:

$$A = -\tfrac{1}{4}\int d^4x F_{\mu\nu}F^{\mu\nu} + \frac{\mu^2}{2}\int d^4x A_\mu A^\mu - m\int d\tau(\dot z_\mu \dot z^\mu)^{1/2} - m_M\int d\tau(\dot z_{M\mu}\dot z_M^\mu)^{1/2} - e\int d\tau A_\mu(z)\dot z^\mu . \quad (2.1)$$

In (2.1), the overdots refer to differentiation with respect to some timelike parameter τ and $z^\mu(\tau)$ and $z_M^\mu(\tau)$ denote the trajectories of the point charge of mass m and the point monopole of mass m_M. $F^{\mu\nu}$ was defined by Dirac in such a way that the vector field has an interaction with the magnetic monopole which makes the resultant equations of motion symmetrical between the electric and magnetic parts when $\mu = 0$. $F^{\mu\nu}$ is given by[8]

$$F^{\mu\nu}(x) = \partial^\mu A^\nu(x) - \partial^\nu A^\mu(x) + {}^\star G^{\mu\nu}(x) , \quad (2.2)$$

where ${}^\star G$ is the dual of G,

$${}^\star G^{\mu\nu} = \tfrac{1}{2}\epsilon^{\mu\nu\lambda\rho}G_{\lambda\rho} , \quad (2.3)$$

and

$$G_{\lambda\rho}(x) = g\int d\tau\, d\sigma \frac{\epsilon(\sigma)}{2}\delta^4(x - y(\tau,\sigma)) \times [\dot y_\lambda(\tau,\sigma)y_\rho'(\tau,\sigma) - y_\lambda'(\tau,\sigma)\dot y_\rho(\tau,\sigma)] . \quad (2.4)$$

The above expression contains the Dirac string variable $y_\mu(\tau,\sigma)$, denoting an arbitrary point on the two-dimensional space-time sheet traced out by the string associated with the pole. The prime means differentiation with respect to the spatial parameter σ. The monopole is located at $\sigma = 0$, so that

$$y^\mu(\tau, 0) \equiv z_M^\mu(\tau) . \quad (2.5)$$

Note that we have made a slight departure from Dirac[3] in introducing the string. The string originating from the pole goes to both plus and minus infinity in the parameter σ. We find this symmetrical way of introducing the string necessary in our formal manipulations.[9] Note further that because of the identity (2.5), $z_M^\mu(\tau)$ is not an independent dynamical variable, but is included in the string variables $y^\mu(\tau,\sigma)$ $[-\infty < \sigma < \infty]$.

From the definition of $F^{\mu\nu}$, we have the identity

$$\partial_\mu {}^\star F^{\mu\nu} = -\partial_\mu G^{\mu\nu} = g\int \dot z_M^\nu(\tau)\delta^4(x - z_M)d\tau . \quad (2.6)$$

The equation for $F^{\mu\nu}$ containing the coordinates $z^\nu(\tau)$ of the charge that can be obtained from (2.1) is

$$\partial_\mu F^{\mu\nu} + \mu^2 A^\nu = e\int d\tau\, \delta^4(x - z(\tau))\, \dot z^\nu . \quad (2.7)$$

The action also leads to other equations involving z_μ, $z_{M\mu}$, and $y_\mu(\tau,\sigma)$. For the development of the Hamiltonian formalism it is convenient to identify τ with the real time t and set $z_0 = z_{M0} = y_0 = t$; with this identification, we obtain the following equations:

$$m\ddot z_i = e\dot z^\mu[\partial_i A_\mu(z) - \partial_\mu A_i(z)] , \quad (2.8a)$$

$$m_M \ddot z_{Mi} = g\dot z_M^\mu\, {}^\star F_{i\mu}(z_M) , \quad (2.8b)$$

and

$$\epsilon_{i\mu\nu\lambda}\partial_\alpha F^{\alpha\mu}(y)\dot y^\nu y'^\lambda = 0 , \quad (2.8c)$$

where i is a spatial index and μ, ν, and λ run from 0 to 3. For simplicity in the expression, we have used nonrelativistic kinematics for the charge and the monopole in (2.8a) and (2.8b). The relativistic case presents no new problem.

