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Progress toward a theory of supergravity*

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As a new approach to supergravity, an action containing only vierbein and Rarita-Schwinger fields ($V_{a\mu}$ and ψ_μ) is presented together with supersymmetry transformations for these fields. The action is explicitly shown to be invariant except for a ψ^5 term in its variation. This term may also vanish, depending on a complicated calculation. (Added note: This term has now been shown to vanish by a computer calculation, so that the action presented here does possess full local supersymmetry.)

Even early in the development of the Fermi-Bose supersymmetry concept, it was thought that the new fermionic symmetry transformation might be important for the theory of gravitation.[1] Two similar but apparently inequivalent approaches to this theory of "supergravity" have been formulated by Arnowitt, Nath, and Zumino[2] and by Zumino.[3] These approaches exploit the geometry of "superspace,"[4] a manifold parametrized by four anticommuting spinor coordinates θ_α in addition to the normal Riemannian coordinates x^μ. The theories are formulated in terms of superfields which contain a very large number of ordinary fields—i.e., vectors, tensors, spinors, etc. Although it is expected that some component fields are merely generalized gauge excitations and not true physical fields, the physical content of the Arnowitt-Nath-Zumino theories has never been spelled out, but there are indications[2] that, as is perhaps desirable, the approaches necessarily bring in gauge vector and spin-$\frac{1}{2}$ particles in addition to tensor and spin-$\frac{3}{2}$ particles.

In this note we report on progress in a very different approach to supergravity in which we commit ourselves from the start to a formulation without superspace in which the only fields in the gravitational supermultiplet are the metric tensor $g_{\mu\nu}(x)$ [or, equivalently, the vierbein field $V_{a\mu}(x)$] and a Rarita-Schwinger field $\psi_\mu(x)$. If fully successful, we would then expect to adjoin matter supermultiplets of lower-spin fields in much the same way as matter fields are treated in conventional gravitation.

There is a theorem[5] in the usual theory of global supersymmetry which demonstrates the existence of irreducible representations of the graded Lie algebra of supersymmetry charges and Poincaré group generators. Some of these representations act in the Hilbert space of helicity states of two massless particles, one neutral boson and one Majorana fermion of adjacent spins J and $J+\frac{1}{2}$ (for any $J=0, \frac{1}{2}, 1, \ldots$). It is therefore known that a representation exists containing states of massless spin-$\frac{3}{2}$ and spin-2 particles, and it was suggested[6] earlier that these particles form the gravitational supermultiplet. The theorem does not guarantee that there exists a corresponding interacting quantum field theory, but it is reasonable to hope that it exists, and this is the basic mathematical motivation for our approach. Many questions can be asked about the physical motivation and consistency of both this treatment and the entire concept of supergravity. We shall discuss some of them at the end of this note, and we proceed now to the formulation.

The starting point of our approach is the generally covariant action[7]

$$I=\int d^4x(\mathcal{L}_2+\mathcal{L}_{3/2})$$
$$=\int d^4x[\tfrac{1}{4}\kappa^{-2}\sqrt{-g}\,R-\tfrac{1}{2}\epsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda(x)\gamma_5\gamma_\mu D_\nu\psi_\rho(x)] \tag{1}$$

describing the interaction of vierbein fields and Rarita-Schwinger[8] fields subject to the Majorana constraint $\psi_\rho(x)=C\bar{\psi}_\rho(x)^T$. The covariant derivative[9]

$$D_\nu\psi_\rho(x)=\partial_\nu\psi_\rho(x)-\Gamma^\sigma_{\nu\rho}\psi_\sigma+\tfrac{1}{2}\omega_{\nu ab}\sigma^{ab}\psi_\rho \tag{2}$$

involves the standard Christoffel symbol (although it cancels in $\mathcal{L}_{3/2}$ because of the tensor density $\epsilon^{\lambda\rho\mu\nu}$) and the vierbein connection

