Regularization of fermion loops in gauge models*

Debojit Barua and Suraj N. Gupta
Department of Physics, Wayne State University, Detroit, Michigan 48202
(Received 23 February 1976)

Fermion loops in gauge models with γ_5 couplings cannot be regularized in a gauge-invariant manner either by the method of auxiliary fields or by dimensional continuation. It is shown that in the absence of a gauge-invariant regularization procedure, the method of auxiliary fields can be employed in such a way that non-gauge-invariance is confined to the renormalization constants. It is thus possible to obtain gauge-invariant renormalized results as well as resolve anomalies without the use of Ward's identities. Explicit calculations are carried out for the self-energy and triangle diagrams with fermion loops in the Appelquist-Quinn model. The present treatment is also compared with naive dimensional regularization for the γ_5 couplings.

I. INTRODUCTION

Regularization by auxiliary fields[1] or by dimensional continuation[2] is widely used for the evaluation of divergent integrals in quantum field theory, and usually one or both of these methods can be employed successfully for this purpose. However, the regularization of fermion loops in gauge models presents a special difficulty, because the auxiliary fields cannot preserve the symmetry of the system, while dimensional continuation cannot be applied to the γ_5 couplings. The aim of the present paper is to show a way out of this difficulty with the use of the method of auxiliary fields, and also examine the situation with the use of dimensional regularization.

Our approach is based on the observation that it is possible to carry out regularization without imposing symmetry constraints on the auxiliary-field couplings, provided that it is ensured that the contributions of the auxiliary fields to a given physical process appear only in renormalization constants. This requirement will be referred to as the *regularizability condition.* The renormalization constants, of course, are to be dropped wherever they occur in the results obtained by our regularization procedure.

One might think that a regularization procedure that generates non-gauge-invariant renormalization constants will prove inconvenient in practice. However, as we shall show by applications to the self-energy and triangle diagrams arising from fermion loops in the Appelquist-Quinn gauge model,[3] our treatment is quite straightforward. It is especially worthwhile to explore the present approach in view of the fact that different prescriptions[2,4,5] suggested for dimensional continuation of the γ_5 couplings are found to yield different results for loops involving such couplings.

Although the treatment given in this paper is recommended for fermion loops in gauge models with γ_5 couplings, it can on occasion be applied to other systems of interacting fields with symmetries.[6] Our treatment can also be used in conjunction with dimensional regularization in physical applications.

After a discussion of the regularizability condition for the auxiliary fields in Sec. II, regularization and renormalization of self-energies resulting from fermion loops in the Appelquist-Quinn model will be carried out in Sec. III. We shall then regularize the triangle diagrams in Sec. IV, apply the regularizability condition, and thus demonstrate the well-known necessity[7,8] of another fermion field without the use of Ward's identities. The treatment of fermion loops with dimensional regularization will be examined in Sec. V.

Our notation is such that $x_\mu = (x_1, x_2, x_3, x_4)$ with $x_4 = ix_0$, and $\epsilon_{\mu\nu\lambda\rho}$ is completely antisymmetrical with $\epsilon_{1234} = i$.

II. REGULARIZABILITY CONDITION

It will be useful to discuss briefly the method of regularization by auxiliary fields[1] and the verification of the regularizability condition.

Let us consider a fermion loop that gives rise to a divergent intergral of the form

$$F(k^{(i)}; M^2) = C_0 \int dq\, R(q, k^{(i)}; M^2), \tag{2.1}$$

where $k^{(i)}$ are the momentum four-vectors of lines attached to the fermion loop, and C_0 is a constant which may involve the fermion mass M. By introducing an arbitrary number of normal and abnormal auxiliary fields with appropriate couplings, it is possible to replace (2.1) by

$$[F(k^{(i)}; M^2)]_{\text{reg}} = C_0 \int dq \Big[R(q, k^{(i)}; M^2) + \sum_\alpha \eta^{(\alpha)} R(q, k^{(i)}; M^{(\alpha)2})\Big], \tag{2.2}$$

where $\eta^{(\alpha)}$ equals 1 or -1, and $M^{(\alpha)}$ is an auxiliary mass ultimately tending to infinity. With the use of only one auxiliary field, (2.2) can be converted into

$$[F(k^{(i)};M^2)]_{\rm reg}=-C_0\int_0^{\xi^2}dz\int dq\,R'(q,k^{(i)};M^2+z), \tag{2.3}$$

