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Equivalent-boson method and free currents in two-dimensional gauge theories*

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In the context of the equivalent-boson method a large variety of quark-gluon theories are studied in two dimensions. A number of simple results are obtained, including the fact that gluon interactions (Abelian and non-Abelian) leave most of the physical currents free (at zero quark mass).

I. INTRODUCTION

As a method for exact solution of certain two-dimensional field theories, the equivalent-boson method has a long history.[1] Coleman's recent work[2] has stimulated new interest in the method by emphasizing a naturally broader role: Even when exact solutions are not forthcoming, the method provides an equivalence map between various fermion and boson field theories. Coleman's equivalence, supplemented by Mandelstam's explicit boson construction of fermions,[3] is in fact adequate to pair *any Abelian* fermion theory with a boson theory. Reference 4 is an extension of these ideas to *non-Abelian* fermions, thus completing the picture: *Any fermion (plus bosons if desired) two-dimensional field theory has its boson-equivalent field theory.*

As a result of this, surprising features of field theories (at least in two dimensions) are revealed. Out of many such surprises, I find the "appearance of quantum numbers out of nothing" to be one of the most striking: In the U(1) correspondence of Coleman and of Mandelstam, charge is given an underlying topological interpretation; from the boson point of view, charge cannot be seen as a Noether symmetry; it arises dynamically and topologically in the formation of the (charged) soliton. In the SU(N) models of Ref. 4, this is strikingly uplifted; full non-Abelian symmetries arise—due entirely to an interplay of topology and quantum mechanics.[5] Extensions of these ideas to four dimensions are problematic, but I am not yet convinced that important parts cannot be carried over. In my opinion, there is hope of understanding four-dimensional internal symmetry as a quantum-topological effect.

In the present paper, I am going to use the equivalent-boson methods to look into the structure of quark-gluon gauge theories, Abelian and non-Abelian. The solution to the simplest of these models, two-dimensional electrodynamics[1,6] is well known, and I have not yet been able to solve the color-sector of the non-Abelian models; yet there are a number of simple results which are easy to see.

Section II is a brief review of the Schwinger model in the formalism of Refs. 3 and 4. Section III is concerned with many quarks and non-Abelian gauge theories. The fact emerges here that, in such models, most of the physical currents are *free* (at zero quark mass). Special cases of this phenomenon have been noticed by Segrè and Weisberger[7] and recently, in a non-Abelian case, by Frishman.[8] With the present methods, we recover their results in a broad picture: For example, in an SU(M)⊗U(N) color-gluon-quark model [M^2-1 gluons; U(N) is ordinary weak-strong symmetry], *all the U(N) currents are free.* If the Abelian gluon is added, then the *traceless SU(N) currents remain free,* while the baryon number current picks up just the dynamics of the Schwinger model. In parallel to the equivalent-boson method, I also give alternate elementary arguments, based on free-current algebra, for the freedom of the various currents. The methods used are peculiar to two dimensions. The results serve to point up inherent difficulty in using these models as analogs of color-gauge models in four dimensions.

II. TWO-DIMENSIONAL ELECTRODYNAMICS

The Lagrangian for electrodynamics is

$$\mathcal{L}=\bar\psi(i\partial\!\!\!/ -gV\!\!\!/)\psi-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\,, \qquad (2.1)$$

where $V\!\!\!/\equiv\gamma^\mu V_\mu$, $F_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu$. In two dimensions, the model has been solved by many people and with many methods.[1,6] Because the model is a prototype for the more complicated models of Sec. III, I will give here an "instant operator solution" in the language of Refs. 3 and 4. The implied equations of motion are

$$(i\partial\!\!\!/ -gV\!\!\!/)\psi=0,\ \ \partial^\mu F_{\mu\nu}=gJ_\nu\,,\ \ J_\nu=\bar\psi\gamma_\nu\psi\,. \qquad (2.2)$$

Following Ref. 4, we go first to the interaction picture. A free Fermi field is represented in terms of a free massless Bose field as

$$\psi_D^r(x) = i^{r-1}\kappa N \exp\left\{-i\sqrt{\pi}\left[\int_{-\infty}^{x} d\xi\, \dot{\phi}_D(\xi) + (-1)^{r-1}\phi_D(x)\right]\right\}; \quad \Box^2\phi_D = 0\,. \tag{2.3}$$