B. Canonical momenta and constraints

To obtain the Hamiltonian, we calculate the canonical momenta $\pi_\mu(x)$, $\eta_i(t,\sigma)$, and $p_i(t)$ corresponding to the variables $A^\mu(x)$, $y^i(t,\sigma)$, and $z^i(t)$. From the Lagrangian corresponding to the action (2.1),

$$L = -\tfrac{1}{4}\int d^3x F_{\mu\nu}F^{\mu\nu} + \frac{\mu^2}{2}\int d^3x A_\mu A^\mu + \tfrac{1}{2}m\dot{\vec z}^2 + \tfrac{1}{2}m_M\dot{\vec z}_M^2 - eA_\mu(z)\dot z^\mu , \quad (2.9)$$

we obtain

$$\pi_\mu(x) = F_{0\mu}(x) , \quad (2.10a)$$

$$\vec\eta(\sigma) = -g\frac{\epsilon(\sigma)}{2}\vec y' \times \vec\pi(y) + m_M\dot{\vec z}_M\delta(\sigma) , \quad (2.10b)$$

and

$$\vec p = m\dot{\vec z} + e\vec A(z) , \quad (2.10c)$$

where we have suppressed the time dependence of the variables. Equations (2.10a) and (2.10b) imply the following primary constraints:

$$\pi_0(x) = 0 \quad (2.11a)$$

and

$$\vec X(\sigma) \equiv \vec\eta(\sigma) + g\frac{\epsilon(\sigma)}{2}[\vec y' \times \vec\pi(y)] = 0 \quad \text{for } \sigma \neq 0 . \quad (2.11b)$$

These are constraint equations since they do not involve time derivatives.

The Hamiltonian is computed from the definition

$$H = \int d\sigma(\vec\eta\cdot\dot{\vec y}) + (\vec p\cdot\dot{\vec z}) + \int d^3x(\vec\pi\cdot\dot{\vec A}) - L \quad (2.12)$$

to which we have the freedom to add arbitrary combinations of constraint terms.

Noticing from (2.10b) that

$$\dot{\vec z}_M = \frac{1}{m_M}\int d\sigma X(\sigma) \equiv \frac{\langle\vec X(0)\rangle}{m_M} \quad (2.13)$$

and using (2.10a)–(2.10c), we write the total Hamiltonian including the constraint terms as

$$H^* = \int d^3x\{\tfrac{1}{4}F_{ij}(x)^2 + \tfrac{1}{2}\vec{\pi}(x)^2 - \tfrac{1}{2}\mu^2 A_\mu(x)A^\mu(x) + [\vec{\nabla}\cdot\vec{\pi}(x) + e\delta^3(x-z)]A_0(x)\}$$
$$+\frac{1}{2m}[\vec{p} - e\vec{A}(z)]^2 + \frac{1}{2m_M}\langle\vec{X}(0)\rangle^2 + \int d^3x v_0(x)\pi_0(x) + \int d\sigma \vec{v}(\sigma)\cdot\vec{X}(\sigma)\,. \quad (2.14)$$

Here $v_0(x)$ and $\vec{v}(\sigma)$ are undetermined Lagrange multipliers. Some constraint terms which arise from rewriting (2.12) are also absorbed in these functions. Since $\vec{X}(\sigma)$ is a constraint for only $\sigma \neq 0$, $v(\sigma)$ will have a zero at $\sigma = 0$. When we require that the constraints (2.11) be preserved in time, that is, that they have vanishing Poisson brackets (PB's) with (2.14), we find the secondary constraint

$$S(x) = \vec{\nabla}\cdot\vec{\pi}(x) + e\delta^3(x-z) - \mu^2 A_0(x) = 0\,. \quad (2.11c)$$

This requirement also determines the explicit form of the components of $\vec{v}(\sigma)$ perpendicular to $\vec{y}'$. We shall not need these forms in the later discussion.

The nonvanishing PB's among the constraints are given by

$$[\pi^0(\vec{x}), S(\vec{x}')]_{PB} = \mu^2\delta^3(x-x') \quad (2.15)$$

and

$$[X_i(\sigma), X_j(\sigma')]_{PB} = -g\frac{\epsilon(\sigma)}{2}\epsilon_{ijk}y_k'(\sigma)[\vec{\nabla}\cdot\vec{\pi}(y)]\delta(\sigma-\sigma'), \quad \sigma, \sigma' \neq 0 \quad (2.16)$$

We now divide all the constraints into two groups.[5] Those which have vanishing PB's with all constraints are called first class and correspond to generators of gauge and coordinate transformations. The others are called second class and can be eliminated from the theory by redefinition of the dynamical variables. Although (2.16) shows that not all X_i are first class, the following combination is first class:

$$K(\sigma) = \vec{X}(\sigma)\cdot\vec{y}'(\sigma)\,. \quad (2.17)$$

The other two linearly independent combinations of the $\vec{X}(\sigma)$ are then second class. Furthermore, (2.15) shows that both $\pi^0(x)$ and $S(x)$ are second class when $\mu \neq 0$. However, in the case $\mu = 0$ corresponding to Dirac's original theory, we see from the same equation (2.15) that $\pi^0(x)$ and $S(x)$ are *first* class.