while the use of three auxiliary fields leads to

$$[F(k^{(i)};M^2)]_{\rm reg} =C_0\int_0^{\xi^2}dz_1\int_0^{\xi^2}dz_2\int dq\,R''(q,k^{(i)};M^2+z_1+z_2), \tag{2.4}$$

where primes denote differentiations with respect to M^2, and ξ^2 tends to infinity. This procedure, of course, can be extended to higher derivatives, if necessary. In practice, it is always advisable to choose the M dependence of the constant C_0 in (2.1) in such a way that $R(q,k^{(i)};M^2)$ is a rational algebraic function of M^2 and requires as few differentiations as possible for convergence over the q space.

In order to achieve maximum freedom in the regularization procedure, we shall adopt the viewpoint that the auxiliary-field couplings required to obtain a result of the form (2.2) need not preserve the symmetry of the system. We can also choose different auxiliary-field couplings for different fermion loops or leave a fermion loop unregularized if regularization is not necessary. However, as mentioned in the preceding section, the regularization procedure must satisfy the regularizability condition, which requires that the contributions of the auxiliary fields appear only in renormalization constants.

It is easy to ascertain whether the regularizability condition is fulfilled in the treatment of the fermion loop. For, let us separate the contribution generated by the auxiliary fields in (2.2), and after dropping the renormalization terms let us denote this contribution as

$$C_0\sum_\alpha \eta^{(\alpha)}\int dq\,R^c(q,k^{(i)};M^{(\alpha)2}), \tag{2.5}$$

where $R^c(q,k^{(i)};M^{(\alpha)2})$ is obtained from $R(q,k^{(i)};M^{(\alpha)2})$ by carrying out the usual expansion in powers of the $k^{(i)}$ and dropping the renormalization terms. Then the criterion for the fulfillment of the regularizability condition can be stated as follows: When $M^{(\alpha)}\to\infty$, the integrand in (2.5) should vanish for finite values of q and converge more rapidly than q^{-4} for infinitely large values of q, so that the integral will vanish altogether. Note that when both $M^{(\alpha)}$ and q tend to infinity, we must treat $M^{(\alpha)}$ as equivalent to q for power-counting purposes.

Thus, possible violations of the regularizability condition can be detected by a power-counting procedure. Indeed, the regularizability condition is violated only in exceptional cases, and then the anomalous situation must be remedied.

III. SELF-ENERGY DIAGRAMS

With the use of the gauge-compensating formalism, the Lagrangian density for the Appelquist-Quinn model can be expressed as

$$\begin{aligned}L=&-\tfrac{1}{2}(\partial_\mu a_\nu)^2-\tfrac{1}{2}m^2a_\mu{}^2-\tfrac{1}{2}(\partial_\mu b)^2-\tfrac{1}{2}m^2b^2-\tfrac{1}{2}(\partial_\mu s)^2-\tfrac{1}{2}\mu^2s^2-\overline{\psi}(\gamma_\mu\partial_\mu+M)\psi-\partial_\mu C^*\partial_\mu C-m^2C^*C\\&-2gma_\mu{}^2s+2ga_\mu(b\partial_\mu s-s\partial_\mu b)-(g\mu^2/m)s(b^2+s^2)-2g^2a_\mu{}^2(b^2+s^2)-(g^2\mu^2/2m^2)(b^2+s^2)^2\\&-ig\overline{\psi}\gamma_\mu\gamma_5\psi a_\mu+(2igM/m)\overline{\psi}\gamma_5\psi b-(2gM/m)\overline{\psi}\psi s-2gmsC^*C,\end{aligned} \tag{3.1}$$

where C is the gauge-compensating field. In general, the model involves two coupling constants f and g, but since we are mainly interested in the γ_5 couplings of the fermion field ψ, we have put $f=0$ to achieve some simplification in calculations.

We shall first investigate the second-order boson self-energy diagrams with fermion loops resulting from the interaction in (3.1). All such diagrams with nonvanishing contributions are shown in Fig. 1.

