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Equivalence theorem and Faddeev-Popov ghosts

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An algebraic proof of the equivalence theorem, to all orders of perturbation theory, is obtained by applying the equations of motion repeatedly in a normal-product algorithm. It is shown that, for certain nonlocal transformations, the equivalence theorem can be maintained by introducing Faddeev-Popov ghosts.

I. GREEN'S FUNCTIONS UNDER A LOCAL TRANSFORMATION

The equivalence theorem in Lagrangian field theory is a very useful concept, which was conceived decades ago.[1] A rigorous proof was given in Ref. 2 for perturbation theory, and since then various authors have discussed different aspects of it.[3] The proof in Ref. 2 is based on formulating the quantized theory in the normal-product algorithm, but unfortunately it consists of laboriously examining a delicate conspiracy among graphs, and it relies on the Haag-Ruelle[4] theorem to conclude the equality of the scattering matrices. In this note, we will present a short and elegant proof that does not depend on a detailed manipulation of graphs, nor on the Haag-Ruelle theorem. We will also extend the equivalence theorem to a certain type of nonlocal transformation and will find that it can only be maintained if we introduce ghosts into the Lagrangian which are not unlike those of Faddeev and Popov.[5]

For simplicity, we will study only the case where there is one scalar field. Generalization to cover more particles and to include spin is obvious. Let the Lagrangian

$$\mathcal{L}_0(\varphi)=\tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi-\tfrac{1}{2}m^2\varphi\,||\,\varphi+\lambda\mathcal{L}_{\text{int}}(\varphi) \tag{1.1}$$

be quantized with "hard mass,"[6] and let its interaction term $\mathcal{L}_{\text{int}}(\varphi)$ be such that the Bogoliubov-Parasiuk-Hepp-Zimmerman[7] (BPHZ) renormalized two-point function has a pole at the physical mass m. The double bars $||$ in the mass term indicate hard quantization for the mass term. Under the local transformation

$$\varphi\to\varphi+F(\varphi), \tag{1.2}$$

where $F(\varphi)$ is a polynomial in φ and its derivatives and where $F(\varphi)\neq-\varphi$, $\mathcal{L}_0(\varphi)$ becomes

$$\mathcal{L}_1(\varphi)=\mathcal{L}_0(\varphi+F(\varphi)), \tag{1.3}$$

$$\begin{aligned}\mathcal{L}_1(\varphi)=&\tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi-\tfrac{1}{2}m^2\varphi\,||\,\varphi+\partial_\mu\varphi\partial^\mu F(\varphi)\\&+\tfrac{1}{2}\partial_\mu F(\varphi)\partial^\mu F(\varphi)-m^2\varphi\,||\,F(\varphi)\\&-\tfrac{1}{2}m^2F(\varphi)\,||\,F(\varphi)+\lambda\mathcal{L}_{\text{int}}(\varphi+F(\varphi)),\end{aligned} \tag{1.4}$$

where we have assigned oversubtractions (indicated by $||$ in the anisotropic product)[8] to those terms that arise from the transformation of the mass term $-\tfrac{1}{2}m^2\varphi\,||\,\varphi$ in $\mathcal{L}_0$.

We will now show that the Green's functions of these two Lagrangians are related. For this purpose, let us define, for each ρ between 0 and 1 inclusive, a Lagrangian $\mathcal{L}_\rho$ to be

$$\mathcal{L}_\rho(\varphi)=\mathcal{L}_0(W_\rho(\varphi)), \tag{1.5}$$

where

$$W_\rho(\varphi)=\varphi+\rho F(\varphi). \tag{1.6}$$

If Q and X are functions of φ and its derivatives, we define

$$\frac{\vec{\delta} Q}{\delta\varphi}X=\sum_{n=0}^{\infty}\frac{1}{n!}\frac{\partial Q}{\partial\partial_{\mu_1}\cdots\partial_{\mu_n}\varphi}\partial_{\mu_1}\cdots\partial_{\mu_n}X, \tag{1.7}$$

$$\frac{\delta Q}{\delta\varphi}\cdot X=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\partial_{\mu_1}\cdots\partial_{\mu_n}\left(\frac{\partial Q}{\partial\partial_{\mu_1}\cdots\partial_{\mu_n}\varphi}X\right), \tag{1.8}$$

$$\frac{\delta Q}{\delta\varphi}=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\partial_{\mu_1}\cdots\partial_{\mu_n}\frac{\partial Q}{\partial\partial_{\mu_1}\cdots\partial_{\mu_n}\varphi}. \tag{1.9}$$

Then if Q is a function of X and its derivatives, and if X is a function of φ and its derivatives

$$\frac{\delta Q(X(\varphi))}{\delta\varphi}=\frac{\delta X}{\delta\varphi}\cdot\frac{\delta Q}{\delta X}. \tag{1.10}$$

The proof is given in part (1) of Appendix.

The classical equation of motion is of great help in determining what the quantized version is. For the Lagrangian $\mathcal{L}_\rho(\varphi)$, the equation of motion is

$$\frac{\delta\mathcal{L}_\rho}{\delta\varphi}=0, \tag{1.11}$$

which because of (1.10) assumes the form

$$\left(1+\rho\frac{\delta F}{\delta\varphi}\right)\cdot\frac{\delta\mathcal{L}_0(W_\rho)}{\delta W_\rho}=0. \tag{1.12}$$

By the well-known normal-product algorithm[7] and with the conventions described in Ref. 9, it can easily be shown that the quantized version of the equation of motion is that, for any functions $Y(\varphi)$,

$\Phi_1(\varphi),\ \Phi_2(\varphi),\ldots,\Phi_s(\varphi)$ of φ and its derivatives,

$$\left\langle T\left[\int d^4y\left\{\frac{\delta\mathcal{L}_0(W_\rho)}{\delta W_\rho}\right|\left|\left(1+\rho\frac{\overrightarrow{\delta}F}{\delta\varphi}\right)Y\right\}(y)X(x)\right]\right\rangle^{\mathcal{L}_\rho}_{N,M}=i\left\langle T\left[\left\{\frac{\overrightarrow{\delta}X}{\delta\varphi}Y\right\}(x)\right]\right\rangle^{\mathcal{L}_\rho}_{N,M},\quad(1.13)$$

where

$$X(x)=\prod_{i=1}^{s}\{\Phi_i(\varphi)\}(x_i),\quad(1.14)$$

$$\left\{\frac{\overrightarrow{\delta}X}{\delta\varphi}Y\right\}(x)=\sum_{i=1}^{s}\left\{\frac{\overrightarrow{\delta}\Phi_i}{\delta\varphi}Y\right\}(x_i)\prod_{j\neq i}\{\Phi_j(\varphi)\}(x_j),\quad(1.15)$$

