Comment on the pseudo-Goldstone phenomenon of Georgi and Pais

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It is shown that supersymmetric theories can provide very clear illustrations of the pseudo-Goldstone phenomenon of Georgi and Pais (i.e., pseudo-Goldstone bosons need *not* be due to an accidental higher symmetry of the potential).

Some time ago it was shown by Weinberg[1] that pseudo-Goldstone bosons (PGB's) will occur for spontaneously broken gauge theories in which the scalar field potential V possesses a higher symmetry than the whole Lagrangian. In general, such particles have a zero mass in the tree approximation but acquire a nonzero mass due to quantum corrections. However, recently Georgi and Pais[2] have demonstrated that it is possible to enlarge the class of theories for which PGB's occur to include some for which the potential has no "accidental" higher symmetry. They show that the number of PGB's is at least as great as n, where n is given by

$$n = D(S_\lambda) - D(G) + D(G_\lambda)\,. \tag{1}$$

The notation is explained in detail in Ref. 2. $D(S_\lambda)$ is the dimension of the space S_λ (the space spanned by the tangents to the surface S in the space of real scalar fields, which gives a potential minimum). $D(G)$ is the dimension of the group G which leaves the Lagrangian invariant. $D(G_\lambda)$ is the dimension of the subgroup of G which leaves elements of S invariant.

For a certain class of theories, S can be generated by acting on any element of S by a group G_{vac} of dimension $D(G_{\text{vac}})$. If $D(G_{\text{vac},\lambda})$ is the dimension of the subgroup which leaves the vacuum invariant, then they argue that

$$D(S_\lambda) = D(G_{\text{vac}}) - D(G_{\text{vac},\lambda})\,. \tag{2}$$

In order to find a renormalizable theory in which V has no accidental higher symmetry, Georgi and Pais exhibit quite a complicated model in which $G = \text{SO}(3)\times\text{SO}(3)\times\text{SO}(3)$ and $G_{\text{vac}} = \text{SO}(3)\times\text{SO}(3)\times\text{SO}(3)\times\text{SO}(3)\times\text{SO}(3)\times\text{SO}(3)$.

It is the purpose of this note to point out that supersymmetry theories[3,4] provide very simple examples of this kind of phenomenon. Indeed, the ease with which PGB's appear is often quite embarrassing. An example of such a model is that of O'Raifeartaigh,[5] which is constructed out of three scalar superfields. Φ_{0-}, Φ_{1+}, and Φ_{2-}, having the discrete symmetry $\Phi_{0-}\to\Phi_{0-}$, $\Phi_{1+}\to-\Phi_{1+}$, and $\Phi_{2-}\to-\Phi_{2-}$. The most general renormalizable, fermion-number-conserving Lagrangian can be written in terms of the superfield notation of Salam and Strathdee[3] as

$$\mathcal{L} = \tfrac{1}{8}(\bar{D}D)^2(|\Phi_{0-}|^2 + |\Phi_{1+}|^2 + |\Phi_{2-}|^2) - \tfrac{1}{2}\bar{D}D[\Phi_{0-}^*(s+g\Phi_{1+}^2) + m\Phi_{2-}^*\Phi_{1+} + \text{H.c.}]\,. \tag{3}$$

$\mathcal{L}$ is not invariant under any continuous group of transformations and thus

$$D(G)=0\;,\quad D(G_\lambda)=0\,. \tag{4}$$

The potential is given by

$$V = |s+gA_1^2|^2 + |mA_2 + 2gA_1^*A_0|^2 + |mA_1|^2\,, \tag{5}$$

which leads to a degenerate minimum

$$|A_1|^2 = -\frac{(m^2+gs)}{g^2}\,,\qquad mA_2 + 2gA_1A_0 = 0\,, \tag{6}$$

where A_0, A_1, A_2 are the complex scalar fields contained in the superfields $\Phi_{0-}, \Phi_{1+}, \Phi_{2-}$ (A_1 is actually real at the minimum). The minimum is invariant under

$$A_0\to\beta A_0\,,\qquad A_2\to\beta A_2\,, \tag{7}$$

where β is arbitrary and complex. G_{vac} is thus $\text{U}(1)\times\text{S}$, where S is the one-dimensional dilation group. V is only invariant under the U(1) transformations and thus G_{vac} is *not* a symmetry of V, and we have an illustration of the type of theories considered by Georgi and Pais.[2] Since $D(G_{\text{vac},\lambda}) = 0$, n, as defined by Eq. (1), is equal to 2 [i.e., $D(G_{\text{vac}})$] and thus we would expect two (real) pseudo-Goldstone fields. This is confirmed by explicit calculation.

It has also been shown[6] that in the case of the O'Raifeartaigh model the PGB's do acquire a (nontachyonic) mass due to one-loop quantum corrections. However, the second point we wish to make is that for *some* supersymmetric models, in fact those for which supersymmetry is unbroken in the tree approximation, quantum corrections do *not* give the PGB's a mass nor do they help to choose the correct vacuum. A simple example of such a theory has recently been examined in detail by Capper and Ramón Medrano.[7] This paper also lists other supersymmetric theories in which PGB's appear.

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