Some remarks on the $U_A(1)$ problem in gauge theories of the strong interaction*

Heinz R. Pagels
The Rockefeller University, New York, New York 10021
(Received 22 September 1975)

Previously proposed solutions to the $U_A(1)$ problem, or ninth axial-vector current problem, are reviewed and a new solution proposed. We point out that in the presence of magnetic gauge charges the anomalous term $F^a_{\alpha\beta}{}^*F^a_{\alpha\beta}$ is not a total divergence. This implies that the $U_A(1)$ symmetry is explicitly broken.

The Lagrangian for a gauge theory of strong interactions is

$$\mathcal{L} = \mathcal{L}_{YM} + i\bar{q}\not{D}q + \mathcal{L}_{SB} .$$

Here $\mathcal{L}_{YM}$ is the action density for an $SU_c(3)$ octet of Yang-Mills colored gauge fields, q is a Fermi field transforming like $\underline{3}$ under $SU_c(3)$ and like $(3,1)+(1,3)$ under the $SU(3)\times SU(3)$ of the strong interactions, D_μ is the $SU_c(3)$ covariant derivative, $\mathcal{L}_{SB}$ is an $SU(3)\times SU(3)$-symmetry breaking term, given by fermion mass terms. We will refer to this theory and its variants as "quantum chromodynamics" (QCD). The symmetry of $\mathcal{L}$ if $\mathcal{L}_{SB}=0$ is $SU_c(3)\times SU(3)\times SU(3)\times U_A(1)\times U(1)$ with $SU_c(3)$ an exact local gauge symmetry. Here U(1) corresponds to baryon number conservation and $U_A(1)$—which is the topic of this paper—to the conservation of the "ninth axial-vector current," an $SU_c(3)$ singlet transforming like $\gamma_\mu\gamma_5\lambda^0$. The assumptions governing this model of strong interactions are (i) only $SU_c(3)$ singlets are in the spectrum of physical states (confinement); (ii) if $\mathcal{L}_{SB}=0$ the $SU(3)\times SU(3)$ symmetry is realized in a Nambu-Goldstone manner with an SU(3) octet of Goldstone pseudoscalar mesons and SU(3) multiplets of states; (iii) if $\mathcal{L}_{SB}=0$ the $U_A(1)$ symmetry is absent; (iv) if $\mathcal{L}_{SB}=0$ the formal scale invariance of $\mathcal{L}$ is absent. This would seem to be a minimal set of requirements if $\mathcal{L}$ is to describe the real world of strong interactions. The manifest advantages of this model of strong interactions have been described by Fritzsch, Gell-Mann, and Leutwyler[1] and by Weinberg.[1]

Our discussion focuses on assumption (iii) above. A logical possibility is that the charge associated with $U_A(1)$ vanishes in the physical sector.[2] This leads to problems in the saturation of equal-time commutators as pointed out by Fritzsch and Gell-Mann[2]; so we assume the charge is nontrivial. If (iii) is not true and the $U_A(1)$ is in fact present if $\mathcal{L}_{SB}=0$ then either there are nine Goldstone bosons or parity doubling of all hadron states. As this latter option is evidently not approximated by hadron states we consider only the first, an extra Goldstone boson. In this instance, as emphasized by Weinberg,[3] if one considers explicit symmetry breaking then there is a state satisfying $m_\eta{}^2 \leq 3m_\pi{}^2$, which is not seen. Further, even if one adds a $\Delta I=1$ mass term to $\mathcal{L}_{SB}$ one finds the $\eta\to 3\pi$ rate too small since the matrix element is related to the divergence of the ninth axial-vector current.[4]