A. Regularization

The contributions of the scattering operator for the self-energy diagrams 1(a) and 1(a′) are

$$S_{1(a)}=\delta(k-k')a_\mu^-(k')a_\nu^+(k)g^2\int dq\,{\rm Tr}[S_F(q)\gamma_\mu\gamma_5S_F(q-k)\gamma_\nu\gamma_5], \tag{3.2}$$

$$S_{1(a')}=\delta(k-k')a_\mu^-(k')a_\mu^+(k)\frac{8g^2M}{\mu^2}\int dq\,{\rm Tr}[S_F(q)], \tag{3.3}$$

with

$$S_F(q)=(iq\cdot\gamma-M)/(q^2+M^2). \tag{3.4}$$

The contribution (3.2), which appears to be quadratically divergent, can be evaluated by regularizing in accordance with (2.4), while it is really unnecessary to regularize or evaluate $S_{1(a')}$ because it evidently contributes only to a renormalization constant. We thus obtain

$$\begin{aligned}S_{aa}&=S_{1(a)}+S_{1(a')}\\&=i\delta(k-k')a_\mu^-(k')a_\nu^+(k)[A_1\delta_{\mu\nu}+B_1(k^2\delta_{\mu\nu}-k_\mu k_\nu)+C_1(k^2+m^2)\delta_{\mu\nu}+\Pi_{\mu\nu}^c(k)],\end{aligned} \tag{3.5}$$

where A_1, B_1, and C_1 are renormalization constants, while the physical term $\Pi_{\mu\nu}^c(k)$ is given by

$$\begin{aligned}\Pi_{\mu\nu}^c(k)=8\pi^2g^2\int_0^1du\bigg\{&u(1-u)(k^2\delta_{\mu\nu}-k_\mu k_\nu)\ln\left(1+\frac{(k^2+m^2)u(1-u)}{M^2-m^2u(1-u)}\right)\\&+\delta_{\mu\nu}M^2\left[\ln\left(1+\frac{(k^2+m^2)u(1-u)}{M^2-m^2u(1-u)}\right)-\frac{(k^2+m^2)u(1-u)}{M^2-m^2u(1-u)}\right]\bigg\}.\end{aligned} \tag{3.6}$$

Similarly, the contributions for the remaining diagrams in Fig. 1 are

$$\begin{aligned}S_{1(b)}&=\delta(k-k')b^-(k')b^+(k)\frac{4g^2M^2}{m^2}\int dq\,\mathrm{Tr}[S_F(q)\gamma_5S_F(q-k)\gamma_5],\\S_{1(b')}&=\delta(k-k')b^-(k')b^+(k)\frac{4g^2M}{m^2}\int dq\,\mathrm{Tr}[S_F(q)],\\S_{1(c)}&=\delta(k-k')[a_\mu^-(k')b^+(k)-b^-(k')a_\mu^+(k)]\frac{2g^2M}{m}\int dq\,\mathrm{Tr}[S_F(q)\gamma_\mu\gamma_5S_F(q-k)\gamma_5],\\S_{1(c')}&=\delta(k-k')[a_\mu^-(k')b^+(k)-b^-(k')a_\mu^+(k)]\frac{4g^2M}{m\mu^2}\int dq\,ik_\mu\,\mathrm{Tr}[S_F(q)],\\S_{1(d)}&=-\delta(k-k')s^-(k')s^+(k)\frac{4g^2M^2}{m^2}\int dq\,\mathrm{Tr}[S_F(q)S_F(q-k)],\\S_{1(d')}&=\delta(k-k')s^-(k')s^+(k)\frac{12g^2M}{m^2}\int dq\,\mathrm{Tr}[S_F(q)],\end{aligned} \tag{3.7}$$

which yield upon regularization results of the form

$$\begin{aligned}S_{bb}&=S_{1(b)}+S_{1(b')}\\&=i\delta(k-k')b^-(k')b^+(k)[A_2+B_2(k^2+m^2)+\Pi^c(k)],\end{aligned} \tag{3.8}$$

$$\begin{aligned}S_{ab}&=S_{1(c)}+S_{1(c')}\\&=i\delta(k-k')[a_\mu^-(k')b^+(k)-b^-(k')a_\mu^+(k)][iA_3k_\mu+\Pi_\mu^c(k)],\end{aligned} \tag{3.9}$$

$$\begin{aligned}S_{ss}&=S_{1(d)}+S_{1(d')}\\&=i\delta(k-k')s^-(k')s^+(k)[A_4+B_4(k^2+\mu^2)+\Pi'^c(k)],\end{aligned} \tag{3.10}$$

where the physical terms $\Pi^c(k)$, $\Pi_\mu^c(k)$, and $\Pi'^c(k)$ are given by

