Comparison of approximate methods for multiple scattering in high-energy collisions. II*

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The scattering in one dimension of a particle by a target of N like particles in a bound state has been studied. The exact result for the transmission probability has been compared with the predictions of the Glauber theory, the Watson optical potential model, and the adiabatic (or fixed scatterer) approximation. The approximate methods work well only at energies well above the binding energy per target particle. The adiabatic approximation works best and the Watson optical potential model is second best. The Watson method is found to work better when the kinematics suggested by Foldy and Walecka are used rather than that suggested by Watson, that is to say, when the two-body t matrix is calculated with the nucleon-target reduced mass instead of the nucleon-nucleon reduced mass.

I. INTRODUCTION

The one-dimensional many-body system has been used extensively to study various approximate multiple scattering formalisms. In particular the one-dimensional many-body system with zero-range interactions is soluble in closed form and thus various models may be compared with the exact solution for such a system. A recent study by Tobocman and Pauli[1] which employed the above model for the three-body case yielded interesting conclusions on both the adiabatic and Glauber approximations. An extension of that study was made by Bajaj and Nogami.[2] It included third-order terms in the Glauber model and considered systems of up to five particles.

In addition to the Glauber and adiabatic models, the Watson optical-potential model has received considerable attention in the literature. There has been some discussion concerning the relative merits of the Glauber approximation and the Watson optical-potential formalism.[3,4] In this work we compare calculations employing the adiabatic, Glauber, and Watson optical-potential models with the exact solution for two-, three-, four-, and ten-particle targets.

The three models tested may be described briefly as follows: The adiabatic (or fixed-scatterer) approximation[5] results from the assumption that the energy of the incident projectile is much greater than the excitation energies of any of the states of the target that play a significant role in the scattering process. This is equivalent to the assumption that the relative motion of the target particles is negligible over the interval of time required for the projectile to complete its interaction with the target.

The Watson optical-potential (WOP) formalism[6] in contrast to the adiabatic approximation assumes target excitation to be unimportant so that target excitation is not permitted to result from the interaction of the projectile with any target particle. On the other hand, the dynamical development of the target is allowed to proceed normally as the projectile finds its way from one target particle to the next.

The Glauber approximation[7] combines the eikonal approximation with the adiabatic approximation. It assumes that the target particles maintain fixed relative positions while the projectile is interacting with the target and that the projectile follows a classical trajectory as it passes through the target. Thus the possibility of large-angle scatterings is ignored and there results a suppression of high-order multiple-scattering processes.

In the work of Tobocman and Pauli[1] it was found that for one-dimensional nucleon-deuteron scattering both the adiabatic and Glauber approximations worked very well down to surprisingly low energies. In this work we extend the comparison to three-, four-, and ten-particle targets where multiple-scattering effects will be much more severe. The Watson formalism is included in the comparison.

We do see that as the number of target particles is increased the discrepancies become greater between the approximate-method predictions and the exact result. Surprisingly, the adiabatic approximation is the best and the Watson formalism only second best. We expected the reverse result. One must conclude that more harm is done in neglecting target excitation than is done in neglecting the dynamical development of the target as the projectile moves from one target member to another.

One of the attractive features of the original Watson formalism[6] was the fact that the optical potential was to be calculated from the projectile–target-particle two-body t matrix, which is closely related to experiment. Foldy and Walecka[5] have suggested using a projectile–target-particle two-body t matrix calculated with the projectile-target reduced mass instead of the projectile–target-particle reduced mass. We have found that the Foldy and Walecka modification does indeed improve the Watson optical-potential approximation with respect to comparisons to exact results.

In Sec. II we formulate the adiabatic approximation, the Watson optical-potential formalism, and the Glauber theory as they apply to the one-dimensional $(N+1)$-body problem. Section III is devoted to a description of how the calculations were performed. The results of the calculation are described in Sec. IV. In Sec. V we discuss the similarities and differences between our one-dimensional model and the physical nucleon-nucleus system.

II. SCATTERING OF A PROJECTILE BY AN N-NUCLEON TARGET IN ONE DIMENSION

The Schrödinger equation for a projectile incident upon an N-nucleon target in one dimension is

$$\left[\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+\sum_{j=1}^{N}V(x-y_j)+H_N(y)-E\right]\Psi(x,y)=0\,, \tag{1}$$

where m is the reduced mass of the projectile and target, x is the separation of the projectile and the center of mass of the target, and y_j is the displacement of the jth target nucleon from the center of mass of the target. H_N is the Hamiltonian for the internal degrees of freedom of the target and $V(x-y_i)$ is the interaction potential of the projectile and the ith particle of the target.