This state of affairs has provoked various attempts to eliminate the extra $U_A(1)$ symmetry. The ninth axial-vector current is well known to have an Adler-Bell-Jackiw anomaly[5] so that if $A_\mu = \bar{q}\gamma_\mu\gamma_5 q$ then

$$\partial_\mu A_\mu = C_A F^a_{\mu\nu}\,{}^*F^a_{\mu\nu} , \tag{1}$$

with C_A a dimensionless nontrivial constant and $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$ is the Yang-Mills $SU_c(3)$ field tensor. The presence of the anomaly does not itself render a solution to the $U_A(1)$ problem since $F^a_{\mu\nu}\,{}^*F^a_{\mu\nu}$ is a divergence of a local operator,

$$\begin{aligned} F^a_{\mu\nu}\,{}^*F^a_{\mu\nu} &= \partial_\mu \xi_\mu , \\ \tfrac{1}{2}\xi_\mu &= \epsilon_{\mu\nu\lambda\delta} A^a_\nu \partial_\lambda A^a_\delta + \tfrac{1}{3} g f^{abc} \epsilon_{\mu\nu\lambda\delta} A^a_\nu A^b_\lambda A^c_\delta , \end{aligned} \tag{2}$$

with A^a_μ the $SU_c(3)$ gauge field. The usual Goldstone theorem requiring a massless pseudoscalar state is unaltered even with the anomalous term. This is because in the Ward identity one uses to prove the Goldstone theorem,[6] although it has an anomalous term, the anomaly vanishes at zero momentum transfer because $F^a_{\mu\nu}\,{}^*F^a_{\mu\nu}$ is a total divergence and this is what is required for the usual conclusion.

Langacker and I (see Ref. 7) suggested that because the scale invariance of the theory is broken (even if $\mathcal{L}_{SB}=0$) the divergence of the current is actually given by

$$\partial_\mu A_\mu = C_A F^a_{\mu\nu}\,{}^*F^a_{\mu\nu} + m\bar{q}i\gamma_5 q \tag{3}$$

with m some scale mass. This Baker-Johnson[8] type anomaly, although it cannot occur in perturbation theory, is a possible solution to the homogeneous Bethe-Salpeter equation for the irreducible vertex for $\partial_\mu A_\mu$. This phase transition to the mode in which axial-vector current conservation is violated is how QED avoids a Goldstone state.[7] For QCD it is not known if this explicit breaking mechanism occurs.

Kogut and Susskind[9] have described an alternate approach which was further developed by Weinberg.[10] They consider the axial-vector current

$$A'_\mu = A_\mu - C_A \xi_\mu \tag{4}$$

so that $\partial_\mu A'_\mu = 0$. The time rate of change of the associated axial charge $Q = \int A'_0 d^3x$ is then

$$\begin{aligned}\dot{Q} &= \int \partial_\mu A'_\mu d^3x + \int \vec{\nabla}\cdot\vec{A}'\, d^3x \\ &= \oint_S \vec{A}'\cdot d\vec{S}\,. \end{aligned}\tag{5}$$

Since A'_μ is an $SU_c(3)$ gauge-dependent quantity [although the integral (5) is gauge-invariant] one can suppose there is a gauge-variant long-range field that produces a nontrivial integral. Then $\dot{Q} \neq 0$, the desired result. This evidently occurs in two-dimensional QED.[9]

An explicit realization of this is the dipole ghost mechanism.[9,10] Consider a matrix element $\langle \alpha | F^a_{\mu\nu} {}^*F^a_{\mu\nu} | \beta \rangle$ with $q = q_\alpha - q_\beta$. This can be made nonvanishing as $q \to 0$, in spite of the fact that $F^a_{\mu\nu} {}^*F^a_{\mu\nu}$ is a total divergence, if there is a zero-mass pseudoscalar that couples to $F^a_{\mu\nu} {}^*F^a_{\mu\nu}$ [see Fig. 1(a)]. While this accomplished the desired effect of explicitly breaking $U_A(1)$ conservation it implies that this zero-mass state couples to other hadrons [Fig. 1(b)]. This undesired pole is to be canceled by a ghost state with negative norm and zero mass, Fig. 1(c), a prescription which does not violate unitarity since the only effect of the ghost is to cancel the zero-mass state in all S-matrix elements. If one supposes that because ξ_μ is gauge-variant the ghost does not couple to matrix elements of ξ_μ (and therefore $F^a_{\mu\nu} {}^*F^a_{\mu\nu}$) one has solved the problem. Like the previous proposal it is not known if this actually happens for QCD.