$$\Pi^c(k)=\frac{8\pi^2g^2M^2}{m^2}\int_0^1du\left[k^2\ln\left(1+\frac{(k^2+m^2)u(1-u)}{M^2-m^2u(1-u)}\right)+\frac{m^2(k^2+m^2)u(1-u)}{M^2-m^2u(1-u)}\right], \tag{3.11}$$

$$\Pi_\mu^c(k)=\frac{8\pi^2g^2M^2}{m}ik_\mu\int_0^1du\ln\left(1+\frac{(k^2+m^2)u(1-u)}{M^2-m^2u(1-u)}\right), \tag{3.12}$$

$$\Pi'^c(k)=\frac{48\pi^2g^2M^2}{m^2}\int_0^1du\left[[M^2+k^2u(1-u)]\ln\left(1+\frac{(k^2+\mu^2)u(1-u)}{M^2-\mu^2u(1-u)}\right)-(k^2+\mu^2)u(1-u)\right]. \tag{3.13}$$

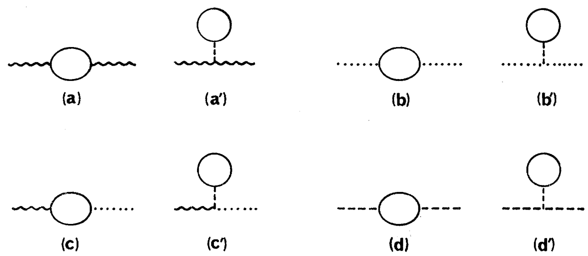

FIG. 1. Self-energy diagrams. Solid lines represent the fermion field ψ, while wavy, dotted, and broken lines represent the boson fields a_μ, b, and s, respectively.

B. Regularizability condition

It will now be verified that our regularization procedure satisfies the regularizability condition. Regularization of (3.2) in accordance with (2.4) corresponds to the introduction of three auxiliary fields, whose contribution is given by

$$\hat{S}_{1(a)} = \delta(k - k')a_\mu^-(k')a_\nu^+(k)g^2 \int dq\,\hat{R}_{\mu\nu}(k,q), \tag{3.14}$$

where

$$\hat{R}_{\mu\nu}(k,q) = \sum_\alpha \eta^{(\alpha)}\mathrm{Tr}[S_F(q;M^{(\alpha)})\gamma_\mu\gamma_5 \times S_F(q-k;M^{(\alpha)})\gamma_\nu\gamma_5], \tag{3.15}$$

with

$$S_F(q;M^{(\alpha)}) = \frac{iq\cdot\gamma - M^{(\alpha)}}{q^2 + M^{(\alpha)2}}. \tag{3.16}$$

We can decompose $\hat{R}_{\mu\nu}(k,q)$ as

$$\hat{R}_{\mu\nu}(k,q) = \hat{R}_{\mu\nu}(0,q) + \tfrac{1}{2}k_\alpha k_\beta\left[\frac{\partial^2\hat{R}_{\mu\nu}(k,q)}{\partial k_\alpha \partial k_\beta}\right]_{k=0} + \hat{R}^c_{\mu\nu}(k,q), \tag{3.17}$$

and it follows from the power-counting argument given in Sec. II that, for $M^{(\alpha)} \to \infty$,

$$\int dq\,\hat{R}^c_{\mu\nu}(k,q) = 0. \tag{3.18}$$

In view of (3.17) and (3.18), the contribution (3.14) affects only the renormalization constants in (3.5).

Similarly, it is easy to see that the regularizability condition is satisfied for the other self-energy diagrams.

C. Renormalization

According to our treatment, renormalization of the regularized self-energy contributions can be achieved simply by dropping the renormalization constants, and the fulfillment of the regularizability condition ensures the gauge invariance of our renormalized results. It should, however, be pointed out that we have identified the renormalization terms in (3.5), (3.8), (3.9), and (3.10) in such a way that they have the same form as the counterterms in the gauge-invariant renormalization procedure of Appelquist, Carazzone, Goldman, and Quinn.[9] Note that although Appelquist *et al.* do not provide a direct counterterm corresponding to the C_1 term in (3.5), this term represents a finite wave-function renormalization, and it can be eliminated by an additional renormalization of the a_μ couplings carried out by these authors.

IV. TRIANGLE DIAGRAMS

The triangle diagrams resulting from the interaction in (3.1) are shown in Fig. 2. We shall apply the regularization procedure to the contributions of all the triangle diagrams, and then discuss the complication caused by the violation of the regularizability condition for the diagram 2(a).