Let $\phi_0(y)$ designate the ground state of the target where

$$(H_N-E_0)\phi_0(y)=0\,. \tag{2}$$

The function $\phi_i(y)$ will designate the ith excited state of the system with energy E_i. The scattering-state wave function for the projectile-plus-target system will have the asymptotic form

$$\Psi(x,y)=\phi_0(y)\exp(ik_0x)+\sum_{n=0}\phi_n\exp(-ik_nx)R_n\,,\quad x<-c \tag{3a}$$

$$\Psi(x,y)=\sum_{n=0}\phi_n(y)\exp(ik_nx)T_n\,,\quad x>c \tag{3b}$$

$$k_n{}^2=\frac{2m}{\hbar^2}(E-E_n)\,, \tag{3c}$$

where c is the range of the projectile-target interaction. The transmission and reflection probabilities are $|1+T_0|^2$ and $|R_0|^2$, respectively. Conservation of flux requires that

$$\sum_{n=0}\frac{k_n}{k_0}(|\delta_{0n}+T_n|^2+|R_n|^2)=1\,. \tag{4}$$

The transmission and reflection amplitudes T_n and R_n are related to the transition operator T by the following expressions:

$$T_n=\delta_{n0}+\frac{i}{2k_n}\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dx'\exp(ik_0x)T_{0n}(x,x')\times\exp(-ik_nx')\,, \tag{5a}$$

$$R_n=\frac{i}{2k_n}\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dx'\exp(ik_0x)T_{0n}(x,x')\exp(ik_nx')\,, \tag{5b}$$

$$T_{0n}(x,x')=\frac{2m}{\hbar^2}\int dy_1dy_2\cdots dy_N\delta(y_1+y_2+\cdots+y_N)\times\phi_0(y)T\phi_n(y)^*\,, \tag{5c}$$

$$T=V+VG_0T\,,\quad V=\sum V(x-y_j)\,, \tag{5d}$$

$$G_0=(E-H_N-K+i\epsilon)^{-1}\,,\quad K=-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\,. \tag{5e}$$

The WOP formalism will be presented in the manner employed by Kerman, McManus, and Thaler.[8] Here the target nucleons are all physically identical, the target states ϕ_n are symmetric under target-nucleon exchange, and the Hamiltonian operator is symmetric under target-nucleon exchange. Projectile–target-nucleon exchange will be ignored. Under these conditions Eq. (5d) becomes

$$T=NV_1(1+\tilde{G}_0T)\,, \tag{6a}$$

$$V_j=V(x-y_j)\,, \tag{6b}$$

$$\tilde{G}_0=aG_0\,, \tag{6c}$$

where a is a projector onto symmetric states. We introduce the operator

$$\tau_1=V_1(1+\tilde{G}_0\tau_1) \tag{7}$$

from which follows

$$V_1=\tau_1(1+\tilde{G}_0\tau_1)^{-1}=(1+\tau_1\tilde{G}_0)^{-1}\tau_1\,. \tag{8}$$

But from Eq. (6) we have

$$V_1=N^{-1}T(1+\tilde{G}_0T)^{-1}\,. \tag{6d}$$

Combining Eqs. (8) and (6d) gives

$$T=N\tau_1+(N-1)\tau_1\tilde{G}_0T=\frac{N}{N-1}T_1\,, \tag{9a}$$

$$T_1=(N-1)\tau_1+(N-1)\tau_1\tilde{G}_0T_1\,. \tag{9b}$$

Up to this point nothing beyond projectile–target-nucleon exchange has been neglected. We now make the impulse approximation by replacing τ_1 by the two-body transition operator t_1. This may be effected by replacing $\tilde{G}_0$ by the operator

$$g_0 = (E - E_0 - K + i\epsilon)^{-1}\,, \quad K = \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\,, \tag{10a}$$

so that

$$t_1 = V_1(1 + g_0 t_1)\,. \tag{10b}$$

Here the struck nucleon maintains its position relative to the other members of the target and is endowed with the mass of the entire target. In addition to neglecting projectile-target-nucleon exchange we are thus also neglecting target internal motion while the projectile interacts with any target particle. Next we approximate $\tilde{G}_0$ in Eq. (9b) by

$$\hat{g}_0 = |\phi_0) g_0 (\phi_0|\,, \tag{10c}$$

so that we obtain

$$T_1 \approx (N-1) t_1 (1 + \hat{g}_0 T_1)\,. \tag{10d}$$

This approximation suppresses any target excitation that might result from the interaction of the projectile with a target particle. Equation (10d) is in consequence an equation for the elastic scattering of the projectile from a potential well defined by

$$U(x) = (N-1)\int dy_1 dy_2 \cdots dy_N \delta(y_1 + y_2 + \cdots + y_N) \times \phi_0(y) t_1 \phi_0(y)\,. \tag{11}$$