and N and M denote the order of perturbation in λ and in ρ, respectively. The double bars $||$ on the left-hand side of (1.13) indicate an anisotropic normal product, which only takes effect for those terms derived from the mass term of $\mathcal{L}_0(W_\rho)$. On the other hand, we may easily show the operator relation

$$\frac{\partial\mathcal{L}_\rho}{\partial\rho}(y)=\left\{\frac{\overrightarrow{\delta}\mathcal{L}_0(W_\rho)}{\delta W_\rho}\right|\left|F\right\}(y),\quad(1.16)$$

which means that

$$\left\langle T\left[\int d^4y\frac{\partial\mathcal{L}_\rho}{\partial\rho}(y)X(x)\right]\right\rangle^{\mathcal{L}_\rho}_{N,M}=\left\langle T\left[\int d^4y\left\{\frac{\delta\mathcal{L}_0(W_\rho)}{\delta W_\rho}\right|\left|F\right\}(y)X(x)\right]\right\rangle^{\mathcal{L}_\rho}_{N,M}.\quad(1.17)$$

Letting Y in the equation of motion (1.13) be F and combining the result with (1.17), we obtain

$$\left\langle T\left[\int d^4y\frac{\partial\mathcal{L}_\rho}{\partial\rho}(y)X(x)\right]\right\rangle^{\mathcal{L}_\rho}_{N,M}=\left\langle T\left[\int d^4y\left\{\frac{\delta\mathcal{L}_0(W_\rho)}{\delta W_\rho}\right|\left|\left(-\rho\frac{\overrightarrow{\delta}F}{\delta\varphi}\right)F\right\}(y)X(x)\right]\right\rangle^{\mathcal{L}_\rho}_{N,M}+i\left\langle T\left[\left\{\frac{\overrightarrow{\delta}X}{\delta\varphi}F\right\}(x)\right]\right\rangle^{\mathcal{L}_\rho}_{N,M}.\quad(1.18)$$

Using the equation of motion (1.13) repeatedly with $Y=(-\rho\overrightarrow{\delta}F/\delta\varphi)F,(-\rho\overrightarrow{\delta}F/\delta\varphi)^2F,\ldots,(-\rho\overrightarrow{\delta}F/\delta\varphi)^{n-1}F$, this equation is converted into

$$\left\langle T\left[\int d^4y\frac{\partial\mathcal{L}_\rho}{\partial\rho}(y)X(x)\right]\right\rangle^{\mathcal{L}_\rho}_{N,M}=\left\langle T\left[\int d^4y\left\{\frac{\delta\mathcal{L}_0(W_\rho)}{\delta W_\rho}\right|\left|\left(-\rho\frac{\overrightarrow{\delta}F}{\delta\varphi}\right)^nF\right\}(y)X(x)\right]\right\rangle^{\mathcal{L}_\rho}_{N,M}+i\left\langle T\left[\left\{\frac{\overrightarrow{\delta}X}{\delta\varphi}\sum_{m=0}^{n-1}\left(-\rho\frac{\overrightarrow{\delta}F}{\delta\varphi}\right)^mF\right\}(x)\right]\right\rangle^{\mathcal{L}_\rho}_{N,M}.\quad(1.19)$$

The above equation is true for all $n\geq 0$ if the second term of the right-hand side is considered to be zero for $n=0$. From (1.19) and from the renormalized Schwinger action principle which is easily established in the normal-product algorithm, we get

$$\frac{\partial}{\partial\rho}\langle T[X(x)]\rangle^{\mathcal{L}_\rho}_{N,M}=i\left\langle T\left[\int d^4y\left\{\frac{\delta\mathcal{L}_0(W_\rho)}{\delta W_\rho}\right|\left|\left(-\rho\frac{\overrightarrow{\delta}F}{\delta\varphi}\right)^nF\right\}(y)X(x)\right]\right\rangle^{\mathcal{L}_\rho}_{N,M-1}-\left\langle T\left[\left\{\frac{\overrightarrow{\delta}X}{\delta\varphi}\sum_{m=0}^{n-1}\left(-\rho\frac{\overrightarrow{\delta}F}{\delta\varphi}\right)^mF\right\}(x)\right]\right\rangle^{\mathcal{L}_\rho}_{N,M-1}+\left\langle T\left[\frac{\partial X}{\partial\rho}(x)\right]\right\rangle^{\mathcal{L}_\rho}_{N,M-1}.\quad(1.20)$$

In Sec. III, we will show that this equation leads to the invariance of the S matrix. Here we will content ourselves with obtaining a transformation of Green's functions. Let

$$X(x)=\prod_{i=1}^{s}\{\Phi_i(W_\rho(\varphi))\}(x_i).\quad(1.21)$$

Then from the identity

$$\frac{\overrightarrow{\delta}Q}{\delta\varphi}Y=\frac{\overrightarrow{\delta}Q}{\delta X}\frac{\overrightarrow{\delta}X}{\delta\varphi}Y,\quad(1.22\text{a})$$

which is proved in part (2) of the Appendix, we obtain

$$\frac{\overrightarrow{\delta}X}{\delta\varphi}Y=\frac{\overrightarrow{\delta}X}{\delta W_\rho}\left(1+\rho\frac{\overrightarrow{\delta}F}{\delta\varphi}\right)Y\quad(1.22\text{b})$$

for any function $Y(\varphi)$. Moreover, it follows directly from (1.21) that

$$\frac{\partial X}{\partial\rho}=\frac{\overrightarrow{\delta}X}{\delta W_\rho}F.\quad(1.23)$$

Letting Y in (1.22b) be

$$\sum_{m=0}^{n-1}\left(-\rho\frac{\overrightarrow{\delta}F}{\delta\varphi}\right)^mF,$$

and combining the result with (1.20), (1.22b), and (1.23), we obtain

$$\frac{\partial}{\partial\rho}\langle T[X(x)]\rangle^{\mathcal{L}_\rho}_{N,M}=i\left\langle T\left[\int d^4y\left\{\frac{\delta\mathcal{L}_0(W_\rho)}{\delta W_\rho}\right|\left|\left(-\rho\frac{\overrightarrow{\delta}F}{\delta\varphi}\right)^nF\right\}(y)X(x)\right]\right\rangle^{\mathcal{L}_\rho}_{N,M-1}+\left\langle T\left[\left\{\frac{\overrightarrow{\delta}X}{\delta W_\rho}\left(-\rho\frac{\overrightarrow{\delta}F}{\delta\varphi}\right)^nF\right\}(x)\right]\right\rangle^{\mathcal{L}_\rho}_{N,M-1},\quad(1.24)$$

for all $n \geq 0$. Letting $n \geq M$, each term on the right-hand side of (1.24) is zero, because its prescribed order in ρ is exceeded. Hence

$$\frac{\partial}{\partial\rho}\left\langle T\left[\prod_{i=1}^{s}\{\Phi_i(W_\rho(\varphi))\}(x_i)\right]\right\rangle_{N,M}^{\mathcal{L}_\rho} = 0. \tag{1.25}$$