Here $r = 1, 2$ is the spinor index, and the subscript D means free fields in the interaction picture. $\kappa = (\mu c/2\pi)^{1/2}$ is the usual normalization. The free current is $J_\mu = -(1/\sqrt{\pi})\epsilon_{\mu\nu}\partial^\nu \phi_D$ ($\epsilon_{01} = +1$). Using the interaction Lagrangian $\mathcal{L}_I = -gJ_\mu V^\mu$, one shows immediately that

$$U^+\dot{\phi}_D U = \dot{\phi} + \frac{g}{\sqrt{\pi}} V_1\,. \tag{2.4}$$

U is the Dyson time-evolution operator, so that the fields on the right, without subscripts, are full Heisenberg-picture fields.

Thus, the equivalent-boson field theory is

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g}{\sqrt{\pi}} V^\mu \epsilon_{\mu\nu}\partial^\nu\phi\,, \tag{2.5}$$

and the fermion is

$$\psi^r(x) = U^+\psi_D^r(x)U = i^{r-1}\kappa N \exp\left\{-i\sqrt{\pi}\left[\int_{-\infty}^{x} d\xi\left(\dot{\phi}(\xi) + \frac{g}{\sqrt{\pi}}V_1(\xi)\right) + i^{r-1}\phi(x)\right]\right\}\,. \tag{2.6}$$

Note that under a gauge transformation $V_\mu \to V_\mu - (1/g)\partial_\mu\Lambda$, $\psi \to e^{i\Lambda}\psi$, as required. It is a simple matter of differentiation to show that (2.6) solves (2.2). Useful in this check are the equations of motion implied by (2.5)

$$\Box^2\phi = \frac{g}{\sqrt{\pi}}F_{10} = \frac{g}{\sqrt{\pi}}\tfrac{1}{2}\epsilon_{\mu\nu}F^{\mu\nu}\,,\qquad \partial_\mu F^{\mu\nu} = -\frac{g}{\sqrt{\pi}}\epsilon^{\nu\mu}\partial_\mu\phi\,. \tag{2.7}$$

To solve this equivalent-boson system, it is common to express V_μ in terms of spin-zero fields[7]

$$V_\mu = \epsilon_{\mu\nu}\partial^\nu\chi + \partial_\mu\lambda, \quad F_{\mu\nu} = -\epsilon_{\mu\nu}\Box^2\chi\,. \tag{2.8}$$

χ and λ will need rescaling to have the dimensions of spin-zero fields. Substituting into (2.5), one obtains

$$\mathcal{L} \simeq \tfrac{1}{2}(\partial_\mu\phi)^2 + \tfrac{1}{2}\Box^2\chi\Box^2\chi - \frac{g}{\sqrt{\pi}}\partial^\mu\phi\partial_\mu\chi\,. \tag{2.9}$$

I have dropped λ by an integration by parts. The diagonalization transformation $\phi \equiv \phi' + (g/\sqrt{\pi})\chi$ results in

$$\mathcal{L} \cong \tfrac{1}{2}(\partial_\mu\phi')^2 + \tfrac{1}{2}\Box^2\chi\left(\Box^2 + \frac{g^2}{\pi}\right)\chi\,, \tag{2.10}$$

where one more integration by parts is implied. The final simple structure of the system is seen in terms of the fields $\hat{\chi}_1, \hat{\chi}_2$,

$$\hat{\chi}_1 \equiv \frac{\sqrt{\pi}}{g}\Box^2\chi, \quad \hat{\chi}_2 \equiv \frac{\sqrt{\pi}}{g}\left(\Box^2 + \frac{g^2}{\pi}\right)\chi\,,\qquad \mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi')^2 - \tfrac{1}{2}\hat{\chi}_1\left(\Box^2 + \frac{g^2}{\pi}\right)\hat{\chi}_1 + \tfrac{1}{2}\hat{\chi}_2\Box^2\hat{\chi}_2\,. \tag{2.11}$$