III. DEVELOPMENT OF THE FORMALISM

For the further discussion of the theory it is convenient to choose dynamical variables which have vanishing PB's with all the first-class constraints. Then there will be no need to impose these constraints as subsidiary conditions on the states, and also the appropriate Lagrange multiplier terms in the Hamiltonian can be dropped. This will be done in the present section.

It is also necessary to take account of the second-class constraints. One way of proceeding is to modify the PB's to get Dirac brackets. An alternative,[10] equivalent approach which we shall adopt here is to add appropriate linear combinations of constraint terms to each of the above variables in such a way that the resultant *starred* variables have vanishing PB's with all second-class constraints. Then we may effectively set all second-class constraints equal to zero.

In view of the difference in the nature of the constraints, we will treat separately the cases when the gauge field has zero mass and when it is massive. We shall begin with the simpler zero-mass case.

To begin with, since the structure of the mass less theory is essentially the same as electrodynamics, we expect and may easily verify that both the "electric" field variable $\vec{\pi}(x)$ and the "magnetic" field variable

$$Q_k = \tfrac{1}{2}\epsilon_{kij}F_{ij} \quad (3.1)$$

have vanishing PB's with the first-class constraints (2.11a), (2.11c), and (2.17). It is convenient to separate these into their transverse and longitudinal parts, which are separately gauge invariant (that is, have vanishing PB's with the first-class constraints). We thus write

$$\begin{aligned} \vec{Q}^T &= \vec{Q} - \frac{\vec{\nabla}}{\nabla^2}(\vec{\nabla}\cdot\vec{Q})\,, \\ \vec{Q}^L &= \vec{Q} - \vec{Q}^T\,, \\ \vec{\pi}^T &= \vec{\pi} - \frac{\vec{\nabla}}{\nabla^2}(\vec{\nabla}\cdot\vec{\pi})\,, \\ \vec{\pi}^L &= \vec{\pi} - \vec{\pi}^T\,. \end{aligned} \quad (3.2)$$

We now introduce, following Schwinger,[11] transverse potentials $\vec{q}^{(1)}$ and $\vec{q}^{(2)}$ for the transverse magnetic and electric fields:

$$\vec{Q}^T = (\vec{\nabla}\times\vec{q}^{(1)}), \quad \vec{\nabla}\cdot\vec{q}^{(1)} = 0\,, \quad (3.3)$$

$$\vec{\pi}^T = -(\vec{\nabla}\times\vec{q}^{(2)}), \quad \vec{\nabla}\cdot\vec{q}^{(2)} = 0\,. \quad (3.4)$$

From the canonical PB's one may deduce the usual PB's between the electric and magnetic fields:

$$[Q_i(x), \pi_j(x')]_{PB} = -\epsilon_{ijk}\frac{\partial}{\partial x_k}\delta^3(x-x'). \tag{3.5}$$

We may find the other PB's in a similar way.

Next, reference to (2.11c) shows that both the canonical momentum $\vec{p}$ of the charged particle and the variable $\vec{A}(x)$ are not gauge invariant. However, these occur in the Hamiltonian only in the gauge-invariant combination $[\vec{p}-e\vec{A}(z)]$. We may write this in the form

$$\vec{p}-e\vec{A}(z)=\vec{p}^{(1)}-e\vec{q}^{(1)}, \tag{3.6}$$

where

$$\vec{p}^{(1)}=\vec{p}-e\vec{A}^L(z)+eg\int d\sigma\,\frac{\epsilon(\sigma)}{2}\,\frac{\vec{y}'\times\vec{\nabla}_y}{\nabla^2}\,\delta^3(\vec{y}-\vec{z}). \tag{3.7}$$