Consequently, the matrix elements

$$\left\langle T\left[\prod_{i=1}^{s}\{\Phi_i(W_\rho(\varphi))\}(x_i)\right]\right\rangle_{N,M}^{\mathcal{L}_\rho}$$

are ρ independent. Choosing successively $\rho=0$ and $\rho=1$, we have proved that, for all N and M,

$$\left\langle T\left[\prod_{i=1}^{s}\{\Phi_i(\varphi)\}(x_i)\right]\right\rangle_{N}^{\mathcal{L}_0} = \left\langle T\left[\prod_{i=1}^{s}\{\Phi_i(\varphi+F)\}(x_i)\right]\right\rangle_{N,M}^{\mathcal{L}_1}. \tag{1.26}$$

On the right-hand side of (1.26), M now refers to the number of F vertices (that is, the order of perturbation in ρ at $\rho=1$).

II. NONLOCAL TRANSFORMATIONS

Let us now generalize the problem of Sec. I by constructing the Lagrangian

$$\mathcal{L}_1(\varphi) = \mathcal{L}_0(\varphi + F(\varphi)), \tag{2.1}$$

where $\mathcal{L}_0(\varphi)$ is given in (1.1) and $F(\varphi)$ is now the nonlocal object

$$\{F(\varphi)\}(x) = \int d^4y\,\Delta_F(x-y;\mu)\{G(\varphi)\}(y). \tag{2.2}$$

In the above equation, $\Delta_F(x-y;\mu)$ is the free propagator for a particle of mass μ and $G(\varphi)$ is a local object; then the Feynman graphs generated by the Lagrangian $\mathcal{L}_1(\varphi)$ can still be renormalized by the BPHZ formalism. The discussion in Sec. I suggests that we may establish the equivalence theorem by introducing, for $0 \leq \rho \leq 1$, the Lagrangian

$$\mathcal{L}_\rho(\varphi) = \mathcal{L}_0(W_\rho), \tag{2.3}$$

where

$$W_\rho(\varphi) = \varphi + \rho F(\varphi), \tag{2.4}$$

and then showing the ρ independence of the matrix elements

$$\left\langle T\left[\prod_{i=1}^{s}\{\Phi_i(W_\rho(\varphi))\}(x_i)\right]\right\rangle_{N,M}^{\mathcal{L}_\rho}.$$

This approach is, in fact, incorrect because of possible Wick contractions in Eq. (1.13) between $\varphi(y)$ and the nonlocal object $Y(y)$. The reader may show that, by applying the techniques of Sec. I for the Lagrangian (2.3), such contradictions generate diagrams with loops as shown in Fig. 1, and that these graphs violate the equivalence between the Lagrangians $\mathcal{L}_0$ and $\mathcal{L}_1$. We may try to fulfill the relation (1.26) by introducing a new Lagrangian $\mathcal{L}_1'(\varphi)$ different from $\mathcal{L}_0(\varphi+F)$. This is inspired by the functional analysis of 't Hooft and Veltman.[10] The new Lagrangian $\mathcal{L}_\rho'(\varphi)$ differs from $\mathcal{L}_0(\varphi+\rho F)$ by the introduction of ghosts fields that interact with the physical fields. This Lagrangian is

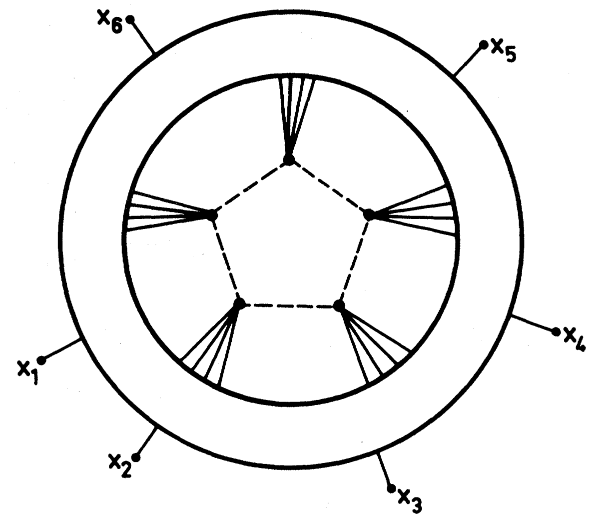

FIG. 1. Unwanted loop diagram with five $\delta G/\delta\phi$ vertices. The dashed line represents the propagator $\Delta_F(x-y;\mu)$.

$$\mathcal{L}_\rho'(\varphi) = \mathcal{L}_\rho(\varphi) + \partial_\mu\bar{c}\,\partial^\mu c - \mu^2\bar{c}\,||\,c - \rho\bar{c}\,\frac{\vec{\delta}G(\varphi)}{\delta\varphi}\,c, \tag{2.5}$$

where c and $\bar{c}$ are scalar fields which satisfy Fermi statistics and are quantized with the Wick contraction

$$c^{\bullet}(x)\bar{c}^{\bullet}(0) = i\Delta_F(x;\mu). \tag{2.6}$$

The reader may prove that $\mathcal{L}_\rho'(\varphi)$ is equivalent to $\mathcal{L}_0(\varphi)$ by the same technique as in Sec. I.