Thus, the approximation employed in Eq. (10) yields the following formulation. The Schrödinger equation

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + U(x) + E_0 - E\right]\Psi(x) = 0 \tag{12}$$

is solved subject to the conditions

$$\Psi(x) = \exp(ik_0 x) + R_0' \exp(-ik_0 x)\,, \quad x < -c \tag{13a}$$

$$= T_0' \exp(ik_0 x)\,, \quad x > c\,. \tag{13b}$$

Then the transition and reflection probabilities are taken to be

$$|T_0|^2 = \left|\frac{N}{N-1} T_0'\right|^2, \tag{14a}$$

$$|R_0|^2 = \left|\frac{N}{N-1} R_0'\right|^2. \tag{14b}$$

We now turn to the adiabatic approximation and the Glauber formalism. The adiabatic approximation consists of setting

$$\Psi(x,y) \approx \phi_0(y)\psi(x,y)\,, \tag{15}$$

where

$$\left[\frac{\partial^2}{\partial x^2} + \omega(x,y) + k_0^2\right]\psi(x,y) = 0\,, \tag{16a}$$

$$k_0^2 = \frac{2m}{\hbar^2}(E - E_0)\,, \tag{16b}$$

$$\omega(x,y) = -\sum_j \frac{2m}{\hbar^2} V(x - y_j) = \nu \sum_j \delta(x - y_j)\,. \tag{16c}$$

Equation (16) is the Schrödinger equation for the scattering of the (reduced mass) projectile by N fixed scatterers. Here we employ zero-range potentials which are nonoverlapping.

The soluton of Eq. (16) for nonoverlapping potentials has been given by Kujawski.[9] We employ an alternative method proposed by Bajaj and Nogami.[10] This method is an iterative procedure that enables one to calculate the transition amplitudes for $n+1$ fixed scatterers from those of n fixed scatterers provided they are nonoverlapping. The following is assumed: (1) The n scatterers are located such that their potentials vanish outside the region $x_n + c_n > x > x_n - d_n$, where x_n is the coordinate of the nth target nucleon. (2) The action of the $(n+1)$th scatterer is confined to the region $x_1 + c_1 > x > x_1 - c_1$. (3) The regions indicated in (1) and (2) do not overlap. Now if the $(n+1)$th scatterer were not present the wave function would be

$$\psi^{(n)} = \exp(ikx) + R^{(n)}\exp(-ikx)\,, \quad x < x_n - d_n \tag{17a}$$

$$= T^{(n)}\exp(ikx)\,, \quad x > x_n + c_n\,. \tag{17b}$$

An independent solution of the n-scatterer Schrödinger equation can be formed from a linear combination of $\psi^{(n)*}$ and $\psi^{(n)}$. We denote this solution by $\chi^{(n)}$:

$$\chi^{(n)} = S^{(n)}\exp(-ikx), \quad x < x_n - d_n \tag{18a}$$

$$= \exp(-ikx) + P^{(n)}\exp(ikx), \quad x > x_n + c_n \tag{18b}$$

where

$$S^{(n)} = (1 - R^{(n)*}R^{(n)})/T^{(n)*}, \tag{18c}$$

$$P^{(n)} = -R^{(n)*}T^{(n)}/T^{(n)*}. \tag{18d}$$

If only the $(n+1)$th scatterer were present we would have equations identical to Eqs. (17) and (18) with superscripts n replaced by 1.

A solution of the $(n+1)$-scatterer problem will now be constructed in terms of the functions $\psi^{(1)}$, $\chi^{(1)}$, and $\psi^{(n)}$. Suppose that $x_1 + c_1 < x_n - d_n$. Then

$$\begin{aligned}\psi^{(n+1)} &= \exp(ikx) + R^{(n+1)}\exp(-ikx), \quad x < x_1 - c_1 \\ &= \psi^{(1)} + \alpha\chi^{(1)}, \quad x_1 + c_1 < x < x_n - d_n\end{aligned} \tag{19a}$$

$$\psi^{(n+1)} = T^{(n+1)}\exp(ikx), \quad x > x_n + c_n$$
$$= \beta\psi^{(n)}, \quad x_1 + c_1 < x < x_n - d_n. \tag{19b}$$