Here $\vec{A}^L$ is the longitudinal part of $\vec{A}$. We note that $\vec{p}^{(1)}$ is gauge invariant and satisfies the canonical equations:

$$\begin{aligned}&[p_i^{(1)}; p_j^{(1)}]_{PB}=0,\\ &[z_i, p_j^{(1)}]_{PB}=\delta_{ij},\\ &[p_i^{(1)}, q_j^{(a)}]=0\quad (a=1,2).\end{aligned} \tag{3.8}$$

Thus $\vec{p}^{(1)}$ may be regarded as the momentum of the charged particle. Similarly, the corresponding quantity for the monopole given in (2.13) may be rewritten as

$$\langle\vec{X}(0)\rangle=\vec{p}^{(2)}-g\vec{q}^{(2)}(z_M), \tag{3.9}$$

where

$$\vec{p}^{(2)}=\int d\sigma\vec{\eta}-g\int d\sigma\,\frac{\epsilon(\sigma)}{2}\,y_m'\vec{\nabla}q_m^{(2)}(y)+g\int d\sigma\,\frac{\epsilon(\sigma)}{2}\,\frac{\vec{y}'\times\vec{\nabla}_y}{\nabla^2}\,[\vec{\nabla}\cdot\vec{\pi}(y)]. \tag{3.10}$$

We have used (3.4), (3.2), and (2.2) in getting (3.10). $\vec{p}^{(2)}$ is gauge invariant and satisfies

$$\begin{aligned}&[p_i^{(2)}, p_j^{(2)}]_{PB}=0,\\ &[z_{Mi}, p_j^{(2)}]_{PB}=\delta_{ij},\\ &[p_i^{(2)}, q_j^{(a)}]_{PB}=0\quad (a=1,2).\end{aligned} \tag{3.11}$$

Further, $\vec{p}^{(2)}$ has zero PB with the variables associated with the charged particle. Therefore $\vec{p}^{(2)}$ may be identified as the momentum of the monopole.

Next we discuss the string variable $\vec{y}(\sigma)$. From (2.17) and (2.11b) we observe that $\vec{y}(\sigma)$ does not have a vanishing PB with the first-class constraint K. To find gauge-invariant coordinates for the string, we note that $K(\sigma)$ has the interpretation of a generator of transformations in the space of the parameter σ. This suggests the introduction of the new parameter

$$h(\sigma)=\int_0^\sigma d\sigma'|\vec{y}'(\sigma')|, \tag{3.12}$$

which has the physical significance of the length measured along the string. Then the new string variable defined by

$$\vec{Y}(\sigma)\equiv\vec{y}(h^{-1}(\sigma)) \tag{3.13}$$

can be seen to have vanishing PB with $K(\sigma)$. To verify this statement one may use the identity

$$[K(\sigma), h^{-1}(\rho)]_{PB}=\delta(\sigma-h^{-1}(\rho)), \tag{3.14}$$

which follows from commuting $K(\sigma)$ with the equation $h(h^{-1}(\rho))=\rho$.

Note that

$$\begin{aligned}\vec{Y}'(\sigma)&\equiv\frac{d}{d\sigma}\vec{y}(h^{-1}(\sigma))\\ &=\vec{y}'(h^{-1}(\sigma))\frac{dh^{-1}(\sigma)}{d\sigma}\\ &=\frac{\vec{y}'(h^{-1}(\sigma))}{|\vec{y}'(h^{-1}(\sigma))|}.\end{aligned} \tag{3.15}$$

Thus $\vec{Y}'(\sigma)$ is a vector of unit magnitude.[12]

Now that we have defined gauge-invariant dynamical variables we will take account of the second-class constraints. The variables $\vec{z}$, $\vec{z}_M$, and $\vec{q}^{(a)}$ all have vanishing PB's with the $X_i(\sigma)$ in (2.11b) and thus are suitable as they stand for observable dynamical variables. The $p^{(a)}$'s, however, should be modified to get the new starred variables as follows:

$$\vec{p}^{(1)*}=\vec{p}^{(1)}+\int d\sigma f(\sigma)\vec{X}(\sigma), \tag{3.16}$$

$$\vec{p}^{(2)*}=\vec{p}^{(2)}-\int d\sigma f(\sigma)\vec{X}(\sigma). \tag{3.17}$$

Here $f(\sigma)$ is defined to be a function which is zero at $\sigma=0$ and unity elsewhere.[13] The presence of $f(\sigma)$ is due to the fact that $X(\sigma)$ is not a constraint for $\sigma=0$. The $\vec{p}^{(a)*}$ variables above, which differ from the $\vec{p}^{(a)}$ variables only by some constraints, can be verified to have vanishing PB's with all constraints. Finally, one may calculate the starred string coordinates $\vec{Y}(\sigma)^*$ which have vanishing PB's with all constraints, but we shall not give them here as we do not need them.

