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Symmetry breaking and scalar bosons*

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There are reasons to suspect that the spontaneous breakdown of the gauge symmetries of the observed weak and electromagnetic interactions may be produced by the vacuum expectation values of *massless* weakly coupled elementary scalar fields. A method is described for finding the broken-symmetry solutions of such theories even when they contain arbitrary numbers of scalar fields with unconstrained couplings. In any such theory, there should exist a number of heavy Higgs bosons, with masses comparable to the intermediate vector bosons, plus one light Higgs boson, or "scalon" with mass of order $\alpha G_F^{-1/2}$. The mass and couplings of the scalon are calculable in terms of other masses, even without knowing all the details of the theory. For an SU(2) $\otimes$ U(1) model with arbitrary numbers of scalar isodoublets, the scalon mass is greater than 5.26 GeV; a likely value is 7–10 GeV. The production and decay of the scalon are briefly considered. Some comments are offered on the relation between the mass scales associated with the weak and strong interactions.

I. INTRODUCTION

A few years ago, Coleman and E. Weinberg[1] (CW) demonstrated that the spontaneous breakdown of gauge symmetries could be produced by the vacuum expectation values of weakly coupled elementary scalar fields *of zero mass*. The vacuum expectation values of the scalar fields would be determined by a balance between the ϕ^4 interaction term and one-loop corrections rather than between the ϕ^4 interaction term and a scalar mass term.

In this paper we wish to reopen the question of whether the spontaneous breakdown of the gauge symmetries associated with the observed weak and electromagnetic interactions is really of the CW type. Our reasons for suspecting that this may be the case are presented in Sec. II. In Sec. III we show how to extend the analysis of CW to a much larger class of gauge theories, theories in which there may be arbitrary numbers of scalar fields with more or less arbitrary interactions. Sections IV and V deal with the observable consequences of this sort of theory.

Our most striking result is that these theories require the existence of an unknown number of heavy Higgs bosons,[2] with about the same mass as the intermediate vector bosons, plus *one* "light" Higgs boson, with mass of order $\alpha G_F^{-1/2}$. The mass and couplings of the light Higgs boson may be calculated in terms of other masses, even without knowing all the details of the underlying gauge model. This light Higgs boson may be considered as the "pseudo-Goldstone boson"[3] associated with scale invariance. That is, the theory is scale-invariant in lowest order, so the spontaneous breaking of scale invariance entails the existence of a scalar particle with vanishing zeroth-order mass; one-loop corrections then break scale invariance, so they give this particle a relatively small mass. We would like for this reason to call this particle a "scalon." The important point for practical purposes is that the mass and couplings of the scalon may be calculated in terms of other masses, even without knowing all the details of the underlying gauge model.

These theories have a great deal of predictive power, which we have only begun to explore. Not only is the spontaneous symmetry breaking described by weakly coupled scalar fields, so that all the familiar perturbative results of gauge models are preserved; in addition, the theory is subject to constraints, which remove many of the free parameters of general gauge theories. One of these constraints is of course the vanishing of the bare scalar masses; the other constraint is a condition on the ϕ^4 couplings, described in Sec. III.

In Sec. VI we offer some speculative remarks about the relations among the various mass scales of physics.

II. EFFECTIVE FIELD THEORIES WITH MASSLESS SCALARS

In this section we present our reasons for suspecting that the spontaneous breakdown of the gauge symmetries of the weak and electromagnetic interactions is produced by the CW mechanism.[1] Our argument is admittedly far from compelling; the reader who finds it totally unconvincing is advised to turn immediately to Sec. III, and take the masslessness of the elementary scalar fields as a mere hypothesis. Nothing in the next three sections depends on the line of argument presented in this section.

It is attractive to suppose that the nonsimple gauge group of the observed weak, electromagnetic, and strong interactions is only a part of a larger, simple group. In order to explain why we only see effects of some of the gauge bosons, we are then compelled to assume[4] that the simple gauge group suffers a spontaneous breakdown, much stronger than the breakdown of, say, SU(2) ⊗U(1); the unobserved vector bosons then get superlarge masses, much larger than μ_W. In such a theory the physics of "ordinary" energies would be described by an effective field theory,[5] involving all those particles which did not get superlarge masses from the superstrong symmetry breaking.

What particles are these? Certainly the gauge bosons corresponding to symmetries which are not spontaneously broken at the superstrong level will not get masses from this symmetry breaking. Also, there may be chiral symmetries which are not broken at the superstrong level, and these symmetries may keep some of the fermions from getting superlarge masses. These gauge bosons and fermions, which appear at ordinary energies as massless fields in the effective Lagrangian, are to be identified with the "observed" intermediate vector bosons, quarks, and leptons.

If these were the only fields in the effective Lagrangian, then the masses of the corresponding particles would have to arise from a further *dynamical* spontaneous breakdown[6] of the remaining gauge and chiral symmetries. That is, in the absence of elementary spin-0 fields, the Goldstone bosons associated with the spontaneous symmetry breakdown of the weak and electromagnetic gauge symmetries would have to be bound states, held together by some sort of gauge interaction which becomes strong at energies of order 300 GeV.

It is certainly possible that the spontaneous symmetry breakdown which gives masses to the W and Z bosons is of this dynamical nature. However, the idea has drawbacks.[7] We do not know what could be the origin of the strong force which produces the Goldstone bosons as bound states; it seems that the ordinary strong interaction is much too weak at energies of order 300 GeV to do the job. [Rather, the ordinary strong interactions are believed to produce the spontaneous breakdown of chiral SU(2)⊗SU(2), for which the pion serves as Goldstone boson.] Also, theories of this type tend to be plagued either with true Goldstone bosons — massless spin-0 particles that are not eliminated by the Higgs mechanism — or exact unbroken global chiral symmetries.[8] Finally, and perhaps most importantly, if the spontaneous symmetry breakdown which gives masses to the W and Z involved strong interactions, then we could not calculate quantities such as μ_Z/μ_W perturbatively, and we would have no understanding of why the relative strengths of the neutral- and charged-current weak interactions are what they are.