A. Regularization

The contribution of the scattering operator for the triangle diagram 2(a) is

$$S_{2(a)} = i\delta(p - k - k')a_\mu^-(k')a_\nu^-(k)a_\lambda^+(p)(2g^3)F_{\mu\nu\lambda 5}(k,k'), \tag{4.1}$$

where

$$F_{\mu\nu\lambda 5}(k,k') = \int dq\,\mathrm{Tr}\left[\frac{i(q-k')\cdot\gamma - M}{(q-k')^2 + M^2}\gamma_\mu\gamma_5\frac{iq\cdot\gamma - M}{q^2+M^2} \times\gamma_\nu\gamma_5\frac{i(q+k)\cdot\gamma - M}{(q+k)^2+M^2}\gamma_\lambda\gamma_5\right]. \tag{4.2}$$

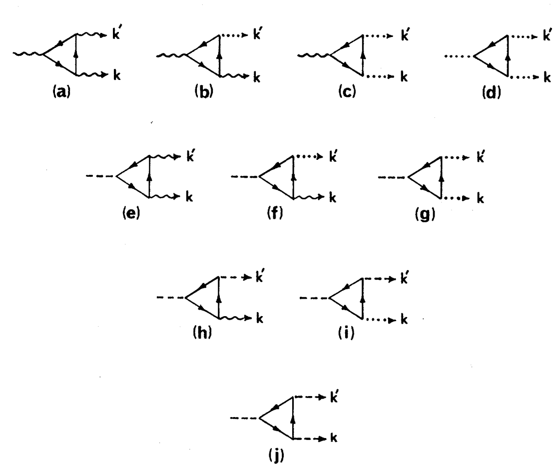

FIG. 2. Triangle diagrams. Corresponding to each diagram there exists another with the direction of the ψ lines reversed.

Since (4.2) appears to be linearly divergent, it is sufficient to regularize it with one auxiliary field in accordance with (2.3), which leads to the finite result

$$\begin{aligned} F_{\mu\nu\lambda 5}(k,k') = {} & i[A_1(k',k)k'_\alpha - A_1(k,k')k_\alpha]\epsilon_{\alpha\mu\nu\lambda} \\ & + i[A_2(k',k)k'_\nu + A_3(k',k)k_\nu]k'_\alpha k_\beta \epsilon_{\alpha\beta\mu\lambda} \\ & - i[A_2(k,k')k_\mu + A_3(k,k')k'_\mu]k'_\alpha k_\beta \epsilon_{\alpha\beta\nu\lambda} \\ & + 8i\pi^2 M^2[I_{00}(k',k) - 2I_{10}(k',k)]k'_\alpha \epsilon_{\alpha\mu\nu\lambda} \\ & - 8i\pi^2 M^2[I_{00}(k,k') - 2I_{10}(k,k')]k_\alpha \epsilon_{\alpha\mu\nu\lambda} \\ & - \tfrac{4}{3} i\pi^2(k'_\alpha - k_\alpha)\epsilon_{\alpha\mu\nu\lambda}, \end{aligned} \tag{4.3}$$

where

$$\begin{aligned} A_1(k,k') &= k\cdot k' A_2(k,k') + k'^2 A_3(k,k'), \\ A_2(k,k') &= 8\pi^2 I_{11}(k,k'), \\ A_3(k,k') &= 8\pi^2[I_{01}(k,k') - I_{02}(k,k')], \end{aligned} \tag{4.4}$$

with

$$\begin{aligned} I_{st}(k,k') &= I_{ts}(k',k) \\ &= \int_0^1 du \int_0^{1-u} dv\, u^s v^t [u(1-u)k^2 + v(1-v)k'^2 \\ &\qquad + 2uv k\cdot k' + M^2]^{-1}. \end{aligned} \tag{4.5}$$

For the diagram 2(b), the contribution of the scattering operator is convergent after trace evaluation, and therefore regularization is not necessary. Moreover, the contributions for the diagrams 2(c), 2(d), 2(h), and 2(i) can be shown to vanish.