Now, by substituting the new variables in the Hamiltonian (2.14) (with $\mu=0$) we get simply

$$H^*=\tfrac{1}{2}\int d^3x(\vec{Q}^2+\vec{\pi}^2)+\frac{[\vec{p}^{(1)*}-e\vec{q}^{(1)}(z)]^2}{2m}+\frac{[\vec{p}^{(2)*}-g\vec{q}^{(2)}(z_M)]^2}{2m_M}, \tag{3.18}$$

where we have dropped some constraint terms. Note that by using the starred variables we have automatically evaluated the Lagrange multiplier terms for the second-class constraints. It is interesting that (3.18) displays a manifest symmetry between the variables associated with the pole and with the charge.

Equation (3.18) is essentially of the same form as that given in the two-potential formalism of Schwinger[12] and Zwanziger[7] (cf. Ref. 6).

We may similarly calculate the angular momentum of the system and express it in terms of the starred variables as

$$\vec{L}^* = \vec{z}\times[\vec{p}^{(1)*} - e\vec{q}^{(1)}(z)] + \vec{z}_M\times[\vec{p}^{(2)*} - g\vec{q}^{(2)}(z_M)] - \int d^3x[\vec{x}\times(\vec{\pi}\times\vec{Q})]\,. \tag{3.19}$$

In (3.18) and (3.19) the independent dynamical variables are $\vec{z}$, $\vec{z}_M$, $\vec{p}^{(a)*}$, and $\vec{q}^{(a)}$. Note that $\vec{Q}^T$ and $\vec{\pi}^T$ are given in terms of $\vec{q}^{(a)}$ by (3.3) and (3.4) while their longitudinal parts are the magnetic and electric fields resulting from point magnetic and electric charges at $\vec{z}_M$ and $\vec{z}$, respectively. Thus the dynamical variables associated with the string do not appear explicitly in the Hamiltonian. Nevertheless, they have not completely disappeared from the theory since they turn up when we consider PB's of these dynamical variables. To see this we use (3.16), (3.17), and (2.16) to evaluate the PB's among the $\vec{p}^{(a)*}$'s. This finally results in the following expressions:

$$[p_i^{(1)*}, p_j^{(1)*}]_{PB} = [p_i^{(2)*}, p_j^{(2)*}]_{PB} = -[p_i^{(1)*}, p_j^{(2)*}]_{PB} = +eg\epsilon_{ijk}\left\{-\frac{1}{4\pi}\frac{(z-z_M)_k}{|\vec{z}-\vec{z}_M|^3} + \int d\sigma\,\frac{\epsilon(\sigma)}{2}[1-f(\sigma)]\,Y_k'(\sigma)\delta^3(z-Y(\sigma))\right\}. \tag{3.20}$$

The PB's among the other dynamical variables appearing in the Hamiltonian do not involve the string variables.

The theory may now be quantized by replacing all PB's of the dynamical variables by $(-i)$ times the corresponding commutators. We can recover essentially the formulation of Lipkin, Weisberger, and Peshkin[14] by imposing the subsidiary condition that the second term of (3.20) vanish on the allowed states. Since $[1-f(\sigma)]$ is zero if σ is not zero, this is achieved by requiring that $\delta^3(z-z_M)$ vanish on the allowed states. It is worthwhile to note, however, that the presence of the second term ensures that the Jacobi identity among the $p_i^{(a)*}$ is fulfilled as an operator relation, unlike the situation in Ref. 14. In verifying this result, it is convenient to use the representation of $f(\sigma)$ given in footnote 13, and to take the limit $\delta\to 0$ at the end. It may also be observed that the use of the infinite rather than the semi-infinite string is necessary to ensure the Jacobi identity.