We will present here a simpler proof, well adapted to this special kind of nonlocal transformation. For this purpose, we will first transform (2.5) into an equivalent Lagrangian which is manifestly local:

$$\begin{aligned} L_\rho(\varphi) = {} & \mathcal{L}_0(W_\rho) + \partial_\mu\bar{n}\,\partial^\mu n - \mu^2\bar{n}\,||\,n + \bar{n}G(\varphi) \\ & + \partial_\mu\bar{c}\,\partial^\mu c - \mu^2\bar{c}\,||\,c - \rho\bar{c}\,\frac{\vec{\delta}G(\varphi)}{\delta\varphi}c, \end{aligned} \tag{2.7}$$

where

$$W_\rho = \varphi + \rho n, \tag{2.8}$$

and where n and $\bar{n}$ are scalar fields satisfying Bose statistics. It is a simple task to show that the Green's functions of normal products not involving n or $\bar{n}$ are the same whether for $\mathcal{L}_\rho(\varphi)$ or $L_\rho(\varphi)$. For any local functions Y, $\Phi_1, \ldots, \Phi_s$ of φ, n, $\bar{n}$, c, and $\bar{c}$ the equations of motion for φ, $\bar{n}$, c, and n are, respectively,

$$\left\langle T\left[\int d^4y\left\{\left(\frac{\delta\mathcal{L}_0(W_\rho)}{\delta W_\rho}\Big|\Big| + \bar{n}\frac{\vec{\delta}G}{\delta\varphi} - \rho\bar{c}\frac{\vec{\delta}}{\delta\varphi}\left(\frac{\vec{\delta}G}{\delta\varphi}c\right)\right)Y\right\}(y)X(x)\right]\right\rangle^{L_\rho}_{N,M}$$
$$= i\sum_{i=1}^{s}(-1)^{(\Phi_1\cdots\Phi_i,\,Y)}\left\langle T\left[\Phi_1(x_1)\cdots\Phi_{i-1}(x_{i-1})\left\{\frac{\vec{\delta}\Phi_i}{\delta\varphi}Y\right\}(x_i)\Phi_{i+1}(x_{i+1})\cdots\Phi_s(x_s)\right]\right\rangle^{L_\rho}_{N,M}, \tag{2.9}$$

$$\left\langle T\left[\int d^4y\left\{\left(\rho\frac{\delta\mathcal{L}_0(W_\rho)}{\delta W_\rho}\Big|\Big| - \bar{n}(\Box + \mu^2\,||)\right)Y\right\}(y)X(x)\right]\right\rangle^{L_\rho}_{N,M}$$
$$= i\sum_{i=1}^{s}(-1)^{(\Phi_1\cdots\Phi_i,\,Y)}\left\langle T\left[\Phi_1(x_1)\cdots\Phi_{i-1}(x_{i-1})\left\{\frac{\vec{\delta}\Phi_i}{\delta n}Y\right\}(x_i)\Phi_{i+1}(x_{i+1})\cdots\Phi_s(x_s)\right]\right\rangle^{L_\rho}_{N,M}, \tag{2.10}$$

$$\left\langle T\left[\int d^4y\left\{\left(-c(\Box + \mu^2\,||) - \rho\frac{\vec{\delta}G}{\delta\varphi}c\right)Y\right\}(y)X(x)\right]\right\rangle^{L_\rho}_{N,M}$$
$$= i\sum_{i=1}^{s}(-1)^{(\Phi_1\cdots\Phi_{i-1},\,cY)}(-1)^{(\Phi_i,\,Y)}\left\langle T\left[\Phi_1(x_1)\cdots\Phi_{i-1}(x_{i-1})\left\{\frac{\vec{\delta}\Phi_i}{\delta\bar{c}}Y\right\}(x_i)\Phi_{i+1}(x_{i+1})\cdots\Phi_s(x_s)\right]\right\rangle^{L_\rho}_{N,M}, \tag{2.11}$$

and

$$\left\langle T\left[\int d^4y\{(-n(\Box + \mu^2\,||) + G)Y\}(y)X(x)\right]\right\rangle^{L_\rho}_{N,M}$$
$$= i\sum_{i=1}^{s}(-1)^{(\Phi_1\cdots\Phi_i,\,Y)}\left\langle T\left[\Phi_1(x_1)\cdots\Phi_{i-1}(x_{i-1})\left\{\frac{\vec{\delta}\Phi_i}{\delta\bar{n}}Y\right\}(x_i)\Phi_{i+1}(x_{i+1})\cdots\Phi_s(x_s)\right]\right\rangle^{L_\rho}_{N,M}, \tag{2.12}$$

where

$$X(x) = \Phi_1(x_1)\Phi_2(x_2)\cdots\Phi_s(x_s), \tag{2.13}$$

$$\delta\Phi/\delta\bar{c} = (-1)^\eta \times \text{ordinary } \delta\Phi/\delta\bar{c}, \tag{2.14}$$

η = number of permutations of fermions to bring $\bar{c}$ out of Φ to its left, and (Φ, Y) = number of permutations of fermions in the transformation

$$\Phi Y \to Y\Phi. \tag{2.15}$$

Letting Y be ρc, $-c$, and $\bar{n}$ in equations (2.9), (2.10), and (2.11), respectively, and adding them,[11] we have

$$\langle T\Delta X(x)\rangle^{L_\rho}_{N,M} = 0, \tag{2.16}$$

where

$$\Delta X(x) = \sum_{i=1}^{s}(-1)^{(\Phi_1\cdots\Phi_i,\,c)}\Phi_1(x_1)\cdots\Phi_{i-1}(x_{i-1})\left[\rho\frac{\vec{\delta}\Phi_i}{\delta\varphi}c - \frac{\vec{\delta}\Phi_i}{\delta n}c + (-1)^{(\Phi_i,\,c)}\frac{\vec{\delta}\Phi_i}{\delta\bar{c}}\bar{n}\right](x_i)\Phi_{i+1}(x_{i+1})\cdots\Phi_s(x_s). \tag{2.17}$$

Meanwhile

$$\frac{\partial L_\rho}{\partial_\rho} = \frac{\vec{\delta}\mathcal{L}_0(W_\rho)}{\delta W_\rho}\Big|\Big|n - \bar{c}\frac{\vec{\delta}G(\varphi)}{\delta\varphi}c, \tag{2.18}$$

so that

$$\left\langle T\left[\int d^4y\frac{\partial L_\rho}{\partial\rho}(y)X(x)\right]\right\rangle^{L_\rho}_{N,M} = \left\langle T\left[\int d^4y\left\{\frac{\delta\mathcal{L}_0(W_\rho)}{\delta W_\rho}\Big|\Big|n - \bar{c}\frac{\vec{\delta}G(\varphi)}{\delta\varphi}c\right\}(y)X(x)\right]\right\rangle^{L_\rho}_{N,M}. \tag{2.19}$$