It is quite straightforward to regularize the contributions for the remaining diagrams 2(e), 2(f), 2(g), and 2(j) by the application of (2.3). We have performed these calculations, and the results are found to be of the form

$$\begin{aligned} S_{2(e)} &= \delta(p-k-k')a_\mu^-(k')a_\nu^-(k)s^+(p)(4g^3M^2/m) \\ &\quad \times F_{\mu\nu}(k,k'), \\ S_{2(f)} &= -\delta(p-k-k')b^-(k')a_\nu^-(k)s^+(p)(8g^3M^2/m^2) \\ &\quad \times F_\nu(k,k'), \\ S_{2(g)} &= \delta(p-k-k')b^-(k')b^-(k)s^+(p)(16g^3M^4/m^3) \\ &\quad \times F(k,k'), \\ S_{2(j)} &= -\delta(p-k-k')s^-(k')s^-(k)s^+(p)(16g^3M^4/m^3) \\ &\quad \times F'(k,k'), \end{aligned} \tag{4.6}$$

with

$$\begin{aligned} F_{\mu\nu}(k,k') &= iA_0\delta_{\mu\nu} + F^c_{\mu\nu}(k,k'), \\ F_\nu(k,k') &= B_0(k_\nu + 2k'_\nu) + F^c_\nu(k,k'), \\ F(k,k') &= iC_0 + F^c(k,k'), \\ F'(k,k') &= iC'_0 + F'^c(k,k'), \end{aligned} \tag{4.7}$$

where A_0, B_0, C_0, and C'_0 are renormalization constants for the $a_\mu{}^2 s$, $a_\mu(b\partial_\mu s - s\partial_\mu b)$, $b^2 s$, and s^3 couplings, respectively. As in the case of the self-energies, the renormalization terms here have the same form as the counterterms of Appelquist *et al.*,[9] who have also discussed the separation of renormalization terms for the three-point functions.

It is now necessary to examine the regularizability condition to see whether we can justify our regularization procedure and achieve renormalization by dropping the renormalization constants.

B. Violation of regularizability condition

Verification of the regularizability condition for the triangle diagrams can be carried out by a treatment analogous to that used for the self-energy contributions in Sec. III. It is then easy to see that the regularizability condition is satisfied for all triangle diagrams except possibly the diagram 2(a). The difficulty in the case of the diagram 2(a) arises from the fact that the integral $F_{\mu\nu\lambda 5}(k,k';M')$ resulting from the auxiliary field appears to be linearly divergent. Therefore, by expanding this integral in powers of k and k', and by applying the power-counting and covariance arguments, one finds that $F_{\mu\nu\lambda 5}(k,k';M')$ could give rise to nonvanishing terms proportional to the first power of k or k'. Such terms cannot be absorbed by renormalization because there is no coupling term in (3.1) involving the product of three a_μ's

We shall now demonstrate by direct calculations that $F_{\mu\nu\lambda 5}(k,k';M')$ indeed gives rise to a nonvanishing contribution as expected from general considerations. By expanding the integral

$$F_{\mu\nu\lambda 5}(k,k';M') = \int dq\,\mathrm{Tr}\left[\frac{i(q-k')\cdot\gamma - M'}{(q-k')^2+M'^2}\gamma_\mu\gamma_5 \frac{iq\cdot\gamma - M'}{q^2+M'^2}\gamma_\nu\gamma_5 \frac{i(q+k)\cdot\gamma - M'}{(q+k)^2+M'^2}\gamma_\lambda\gamma_5\right] \tag{4.8}$$

in powers of k and k' with the use of the relation

$$\frac{1}{(q\pm l)^2+M'^2}=\frac{1}{q^2+M'^2}\mp\frac{2l_\mu q_\mu}{(q^2+M'^2)^2}+\cdots, \quad (4.9)$$

dropping terms that vanish for $M'\to\infty$, and simplifying by symmetrization over the q space and trace evaluation, we obtain

$$F_{\mu\nu\lambda5}(k,k';M')=(k_\alpha-k'_\alpha)\epsilon_{\alpha\mu\nu\lambda}\times\int dq\left[\frac{2q^2-4M'^2}{(q^2+M'^2)^3}-\frac{2q^4-6M'^2q^2}{(q^2+M'^2)^4}\right]=\tfrac{2}{3}i\pi^2(k_\alpha-k'_\alpha)\epsilon_{\alpha\mu\nu\lambda}, \quad (4.10)$$

which confirms the violation of the regularizability condition for the triangle diagram 2(a).