To try to understand the role of the string in this theory we may compute its time evolution as follows:

$$\dot{\vec{Y}}(\sigma) = [\vec{Y}(\sigma), H^*]_{PB} = \dot{\vec{z}} - \vec{Y}'(\sigma)[(\dot{\vec{z}} - \dot{\vec{z}}_M)\cdot\vec{Y}'(0)]\,. \tag{3.21}$$

From (3.21) we see that the string *does* evolve in time. Thus we cannot, for example, completely eliminate the string by keeping it fixed in a given direction. This intriguing feature of the string variable has to be further explored before a full quantum-mechanical Hamiltonian description of Dirac's theory can be developed.

It may be helpful, at this point, to contrast our treatment with that of Dirac[3] who imposes the requirement that the world sheet of the string never intersects the charged particle—formally $j^\mu(y) \equiv \partial_\alpha F^{\alpha\mu}(y) = 0$. He adopts this condition which is a possible solution of the following equation:

$$\epsilon_{\lambda\rho\mu\nu}\dot{y}^\rho y'^\mu \partial_\alpha F^{\alpha\nu}(y) = 0\,.$$

This equation follows from varying the string coordinate in the action. On the other hand, we have allowed the more general equation above to hold rather than its special solution $j^\mu(y)=0$, which does not follow from the action principle. If we impose $j^\mu(y)=0$, using (2.11c) we find that the right-hand side of (2.16) vanishes so that all the X_i's become first class. Then all the string variables become gauge variables for $\mu=0$ and can be completely eliminated from the theory.

We have also worked out the Hamiltonian formalism for the massive case ($\mu\neq 0$). Then the Hamiltonian in terms of starred variables turns out to be

$$H^* = \frac{1}{2}\int d^3x \left\{ \vec{Q}^2 + \vec{\pi}^2 + \mu^2 \vec{A}^{*2} + \frac{1}{2\mu^2}[\vec{\nabla}\cdot\vec{\pi} + e\delta^3(x-z)]^2 \right\} + \frac{1}{2m}[\vec{p} - e\vec{A}^*(z)]^2 + \frac{1}{2m_M}\left[\int d\sigma\, \vec{X}(\sigma)\right]^{*2}, \tag{3.22}$$

where

$$\vec{A}^*(x) = \vec{A}(x) + \int d\sigma\, f(\sigma)\, \frac{\delta^3(x - y(\sigma))}{\vec{\nabla}\cdot\vec{\pi}(y)}\, \vec{X}(\sigma),$$

$$\left[\int d\sigma\, \vec{X}(\sigma)\right]^* = \int d\sigma\, \vec{X}(\sigma)[1 - f(\sigma)].$$

In this case only $K(\sigma)$ is first class. There is an important distinction between (3.22) and (3.18) in that the string variables are present in (3.22) through the $\mu^2 A^{*2}$ term.

Although the quantization of the massive theory is difficult, owing to the nontrivial presence of the string coordinates in the PB's of dynamical variables, one can get some insight into this theory by considering its *static limit* when several monopoles (and no electric charges) are present and the monopoles are infinitely massive [to treat several monopoles, replace $g\frac{1}{2}\epsilon(\sigma)$ by $\sum_N g_N \frac{1}{2}\epsilon(\sigma - \sigma_N)$ in (2.4) and all subsequent equations. This corresponds to monopole N at position $\sigma = \sigma_N$]. By setting to zero the PB's of dynamical variables with H^*, one finds that the theory corresponds to static monopoles separated by straight strings and bound by Yukawa as well as linear potentials. Actually the coefficient of the linear term turns out to be infinite so that the theory requires regularization of some sort for consistency. Such a situation has been considered as a model for quark binding by Nambu[4] and other authors (see the references in the following paper[15]). The equation for the static energy is the same as that given in Ref. 6. More details of these considerations are also given in Ref. 15.

IV. CONCLUDING REMARKS

There have been essentially two distinct covariant approaches to the theory of magnetic monopoles. One is the two-potential formalism[7,11] and the other is described by Dirac's string action.[3] In this paper we have shown that the Hamiltonian corresponding to Dirac's action is essentially the same as that in the two-potential formalisms. In particular, the string variables can be absorbed in other dynamical variables, so that they are not manifestly present in either the Hamiltonian or the total angular momentum. They can also be eliminated from the Poisson brackets of these variables by imposing appropriate restrictions on the states in the quantized theory. However, even though the system of charges, monopoles, and fields can be described without mentioning the string, it is interesting that the time dependence of the string is not arbitrary, but is predicted by the theory.