$$\omega_{\nu ab}=\tfrac{1}{2}[V_a{}^\mu(\partial_\nu V_{b\mu}-\partial_\mu V_{b\nu})+V_a{}^\rho V_b{}^\sigma(\partial_\sigma V_{c\rho})V^c{}_\nu]$$
$$-(a\leftrightarrow b)\,, \tag{3}$$

while

$$\sigma_{ab}=\tfrac{1}{4}[\gamma_a,\gamma_b].$$

It is hoped that this Lagrangian, after possible addition of subsidiary field terms, has a hidden supersymmetry just as the Lagrangian for Yang-Mills fields interacting with Majorana fermions in the adjoint representation is known to have a hidden supersymmetry.[10]

To investigate a possible supersymmetry, we write down trial expressions for the transformation law:

$$\delta\psi_\mu(x) = \kappa^{-1} D_\mu \epsilon(x), \tag{4}$$

$$\delta V^a{}_\mu(x) = i\kappa\bar{\epsilon}(x)\gamma^a\psi_\mu(x), \tag{5}$$

$$\delta g_{\mu\nu}(x) = i\kappa\bar{\epsilon}(x)[\gamma_\mu\psi_\nu(x) + \gamma_\nu\psi_\mu(x)], \tag{6}$$

where the supersymmetry parameter is taken to be an arbitrary Majorana spinor field $\epsilon(x)$ of dimension $l^{1/2}$. It is a big step to commit ourselves to local supersymmetry (which has not previously been achieved in any explicit model in four-dimensional[11] space-time), but it is necessary to do this because we are already committed to general coordinate invariance via the ansatz for the Lagrangian and must therefore eschew such coordinate-dependent notions as a constant, space-time-independent, spinor ϵ.

To motivate our trial expressions we note, as essentially pointed out long ago,[8] that the free massless Rarita-Schwinger Lagrangian is invariant under the gauge-type transformation $\delta\psi_\mu = \partial_\mu\epsilon(x)$. We are therefore attempting to interpret supersymmetry as the curved-space generalization of the old Rarita-Schwinger gauge transformation. The trial expression for $\delta V_{a\mu}$ resembles the supersymmetry transformation[10] law of the Yang-Mills gauge field $\delta B_\mu{}^a$, but it is basically motivated by good results.

We now discuss the possible invariance of our action under the trial transformation laws and shall find, to speak cryptically for a moment, that the action is partly invariant and can be modified to be more invariant. The commutator of two supersymmetry transformations is also important and will be discussed.

The variation of the action integral can be written in functional notation as

$$\delta I = \int d^4x \left(\frac{\delta\mathcal{L}_2}{\delta g_{\mu\nu}}\delta g_{\mu\nu} + \frac{\delta\mathcal{L}_{3/2}}{\delta\psi_\rho}\delta\psi_\rho + \frac{\delta\mathcal{L}_{3/2}}{\delta V_{a\mu}}\delta V_{a\mu} \right). \tag{7}$$

Since the first two terms are linear in ψ and the last term is cubic, the two sets must vanish separately if we are to have invariance.

To investigate the linear term we note the textbook result

$$\delta I_2 = -\tfrac{1}{4}\kappa^{-2} \int d^4x\sqrt{-g}\,(R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R)\delta g_{\mu\nu}(x) \tag{8}$$

for arbitrary variations of the metric in which we insert the specific variation (6). The linear term in $\delta I_{3/2}$ can be shown to cancel (8) by a heartwarming calculation which we can only outline here. After partial integration and use of the Majorana property this linear term can be written as

$$\delta I_{3/2} = \kappa^{-1} \int d^4x\, \epsilon^{\lambda\rho\mu\nu}\bar{\epsilon}\,\overleftarrow{D}_\lambda\overleftarrow{D}_\nu\gamma_5\gamma_\mu\psi_\rho. \tag{9}$$

The Ricci identity[12] is then used to bring in the full curvature tensor $R_{\nu\lambda ab}$. After some Dirac algebra and use of the cyclic property $\epsilon^{\mu\nu\lambda\sigma}R_{\nu\lambda\sigma b} = 0$, one finds that the only surviving term involves the Einstein tensor and exactly cancels (8). We consider this cancellation a very positive indicator for our approach because the intimate structure of the Dirac algebra and the apparatus of Riemannian geometry enter the proof.