The above yields the linear combination of functions that fulfills the asymptotic boundary conditions and is a solution of the Schrödinger equation. The coefficients α and β are determined by the requirement that the above two equations are in agreement in the region $x_1 + c_1 < x < x_n - d_n$. These coefficients in turn determine the transmission and reflection amplitudes:

$$T^{(n+1)} = T^{(1)}T^{(n)}/(1 + D^{(n+1)}), \tag{20a}$$

$$R^{(n+1)} = (R^{(1)} + R^{(n)}T^{(1)}T^{(1)*-1})/(1 + D^{(n+1)}), \tag{20b}$$

with

$$D^{(n+1)} = R^{(n)}R^{(1)*}R^{(1)}R^{(1)*-1}. \tag{20c}$$

Equation (20) provides a straightforward step-by-step method for calculating the transmission and reflection amplitudes, $T^{(N)}(y)$ and $R^{(N)}(y)$, for the scattering of the projectile from the N target nucleons fixed at positions $y_1, y_2, \ldots, y_N$. The projectile has reduced mass m and relative motion kinetic energy $\hbar^2 k_0^2/2m$. These amplitudes must then be averaged over the ground-state density distribution to yield the adiabatic approximation

$$T_0^{\mathrm{AD}} = \int dy_1 dy_2 \cdots dy_N \delta(y_1 + y_2 + \cdots + y_N) \times \phi_0(y)^2 T^{(N)}(y), \tag{21a}$$

$$R_0^{\mathrm{AD}} = \int dy_1 dy_2 \cdots dy_N \delta(y_1 + y_2 + \cdots + y_N) \times \phi_0(y)^2 R^{(N)}(y). \tag{21b}$$

The Glauber approximation in this context consists of neglecting backward scattering within the adiabatic approximation so that

$$T_0^{\mathrm{GL}} = \int dy_1 dy_2 \cdots dy_N \delta(y_1 + y_2 + \cdots + y_N) \times \phi_0(y)^2 T^{(1)}(y_1)T^{(1)}(y_2)\cdots T^{(1)}(y_N), \tag{22a}$$

$$R_0^{\mathrm{GL}} = 0. \tag{22b}$$

III. DETAILS OF CALCULATION

In order to calculate the adiabatic transmission amplitude we approximate Eq. (21a) by the expression

$$T_0^{\mathrm{AD}} \cong \sum_{j=1}^{n} p_j T^{(N)}(y^j) \Big/ \sum_{j=1}^{n} p_j, \tag{23}$$

where the subscript j identifies a set of N random numbers between zero and one, namely $a_i^{(j)}$ where $1 \le i \le N$. We set

$$y_i^{(j)} = c(2a_i^{(j)} - 1), \quad 1 \le i \le N-1$$
$$y_N^{(j)} = -\sum_{i=1}^{N-1} y_i^{(j)}, \tag{24}$$

where c is a number chosen so that the target-density distribution $\rho(x)$ is negligible when $x > c$. The coefficient p_i is set equal to one if the condition

$$a_N^{(j)} < \phi_0(y)^2 \tag{25}$$

is met, and set equal to zero otherwise. For a sufficiently large number n of sets of N random numbers Eq. (23) is a good approximation to Eq. (21a) for the target ground-state density $\phi_0(y)^2$.

In order to calculate the Glauber approximation transmission amplitude Eq. (22a) is approximated by

$$T_0^{\mathrm{GL}} \cong \sum_{j=1}^{n} p_j T^{(1)}(y_1^{(j)}) \cdots T^{(1)}(y_N^{(j)}) \Big/ \sum_{j=1}^{n} p_j. \tag{26}$$

For the Watson optical potential we write

$$U(x) = \frac{(N-1)\int dy_1 dy_2 \cdots dy_N \delta(y_1 + y_2 + \cdots + y_N)\, \phi_0(y)^2\, t(x - y_1)}{\int dy_1 dy_2 \cdots dy_N \delta(y_1 + y_2 + \cdots + y_N)\, \phi_0(y)^2}. \tag{27}$$