These difficulties lead us to ask whether the effective Lagrangian could perhaps include fields of spin 0 as well as spin $\frac{1}{2}$ and spin 1. We would like to believe that the original Lagrangian did not involve any enormous mass ratios, so that the superstrong symmetry breakdown would either give any particle a superlarge mass, or else leave it massless. Hence it seems natural to suppose that those scalars which did not get superlarge masses from the superstrong symmetry breakdown appear in the effective Lagrangian at ordinary energies as particles of zero bare mass,[9] like the fermions and gauge bosons.

The trouble with this suggestion is that no one has been able to suggest any satisfactory reason why any scalars (aside from Goldstone bosons, which do not count because of their derivative couplings) should escape getting superheavy masses from the superstrong spontaneous symmetry breakdown.[10] One possibility is that the superstrong symmetry breakdown leaves both a chiral symmetry and a supersymmetry[11] unbroken, so that there is a multiplet including massless scalars and fermions. Unfortunately, the subsequent ordinary breakdown which gives masses to the intermediate vector bosons would then produce Goldstone fermions.

However, we do not really know anything about the mechanism for the superstrong symmetry breakdown — it might even involve strong gravitational forces[12] — so we are free at least to imagine that it might have left some scalars massless, along with the massless bosons and fermions. We must assume that these scalars have flavor but not color, so that they would not have strong interactions at ordinary energies, and we would not lose the advantages of a pure quark-gluon theory of strong interactions.[13] These scalars would provide the Goldstone bosons needed for the subsequent ordinary symmetry breaking, without the need for strong forces, so that they would provide an escape from the difficulties associated with dynamical symmetry breaking mentioned above. (Of course, since they are assumed to have flavor but not color, their vacuum expectation values could not break the gauge group of the strong interactions, and would therefore leave the gluons massless.)

Whatever misgivings one may have about this rather speculative line of reasoning, it at least leads us to study a well-defined class of theories: gauge field theories with massless scalars and fermions. Our concern here will almost entirely be with the "ordinary" spontaneous symmetry breakdown in such theories. However, the methods we use might also be relevant to the superstrong symmetry breakdown itself (as was originally suggested in Ref. 14) and have up to now been applied chiefly to deal with the superstrong symmetry breaking.[14,15] Indeed, even if we put scalar mass terms into the Lagrangian for a gauge theory, the theory will look like a massless field theory at superlarge values of the scalar fields.[14] The trouble with the assumption that the superstrong symmetry breaking is of the CW type[1] is the same trouble as for any theory of superstrong symmetry breaking: Even with no mass terms in the Lagrangian, in all cases we have examined a superstrong symmetry breakdown would give all non-Goldstone scalars superheavy masses,[10] so that none of them would be available to break the remnant gauge symmetries of the observed weak and electromagnetic interactions.

We will return in Sec. VI to the problem of the relation between the scale of the "ordinary" symmetry breaking — say 300 GeV — and the scale of the superstrong symmetry breaking.

III. SPONTANEOUS SYMMETRY BREAKING IN MASSLESS THEORIES

In their original analysis of spontaneous symmetry breaking in massless field theories, Coleman and E. Weinberg[1] dealt for the most part with simple theories having only a single Higgs boson.[2] For reasons which we hope to make clear, it is not entirely trivial to apply their analysis to more general models having arbitrary numbers of elementary scalars whose couplings are weak but otherwise more or less arbitrary. Recently a method for dealing with such general theories has been developed by one of us.[14] In this method, renormalization-group equations were used directly to find the minimum of the potential. In this section we will present a simplified and generalized version[15] of this method, in which the renormalization group is used only to justify the imposition of certain constraints on the couplings; the search for local minima of the potential is then carried out by means of ordinary perturbation theory.

We consider a general renormalizable gauge theory with an arbitrary multiplet of weakly-coupled real color-neutral scalar fields. Spontaneous symmetry breaking in such theories may be explored by studying an effective potential[16] $V(\Phi)$, defined as a function of a c-number real scalar field multiplet Φ_i. We define the potential so that

$$V(0)=0. \tag{3.1}$$

The theory is assumed to be massless, in the sense that[1]

$$\left.\frac{\partial^2 V(\Phi)}{\partial\Phi_i\partial\Phi_j}\right|_{\Phi=0}=0. \tag{3.2}$$

For simplicity, we suppose there is some symmetry which excludes terms in $V(\Phi)$ odd in Φ; then the only coupling constants that we need to define are the Φ^4 couplings

$$\left.\frac{\partial^4 V(\Phi)}{\partial\Phi_i\partial\Phi_j\partial\Phi_k\partial\Phi_l}\right|_{\Phi\sim\Lambda}\equiv f_{ijkl}. \tag{3.3}$$

By "$\Phi\sim\Lambda$" we mean that the fields here are put at a renormalization point characterized by a mass scale Λ; this will be made precise below.

For definiteness, it will also be assumed that for generic values of Λ the nonzero components of f_{ijkl} are all of order e^2, where $e\ll 1$ is a typical gauge coupling constant. An expansion in powers of e^2 is then the same as an expansion in the number of loops. Actually, for our purposes it would be adequate merely to assume that the components of f_{ijkl} are of a common order of magnitude f, with f in the range

$$1\gg f\gg e^4.$$

Only small changes in our discussion would be required to deal with this more general case.