We turn now to the cubic term, which is essentially determined by the energy-momentum tensor of the spin-$\frac{3}{2}$ field. After some Dirac algebra one obtains

$$\Delta\mathcal{L}_{3/2} = \tfrac{1}{4}i\kappa\epsilon^{\lambda\rho\mu\nu}(\bar{\psi}_\lambda\gamma_d\,\psi_\rho)\{(\bar{\epsilon}\gamma_5\gamma^d D_\nu\psi_\mu) + \tfrac{1}{2}i\epsilon_\mu{}^{bcd}[D_\nu(\bar{\epsilon}\gamma_c\psi_b) - D_b(\bar{\epsilon}\gamma_c\psi_\nu) + D_c(\bar{\epsilon}\gamma_\nu\psi_b)]\}. \tag{10}$$

To be strictly correct, one must pay careful attention to the covariant derivatives which appear with both Riemann (Greek) and vierbein or local Lorentz (Latin) indices. However, it is usually sufficient to work locally in a flat coordinate system and obtain final results by invoking general covariance. For full invariance $\int d^4x\,\Delta\mathcal{L}_{3/2}$ must vanish for all possible trial functions $\epsilon(x)$, $V_{a\mu}(x)$, and $\psi_\mu(x)$, and one can show that this does not happen by choosing the trial functions $V_{a\mu}(x) = \eta_{a\mu}$ (global flat Lorentz metric) and $\psi_\mu(x) = \partial_\mu\theta(x)$ with suitable spinor[13] fields $\theta(x)$ and $\epsilon(x)$. We have now learned that the action (1) is invariant under the transformations (4)–(6) up to, but not including, terms cubic in ψ, and we will show below that this situation can be improved by modification of (1) and (4)–(6).

Let us examine the commutator of two supersymmetry transformations, using (4) and (5). After straightforward Dirac and vierbein algebra, we find

$$[\delta_1, \delta_2] V_{a\mu}(x) = D_\mu \xi_a(x), \qquad (11)$$

where

$$\xi_a(x) = i\bar{\epsilon}_2(x)\gamma_a \epsilon_1(x). \qquad (12)$$

The additional result

$$[\delta_1, \delta_2] g_{\mu\nu}(x) = D_\mu \xi_\nu(x) + D_\nu \xi_\mu(x) \qquad (13)$$

follows immediately if we use the relation $g_{\mu\nu} = V_{a\mu} V^a{}_\nu$ between vierbein field and metric tensor. Equations (11) and (13) exactly describe general coordinate transformations with infintesimal parameter $\xi_\mu(x)$, and suggest that the conventional relation between global supersymmetry transformations and translations generalizes very elegantly in the local case considered here. We postpone discussion of $[\delta_1, \delta_2]\psi_\mu(x)$ because this commutator simplifies after the modification to which we previously alluded.

We now show that the partial invariance of the action can be improved by modifying the action (1) and the transformation laws (4)–(6). It is natural to try to include auxiliary fields (i.e., no derivatives) in the Lagrangian, because such fields have always appeared in conventional supersymmetry. However, our attempts to do this met with little success, and we now believe that the auxiliary field structure is more complicated in supergravity than in previous supersymmetry models.