This last step is a special case of the general approach (to higher-derivative Lagrangians) of Ref. 9. All three fields are free. $\hat{\chi}_1$ has mass (squared) g^2/π, and $\hat{\chi}_2$ has negative metric. With the summarizing relations

$$V_\mu = \frac{\sqrt{\pi}}{g}\epsilon_{\mu\nu}\partial^\nu(\hat{\chi}_2 - \hat{\chi}_1) + \partial_\mu\lambda\,,\qquad \phi = \phi' + \hat{\chi}_2 - \hat{\chi}_1\,, \tag{2.12}$$

the Fermi field (2.6) is easily expressed in terms of the free fields $(\phi', \hat{\chi}_1, \hat{\chi}_2)$. Adding a Fermi-mass term to (2.1) will, as usual, result in an extra term in (2.11) proportional to[10] $\cos(\phi' + \hat{\chi}_2 - \hat{\chi}_1)$.

The prescription for the gauge-invariant current

$$J_{\rm GI}^\mu(x) = \lim_{x\to y} : \bar{\psi}(x)\gamma^\mu \exp\left[+ig\int_x^y V_1(\xi)d\xi\right]\psi(y):\tag{2.13}$$

is easily carried out with the representation (2.6) following Refs. 3 and 4. The algebra takes only a few lines and the result is proportional to

$$J_{\rm GI}^\mu = -\frac{1}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_\nu\phi\,. \tag{2.14}$$

This result is easy to anticipate: ϕ, the field that makes up the *free* fermion, is the only scalar in the problem not involved in gauge transformations. The gauge-invariant axial-vector analog

$$J_{\rm GI}^{\mu 5} = \frac{1}{\sqrt{\pi}}\partial^\mu\phi, \quad \partial_\mu J_{\rm GI}^{\mu 5} = \frac{g}{2\pi}\epsilon_{\mu\nu}F^{\mu\nu} \tag{2.15a}$$

is *not* conserved by virtue of (2.7). This of course is the axial-vector anomaly (here associated with the relevant single-Fermi-loop vacuum polarization graph). Conserved but *not* gauge-invariant currents can be constructed by writing $\phi \to \phi'$ in (2.14) and (2.15a):

$$J_\mu^c = -\frac{1}{\sqrt{\pi}} \epsilon_{\mu\nu} \partial^\nu \phi', \quad J_\mu^{5c} = \frac{1}{\sqrt{\pi}} \partial_\mu \phi' . \qquad (2.15b)$$

Note also that, in this spin-zero representation, Maxwell's equations are satisfied only up to a choice of gauge: One calculates from (2.11) and (2.12)

$$\partial^\mu F_{\mu\nu} = \frac{g}{\sqrt{\pi}} \epsilon_{\nu\mu} \partial^\mu \hat{\chi}_1 \qquad (2.16a)$$

or

$$\partial^\mu F_{\mu\nu} - g J_\nu^{\mathrm{GI}} = \frac{g}{\sqrt{\pi}} \epsilon_{\nu\mu} \partial^\mu (\phi' + \hat{\chi}_2) . \qquad (2.16b)$$

This is in agreement with Lowenstein and Swieca.[11] The fields ϕ' and $\hat{\chi}_2$ must cancel in all physical matrix elements, leaving no massless physical excitations. It is clear that gauge-invariant quantities (such as $\bar{\psi}\psi$, etc.) are functions of $\phi' + \hat{\chi}_2$. Then, because these have opposite metric, the cancellation in the physical sector is complete.

The well-known structure brought out in the discussion above is the "Schwinger mechanism" for eliminating the Goldstone boson associated with a "spontaneous breakdown" of the U(1) axial symmetry. This is called "seizing" in Ref. 1. Note that in this covariant gauge $\langle h | Q_5 | h' \rangle = 0$. Here, $|h\rangle, |h'\rangle$ are physical states constructed out of $\hat{\chi}_1$ only, while $Q_5 \equiv \int dx J_0^{5c}(x)$. This vanishing of the axial charges in the physical subspace is equivalent to $\Box^2 J_\mu^{5c} = 0$. The original algebra of bilinears is easily checked. In actual fact, however, such statements are quite gauge-dependent: In noncovariant gauges, it can be argued that no definite value can be assigned to $\langle h | Q_5 | h' \rangle$.[1,11]

III. REMARKS ON THEORIES WITH MANY QUARKS AND NON-ABELIAN GLUONS

In this section, our exposition will be a cataloging of various (zero-mass quark) models, studying progressively more complicated quark and gluon systems.