The situation changes drastically if the gauge field is given a mass. In this paper we have added such a mass term directly, but our analysis would presumably go through if such a mass term is considered as arising from spontaneous symmetry breaking. The string variables cannot now be eliminated from the Hamiltonian and interact in a crucial way with the rest of the system. In the static limit of such a Hamiltonian, we see that the interaction potential of the monopoles has both a Yukawa-type term and a term proportional to the distance between the interacting monopoles. Furthermore, it is possible that the excitation spectrum of the string may be similar to that of the dual string.[16] These two properties make such a theory a good candidate for the dynamics of mesons which would arise as bound states of the unobservable quarks (monopoles of this theory) and which would interact with the gluon field. To bring these conjectures to a definite shape, it is necessary to first regularize the infinities that occur in our expressions.

The infinities that are present in the theory are associated with the well-known divergences due to the self-energies of the point particles and also the self-energy due to an infinitely thin string. A natural way of regularization of the latter infinity may be by giving a finite lateral extension to the string. As other authors have argued,[17] the finite lateral extension itself may be connected with the spontaneous symmetry breaking. However, in the following paper,[15] we approach this problem somewhat phenomenologically. Given the presence of a one-dimensional string in the theory, we discuss a covariant procedure of eliminating the infinities. We then apply the static limit of the regularized theory to meson spectroscopy.

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[1]For a comprehensive review and a list of earlier references, see V. I. Strazhev and L. M. Tomilichik, Fiz. Elem. Chastits At. Yad. 4, 187 (1973) [Sov. J. Part. Nucl. 4, 78 (1973)].

[2]A. O. Barut and G. L. Bornzin, Nucl. Phys. B81, 477 (1974), and references therein; Y. Nambu, in *Pro- of The John Hopkins Workshop on Current Problems in High Energy Particle Theory*, edited by G. Domokos and S. Kövesi-Domokos (Johns Hopkins Univ. Press, Baltimore, 1974), p. 1.

[3]P. A. M. Dirac, Phys. Rev. 74, 817 (1948).

[4]Y. Nambu, Phys. Rev. D 10, 4262 (1974).

[5]P. A. M. Dirac, *Lectures on Quantum Mechanics*, lectures given at Belfer Graduate School of Science, Yeshiva University, 1964 (Academic, New York, 1965). See also J. Weinberg, Univ. of California, Berkeley, Ph.D. thesis, 1943 (unpublished).

[6]A. P. Balachandran, H. Rupertsberger, and J. Schechter, Phys. Rev. D 11, 2260 (1975). See also A. Jevicki and P. Senjanovic, *ibid.* 11, 860 (1975); A. Patkós, Budapest report, 1975 (unpublished).

[7]D. Zwanziger, Phys. Rev. D 3, 885 (1971).

[8]We use the metric $g^{00}=-g^{11}=-g^{22}=-g^{33}=1$, and choose $\epsilon_{0123}=1$. When the equations involve only spatial indices, we will use either lowered indices or the vector notation.

[9]See Ref. 6 for a discussion on the need for the infinite string rather than the semi-infinite string for consistent quantization of Zwanziger's Lagrangian.

[10]P. G. Bergmann and I. Goldberg, Phys. Rev. 98, 531 (1955).

[11]J. Schwinger, Phys. Rev. 144, 1087 (1966); 151, 1048 (1966); 151, 1055 (1966).

[12]The theory can be rewritten in terms of $\vec{Y}(\sigma)$ instead of $\vec{y}(\sigma)$, which in turn can be obtained by integrating $\vec{Y}'(\sigma)$. Then, (3.15) implies that there are, in effect, only two degrees of freedom associated with the string variable.

[13]One possible explicit representation for $f(\sigma)$ is given by $f(\sigma)=\lim_{\delta\to 0^+}[\theta(\sigma-\delta)+\theta(-\sigma-\delta)]$.

[14]H. J. Lipkin, W. I. Weisberger, and M. Peshkin, Ann. Phys. (N.Y.) 53, 203 (1969); M. Peshkin, *ibid.* 66, 542 (1971).

[15]A. P. Balachandran, R. Ramachandran, J. Schechter, Kameshwar C. Wali, and Heinz Rupertsberger, following paper, Phys. Rev. D 13, 361 (1976).

[16]For discussion of the nature of trajectories in this context, see P. H. Frampton, Phys. Rev. D 12, 538 (1975).

[17]H. B. Nielsen and P. Olesen, Nucl. Phys. B61, 45 (1973); See also G. Parisi, Phys. Rev. D 11, 970 (1975).