A different strategy succeeds. Namely, we try to cancel the noninvariant cubic term $\Delta\mathcal{L}_{3/2}$ by adding terms bilinear in ψ to the trial[14] expression (4) for $\delta\psi_\mu$ and adding a quartic term to the Lagrangian. The modified transformation[15] is

$$\delta\psi_\mu(x) = \kappa^{-1} D_\mu \epsilon + \tfrac{1}{4} i\kappa (2\bar{\psi}_\mu \gamma_a \psi_b + \bar{\psi}_a \gamma_\mu \psi_b)\sigma^{ab}\epsilon \qquad (14)$$

and the quartic modification of the Lagrangian is

$$\mathcal{L}_4 = -\tfrac{1}{32}\kappa^2(-g)^{-1/2}(\epsilon^{\tau\alpha\beta\nu}\epsilon_\tau{}^{\gamma\delta\mu} + \epsilon^{\tau\alpha\mu\nu}\epsilon_\tau{}^{\gamma\delta\beta} - \epsilon^{\tau\beta\mu\nu}\epsilon_\tau{}^{\gamma\delta\alpha}) \times (\bar{\psi}_\alpha\gamma_\mu\psi_\beta)(\bar{\psi}_\gamma\gamma_\nu\psi_\delta). \qquad (15)$$

One can verify that the new action

$$I' = \int d^4x (\mathcal{L}_2 + \mathcal{L}_{3/2} + \mathcal{L}_4) \qquad (16)$$

is invariant up to, but not including, terms of order ψ^5 under the transformations (5) and (14). In order to verify this result, note that (i) the new term in $\delta\psi_\mu$ cancels those terms in (10) where covariant derivatives are applied to ψ fields, (ii) the cubic part of $\delta\mathcal{L}_4$ cancels those terms in (10) with derivatives of $\epsilon(x)$, and (iii) there is the curious identity

$$\epsilon^{\mu abc}\epsilon_\mu{}^{def} - \epsilon^{\mu dbc}\epsilon_\mu{}^{aef} = \epsilon^{\mu adc}\epsilon_\mu{}^{bef} + \epsilon^{\mu abd}\epsilon_\mu{}^{cef}. \qquad (17)$$

The quintic term in $\delta I'$ is very complicated, and we are studying it. It is conceivable that it vanishes, because it has a superficial resemblence to a term known to vanish in the combined non-Abelian supersymmetric models.[16] Since one can show (essentially by dimensional analysis) that further modification involving order-ψ^6 terms in I' and order-ψ^4 terms in $\delta\psi_\mu$ cannot cancel the quintic term, it is critical to verify whether it vanishes.

We now discuss the commutators of the supersymmetry transformations (5) and (14). The Jacobi identity and the Poisson bracket structure of symmetries guarantee that $[\delta_1, \delta_2]I'$ vanishes up to but not including terms of order ψ^4, but it is still useful to examine the explicit variation of the fields. The previous expression (11) for $[\delta_1, \delta_2]V_{a\mu}$ now acquires an additional term quadratic in ψ which is awkward to write. The change in $g_{\mu\nu}$ is somewhat simpler and is given by

$$[\delta_1, \delta_2] g_{\mu\nu} = D_\mu \xi_\nu + D_\nu \xi_\mu - i\kappa^2 \bar{\psi} \cdot \xi (\gamma_\mu \psi_\nu + \gamma_\nu \psi_\mu). \qquad (18)$$

In a moment we will discuss the new term by which (18) differs from a general coordinate transformation. For the field ψ_μ we find, using a Fierz transformation and some tensor and spinor calculus, the expression

$$[\delta_1, \delta_2]\psi_\mu = \xi^\rho D_\rho \psi_\mu + (D_\mu \xi^\rho)\psi_\rho - D_\mu(\xi \cdot \psi) + \mathcal{R}. \qquad (19)$$

The first two terms are the expected general coordinate transformation. The third term will be discussed shortly. The remainder $\mathcal{R}$ contains terms linear in ψ which would vanish if the minimal curved-space Rarita-Schwinger equation

$$\epsilon^{\lambda\rho\mu\nu}\gamma_5\gamma_\mu D_\nu \psi_\rho = 0 \qquad (20)$$

were used. There are additional terms in $\mathcal{R}$ of order ψ^3 which are quite complicated and which we have not yet studied.