The form chosen for the projectile-target-nucleon interaction is zero-range:

$$v(x - y_1) = \frac{2m}{\hbar^2} V(x - y_i) = -\nu\delta(x - y_i). \tag{28}$$

Therefore, the assumption that no overlap exists for the various projectile-nucleon interactions is rigorously met. By substituting the above potential into Eq. (10b) and solving for t we obtain

$$t(x - y_j) = -\frac{\hbar^2}{2m}\left[\frac{v\,\delta(x - y_j)}{1 + v g_0(y_j, y_j)}\right] = -\frac{\hbar^2}{2m}\hat{t}(x - y_j), \tag{29a}$$

where

$$g_0(x, x') = \frac{\hbar^2}{2m}\sum_\alpha\sum_\beta \chi_\alpha(x)\,\langle\chi_\alpha|(E - E_0 - K + i\epsilon)^{-1}|\chi_\beta\rangle \times \chi_\beta(x') = \exp[i k_0(x_> - x_<)]/2ik_0, \tag{29b}$$

where the χ_α's are any complete set. Thus we have that

$$t(x - y_j) = -\frac{\hbar^2}{2m}\left[\frac{\nu\,\delta(x - y_j)}{1 - i v/2k_0}\right] \tag{29c}$$

The optical potential of Eq. (27) is then given by

$$U(x) = -\frac{\hbar^2}{2m}\frac{\nu(N-1)}{(1-i\nu/2k_0)}\frac{\gamma(x)}{\int_{-\infty}^{\infty} dx\gamma(x)}, \quad (30a)$$

$$\gamma(x) = \int dy_3 dy_4 \cdots dy_N\, \phi_0\left(x, -x-\sum_3^N y_j, y_3, y_4, \ldots, y_N\right)^2. \quad (30b)$$

The multiple integral is then approximated by a sum over n sets of N-2 random numbers $y_s^{(j)}$, $3 \leq s \leq N$, between $-c$ and c:

$$\gamma(x) \approx \sum_{j=1}^{N} \phi_0\left(x, -x-\sum_3^N y_s^{(j)}, y_3^{(j)}, y_4^{(j)}, \ldots, y_N^{(j)}\right)^2. \quad (30c)$$

This result is then used in conjunction with Eqs. (12), (13), and (14) to calculate the transmission probability.

In order to obtain the adiabatic approximation it is necessary to obtain the single-scatterer reflection and transmission amplitudes

$$T^{(1)}(y_j) = 1 + \frac{i}{2k_0}\int_{-\infty}^{\infty} dx \exp(ik_0 x)\,\hat{t}(x-y_j) \times \exp(-ik_0 x) = 1 - i\nu/(2k_0 - i\nu), \quad (31a)$$

$$R^{(1)} = \frac{i}{2k_0}\int_{-\infty}^{\infty} dx \exp(ik_0 x)\,\hat{t}(x-y_j)\exp(ik_0 x) = -i\nu\exp(2ik_0 y_j)/(2k_0 - i\nu). \quad (31b)$$

The above two expressions are employed in Eq. (20) to obtain $T^{(N)}(y_1, y_2, \ldots, y_N)$. This in turn is used in Eq. (23) to calculate the adiabatic approximation of the transmission amplitude. In order to obtain the transmission amplitude from the Glauber theory one substitutes Eq. (31a) into Eq. (26) and arrives at the appropriate expression.

IV. RESULTS OF CALCULATION

The exact transmission and reflection amplitudes for a particle incident on an N-body system in which the particles interact via a zero-range potential $2mV(x) = -\hbar^2\nu\delta(x)$ is given by McGuire[11]:

$$T_{\text{exact}} = \frac{4ik_0 - \nu(N-1)}{4ik_0 + \nu(N+1)}, \quad (32a)$$

$$R_{\text{exact}} = 0, \quad (32b)$$

where k_0 is the wave number of the projectile in the center-of-mass system and m is the reduced mass of the projectile with respect to the N-particle system. In Figs. 1 we compare calculations done for the transmission probability using the WOP, adiabatic, and Glauber formalisms with the exact calculation for two-, three-, four-, and ten-particle targets. The density distribution functions employed in these four figures were obtained from the exact-solution ground-state wave function,

$$\phi_0(y) = C\exp\left[-\frac{\bar{c}}{2}\sum_{i>j}(|y_i - y_j|)\right], \quad (33)$$

where C is a normalization constant, $\bar{c} = m\nu/\hbar^2$, and y_i is the coordinate of the ith nucleon. In all cases calculations were done from 0 to 100-MeV lab energies.

Comparison of the approximate results for the transmission probability with the exact result reveals that the adiabatic approximation works better than the WOP formalism while the WOP formalism works better than the Glauber theory. All the approximate methods do poorly for the reflection probability unless the incident energy is very high.

The approximate methods give better results as the incident energy is increased and the number of particles in the target is decreased. This is to be expected in consequence of the adiabaticity assumptions. This trend is emphasized in our model because there is no saturation in binding.