There is an alternate to these proposals. Our observation is that *if there are colored* $SU_c(3)$ *magnetic charges then* $F^a_{\mu\nu} {}^*F^a_{\mu\nu}$ *is not a total divergence and the* $U_A(1)$ *symmetry is explicitly broken.* By the presence of colored magnetic charges we mean that

$$D_\mu {}^*F^a_{\mu\nu} = {}^*J^a_\nu \tag{6}$$

does not vanish everywhere. Then the formal manipulations one uses to prove $F^a_{\mu\nu} {}^*F^a_{\mu\nu} = \partial_\mu \xi_\mu$ are invalid, a consequence of (nonphysical) string singularities in A^a_μ, and the matrix elements of $F^a_{\mu\nu} {}^*F^a_{\mu\nu}$ do not vanish for zero momentum transfer. The $U_A(1)$ symmetry is explicitly broken. Because there is no anomaly in the SU(3) octet of axial-vector currents these are conserved.

Many people[11] have suggested that magnetic charges have a role in the confinement problem. Most such proposals are based on classical chromodynamics (CCD) usually supplemented with scalar fields with a judicious representation content.[12] Magnetic charges are presumably confined along with all $SU_c(3)$ nonsinglets.[13] The main point is that if magnetic confinement with magnetic charges is a solution to QCD and (6) is nonvanishing then it also will solve the $U_A(1)$ problem. This, like the previous proposals, must remain speculation until more is known about QCD. The novel feature of our proposal is that there may be a direct connection between magnetic confinement mechanisms and the solution to the $U_A(1)$ problem.

The simplest illustration of this idea is for the classical electrodynamics of the Dirac monopole.[14] Then with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$F_{\mu\nu} {}^*F_{\mu\nu} = \partial_\mu (A_\nu {}^*F_{\mu\nu}) - A_\nu {}^*J_\nu \tag{7}$$

is not a total divergence if the magnetic current ${}^*J_\nu = \partial_\mu {}^*F_{\mu\nu}$ is nonvanishing. This is possible if the electromagnetic potential A_λ is singular along a string so that $(\partial_\mu \partial_\nu - \partial_\nu \partial_\mu) A_\lambda \neq 0$. As Dirac pointed out such a singularity has no observable consequences if the quantization condition $eg = n/2$ for electric and magnetic charge is obeyed. In a possible generalization of Dirac's treatment to classical chromodynamics with colored magnetic charges one finds that $F^a_{\mu\nu} {}^*F^a_{\mu\nu}$ is not a total divergence.[15]

Schwinger and others[16] have developed a quantum

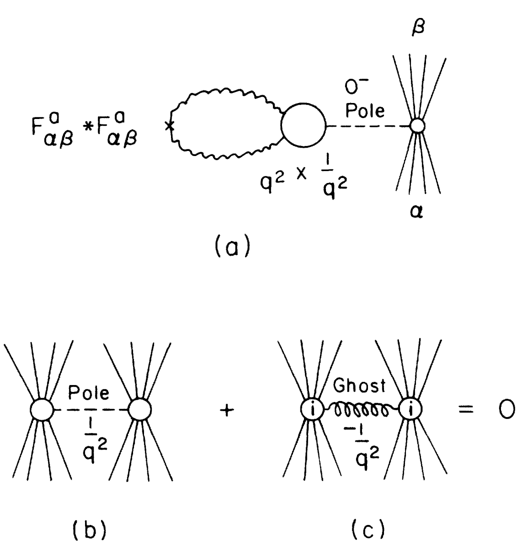

FIG. 1. The dipole ghost mechanism.

theory of real magnetic monopoles. In Schwinger's version one can verify that the operator $F_{\mu\nu} {}^*F_{\mu\nu} = \vec{E}\cdot\vec{H}$ is not a total divergence if both electric and magnetic charges are present. How much of this is actually valid for QCD with confined charges is a matter of speculation.

*Work supported in part by the U. S. Energy Research and Development Administration under Contract No. E(11-1)-2232B.

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