Under these assumptions, the potential is dominated [for $\ln(\Phi/\Lambda)$ not too large] by the zero-loop term

$$V_0(\Phi)=\tfrac{1}{24}f_{ijkl}\Phi_i\Phi_j\Phi_k\Phi_l. \tag{3.4}$$

In general, the only stationary point of this term would be at $\Phi=0$. The one-loop corrections are of order $e^4\Phi^4\ln(\Phi/\Lambda)$, and it might appear that these could shift the stationary point of $V(\Phi)$ to a symmetry-breaking point where $\ln(\Phi/\Lambda)$ is of order $1/e^2$. However, such points would be outside the range of validity of perturbation theory. Thus, a direct use of perturbation theory fails to reveal whether the symmetries of this theory are spontaneously broken.

However, suppose that before we start to use perturbation theory, we choose the renormalization scale Λ to have a value Λ_W at which $V_0(\Phi)$ *does* have a nontrivial minimum on some ray $\Phi_i=n_i\phi$. We do this by adjusting Λ so that the minimum value of the real continuous function $V_0(N)$ on the unit sphere[17] $N_iN_i=1$ is zero:

$$\min_{N_iN_i=1}(f_{ijkl}N_iN_jN_kN_l)=0. \tag{3.5}$$

If this minimum is attained for some specific unit vector $N_i=n_i$, then $V_0(\Phi)$ will attain a minimum value of zero everywhere on the ray $\Phi_i=n_i\phi$.

In the work of CW,[1] there was only a single ϕ^4 coupling constant f, and the constraint which corresponded to our Eq. (3.5) was simply that f (or at least the part of f of order e^2) vanished. It is crucial here that even in more general theories with many free parameters in the f_{ijkl}, we are only imposing *one* constraint on the f's, that the single quantity on the left-hand side of (3.5) must vanish. With a single renormalization scale parameter Λ at our disposal, we expect even for generic initial values of the f_{ijkl}, it should be possible to find a value of Λ at which Eq. (3.5) is satisfied. [This was shown for a variety of special cases in Ref. 14 by integration of the renormalization-group equations. More generally, if we suppose that there are F free parameters in the f_{ijkl}, then the constraint (3.5) restricts these parameters to an $(F-1)$-dimensional surface, but by changing Λ away from Λ_W we can move the free parameters in f_{ijkl} off this surface and fill out at least a finite F-dimensional region.]

In order for the minimum (3.5) to be attained at a unit vector n_i, it is obviously necessary that

$$f_{ijkl}n_j n_k n_l = 0. \tag{3.6}$$

However, this only implies that $V_0(N)$ vanishes and is stationary at $N=n$; we are also demanding that it is a minimum there. This further requires that

$$f_{ijkl}u_i u_j n_k n_l \geq 0 \tag{3.7}$$

for all vectors u_i. Thus all eigenvalues of the matrix

$$P_{ij} \equiv \tfrac{1}{2} f_{ijkl} n_k n_l \tag{3.8}$$

are either positive or zero.

As an example of these constraints on the f's, consider Φ's transforming as the symmetric second-rank tensor representation of SU(n). Then Φ is an $n \times n$ complex symmetric matrix and

$$V_0(\Phi) = f_1(\mathrm{Tr}\Phi\Phi^\dagger)^2 + f_2 \mathrm{Tr}(\Phi\Phi^\dagger)^2.$$

It is shown in Ref. 14 that if we restrict the Φ's to a hypersphere, the stationary points of V_0 for which $V_0 = 0$ occur for values of the f's such that $-f_2/f_1$ equals an integer between 1 and n; the minimum on the hypersphere has $V_0=0$ for nonzero f's if and only if $f_2 > 0$ and $-f_2/f_1 = n$, or $f_2 < 0$ and $-f_2/f_1 = 1$.

The minimum of our zeroth-order potential is attained along a ray $\Phi_i = \phi n_i$. When we turn on the higher-order terms $\delta V(\Phi)$ in the potential, we give the potential a small curvature in the radial direction, which picks out a definite value $\langle\phi\rangle$ of ϕ at the minimum, and we also produce a small shift in the direction of Φ_i at this minimum. The condition for a stationary point at $n_i\langle\phi\rangle + \delta\Phi_i$ is

$$0 = \left[\frac{\partial}{\partial\Phi_i}(V_0(\Phi) + \delta V(\Phi))\right]_{n\langle\phi\rangle + \delta\Phi},$$

or to first order in small quantities

$$0 = P_{ij}\delta\Phi_j\langle\phi\rangle^2 + \left[\frac{\partial\,\delta V(\Phi)}{\partial\Phi_i}\right]_{n\langle\phi\rangle}, \tag{3.9}$$

where P is the matrix (3.8). This uniquely determines $\delta\Phi$ (and shows that it is of order $\delta V/V_0 \sim e^2$) except for possible terms in directions along eigenvectors of P with eigenvalue zero. It is therefore important for us to identify all the zero eigenvalues of P.

One such eigenvector is n itself; according to (3.6) we have

$$P_{ij}n_j = 0. \tag{3.10}$$

Also if the theory has any continuous symmetry

$$\delta\Phi_i = \epsilon\Theta_{ij}\Phi_j, \quad \Theta_{ij} = -\Theta_{ji} \tag{3.11}$$

then by following the usual proof of Goldstone's theorem[18] we see that

$$P_{ij}(\Theta n)_j = 0. \tag{3.12}$$

In general, there is no reason to expect that P should have any other eigenvectors with eigenvalue zero, and *we shall assume that it does not.* Since P is a positive matrix of order e^2, this means that *all eigenvalues of P are positive-definite and of order e^2, except for the zero eigenvalues associated with eigenvectors n and Θn.* In particular, this implies that the unit vectors n at which the minimum (3.5) is attained form at most a discrete set, aside from possible symmetry rotations,[19] for if $V_0(n+\epsilon u)$ vanished over a range of infinitesimal ϵ for some vector u orthogonal to n, then u would be an eigenvector of P with eigenvalue zero.