The binding energy of the target resulting from the zero-range interactions increases as $N(N^2-1)$ with the number N of target nucleons.[11] The interaction strength was chosen so that the two-body binding energy would be 2.2 MeV with particle masses taken equal to the nucleon mass. Thus the binding energy of the three-nucleon system is 8.8 MeV, that of the four-nucleon system is 22.0 MeV, and that of the ten-nucleon system is 363 MeV. We see that the adiabatic approximation works well if the incident energy is greater than about three times the binding energy per nucleon.

The lack in saturation of binding leads to an absence of saturation of density. As the number of target nucleons is increased there is a sharp increase in target nucleon density. It is not obvious what effect this change in density might have on the predictions of the adiabatic approximation and the WOP formalisms. The Glauber predictions are independent of density. To study the effect of target density we did a series of calculations using a target-particle density of the form

$$\hat{\phi}_0(y)^2 = \prod_{i=1}^{N} p(y_i), \quad (34a)$$

$$p(y) = 1, \quad |y| < R - S, \quad (34b)$$

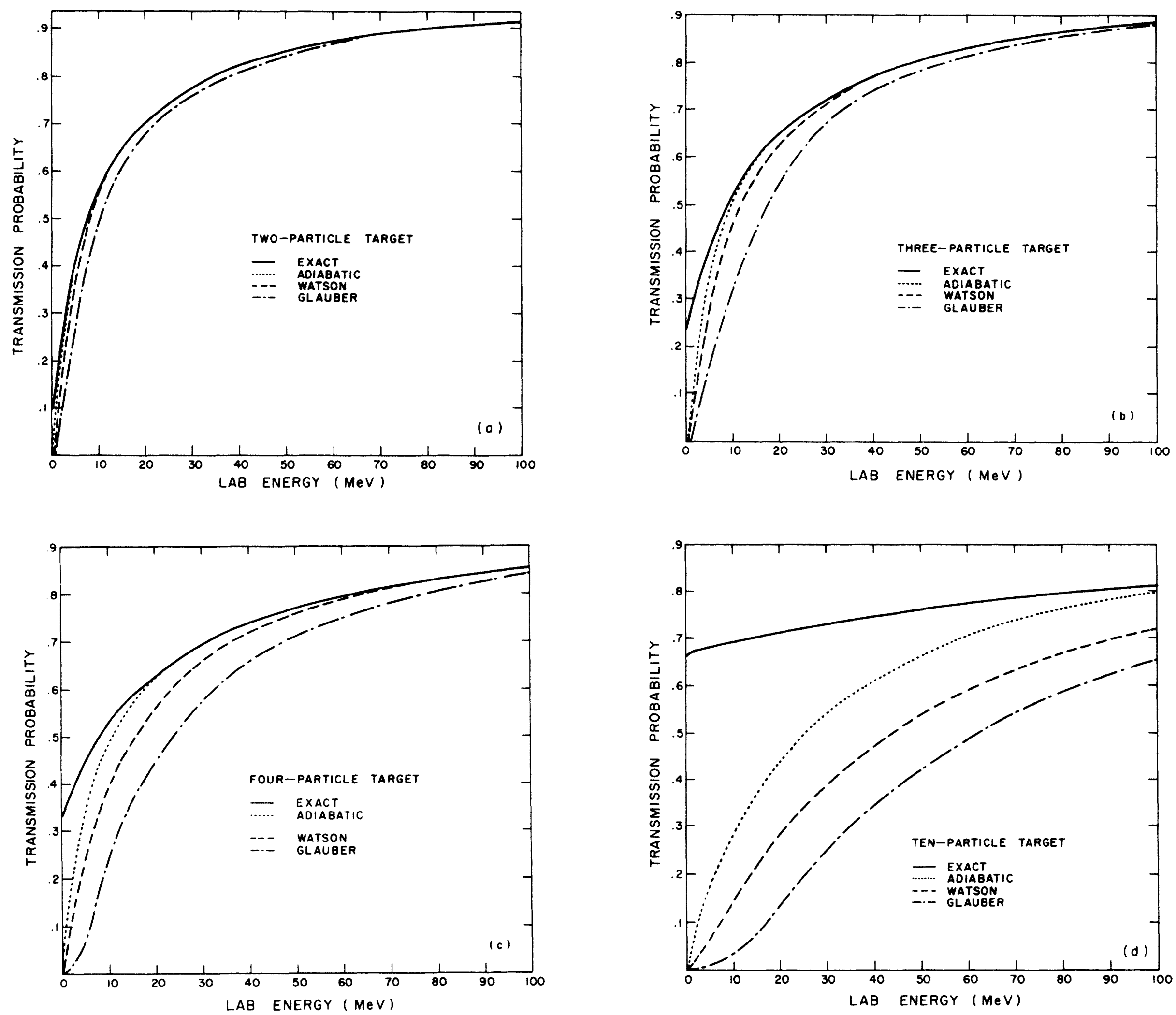

FIG. 1. Transmission probability for a particle impinging on a bound state of N (=2,3,4,10) particles in one dimension. The particles are distinguishable, have mass of 1 amu each, and interact with each other via identical zero-range potentials. The strength of the interaction is such as to produce a binding energy of 2.2, 8.8, 22.0, and 363 MeV for the two-, three-, four-, and ten-particle targets, respectively. The exact result is compared with that given by the adiabatic approximation, the Watson optical potential theory, and the Glauber theory.

$$p(y) = \exp\left[-\frac{(x-R+S)^2}{S^2} ln 2 \right], \quad |y| > R-S$$

$$R = N \times 0.5 \text{ F} \qquad (34c)$$
$$S = 0.5 \text{ F}$$

in place of Eq. (33).

For the two-, three-, and four-target-particle cases the WOP formalism is almost completely insensitive to the change in the target density while the adiabatic approximation is affected only rather weakly. For the ten-particle case where the difference in density is very large both formalisms show a much reduced transmission probability for the smaller-density case.

The Watson theory two-body transition operator t shown in Eq. (29) is not the simple nucleon-nucleon transition operator because the projectile-target reduced mass $m = NM/(N+1)$ is used instead of the nucleon-nucleon reduced mass of $M/2$. We tried a series of WOP calculations comparing the consequences of using various different types of kinematics in the approximate Green's function. Using the kinematics suggested by Watson, the Green's function operator G_0 appearing in Eq. (7) would be approximated by