Letting Y be n and $\bar{n}$ in (2.10) and (2.12), respectively, and combining them with (2.19),

$$\left\langle T\left[\int d^4y\frac{\partial L_\rho}{\partial\rho}(y)X(x)\right]\right\rangle^{L_\rho}_{N,M} = \frac{i}{\rho}\left\langle T\left[\left\{\frac{\vec{\delta}X}{\delta n}n - \frac{\vec{\delta}X}{\delta\bar{n}}\bar{n}\right\}(x)\right]\right\rangle^{L_\rho}_{N,M} + \left\langle T\left[\int d^4y\left\{\frac{G(\varphi)\bar{n}}{\rho} - \bar{c}\frac{\vec{\delta}G(\varphi)}{\delta\varphi}c\right\}(y)X(x)\right]\right\rangle^{L_\rho}_{N,M}, \tag{2.20}$$

where

$$\frac{\vec{\delta}X}{\delta n}n = \sum_{i=1}^{s}\Phi_1\cdots\Phi_{i-1}\left(\frac{\vec{\delta}\Phi_i}{\delta n}n\right)\Phi_{i+1}\cdots\Phi_s, \tag{2.21}$$

and similarly for $\bar{n}$. Letting X be $(1/\rho)\{\bar{c}G(\varphi)\}(y)X(x)$ in (2.16) and combining it with (2.20), together with the Schwinger principle, we obtain

$$\frac{\partial}{\partial\rho}\langle T[X(x)]\rangle^{L_\rho}_{N,M}=\left\langle T\left[\left\{\frac{\partial X}{\partial\rho}-\frac{1}{\rho}\left(\frac{\vec{\delta}X}{\delta n}n-\frac{\vec{\delta}X}{\delta\bar{n}}\bar{n}\right)\right\}(x)\right]\right\rangle^{L_\rho}_{N,M}+\frac{i}{\rho}\left\langle T\left[\int d^4y\{\bar{c}G(\varphi)\}(y)\Delta X(x)\right]\right\rangle^{L_\rho}_{N,M}, \tag{2.22}$$

from which we will in Sec. III show the invariance of the scattering matrix.

Let the Φ's be functions of $W_\rho(\varphi)$ and its derivatives. Then

$$\frac{\partial}{\partial\rho}\langle T[X(x)]\rangle^{L_\rho}_{N,M}=\left\langle T\left[\left\{\left(\frac{\vec{\delta}X}{\delta W_\rho}-\frac{1}{\rho}\frac{\vec{\delta}X}{\delta n}\right)n\right\}(x)\right]\right\rangle^{L_\rho}_{N,M}+\frac{i}{\rho}\left\langle T\left[\int d^4y\{\bar{c}G(\varphi)\}(y)\left\{\left(\rho\frac{\vec{\delta}X}{\delta\varphi}-\frac{\vec{\delta}X}{\delta n}\right)c\right\}(x)\right]\right\rangle^{L_\rho}_{N,M}. \tag{2.23}$$

Since $\vec{\delta}X/\delta n=\rho\vec{\delta}X/\delta W_\rho$ and $\vec{\delta}X/\delta\varphi=\vec{\delta}X/\delta W_\rho$ this becomes

$$\frac{\partial}{\partial\rho}\langle T[X(x)]\rangle^{L_\rho}_{N,M}=0, \tag{2.24}$$

completely analogous to (1.25) for local transformations since the external legs $\varphi(x)+\rho n(x)$ are equivalent by (2.12) to the external legs

$$\varphi(x)+\rho\int d^4y\,\Delta_F(x-y;\mu)\{G(\varphi)\}(y)$$

in a T product which does not contain any external legs $\bar{n}$. Consequently, the introduction of the Faddeev-Popov ghosts into the Lagrangian does indeed allow us to maintain the equivalence theorem.

III. INVARIANCE OF THE S MATRIX

A. Local transformation

The relation (1.25) for the two-point function means

$$\langle T[\{\varphi+\rho F(\varphi)\}(x)\{\varphi+\rho F(\varphi)\}(0)]\rangle^{\mathcal{L}_\rho}_{N,M}=\langle T[\varphi(x)\varphi(0)]\rangle^{\mathcal{L}_0}_{N}. \tag{3.1}$$

Let the Lagrangian $\mathcal{L}_0$ be such that its two-point function has a simple pole at $p^2=m^2$ with residue i. Then the above equation tells us that

$$(p^2-m^2)\langle T[\{\tilde{\varphi}+\rho\tilde{F}(\varphi)\}(p)\{\tilde{\varphi}+\rho\tilde{F}(\varphi)\}(-p)]\rangle^{\mathcal{L}_\rho}_{N,M}\big|_{p^2=m^2}=i, \tag{3.2}$$

where the tilde denotes the Fourier transform of the fields. Let $D(\rho,p^2)$ be the (one-particle irreducible) vertex function of $F(\varphi)$ and φ at momentum p for the Lagrangian $\mathcal{L}_\rho$. Then

$$\langle T[\tilde{F}(\varphi)(p)\tilde{\varphi}(-p)]\rangle^{\mathcal{L}_\rho}_{N,M}=\{D(\rho,p^2)\langle T[\tilde{\varphi}(p)\tilde{\varphi}(-p)]\rangle^{\mathcal{L}_\rho}\}_{N,M}. \tag{3.3}$$

Let $E(\rho,p^2)$ be the (one-particle irreducible) vertex function of $F(\varphi)$ and $F(\varphi)$ at momentum p for the Lagrangian $\mathcal{L}_\rho$. Then

$$\langle T[\tilde{F}(\varphi)(p)\tilde{F}(\varphi)(-p)]\rangle^{\mathcal{L}_\rho}_{N,M}=E(\rho,p^2)_{N,M}+\{D^2(\rho,p^2)\langle T[\tilde{\varphi}(\rho)\tilde{\varphi}(-p)]\rangle^{\mathcal{L}_\rho}\}_{N,M}. \tag{3.4}$$

The spectral properties of the functions $D(\rho,p^2)$ and $E(\rho,p^2)$ ensure their reality and continuity at $p^2=m^2$. Then (3.2), (3.3), and (3.4), give

$$(p^2-m^2)\{[1+\rho D(\rho,m^2)]^2\langle T[\tilde{\varphi}(p)\tilde{\varphi}(-p)]\rangle^{\mathcal{L}_\rho}\}_{N,M}\big|_{p^2=m^2}=i. \tag{3.5}$$