It now follows that apart from possible symmetry transformations $\delta\Phi \to \Phi + \epsilon\Theta\Phi$, all components of $\delta\Phi$ are uniquely determined by Eq. (3.9), except for the component along the n direction. Instead of using (3.9) to determine the component of $\delta\Phi$ along the n direction, we must use it to determine $\langle\phi\rangle$ itself. Contracting (3.9) with n_i and using (3.10), we find

$$0 = n_i\left[\frac{\partial}{\partial\Phi_i}\delta V(\Phi)\right]_{n\langle\phi\rangle} = \left[\frac{\partial\,\delta V(n\phi)}{\partial\phi}\right]_{\langle\phi\rangle}. \tag{3.13}$$

This is the basic equation we will use below to find $\langle\phi\rangle$.

We have seen that the potential does have a stationary point near $n\langle\phi\rangle$ for small perturbation $\delta V(\Phi)$. But is this a minimum? We will show in the next section that the matrix of second derivatives of the potential at its stationary point is positive-definite, aside from zero eigenvalues in "Goldstone" directions Θn. Thus, the stationary point is at least a local minimum. In order to show that it is an absolute minimum, we would need to use the renormalization-group equations to search for all the values of the renormalization scale parameter Λ where (3.5) is satisfied, and then compare the minimum value of the potential calculated for each Λ, as done for special cases in Ref. 14. We will not attempt this here. However, we will be able to show below that V is less at the local minimum than at $\Phi = 0$, so whichever local minimum is deepest, it is definitely one with broken symmetry.

Let us now use for δV the one-loop terms in a general renormalizable gauge theory. By adopting suitable renormalization conventions[20] for the Φ^4 couplings f_{ijkl}, we can write the one-loop potential along the ray $\Phi = n\phi$ in the form[21]

$$\delta V(n\phi) = \frac{1}{64\pi^2}\{3\,\mathrm{Tr}[\mu_\phi{}^4 \ln(\mu_\phi{}^2/\Lambda_W{}^2)] + \mathrm{Tr}[M_\phi{}^4 \ln(M_\phi{}^2/\Lambda_W{}^2)] - 4\,\mathrm{Tr}[m_\phi{}^4 \ln(m_\phi{}^2/\Lambda_W{}^2)]\}, \quad (3.14)$$

where μ_ϕ, M_ϕ, and m_ϕ are the zeroth-order vector, scalar, and spinor mass matrices for a scalar field vacuum expectation value $n\phi$. This expression now makes precise what we mean by the renormalization scale Λ_W. (We have assumed that the scalars have no color, so the strong interactions would only affect the fermion term, which we shall eventually drop anyway.)

We are working in a massless theory, so the matrices $\mu_\phi{}^2$, $M_\phi{}^2$, and $m_\phi{}^2$ are all simply proportional to ϕ^2:

$$\mu_\phi{}^2 = \mu^2\phi^2/\langle\phi\rangle^2, \quad (3.15)$$

$$M_\phi{}^2 = M^2\phi^2/\langle\phi\rangle^2, \quad (3.16)$$

$$m_\phi{}^2 = m^2\phi^2/\langle\phi\rangle^2, \quad (3.17)$$

where μ, M, and m are the true mass matrices, evaluated for $\phi = \langle\phi\rangle$. Specifically,

$$(\mu^2)_{\alpha\beta} = \langle\phi\rangle^2 n_i n_j (\Theta_\alpha \Theta_\beta)_{ij}, \quad (3.18)$$

$$(M^2)_{ij} = \tfrac{1}{2}\langle\phi\rangle^2 f_{ijkl} n_k n_l, \quad (3.19)$$

$$(m)_{pq} = \langle\phi\rangle (\Gamma_i)_{pq} n_i, \quad (3.20)$$

where Θ_α and Γ_i are the gauge and Yukawa coupling matrices.[22] The one-loop potential may then be put in the form

$$\delta V(n\phi) = A\phi^4 + B\phi^4 \ln(\phi^2/\Lambda_W{}^2), \quad (3.21)$$

where A and B are the dimensionless constants

$$A = \frac{1}{64\pi^2\langle\phi\rangle^4}\{3\,\mathrm{Tr}[\mu^4 \ln(\mu^2/\langle\phi\rangle^2)] + \mathrm{Tr}[M^4 \ln(M^2/\langle\phi\rangle^2)] - 4\,\mathrm{Tr}[m^4 \ln(m^2/\langle\phi\rangle^2)]\}, \quad (3.22)$$

$$B = \frac{1}{64\pi^2\langle\phi\rangle^4}(3\,\mathrm{Tr}\,\mu^4 + \mathrm{Tr}\,M^4 - 4\,\mathrm{Tr}\,m^4)\ . \quad (3.23)$$

We see immediately that the potential (3.21) has a nontrivial stationary point [in the sense of Eq. (3.13)], at a value of $\langle\phi\rangle$ given by

$$\ln(\langle\phi\rangle^2/\Lambda_W{}^2) = -\tfrac{1}{2} - A/B. \quad (3.24)$$

Note that A and B are both of order e^4, so the logarithm is of order unity, and perturbation theory should be valid. If we had not chosen a renormalization point satisfying (3.5), then A would have a term of order e^2, and the logarithm would be of order $1/e^2$.