$$g_0 = \left(E - E_0 + \frac{\hbar^2}{M}\frac{\partial^2}{\partial x^2} + i\epsilon \right)^{-1} \qquad (35)$$

instead of the Foldy and Walecka choice shown in Eq. (10a). Then in place of Eq. (29c) we would get

$$t(x-y_j) = -\frac{\hbar^2}{2m}\left\{\frac{\nu\,\delta(x-y_j)}{1-(i\nu/2k_0)[(N+1)/2N]^{1/2}}\right\}. \tag{36}$$

This is the true nucleon-nucleon t matrix for our model. For comparison we also tried

$$g_0 = \left[E - E_0 + \frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2} + i\epsilon\right]^{-1} \tag{37}$$

which gives

$$t(x-y_j) = -\frac{\hbar^2}{2m}\left\{\frac{\nu\,\delta(x-y_j)}{1-(i\nu/2k_0)[(N+1)/N]^{1/2}}\right\}. \tag{38}$$

Equation (29c) is the result of using a reduced mass of $m = NM/(N+1)$, Eq. (36) is the result of using a reduced mass of $M/2$, and Eq. (38) is the result of using a reduced mass of M in the two-body propagator g_0. The reduced mass $m = NM/(N+1)$, suggested by Foldy and Walecka, gave the best result. This is shown in Fig. 2.

An interesting aspect of our results is the fact that the approximate transmission probabilities all vanish in the zero-energy limit while the exact transmission probability approaches a finite value. The behavior of the exact result is a consequence of the extreme symmetry of the model. The masses of all particles are the same and the mutual interactions are all identical.

A similar effect was found by Dodd,[12] who used the Faddeev formalism to study the one-dimensional three-body system. The exact zero-energy transmission probability became zero when the mutual interactions between different pairs of particles were no longer the same.

Our model is particularly unfavorable for the Glauber theory since backward scatterings play a much more important role in this model than they would in a three-dimensional system with finite-range interactions. In the one-dimensional system the projectile collides head-on with every member of the target at least once. In a three-dimensional system the projectile will miss many members of the target and many of the collisions it does make will be just glancing ones. Thus in practical applications the Glauber theory may give results comparable in quality to the WOP formalism.

We find that the adiabatic approximation of neglecting dynamical development of the target over the duration of the scattering process is a less severe restriction that the Watson approximation of neglecting excitation of the target. The adiabatic approximation is generally too difficult to implement in realistic cases. When the eikonal assumption is added to the adiabatic one giving the Glauber formalism we find that the combination works less well than the Watson formalism.