U(1) ⊗ U(N) model

By this, I mean an Abelian gluon coupled to a U(N) multiplet of quarks (say by baryon number). Here we need the generalized fermion-boson equivalence of Ref. 4. For free quarks,

$$\psi_a^r(x) \simeq i^{r-1} \kappa \times N \exp\left\{ -i\sqrt{\pi} \left[\int_{-\infty}^{x} d\xi\, \dot{\phi}_a(\xi) + (-1)^{r-1} \phi_a(x) \right] \right\}, \qquad (3.1)$$

where $a = 1, \ldots, N$, $\Box^2 \phi_a = 0$. Here I have omitted the Klein transformation operators ξ of Ref. 4. For calculation of charged currents, the details of their use are found in Ref. 4. The baryon number current is[12]

$$\begin{aligned} J_\mu &= :\bar{\psi}\gamma_\mu \psi: \\ &= -\frac{1}{\sqrt{\pi}} \epsilon_{\mu\nu} \partial^\nu \sum_a \phi_a \\ &= -\left(\frac{N}{\pi}\right)^{1/2} \epsilon_{\mu\nu} \partial^\nu \phi_+ , \\ \phi_+ &\equiv \frac{1}{\sqrt{N}} \sum_a \phi_a . \end{aligned} \qquad (3.2)$$

The *other* (diagonal) currents are

$$\begin{aligned} J_\mu^{\alpha(D)} &= :\bar{\psi} \frac{\lambda^{\alpha(D)}}{2} \gamma_\mu \psi: \\ &= -\frac{1}{2\sqrt{\pi}} \epsilon_{\mu\nu} \partial^\nu \sum_d \lambda_d(\alpha) \phi_d , \\ &\sum_d \lambda_d(\alpha) = 0 , \end{aligned} \qquad (3.3)$$

where $\lambda_d(\alpha)$ are the eigenvalues of the diagonal λ matrices of U(N) $[\lambda^{\alpha(D)}]$. The charged (off-diagonal) currents come out ($J_\pm \equiv J_0 \pm J_1$),

$$J_{\pm(x)}^{\alpha(O)} = \left(\frac{\mu c}{\pi}\right) \sum_{cd} \left(\frac{\lambda_\alpha^{(O)}}{2}\right)_{cd} N \exp\left\{ i\sqrt{\pi} \left[\int_{-\infty}^{x} d\xi (\dot{\phi}_c(\xi) - \dot{\phi}_d(\xi)) \pm (\phi_c(x) - \phi_d(x)) \right] \right\} . \qquad (3.4)$$

If, as in Ref. 4, we imagine orthogonalizing ϕ_a into $\phi_+, \{\Phi_a\}, a = 1, \ldots, N-1$, then only ϕ_+ is excited by the interaction

$$\mathcal{L}_I = -g J_\mu V^\mu = +g\left(\frac{N}{\pi}\right)^{1/2} \epsilon_{\mu\nu} \partial^\nu \phi_+ V^\mu . \qquad (3.5)$$

The equivalent-boson Lagrangian is

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + g\left(\frac{N}{\pi}\right)^{1/2} \epsilon_{\mu\nu} \partial^\nu \phi_+ V^\mu + \tfrac{1}{2}(\partial_\mu \phi_+)^2 + \tfrac{1}{2} \sum_{a=1}^{N-1} (\partial_\mu \Phi_a)^2 . \qquad (3.6)$$