C. Regularizability condition with two fermion fields

From an examination of the Ward identities relating the contributions of different triangle diagrams, it was concluded by Gross and Jackiw[7] that the gauge model requires another fermion field χ with opposite γ_5 coupling to the a_μ field. The approach of Gross and Jackiw is similar to that used by earlier authors[10] for the treatment of triangle anomalies. We shall here demonstrate the necessity for this additional field solely on the basis of the regularizability condition.

With the introduction of the fermion field χ, the Lagrangian density (3.1) acquires the additional terms

$$L'=-\bar\chi(\gamma_\mu\partial_\mu+\kappa)\chi+ig\bar\chi\gamma_\mu\gamma_5\chi a_\mu-(2ig\kappa/m)\bar\chi\gamma_5\chi b-(2g\kappa/m)\bar\chi\chi s, \quad (4.11)$$

where the χ couplings have been obtained from the ψ couplings in (3.1) by the replacement

$$\psi\to\chi,\quad M\to\kappa,\quad g\to-g,\quad s\to-s, \quad (4.12)$$

and it is to be noted that the boson-boson couplings in (3.1) remain unchanged under (4.12).

In order to show that the additional field provides a solution to the violation of the regularizability condition for the diagram 2(a), let us consider the diagram 2(a) and a similar diagram involving the χ fermion field. The contribution of the scattering operator for these two diagrams is given by

$$S_{2(a)}(M)+S_{2(a)}(\kappa)=i\delta(p-k-k')a_\mu^-(k')a_\nu^-(k)a_\lambda^+(p)(2g^3)\times[F_{\mu\nu\lambda5}(k,k';M)-F_{\mu\nu\lambda5}(k,k';\kappa)], \quad (4.13)$$

while the contribution resulting from the auxiliary fields required for the regularization of (4.13) is

$$i\delta(p-k-k')a_\mu^-(k')a_\nu^-(k)a_\lambda^+(p)(2g^3)\times[-F_{\mu\nu\lambda5}(k,k';M')+F_{\mu\nu\lambda5}(k,k',\kappa')],$$

which vanishes in view of (4.10), and therefore the regularizability condition is satisfied.

V. COMPARISON WITH DIMENSIONAL REGULARIZATION

Although various suggestions[2,4,5] have been made for dimensional regularization of fermion loops with γ_5 couplings, the usual treatment essentially involves the following two operations.

(1) The contributions of fermion loops are evaluated by excluding the γ matrices from dimensional continuation, which we shall refer to as naive dimensional regularization.

(2) Since this procedure is not manifestly gauge invariant, it becomes necessary to verify that the results obtained for the fermion loops satisfy the Ward identities.

We have evaluated the self-energy contributions, given by (3.2), (3.3), and (3.7), also by naive dimensional regularization, and the physical terms in all the self-energy contributions are found to be *identical* to those obtained in Sec. III. Further, the renormalization constants appearing in (3.5), (3.8), (3.9), and (3.10) are given by

$$\begin{aligned}
A_1&=-\alpha-\beta+8\pi^2g^2M^2\int_0^1du\ln\left(1-\frac{m^2(u-u^2)}{M^2}\right),\qquad B_1=-\frac{\alpha}{6M^2}+8\pi^2g^2\int_0^1du(u-u^2)\ln\left(1-\frac{m^2(u-u^2)}{M^2}\right),\\
C_1&=8\pi^2g^2M^2\int_0^1du\,\frac{u-u^2}{M^2-m^2(u-u^2)},\qquad A_2=\alpha-8\pi^2g^2M^2\int_0^1du\ln\left(1-\frac{m^2(u-u^2)}{M^2}\right),\\
B_2&=-\frac{\alpha}{m^2}+\frac{8\pi^2g^2M^2}{m^2}\int_0^1du\left[\ln\left(1-\frac{m^2(u-u^2)}{M^2}\right)-\frac{m^2(u-u^2)}{M^2-m^2(u-u^2)}\right],\\
A_3&=-\frac{\alpha}{m}-\frac{\beta}{2m}+\frac{8\pi^2g^2M^2}{m}\int_0^1du\ln\left(1-\frac{m^2(u-u^2)}{M^2}\right),\\
A_4&=\frac{\alpha\mu^2}{m^2}+\frac{16\pi^2g^2M^2(6M^2-\mu^2)}{3m^2}+\frac{48\pi^2g^2M^2\mu^2}{m^2}\int_0^1du(u-2u^2)\ln\left(1-\frac{\mu^2(u-u^2)}{M^2}\right),\\
B_4&=-\frac{\alpha}{m^2}+\frac{16\pi^2g^2M^2}{3m^2}+\frac{48\pi^2g^2M^2}{m^2}\int_0^1du(u-u^2)\ln\left(1-\frac{\mu^2(u-u^2)}{M^2}\right),
\end{aligned} \quad (5.1)$$