Let us define $Z(\rho)$ to be the inverse of $[1+\rho D(\rho,m^2)]$ in the sense of formal power series in ρ and in the coupling constants. Then (3.5) reads

$$(p^2-m^2)\left\{\frac{1}{Z^2(\rho)}\langle T[\tilde{\varphi}(p)\tilde{\varphi}(-p)]\rangle^{\mathcal{L}_\rho}\right\}_{N,M}\Bigg|_{p^2=m^2}=i, \tag{3.6}$$

which means that the counterterms in $\mathcal{L}_\rho$ obtained from these in $\mathcal{L}_0$ by replacing φ by $\varphi+\rho F$, guarantee for any ρ the presence of a simple pole for the propagator at $p^2=m^2$. However, since its residue is not i anymore, the definition of the S matrix necessitates the introduction of a wave-function renormalization constant $Z(\rho)$. The S-matrix elements for a total number s of incoming and outgoing particles are

$$S_{N,M}(\rho,p_1,\ldots,p_s)=\left\{\prod_{i=1}^{s}(p_i^{\,2}-m^2)\left\langle T\left[\prod_{i=1}^{s}\frac{\tilde{\varphi}(p_i)}{Z(\rho)}\right]\right\rangle^{\mathcal{L}_\rho}\right\}_{N,M}\Bigg|_{p_i^2=m^2}. \tag{3.7}$$

We will now show that $S_{N,M}(\rho,p_1,\ldots,p_s)$ is in fact ρ independent. By differentiating (3.6), with respect to ρ, we obtain

$$(p^2-m^2)\left\{\frac{2}{Z^3(\rho)}\frac{\partial Z(\rho)}{\partial\rho}\langle T[\tilde\varphi(p)\tilde\varphi(-p)]\rangle^{\mathfrak{L}_\rho}\right\}_{N,M-1}\Big|_{p^2=m^2}=(p^2-m^2)\left\{\frac{1}{Z^2(\rho)}\frac{\partial}{\partial\rho}\langle T[\tilde\varphi(p)\tilde\varphi(-p)]\rangle^{\mathfrak{L}_\rho}\right\}_{N,M-1}\Big|_{p^2=m^2} \quad (3.8)$$

which, together with (1.20), gives

$$(p^2-m^2)\left\{\frac{1}{Z(\rho)}\frac{\partial Z(\rho)}{\partial\rho}\langle T[\tilde\varphi(p)\tilde\varphi(-p)]\rangle^{\mathfrak{L}_\rho}\right\}_{N,M-1}\Big|_{p^2=m^2}=-(p^2-m^2)\langle T[\tilde B(p)\tilde\varphi(-p)]\rangle^{\mathfrak{L}_\rho}_{N,M-1}\big|_{p^2=m^2}\,. \quad (3.9)$$

In (3.9)

$$B(\varphi)=\sum_{m=0}^{n-1}\left(-\rho\frac{\vec\delta F(\varphi)}{\delta\varphi}\right)^m F(\varphi), \quad (3.10)$$

where $n\geq M$. Let $V(\rho,p^2)$ be the (one-particle irreducible) vertex function of $B(\varphi)$ and φ at momentum p for the Lagrangian $\mathfrak{L}_\rho$. Then, similarly to (3.3),

$$\langle T[\tilde B(\varphi)(p)\tilde\varphi(-p)]\rangle^{\mathfrak{L}_\rho}_{N,M}=\{V(\rho,p^2)\langle T[\tilde\varphi(p)\tilde\varphi(-p)]\rangle^{\mathfrak{L}_\rho}\}_{N,M}\,. \quad (3.3')$$

Using this and (3.9) we obtain

$$[1/Z(\rho)]\partial Z(\rho)/\partial\rho=-V(\rho,m^2), \quad (3.11)$$

in the sense of formal power series. Also,

$$\frac{\partial}{\partial\rho}\left\langle T\left[\prod_{i=1}^{s}\tilde\varphi(p_i)\right]\right\rangle^{\mathfrak{L}_\rho}_{N,M}=-\sum_{i=1}^{s}\langle T[\tilde\varphi(p_1)\cdots\tilde\varphi(p_{i-1})\tilde B(p_i)\tilde\varphi(p_{i+1})\cdots\tilde\varphi(p_s)]\rangle^{\mathfrak{L}_\rho}_{N,M-1}. \quad (3.12)$$

Each of the matrix elements on the right-hand side of (3.12) can be decomposed into two parts: a term $W_i(p_1,\ldots p_s)$ which has no pole at $p_i^2=m^2$, and another which has one. Thus

$$\langle T[\tilde\varphi(p_1)\cdots\tilde\varphi(p_{i-1})\tilde B(p_i)\tilde\varphi(p_{i+1})\cdots\tilde\varphi(p_s)]\rangle^{\mathfrak{L}_\rho}_{N,M-1}=W_i(p_1,\ldots,p_s)_{N,M-1}+\left\{V(\rho,p_i^2)\left\langle T\left[\prod_{j=1}^{s}\tilde\varphi(p_j)\right]\right\rangle^{\mathfrak{L}_\rho}\right\}_{N,M-1}. \quad (3.13)$$

Consequently,

$$\prod_{i=1}^{s}(p_i^2-m^2)\frac{\partial}{\partial\rho}\left\langle T\left[\prod_{i=1}^{s}\tilde\varphi(p_i)\right]\right\rangle^{\mathfrak{L}_\rho}_{N,M}\Big|_{p_i^2=m^2}=-\prod_{i=1}^{s}(p_i^2-m^2)s\left\{V(\rho,m^2)\left\langle T\left[\prod_{i=1}^{s}\tilde\varphi(p_i)\right]\right\rangle^{\mathfrak{L}_\rho}\right\}_{N,M-1}\Big|_{p_i^2=m^2}. \quad (3.14)$$

Then, from (3.7), (3.11), and (3.14) we have

$$\frac{\partial}{\partial\rho}S_{N,M}(\rho,p_1,\ldots,p_s)=0. \quad (3.15)$$

Hence the S matrix is invariant under the local transformation (1.2).

B. Nonlocal transformation

The proof of the invariance of the S matrix under nonlocal transformations follows closely from that under local transformations. The relation (2.24) for the two-point function is

$$\langle T[\{\varphi+\rho n\}(x)\{\varphi+\rho n\}(y)]\rangle^{L_\rho}_{N,M}=\langle T[\varphi(x)\varphi(y)]\rangle^{L_0}_{N}\,, \quad (3.16)$$

and consequently from the normalization used in L_0 (or in $\mathfrak{L}_0$)

$$(p^2-m^2)\langle T[\{\tilde\varphi+\rho\tilde n\}(p)\{\tilde\varphi+\rho\tilde n\}(-p)\}]\rangle^{L_\rho}_{N,M}\big|_{p^2=m^2}=i. \quad (3.17)$$