Because $\{\Phi_a\}$ remain free and massless, the solution to this system follows trivially the lines of Sec. II. That there are in fact N-1 free massless Bose fields in this model ($\{\Phi_a\}$) was previously noted by Segrè and Weisberger.[7] In fact, it is easy to see that all the N^2-1 (traceless) SU(N) currents (3.3) and (3.4) are free in this model: The ϕ's of which they are comprised are orthogonal to ϕ_+; hence they can be expressed entirely in terms of $\{\Phi_a\}$ (and not ϕ_+). All remarks apply equally to axial-vector currents. The Abelian gluon excites only the baryon number current and its dynamics is that of the Schwinger model. The resulting (mass)^2 of the gluon (or ϕ_+) is g^2N/π. The scalar and pseudoscalar densities involve ϕ_+ as well as Φ_a, so they are not free.

As it turns out, there is an elementary argument that the SU(N) currents are free in this model. One need only notice that for free currents (in terms of free quarks),

$$[J_\mu(x,t), J_\nu^\alpha(x',t')]=0 \tag{3.7}$$

for all x, x' and t, t'. This a well-known, easily verified, fact peculiar to two dimensions. Since the interaction is only a function of J_μ, it is clear that the SU(N) currents J_μ^α commute with the interaction, and hence remain free. When phenomena like these occur, questions of quark confinement may need closer examination.

SU(M)⊗U(1) theories

Here we mean theories with M^2-1 color gluons [SU(M)] coupling to the appropriate colored quarks, ordinary SU(N) being temporarily suppressed. The (free) color currents have the form of (3.3), (3.4); the off-diagonal currents, as discussed in Ref. 4, appear spatially nonlocal. Thus $\mathcal{L}_1 = -gV_\mu^\alpha J_\alpha^\mu$ is a monstrous interaction, that I have made little progress in simplifying.[13] However, one conclusion is immediate: The currents J_μ^α involve only Φ_a, and *not* ϕ_+. Thus the tables are turned in this model. ϕ_+ is *not* excited: *The baryon number currents* (color singlet) *are free.*[8] *Non-Abelian gluons* [*SU(M)*] *do not seize.*

U(M)⊗U(1) theories

Here I mean to add the Abelian gluon, making M^2 gluons in all. This is the model 't Hooft[14] examined in the large-N limit. In this model, one sees immediately that the SU(M) and U(1) sectors decouple in the equivalent-boson Lagrangian. *The dynamics of the baryon number* (ϕ_+) *currents is precisely that of the Schwinger model.* I cannot prove that any currents are free. Seizing occurs in the baryon number currents, but it is entirely due to the Abelian gluon.

SU(M)⊗U(N) theories

Again we consider just the purely non-Abelian [SU(M)] gluons, this time with a full complement of ordinary quark quantum numbers [U(N)]. We take the quarks as an $M\times N$ matrix

$$\psi=\begin{bmatrix}\psi_{11}&\cdots&\psi_{1N}\\ \psi_{21}&&\psi_{2N}\\ \cdot&&\cdot\\ \cdot&&\cdot\\ \cdot&&\cdot\\ \psi_{M1}&\cdots&\psi_{MN}\end{bmatrix},$$

$$\psi^\dagger=\begin{bmatrix}\psi_{11}^\dagger&\cdots&\psi_{M1}^\dagger\\ \cdot&&\cdot\\ \cdot&&\cdot\\ \cdot&&\cdot\\ \psi_{1N}^\dagger&\cdots&\psi_{MN}^\dagger\end{bmatrix} \equiv\begin{bmatrix}(\psi^\dagger)_{11}&\cdots&(\psi^\dagger)_{1M}\\ \cdot&&\cdot\\ \cdot&&\cdot\\ \cdot&&\cdot\\ (\psi^\dagger)_{N1}&\cdots&(\psi^\dagger)_{NM}\end{bmatrix}, \tag{3.8}$$

so that the color-singlet and "-octet" currents are, respectively,

$$J_\mu^a=:\mathrm{Tr}\left(\frac{\Lambda^a}{2}\bar\psi\gamma_\mu\psi\right):,\quad \mathcal{J}_\mu^\alpha=:\mathrm{Tr}\left(\bar\psi\frac{\lambda^\alpha}{2}\gamma_\mu\psi\right):, \tag{3.9}$$

where Λ^a are $N\times N$ matrices and λ^α are $M\times M$. Before going to the equivalent-boson representation, we observe that for free quark currents

$$[J_\mu^a(x,t), \mathcal{J}_\nu^\alpha(x',t')]=0 \tag{3.10}$$

for all x, x' and t, t', an extension of (3.7). This statement is true as long as $\lambda^\alpha, \Lambda^a$ are not *both* proportional to unity. Thus, we can be certain that the *SU(M) gluon interaction will fail to excite any of the U(N) currents.* All U(N) currents are free in these models, and seizing does not occur.