This stationary point is definitely *not* a minimum unless $B > 0$, because (3.21) decreases without limit for $\phi \to \infty$ if $B < 0$, while V is still a pure quartic if $B = 0$. From Eq. (3.23) we see that B *is* positive-definite if there are no fermions in the theory, and it remains positive-definite even in the presence of fermions as long as the Yukawa coupling constants are not too large. We will assume from now on that this is the case:

$$B > 0. \quad (3.25)$$

Using (3.24) in (3.21), we see immediately that the potential at its stationary point is less than its value $V(0) = 0$ at the origin:

$$V(n\langle\phi\rangle) = \delta V(n\langle\phi\rangle) = -\tfrac{1}{2}B\langle\phi\rangle^4 < 0. \quad (3.26)$$

The proof that this stationary point is a local minimum is completed in the next section.

As promised, Eq. (3.24) shows that the scale of the symmetry-breaking parameter $\langle\phi\rangle$ is set by the renormalization scale Λ_W.

IV. SCALAR-BOSON MASSES

In zeroth order, the squared masses of the scalar bosons are given by the eigenvalues of the matrix

$$(M^2)_{ij} = \left[\frac{\partial^2 V_0(\Phi)}{\partial\Phi_i \partial\Phi_j}\right]_{n\langle\phi\rangle} = P_{ij}\langle\phi\rangle^2. \quad (4.1)$$

According to the assumptions explained in the preceding section, this matrix has a set of positive-definite eigenvalues of order $e^2\langle\phi\rangle^2$, plus a set of zero eigenvalues with eigenvectors $(\Theta n)_j$ corresponding to Goldstone bosons,[18] plus one zero eigenvalue with eigenvector n_j. That is, in zeroth order we find a set of Higgs bosons with masses of order $e\langle\phi\rangle$, about as large as a typical intermediate-vector-boson mass, plus a set of massless Goldstone bosons, plus one light Higgs boson, or "scalon," with vanishing zeroth-order mass.

Turning on a small perturbation $\delta V(\Phi)$ will shift the mass matrix to

$$(M^2 + \delta M^2)_{ij} \equiv \left\{\frac{\partial^2}{\partial\Phi_i \partial\Phi_j}[V_0(\Phi) + \delta V(\Phi)]\right\}_{n\langle\phi\rangle + \delta\Phi}. \quad (4.2)$$

To first order in small quantities, this gives

$$(\delta M^2)_{ij}=\left[\frac{\partial^2\delta V(\phi)}{\partial\Phi_i\partial\Phi_j}\right]_{n\langle\phi\rangle}+f_{ijkl}n_k\delta\Phi_l\langle\phi\rangle, \qquad (4.3)$$

with $\delta\Phi$ given by (3.9). We will find below that all eigenvalues of the matrix (4.2) are positive-definite, except for the zero eigenvalues in Goldstone directions Θn; this will then confirm that the stationary point $n\langle\phi\rangle+\delta\Phi$ is indeed a local minimum of the potential.

We cannot calculate the masses of the heavy Higgs bosons without more information about the details of the theory. However, we note that the eigenvalues of M^2 corresponding to these particles are positive-definite, so that the corresponding eigenvalues of $M^2+\delta M^2$ are also positive-definite, as long as δM^2 is a small perturbation.

The eigenvectors Θn of M^2 are as well eigenvectors of $M^2+\delta M^2$ with zero eigenvalue, provided δV is, like V_0, invariant under Θ.[19] Thus the Goldstone bosons simply remain massless and need not concern us further here. (In a realistic model they would have to all be eliminated by the Higgs mechanism.)

This leaves the scalon. First-order perturbation theory tells us that we must calculate its mass by taking the expectation value of δM^2 with respect to the unperturbed eigenvector n_i. Using (3.6), this gives

$$\begin{aligned}M_S{}^2&=n_in_j(\delta M^2)_{ij}\\&=n_in_j\left[\frac{\partial^2\delta V(\Phi)}{\partial\Phi_i\partial\Phi_j}\right]_{n\langle\phi\rangle}\\&=\left[\frac{d^2}{d\phi^2}V(n\phi)\right]_{\langle\phi\rangle}.\end{aligned} \qquad (4.4)$$

We can therefore calculate $M_S{}^2$ from the second derivative of the one-loop potential (3.21) along the ray $\Phi=n\phi$:

$$M_S{}^2=12\langle\phi\rangle^2[B\ln(\langle\phi\rangle^2/\Lambda_W{}^2)+\tfrac{7}{6}B+A].$$

Using (3.24) to eliminate Λ_W, this gives

$$M_S{}^2=8B\langle\phi\rangle^2. \qquad (4.5)$$

As promised, this is positive-definite, thus completing the proof that we have found a local minimum. We may write Eq. (3.23) for B as a sum over vector bosons V, *heavy* Higgs bosons H, and fermions F, and (4.5) then becomes

$$M_S{}^2=\frac{1}{8\pi^2\langle\phi\rangle^2}\left(3\sum_V\mu_V{}^4+\sum_H M_H{}^4-4\sum_F m_F{}^4\right). \qquad (4.6)$$

For "simple" theories with only a single Higgs boson, Eq. (4.6) gives the same result as calculated by Coleman and E. Weinberg.[1] Also, for such theories, this value for the Higgs mass is greater than a recently derived lower bound[23] by just a factor $\sqrt{2}$. The remarkable new thing we have learned here is that in more general theories with many scalar fields, there will be some unknown number of Higgs bosons with masses of order μ_W, plus *one* scalon with mass given by Eq. (4.6).