Let $D(\rho,p^2)$ denote the one-φ-particle irreducible vertex function of n and φ at momentum p for the Lagrangian L_ρ, and let $E(\rho,p^2)$ denote the one-φ-particle irreducible vertex function of n and n at momentum p for the Lagrangian L_ρ. Then the relations (3.5) and (3.6) and the definition (3.7) of the S matrix (between physical states) are still valid (with $\mathfrak{L}_\rho$ replaced by L_ρ) if, again,

$$Z(\rho)=[1+\rho D(\rho,m^2)]^{-1} \quad (3.18)$$

in the sense of a formal power series. Now from (2.22), the analog of (3.9) is

$$(p^2-m^2)\left\{\frac{1}{Z(\rho)}\frac{\partial Z(\rho)}{\partial\rho}\langle T[\tilde\varphi(p)\tilde\varphi(-p)]\rangle^{L_\rho}\right\}_{N,M-1}\Big|_{p^2=m^2}=i(p^2-m^2)\left(\left\langle T\left[\int d^4z\{\bar cG\}(z)\tilde c(p)\tilde\varphi(-p)\right]\right\rangle^{L_\rho}\right)_{N,M-1}\Big|_{p^2=m^2}. \quad (3.19)$$

Then let us denote by $V(\rho,p^2)$ the one-φ-particle irreducible vertex function for $\int d^4z\{\bar{c}G\}(z),\bar{c}(p),\tilde{\varphi}(-p)$ in the Lagrangain L_ρ

$$\left\langle T\left[\int d^4z\{\bar{c}G\}(z)\bar{c}(p)\varphi(-p)\right]\right\rangle_{N,M-1}^{L_\rho}=\{V(\rho,p^2)\langle T[\varphi(p)\varphi(-p)]\rangle^{L_\rho}\}_{N,M-1}. \tag{3.20}$$

Consequently

$$[1/Z(\rho)]\partial Z(\rho)/\partial\rho=iV(\rho,m^2), \tag{3.21}$$

in the sense of a formal power series. On the other hand,

$$\frac{\partial}{\partial\rho}\left\langle T\left[\prod_{i=1}^{s}\varphi(p_i)\right]\right\rangle_{N,M}^{L_\rho}=i\sum_{i=1}^{s}\left\langle T\left[\int d^4z\{\bar{c}G\}(z)\varphi(p_1)\cdots\varphi(p_{i-1})\bar{c}(p_i)\varphi(p_{i+1})\cdots\varphi(p_s)\right]\right\rangle_{N,M-1}^{L_\rho}. \tag{3.22}$$

Then the argument follows from what was done in the local case with the normal product $\tilde{B}(p_i)$ replaced by the two-legged object $\int d^4z\{\bar{c}G\}(z)\bar{c}(p_i)$. The result is again the independence of the S matrix on ρ (3.15).

Hence $\mathcal{L}_0(\varphi)$ [that is, $L_0(\varphi)$] and $\mathcal{L}_1'(\varphi)$ [that is, $L_1(\varphi)$] have the same S matrices between physical states.

IV. CONCLUSION

An algebraic proof of the equivalence theorem in all orders of perturbation, for local transformations, in the normal-product algorithm, has been developed in Sec. I and Sec. III A.

This elegant proof does not require a tedious manipulation of Feynman diagrams, since the equations of motion and the Schwinger principle have already extracted the relevant properties from them. The invariance of the S matrix under the transformation is also obtained rigorously in all orders of perturbation without recourse to the Haag-Ruelle construction,[4] as opposed to Refs. 2 and 3. Although this proof applies to a polynomial transformation of the fields, it can be easily extended to cover transformations that are power series in the fields φ at $\varphi=0$.

This proof breaks down for the nonlocal transformations. The nonlocal transformations generate a set of unwanted diagrams reminiscent of those described by 't Hooft in gauge field theory and which were canceled there, by the contribution from Faddeev-Popov ghosts.[5]

Incorporating a ghost part into the transformed Lagrangian to cancel these unwanted diagrams, we then restore the equivalence between the Green's functions of the original Lagrangian and those of the resulting Lagrangian, and also the equality of their S-matrix elements between physical states.

An extension of these nonlocal transformation to a vector field will be an important tool for studying gauge invariance.

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APPENDIX

(1) Proof of Eq. (1.10). The content of (1.10) can be stated as follows: Given a function φ of X and its space-time derivatives, and a function X of φ and its space-time derivatives, then

$$\frac{\delta Q[X(\varphi)]}{\delta\varphi}=\frac{\delta X}{\delta\varphi}\cdot\frac{\delta Q}{\delta X}, \tag{A1}$$

where the notations $(\delta/\delta\varphi)\cdot$ and $(\delta/\delta\varphi)$ are defined in (1.8) and (1.9), respectively. In this proof we shall use Einstein's notation $X_{,\mu_1\cdots\mu_n}$ to stand for $\partial X(x)/\partial x_{\mu_1}\cdots\partial x_{\mu_n}$, and dummy indices are always summed over from 0 to 3. From the identity

$$X_{,\mu}=\sum_{k=0}^{\infty}\frac{1}{k!}\,\frac{\partial X}{\partial\varphi_{,\alpha_1\cdots\alpha_k}}\,\varphi_{,\alpha_1\cdots\alpha_k\mu} \tag{A2}$$

we get

$$\frac{\partial X_{,\mu}}{\partial\varphi_{,\nu_1\cdots\nu_m}}=\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{\partial}{\partial\varphi_{,\alpha_1\cdots\alpha_k}}\,\frac{\partial X}{\partial\varphi_{,\nu_1\cdots\nu_m}}\right)\varphi_{,\alpha_1\cdots\alpha_k\mu}+\sum_{i=1}^{m}\delta_\mu^{\nu_i}\,\frac{\partial X}{\partial\varphi_{,\nu_1\cdots\hat{\nu}_i\cdots\nu_m}}. \tag{A3}$$

The second sum on the right-hand side of (A3) is zero for $m=0$; $\hat{\nu}_i$ means that this index is omitted. Introducing another function Y and using (A3), we have

$$\left(Y\frac{\partial X_{,\mu}}{\partial\varphi_{,\nu_1\cdots\nu_m}}\right)_{,\nu_1\cdots\nu_m} = \left(Y\left\{\frac{\partial X}{\partial\varphi_{,\nu_1\cdots\nu_m}}\right\}_{,\mu}\right)_{,\nu_1\cdots\nu_m} + m\left(Y\frac{\partial X}{\partial\varphi_{,\nu_1\cdots\nu_{m-1}}}\right)_{,\nu_1\cdots\nu_{m-1}\mu}. \tag{A4}$$