In fact, I have not yet been able to extract much more than this result from the equivalent-boson representation. However, for the reader's reference, I will indicate how one sets up the problem. For the free quarks, we have from Ref. 4

$$\psi^r_{ab} \simeq i^{r-1}\kappa N \exp\left\{-i\sqrt{\pi}\left[\int_{-\infty}^{x} d\xi\, \dot{\phi}_{ab}(\xi) + (-1)^{r-1}\phi_{ab}(x)\right]\right\}, \tag{3.11}$$

where $a=1,\ldots,M$ is the color index, $b=1,\ldots,N$ is the ordinary SU(N) index. Here again the approximate sign means I have suppressed the ξ's of Ref. 4. For the color-octet currents, one obtains

$$\begin{aligned} \mathcal{J}^{\alpha(D)}_{\mu} &= -\frac{1}{2\sqrt{\pi}}\epsilon_{\mu\nu}\partial^{\nu}\sum_{d}\lambda_d(\alpha)\sum_{b}\phi_{db}, \quad \sum_{d}\lambda_d(\alpha)=0 \\ \mathcal{J}^{\alpha(O)}_{\mu\pm} &= \frac{\mu c}{\pi}\sum_{bcd}\left(\frac{\lambda^{(O)}_{\alpha}}{2}\right)_{cd} N\exp\left\{+i\sqrt{\pi}\left[\int_{-\infty}^{x} d\xi(\dot{\phi}_{cb}(\xi)-\dot{\phi}_{db}(\xi)) \pm (\phi_{cb}(x)-\phi_{db}(x))\right]\right\}, \end{aligned} \tag{3.12}$$

where (D) and (O) refer, as above, to neutral (diagonal) and charged (off-diagonal). For the color singlets,

$$\begin{aligned} J^{a(D)}_{\mu} &= -\frac{1}{2\sqrt{\pi}}\epsilon_{\mu\nu}\partial^{\nu}\sum_{d}\Lambda_d(a)\sum_{b}\phi_{bd}, \\ J^{a(O)}_{\pm} &= \frac{\mu c}{\pi}\sum_{bcd}\left(\frac{\Lambda^{(O)}_{a}}{2}\right)_{cd} N\exp\left\{i\sqrt{\pi}\left[\int_{-\infty}^{x} d\xi(\dot{\phi}_{bc}(\xi)-\dot{\phi}_{bd}(\xi)) \pm (\phi_{bc}(x)-\phi_{bd}(x))\right]\right\}. \end{aligned} \tag{3.13}$$

Now the interaction term $\mathcal{L}_I = -gV^{\alpha}_{\mu}\mathcal{J}^{\mu}_{\alpha}$ (remember, no Abelian gluon) involves and excites the set of $(M-1)N$ basis ϕ's

$$\phi_{1b}-\phi_{2b},\ \phi_{2b}-\phi_{3b},\ \ldots,\ \phi_{M-1,b}-\phi_{M,b} : b=1,\ \ldots,\ N. \tag{3.14}$$

The entire mode space is spanned by the MN-dimensional basis ϕ_{ab}, so the number of unexcited modes is $MN-(M-1)N=N$. *There are N free massless scalar fields left in the theory.* It is easy to see that the neutral U(N) currents (there are precisely N of them) are orthogonal to (3.12). These then are a basis for the N-dimensional unexcited subspace. Further, by SU(N) symmetry, *all the U(N) currents are free.* This is in complete agreement with the elementary argument above. As before, scalar and pseudoscalar densities are not necessarily free.