For instance, consider a more or less realistic model based on the gauge group SU(2)⊗U(1).[24] In order to preserve the usual prediction for m_Z/m_W in terms of the mixing angle Θ, suppose that the scalar fields form an arbitrary number of doublets

$$\chi_l\equiv\begin{pmatrix}\chi_l^+\\ \chi_l^0\end{pmatrix} \qquad (4.7)$$

and their complex conjugates. The vacuum expectation value may be written as a unit vector n times a modulus $\langle\phi\rangle$:

$$\langle\chi_l\rangle=n_l\langle\phi\rangle,\quad \sum_l(n_l^\dagger n_l)=1.$$

We assume of course that charge conservation is not spontaneously broken, so the unit direction vector at the stationary point has the form

$$n_l=\begin{pmatrix}0\\ \lambda_l\end{pmatrix},\quad \sum_l|\lambda_l|^2=1.$$

To determine $\langle\phi\rangle$, we note that the W mass is given in this theory by

$$\begin{aligned}m_W{}^2&=\tfrac14 g^2\sum_l|(t_1+it_2)\langle\chi_l\rangle|^2\\&=\tfrac14 g^2\sum_l|\langle\chi_l^0\rangle|^2\\&=\tfrac14 g^2\langle\phi\rangle^2,\end{aligned}$$

while the Fermi coupling constant is

$$G_F/\sqrt{2}=g^2/8m_W{}^2.$$

From these two relations, we find that

$$\langle\phi\rangle=2^{-1/4}G_F{}^{-1/2}=247\text{ GeV}$$

just as with a single scalar doublet.[24] Equation (4.6) now becomes

$$M_S{}^2=\frac{3\sqrt{2}G_F}{8\pi^2}\left(2m_W{}^4+m_Z{}^4+\tfrac13\sum_H M_H{}^4-\tfrac43\sum_F m_F{}^4\right). \qquad (4.8)$$

As already remarked in the preceding section, all known fermions are much lighter than the intermediate vector bosons, so it is reasonable to drop the sum over fermions in Eq. (4.8). We can write m_W and m_Z in terms of the weak mixing angle θ,

$$m_W{}^2 = \frac{e^2\sqrt{2}}{8G_F \sin^2\theta},$$

$$m_Z{}^2 = \frac{e^2\sqrt{2}}{8G_F \sin^2\theta \cos^2\theta},$$

so (4.8) becomes

$$M_S{}^2 = \frac{3\alpha^2}{8\sqrt{2}\,G_F}\left(\frac{2+\sec^4\theta}{\sin^4\theta}\right) \times \left[1 + \frac{1}{3(2m_W{}^4 + m_Z{}^4)}\sum_H M_H{}^4\right]. \tag{4.9}$$

This is minimized by dropping the sum over Higgs bosons and setting $\theta = 49.38°$; thus we have the lower bound

$$M_S \geq 2.4585\alpha G_F{}^{-1/2} = 5.261 \text{ GeV}. \tag{4.10}$$

If we assume (without any real justification) that the "heavy" Higgs bosons are somewhat lighter than the intermediate vector bosons, and if we take for θ the experimentally favored value $\theta \simeq 35°$, then (4.9) gives a scalon mass of 7 GeV. If the Higgs boson contribution in (4.8) is about the same as the vector-boson contribution, then this estimate becomes 10 GeV.

V. SCALON COUPLINGS

The class of theories considered in this paper allows us not only to calculate the mass of the scalon, but also to calculate its couplings. This is because we know that the scalon corresponds to an eigenvector of the zeroth-order scalar mass matrix M^2, which is in the same direction n_i as the field vacuum expectation value $\langle\Phi_i\rangle$.

In general, the couplings of the scalar fields in any renormalizable gauge theory are given by an effective Lagrangian[22]

$$-\tfrac{1}{24}f_{ijkl}\Phi_i\Phi_j\Phi_k\Phi_l - \bar\psi\Gamma_i\psi\Phi_i - i(\partial_\mu\Phi_i)\Phi_j(\Theta_\alpha)_{ij}A^\mu_\alpha - \tfrac{1}{2}\Phi_i\Phi_j(\Theta_\alpha\Theta_\beta)_{ij}A^\nu_\alpha A_{\beta\nu}, \tag{5.1}$$

where Γ_i and Θ_α are the Yukawa and gauge coupling matrices. We can define fields S and H_i for the scalon and the heavy Higgs bosons by writing

$$\Phi_i \equiv n_i\langle\phi\rangle + n_i S + H_i + \text{Goldstone bosons}, \tag{5.2}$$

with the heavy Higgs boson field H_i defined as that part of Φ_i orthogonal to n_i and to the Goldstone directions $(\Theta_\alpha n)_i$:

$$H_i n_i = H_i(\Theta_\alpha n)_i = 0. \tag{5.3}$$

The terms in (5.1) which involve S and do *not* involve Goldstone bosons are then

$$-\tfrac{1}{4}f_{ijkl}n_i n_j H_k H_l(S^2 + 2S\langle\phi\rangle) - \bar\psi\Gamma_i\psi n_i S - \tfrac{1}{2}n_i n_j(\Theta_\alpha\Theta_\beta)_{ij}A^\nu_\alpha A_{\beta\nu}(S^2 + 2S\langle\phi\rangle) - n_i H_j(\Theta_\alpha\Theta_\beta)_{ij}A^\mu_\alpha A_{\beta\mu}S\ . \tag{5.4}$$

Referring back to Eqs. (3.18), (3.19), and (3.20), we see that the first three terms of Eq. (5.4) may be written in terms of the vector, scalar, and spinor mass matrices μ, M, and m. The effective Lagrangian for the scalon field is then

$$-\tfrac{1}{2}M^2{}_{kl}H_k H_l\left(\frac{S^2}{\langle\phi\rangle^2} + \frac{2S}{\langle\phi\rangle}\right) - \bar\psi m\psi\frac{S}{\langle\phi\rangle} - \tfrac{1}{2}\mu^2{}_{\alpha\beta}A^\nu_\alpha A_{\beta\nu}\left(\frac{S^2}{\langle\phi\rangle^2} + \frac{2S}{\langle\phi\rangle}\right) - (\Theta_\alpha\Theta_\beta n)_j H_j A^\mu_\alpha A_{\beta\mu}S. \tag{5.5}$$