Given the function $Q[X(\varphi)]$, we can write

$$\frac{\partial Q}{\partial\varphi_{,\nu_1\cdots\nu_m}} = \sum_{n=0}^{\infty}\frac{1}{n!}\frac{\partial Q}{\partial X_{,\mu_1\cdots\mu_n}}\frac{\partial X_{,\mu_1\cdots\mu_n}}{\partial\varphi_{,\nu_1\cdots\nu_m}}, \tag{A5}$$

and using the definition (1.9) we get

$$\frac{\delta Q}{\delta\varphi} = \sum_{n,m=0}^{\infty}\frac{(-1)^m}{n!\,m!}\left\{\frac{\partial Q}{\partial X_{,\mu_1\cdots\mu_n}}\frac{\partial X_{,\mu_1\cdots\mu_n}}{\partial\varphi_{,\nu_1\cdots\nu_m}}\right\}_{,\nu_1\cdots\nu_m}. \tag{A6}$$

By a simple application of (A4) with Y replaced by $\partial Q/\partial X_{,\mu_1\cdots\mu_n}$, X by $X_{,\mu_2\cdots\mu_n}$, and μ by μ_1, (A6) becomes

$$\frac{\delta Q}{\delta\varphi} = \sum_{n,m=0}^{\infty}\frac{(-1)^m}{n!\,m!}\left[\left\{\frac{\partial Q}{\partial X_{,\mu_1\cdots\mu_n}}\left(\frac{\partial X_{,\mu_2\cdots\mu_n}}{\partial\varphi_{,\nu_1\cdots\nu_m}}\right)_{,\mu_1}\right\}_{,\nu_1\cdots\nu_m} + m\left\{\frac{\partial Q}{\partial X_{,\mu_1\cdots\mu_n}}\frac{\partial X_{,\mu_2\cdots\mu_n}}{\partial\varphi_{,\nu_1\cdots\nu_{m-1}}}\right\}_{,\nu_1\cdots\nu_{m-1}\mu_1}\right]. \tag{A7}$$

After application of the μ_1 derivative in the second bracket [ ] of (A7), and after simplification of the first bracket [ ], we obtain

$$\frac{\delta Q}{\delta\varphi} = \sum_{n,m=0}^{\infty}\frac{(-1)^{m+1}}{n!\,m!}\left\{\left(\frac{\partial Q}{\partial X_{,\mu_1\cdots\mu_n}}\right)_{,\mu_1}\frac{\partial X_{,\mu_2\cdots\mu_n}}{\partial\varphi_{,\nu_1\cdots\nu_m}}\right\}_{,\nu_1\cdots\nu_m}. \tag{A8}$$

In (A8), we may substitute (A4) again with Y replaced by $(\partial Q/\partial X_{,\mu_1\cdots\mu_n})_{,\mu_1}$, X by $X_{,\mu_3\cdots\mu_n}$, and μ by μ_2. In the result of the above, we may again apply (A4) repeatedly with μ replaced by $\mu_2, \mu_3, \ldots, \mu_n$; we finally obtain

$$\frac{\delta Q}{\delta\varphi} = \sum_{n,m=0}^{\infty}\frac{(-1)^{n+m}}{n!\,m!}\left\{\left(\frac{\partial Q}{\partial X_{,\mu_1\cdots\mu_n}}\right)_{,\mu_1\cdots\mu_n}\frac{\partial X}{\partial\varphi_{,\nu_1\cdots\nu_m}}\right\}_{,\nu_1\cdots\nu_m}, \tag{A9}$$

which by (1.8) and (1.9) proves (A1).

(2) Proof of Eq. (1.22a). Multiplying (A3) by the function Y and using (A2), we obtain

$$\frac{\partial X_{,\mu}}{\partial\varphi_{,\nu_1\cdots\nu_m}}Y = \left(\frac{\partial X}{\partial\varphi_{,\nu_1\cdots\nu_m}}\right)_{,\mu}Y + \sum_{i=1}^{m}\delta_\mu^{\nu_i}\left(\frac{\partial X}{\partial\varphi_{,\nu_1\cdots\hat{\nu}_i\cdots\nu_m}}\right)Y. \tag{A10}$$

From the definition (1.7),

$$\frac{\vec{\delta}Q}{\delta\varphi}Y = \sum_{n=0}^{\infty}\frac{1}{n!}\frac{\partial Q}{\partial X_{,\mu_1\cdots\mu_n}}\frac{\vec{\delta}X_{,\mu_1\cdots\mu_n}}{\delta\varphi}Y. \tag{A11}$$

Choosing in (A10) Y, X, and μ to be $Y_{,\nu_1\cdots\nu_m}$, $X_{,\mu_2\cdots\mu_n}$, and μ_1, this can be rewritten as

$$\frac{\vec{\delta}Q}{\delta\varphi}Y = \sum_{n=0}^{\infty}\frac{1}{n!}\frac{\partial Q}{\partial X_{,\mu_1\cdots\mu_n}}\left\{\sum_{m=0}^{\infty}\frac{1}{m!}\left(\frac{\partial X_{,\mu_2\cdots\mu_n}}{\partial\varphi_{,\nu_1\cdots\nu_m}}\right)_{,\mu_1}Y_{,\nu_1\cdots\nu_m} + \frac{\vec{\delta}X_{,\mu_2\cdots\mu_n}}{\delta\varphi}Y_{,\mu_1}\right\}, \tag{A12}$$

that is,

$$\frac{\vec{\delta}Q}{\delta\varphi}Y = \sum_{n=0}^{\infty}\frac{1}{n!}\frac{\partial Q}{\partial X_{,\mu_1\cdots\mu_n}}\left\{\frac{\vec{\delta}X_{,\mu_2\cdots\mu_n}}{\delta\varphi}Y\right\}. \tag{A13}$$

By reiteration of the above procedure choosing μ successively to be $\mu_2, \ldots, \mu_n$, we obtain

$$\frac{\vec{\delta}Q}{\delta\varphi}Y = \frac{\vec{\delta}Q}{\delta X}\frac{\vec{\delta}X}{\delta\varphi}Y. \tag{A14}$$

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$$\langle T[\{\gamma\varphi\}(x)\Phi_1(x_1)\cdots\Phi_s(x_s)]\rangle = \left\langle T\left[\exp\left(i\int d^4z\{\mathcal{L}_I(\varphi^{(0)})\}(z)\right) \times\{\gamma^{(0)}\varphi^{(0)}\}(x)\Phi_1^{(0)}(x_1)\cdots\Phi_s^{(0)}(x_s)\right]\right\rangle,$$

where the superscript (0) means functions of free fields, and

$$\langle T[\varphi^{(0)}(x)\varphi^{(0)}(y)]\rangle = i\Delta_F(x-y;m) = i\int\frac{d^4k}{(2\pi)^4}\frac{e^{-ik(x-y)}}{k^2-m^2+i\epsilon}.$$

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