V. RELEVANCE OF THE MODEL NUCLEON-NUCLEUS SCATTERING

Our calculations have been done for the symmetrical, one-dimensional, N-body system with zero-range interactions. Of course, this model differs in many important respects from physical nucleon-nucleus scattering; otherwise it would not be soluble. On the other hand, this model is

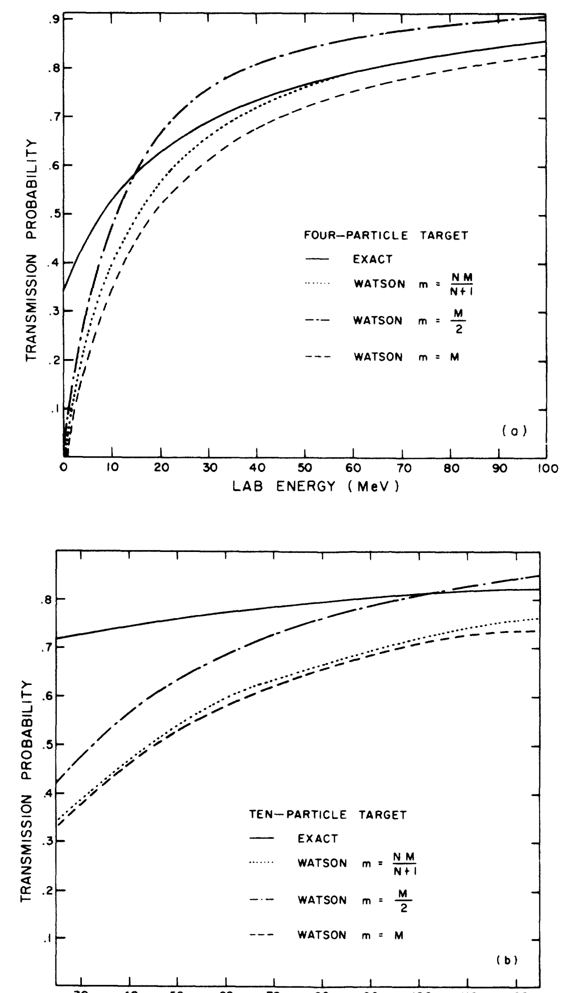

FIG. 2. Transmission probability for a particle impinging on a bound state of N (= 4,10) particles in one dimension. The parameters are the same as for Fig. 1. The exact result is compared with that given by the Watson optical potential theory using a two-body t matrix calculated with a reduced mass of $NM/(N+1)$, $M/2$, and M.

a true many-body system which manifests multiple-scattering processes. Let us survey the significant differences between our model and the nucleon-nucleus system.

Our model lacks the bound excited states of a physical nucleus. Thus one might suppose that it cannot manifest inelastic scattering processes. However, the model does have unbound excited states. These excited states are coupled to the ground state by the perturbation produced by the projectile. Moreover, these excited states are not degenerate with the ground state. If this were not the case the adiabatic approximation would agree with exact calculation at all energies.

It is true that these unbound excited states, the breakup channels, receive no flux, which is not realistic. This fact cannot be regarded as the reason for the success of the adiabatic approximation for our model. The breakup channels are dynamically coupled to the incident channel. The fact that they receive no flux is the consequence of a delicate destructive interference phenomenon. If the strength of the interaction of the projectile with each of the target particles were not exactly equal to each other, then this interference would not occur and there would be nonvanishing breakup flux.

A second difference between our model and the nucleon-nucleus system is the fact that the model predicts that the target becomes more transparent to the projectile as the energy is increased. However, the fact that nuclei become more opaque to nucleons with increasing energy is due to meson production rather than to any multiple-scattering effect. We could have made the model exhibit this sort of effect by making the two-body interaction potential complex and energy-dependent, but this phenomenon is extraneous to the multiple-scattering effects which we wanted to study.

Quasielastic knockout is an important effect in nucleon-nucleus scattering which is absent in our model. However, our model does show elastic knockout which is very similar to quasielastic knockout. This process drains flux out of the elastic channel just as quasielastic knockout does. That is the main effect as far as the elastic scattering is concerned.

Finally, there is the question of the binding energy. Admittedly, the model binding energies get very large when there are more than just a few particles in the target. The two-, three-, and four-particle targets do not have outrageous binding. The ten-particle target does have a very large binding. The main consequence of this is that the approximate methods we tested required greater incident energy in order to be valid.

As we have said, there are differences and similarities between the symmetric one-dimensional N-body system with zero-range interactions and the nucleon-nucleus system. Are the differences crucial? We cannot say with certainty. We have found that the adiabatic approximation works very well for the one-dimensional model and that the Watson optical potential works better with the Foldy-Walecka kinematics than with the Watson kinematics. We can see no obvious reason why these conclusions should be valid for the model and fail to be valid for the nucleon-nucleus system.

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