In particular, we note that the coupling constant for the trilinear interaction SNN of the light Higgs boson to an elementary particle N of definite mass is[29]

$$g_{SNN} = \begin{cases} M_N^2/\langle\phi\rangle & \text{Higgs bosons} \\ m_N/\langle\phi\rangle & \text{fermions} \\ \mu_N^2/\langle\phi\rangle & \text{gauge bosons.} \end{cases} \tag{5.6}$$

One immediate consequence of these results is that the scalon decays preferentially into the heaviest possible particles. (This is *not* necessarily true for the heavy Higgs bosons.) A scalon with mass above 5 GeV would decay into the heavy lepton indicated by recent experiments,[25] and into the charmed quark, and thence to charmed hadrons. Taking the mass of both heavy leptons and charmed quarks as 1.7 GeV, the branching ratio for scalon decay into $\mu^+\mu^-$ pairs (the clearest signature) is very small[26]:

$$\frac{S\to\mu^+ + \mu^-}{S\to\text{all}} \approx \frac{m_\mu^2}{4(1.7\text{ GeV})^2} \approx 10^{-3}. \tag{5.7}$$

Another consequence of these results is that production of the scalon will tend to be dominated by graphs in which it is emitted from heavy-particle lines. For instance, in the production of a scalon in a neutrino reaction at a center-of-mass energy in the range $M_S \ll E \ll \mu_W$, the dominant graph is the one in which the scalon is emitted from the virtual-W line.[27] The factor $\mu_W{}^2$ in the SWW coupling is simply canceled by the $1/\mu_W{}^2$ in the extra W propagator. In consequence, the invariant matrix element for production of the scalon can be obtained from the matrix element for the neutrino reaction without scalon production by dividing by $\langle\phi\rangle$ and changing the value of q^2 at the hadron vertex. Detailed calculations of the production rate are in progress.[28]

Finally, exchange of virtual scalons between two particles of mass m_1, m_2 produces a scalar weak interaction with effective Fermi coupling constant

$$\begin{aligned} G_S &= \left(\frac{m_1}{\langle\phi\rangle}\right)\left(\frac{m_2}{\langle\phi\rangle}\right)\frac{1}{M_S{}^2} \\ &= O(G_F m_1 m_2/M_S{}^2). \end{aligned}$$

If m_1 and m_2 are not much less than M_S, then G_S is not much less than G_F, so for hadrons or heavy leptons this scalar weak interaction may be nearly as strong as the usual vector and axial-vector interactions. However, its effects are insipid: It does not break strangeness or parity or isospin conservation, or transfer charge from leptons to hadrons, or affect neutrinos at all. It is hard to see how it could be detected.

VI. SCALE RELATIONS

We now want to consider the relation of the renormalization scale Λ_W to other scales in physics.

The constraint on f_{ijkl} which defines our renormalization scales was described in terms of the values of the minimum value $F(\Lambda)$ of the quantity $f_{ijkl}(\Lambda)N_iN_jN_kN_l$ on the hypersphere $\Sigma N_i N_i = 1$; our constraint is that $F=0$ at $\Lambda = \Lambda_W$. In general, the Λ dependence of f_{ijkl} will be governed by a renormalization-group equation, which gives

$$\Lambda\frac{d}{d\Lambda}F(\Lambda) = O(e^4). \tag{6.1}$$

We do not know what determines the values of f_{ijkl} at any scale, but it seems reasonable to suppose that the couplings of the scalars, left over after the superstrong symmetry breaking, are determined to have some definite values of order e^2 at a renormalization scale $\overline{\Lambda}$ characteristic of the superstrong symmetry breakdown. There is no reason $F(\overline{\Lambda})$ should vanish, so we may assume it to be of order e^2. Then Eq. (6.1) shows that the change in the logarithm of the renormalization scale required to make $F(\Lambda_W)$ vanish will be order e^2/e^4:

$$\ln(\overline{\Lambda}/\Lambda_W) = O(1/e^2). \tag{6.2}$$

This indicates that the ordinary physical mass scale determined by Λ_W is likely to be enormously different from the scale of the superstrong symmetry breaking.

It is interesting that (6.2) is much like the relation[12] between $\overline{\Lambda}$ and the scale Λ_S of the gluon gauge coupling constant in superunified theories. If we assume that the strong, weak, and electromagnetic interactions all arise from a single simple gauge group, then the strong gauge coupling constant g_S is of order e for renormalization scales $\overline{\Lambda}$, and grows (for asymptotically free theories) as Λ decreases, with

$$\Lambda\frac{d}{d\Lambda}g_S = O(g_S{}^3) \quad \text{for } g_S \ll 1. \tag{6.3}$$

Hence the scale Λ_S at which g_S begins to be much larger than $O(e)$ is determined by

$$\ln(\overline{\Lambda}/\Lambda_S) = O(1/e^2) \tag{6.4}$$

indicating that $\ln(\overline{\Lambda}/\Lambda_S)$ is of the same order as $\ln(\overline{\Lambda}/\Lambda_W)$. Of course this does not mean that Λ_S and Λ_W are comparable — a factor of 2 in a large logarithm can make a very large difference. In fact, Λ_W is of order $\langle\phi\rangle$, or 300 GeV, while electroproduction experiments indicate that Λ_S is of order 300 MeV. In a sense, this is surprisingly close: if $\overline{\Lambda}/\Lambda_W$ is of order[12] 10^{15}, and Λ_W/Λ_S is of order 10^3, then the two logarithms (6.2) and (6.4) differ by only 20%. At any rate, since we are not assuming here that the weak gauge symmetry is dynamically broken by the strong interactions, there is no reason to expect that Λ_S and Λ_W should be very close.