U(M)⊗U(N) theories

Relative to the preceding case, we now add the final Abelian gluon. This will excite one more scalar degree of freedom, being $(1/\sqrt{NM})\sum_{ab}\equiv\phi_+$. Thus there are $N-1$ massless free scalar fields in the models. The re are the neutral traceless ($\mathrm{Tr}\,\Lambda=0$) color-singlet currents. *All SU(N) currents (N^2-1) are free in the model;* the baryon number currents have seized—owing as always only to the dynamics of the Abelian gluon. Such a conclusion also follows immediately from the elementary argument based on (3.10).

Nonzero quark mass

As is well known,[2-4] Fermi-mass terms are "strong" in two dimensions. E.g., in the presence of a common mass term $m\bar{\psi}\psi$, all the ϕ's are coupled,[4] and I cannot prove that any currents are free. Reference 8 gives some discussion of smoothness as $m\to 0$.

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[1]For a recent application, and a list of references, see J. Kogut and L. Susskind, Phys. Rev. D 11, 3594 (1975).

[2]S. Coleman, Phys. Rev. D 11, 2088 (1975).

[3]S. Mandelstam, Phys. Rev. D 11, 3026 (1975).

[4]M. B. Halpern, Phys. Rev. D 12, 1684 (1975).

[5]I have been able to find soliton-like solutions to the classical boson sector of the SU(2) model of Ref. 4. They exhibit two conserved quantum numbers (I_3 and B); yet I cannot find an isospin in the classical model. Thus, I believe that the non-Abelian character of the group is both quantum mechanical and topological. These conclusions were obtained in collaboration with Warren Siegel.

[6]M. Kaku, Phys. Rev. D 12, 2330 (1975).

[7]G. Segrè and W. Weisberger, Phys. Rev. D 10, 1767

(1974).
[8]Y. Frishman, CERN report, 1975 (unpublished).
[9]A. Pais and G. E. Uhlenbeck, Phys. Rev. 79, 145 (1950).
[10]For the classical system with the cosine interaction, the soliton problem is easily reduced to quadratures. With the charge defined as in the text, it is easy to see that such solutions have zero charge, in distinction to the sine-Gordon soliton.
[11]J. Lowenstein and J. Swieca, Ann. Phys. (N.Y.) 68, 172 (1971). From our point of view the shift from Eq. (2.7) to Eq. (2.16) is due to our substituting directly into the Lagrangian, and integrating by parts. The difference is pure gauge.
[12]The relevant identities implicit in Refs. 3 and 4 are

$$[\phi^{(+)}(x),\ \phi^{(-)}(y)] \cong \left[\int_{-\infty}^{x} d\xi\, \dot\phi^{(+)}(\xi),\ \int_{-\infty}^{y} d\eta\, \dot\phi^{(-)}(\eta)\right] + O\big((x-y)^2\big)$$
$$\cong -\frac{1}{4\pi}\ln C\mu(x-y+i\epsilon) - \frac{1}{4\pi}\ln C\mu(x-y-i\epsilon),$$

$$\left[\phi^{(+)}(x),\ \int_{-\infty}^{y} d\eta\, \dot\phi^{(-)}(\eta)\right] \cong \frac{1}{4\pi}\ln C\mu(x-y+i\epsilon) - \frac{1}{4\pi}\ln C\mu(x-y-i\epsilon),$$

$$\left[\int_{-\infty}^{x} d\xi\, \dot\phi^{(+)}(\xi),\ \phi^{(-)}(y)\right] \cong \frac{1}{4\pi}\ln C\mu(y-x-i\epsilon) - \frac{1}{4\pi}\ln C\mu(y-x+i\epsilon).$$

Here (±) means positive- and negative-frequency parts. Finally, $\ln(y-x\pm i\epsilon) = \ln(x-y\mp i\epsilon) \pm i\pi$.
[13]The gluon interaction is one of many whose boson-equivalent field theories are spatially nonlocal. In such models, I believe the Lorentz group itself arises through the interplay of topology and quantum mechanics.
[14]G. 't Hooft, Nucl. Phys. B75, 461 (1974).