where

$$\alpha = \lim_{n\to 4} 8g^2M^2 \frac{\pi^{n/2}\Gamma(2-n/2)}{(M^2)^{2-n/2}}, \quad \beta = \lim_{n\to 4} \frac{32g^2M^2}{\mu^2} \frac{\pi^{n/2}\Gamma(1-n/2)}{(M^2)^{1-n/2}}. \tag{5.2}$$

Instead of examining the regularizability condition, it is now necessary to derive and verify the Ward identities for fermion loops appearing in the self-energy contributions. These identities are expressible as[11]

$$ik_\nu \Pi_{\mu\nu}(k) = m\,\Pi_\mu(k),$$
$$-ik_\mu \Pi_\mu(k) = m[\Pi(k) - (2g/m)T(k)], \tag{5.3}$$

where $\Pi_{\mu\nu}(k)$, $\Pi_\mu(k)$, and $\Pi(k)$ represent the contributions of fermion loops in the self-energy diagrams 1(a), 1(c), and 1(b), while $T(k)$ represents the contribution of the tadpole fermion loop appearing in some accompanying diagrams. We have verified that the relations (5.3) are satisfied by the results obtained by naive dimensional regularization, so that the physical self-energy terms obtained by this procedure are gauge invariant.

The triangle diagrams in Fig. 2 can also be evaluated by naive dimensional regularization. It is then again necessary to derive and verify the Ward identities for the triangle diagrams, and any anomaly found in this manner must be resolved.

The treatment described in the preceding sections appears to be simpler than that outlined in this section, because an examination of the regularizability condition is much easier than the derivation and verification of Ward's identities. In any event, our treatment with auxiliary fields provides a practical alternative procedure for the evaluation of fermion loops with γ_5 couplings.

*Work supported in part by the U. S. Energy Research and Development Administration under Contract No. E(11-1)-2302.

[1]S. N. Gupta, Proc. Phys. Soc. London A66, 129 (1953).

[2]G. 't Hooft and M. Veltman, Nucl. Phys. B44, 189 (1972).

[3]T. Appelquist and H. Quinn, Phys. Lett. 39B, 229 (1972).

[4]W. A. Bardeen, R. Gastmans, and B. Lautrup, Nucl. Phys. B46, 319 (1972).

[5]D. A. Akyeampong and R. Delbourgo, Nuovo Cimento 17A, 578 (1973).

[6]See D. Barua and S. N. Gupta, Phys. Rev. D 10, 1945 (1974), for the regularization of nonlinear pion-pion couplings by the method of auxiliary fields. Although regularization is carried out without imposing the constraints of chiral symmetry on the auxiliary-field couplings, it can be verified that the regularizability condition is fulfilled.

[7]D. J. Gross and R. Jackiw, Phys. Rev. D 6, 477 (1972).

[8]See also C. Bouchiat, J. Iliopoulos, and P. Meyer, Phys. Lett. 38B, 519 (1972); C. P. Korthals Altes and M. Perrottet, *ibid.* 39B, 546 (1972).

[9]T. Appelquist, J. Carazzone, T. Goldman, and H. R. Quinn, Phys. Rev. D 8, 1747 (1973).

[10]For a review of the earlier work on triangle anomalies, see S. L. Adler, in *Lectures on Elementary Particles and Quantum Field Theory*, edited by S. Deser, M. Grisaru, and H. Pendleton (MIT Press, Cambridge, Mass., 1970), p. 1; S. B. Treiman, R. Jackiw, and D. J. Gross, *Lectures on Current Algebra and Its Applications* (Princeton Univ. Press, Princeton, N. J. 1972), p. 97.

[11]Note that infinitesimal gauge transformations for the system under consideration are $a_\mu \to a_\mu + m^{-1}\partial_\mu \Lambda$, $b \to b - \Lambda - 2gm^{-1}s\Lambda$, $s \to s + 2gm^{-1}b\Lambda$, and $\psi \to \psi - igm^{-1}\gamma_5 \Lambda \psi$.