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[1]S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).

[2]By "Higgs bosons" we mean in general all the weakly coupled elementary scalar particles, except for those corresponding to Goldstone bosons.

[3]By "pseudo-Goldstone bosons" we mean in general the Goldstone bosons of accidental symmetries, which are only exact in zeroth order. See S. Weinberg, Phys. Rev. Lett. 29, 1698 (1972) and Phys. Rev. D 7, 2887 (1973). For a generalization, see H. Georgi and A. Pais, Phys.

Rev. D 12, 508 (1975).

[4]S. Weinberg, Phys. Rev. D 5, 1962 (1972). For a simple gauge theory of weak, electromagnetic, and strong interactions, see H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).

[5]T. Appelquist and J. Carazzone, Phys. Rev. D 11, 2856 (1975).

[6]Dynamical symmetry breaking was first discussed in the context of modern gauge theories by R. Jackiw and K. Johnson, Phys. Rev. D 8, 2386 (1973); J. M. Cornwall and R. E. Norton, *ibid.* 8, 3338 (1973). For further references, see Ref. 7. By "dynamical symmetry breaking" we mean spontaneous symmetry breaking in which the Goldstone bosons are bound states. Thus we do *not* put theories of the CW type (Ref. 1) in this category.

[7]S. Weinberg, Phys. Rev. D 13, 974 (1976).

[8]This is illustrated in the model described in Sec. IX of Ref. 7.

[9]Note that a theory with zero bare mass is strictly renormalizable in the sense that there are no ultraviolet divergences which require mass counterterms, provided we use the dimensional regularization method of G. 't Hooft and M. Veltman, Nucl. Phys. B44, 1189 (1972).

[10]The frustrating search for a mechanism which would allow some scalars to escape getting superheavy masses from the superstrong symmetry breaking is described by S. Weinberg, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974,* edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. III-59, Sec. II; in *Gauge Theories and Modern Field Theory,* edited by R. Arnowitt and P. Nath (M.I.T. Press, Cambridge, Mass., 1976).

[11]J. Wess and B. Zumino, Nucl. Phys. B70, 39 (1974).

[12]H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).

[13]See e.g., S. Weinberg, Rev. Mod. Phys. 46, 255 (1974), Sec. VIII.

[14]E. Gildener, Harvard University Ph.D. thesis, 1975 (unpublished); Phys. Rev. D 13, 1025 (1976).

[15]Our simplified approach is briefly described by S. Weinberg, in *Gauge Theories and Modern Field Theory,* Ref. 10.

[16]Effective potentials were introduced for this purpose by J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962); G. Jona-Lasinio, Nuovo Cimento 34, 1790 (1964). Also see Ref. 1. We are tacitly adopting the Landau gauge here, but all observable quantities are of course gauge-independent; see J. Iliopoulos and N. Papanicolaou, Report No. PTENS 75/12 (unpublished), and earlier references cited therein.

[17]The potential $V_0(\Phi)$ has a minimum on any closed compact surface, so it is not necessary to use a *sphere* in imposing this constraint; for instance, an ellipsoid would serve as well. It is convenient to use a sphere because we normalize the scalar fields so that the kinematic term in the Lagrangian is $-\frac{1}{2}\partial_\mu\Phi_i\partial^\mu\Phi_i$.

[18]J. Goldstone, A. Salam, and S. Weinberg, Ref. 16, Sec. II.

[19]We are excluding here the possibility that $V_0(\Phi)$ has some accidental symmetry (other than scale invariance) which is not shared by $\delta V(\Phi)$. Thus we assume that the theory has no pseudo-Goldstone bosons other than the "scalon" itself. (See Ref. 3.) If the theory did involve pseudo-Goldstone bosons, they would mix with the scalon, altering our prediction of the scalon mass.

[20]See Ref. 14 for a general discussion of the choice of renormalization point.

[21]See Ref. 1. This result was subsequently obtained by a different method by S. Weinberg, Phys. Rev. D 7, 2887 (1973).

[22]The notation here is the same as used by S. Weinberg, Ref. 21.

[23]S. Weinberg, Phys. Rev. Lett. 36, 294 (1976). Also see A. D. Linde, report, Lebedev Physical Institute, 1975 (unpublished).

[24]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Physics, Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8),* edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[25]G. Feldman, talk given at the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford University, 1975 (unpublished); invited talk at 1975 Meeting of APS Division of Particles and Fields, Seattle, 1975 (unpublished).

[26]J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Report No. Ref. TH. 2093-CERN (unpublished). This reference provides an extensive discussion of the production and decays of Higgs bosons.

[27]D. Cline (private communication).

[28]J. LoSecco (unpublished). (A similar calculation has been included in a supplement to Ref. 26.)

[29]Equation (5.6) gives the same sort of result as would be expected if the scalon is treated as the Goldstone boson associated with the spontaneous breakdown of scale invariance. However, it must be stressed that we have *not* ignored the radiative corrections which produce anomalies in the conservation of the dilation current; rather it is these anomalies that make the scalon a pseudo-Goldstone boson and give it a mass of order $\alpha G_F^{-1/2}$. As remarked in Sec. III, the one-loop terms in the potential produce small shifts in the direction of the scalar-field vacuum expectation value, so the simple formulas (5.6) for the scalon couplings are only correct to lowest order in α.