We now try to give a tentative interpretation of the algebra of supersymmetry transformations pending further investigation. The third terms in (18) and (19) are quite simple. They are just the original supersymmetry transformation (6) and (4), with field-dependent parameter $\epsilon'(x) = -\kappa\xi \cdot \psi$. The action is separately invariant under this ϵ' transformation, as the earlier discussion surrounding (8) and (9) shows. The $\epsilon'(x)$ terms are a local symmetry of the action even when ϵ_1 and ϵ_2 are constant and are similar to the field-dependent gauge transformation known to occur in the supersymmetry algebra of combined gauge and supersymmetric field theories.[16] Such terms should not affect the algebra of physical states and the transformation law (Ward identities) of observable Smatrix elements. Terms in a transformation of fields which vanish as a consequence of the equations of motion also do not affect the algebra of physical states, and the terms in $\mathcal{R}$ seem to be of this character. It therefore seems very likely that the algebra of field transformations (5), (14), (18), and (19) reduces to the standard supersymmetry algebra when applied to physical states for constant ϵ_1, ϵ_2. We have verified this by explicit calculation using the linearized limit of the transformations and the free spin-2 and spin-$\frac{3}{2}$ particle states of the linearized action obtained by the substitution in (1) of

$$V_{a\mu}(x) = \eta_{a\mu} + \sqrt{8}\,\kappa v_{a\mu}(x), \qquad g_{\mu\nu}(x) = \eta_{\mu\nu} + \sqrt{8}\,\kappa(v_{\mu\nu} + v_{\nu\mu}) \tag{21}$$

and the subsequent limit $\kappa \to 0$.

It is clear that this approach to supergravity requires complicated and critical calculations before its full success will be known. The results so far obtained are hopeful, and we would like to make some final comments predicated upon success.

We have used only vierbein and spin-$\frac{3}{2}$ fields in our construction, but there may exist an equivalent formulation using auxiliary fields, for which the Lagrangian would reduce to the form given here when the auxiliary variables are eliminated. Previous supersymmetry models suggest that this is the case and that the auxiliary field would facilitate the extension of the present theory to include supermultiplets of lower-spin particles.

It is useful to pose and tentatively answer at this stage some questions concerning the physical applicability of supergravity theory. The action (16) has a global chiral symmetry ($\delta\psi_\rho = i\,\theta\gamma_5\psi_\rho$) implying that the spin-$\frac{3}{2}$ particle is massless. It is doubtful that such a particle exists in Nature even though its coupling strength is very weak. This means that we must hope for spontaneous breakdown of the local supersymmetry leading to a massive spin-$\frac{3}{2}$ particle via a "super-Higgs" mechanism.[17] This may occur only after coupling to lower-spin supermultiplets. Such a scenario for supersymmetry breakdown would be attractive because fermion and boson masses are in general unequal in nature and because there is no candidate for a Goldstone neutrino,[18] which would be required if spontaneous breakdown occurs in the absence of a super-Higgs effect.

The present renormalizability situation[19] in quantum gravity gives added motivation for the study of supergravity theory. One may hope that the added local fermionic invariance improves the situation, at least for one-loop diagrams. It is curious that even with the present partially invariant action (16) one can contemplate a study of the ultraviolet structure of one-loop Feynman diagrams with up to four external spin-$\frac{3}{2}$ particles.

Added note. The quintic term mentioned in the text has now been shown to vanish by a computer calculation, and we briefly outline the method. The quintic term is given by

$$\frac{\delta\mathcal{L}_4}{\delta V_{a\mu}}\,\delta V_{a\mu} + \frac{\partial\mathcal{L}_4}{\delta\psi_\rho}\,\delta\psi_\rho\,,$$

where only the ψ^2 part of Eq. (14) is used. This expression is complicated, and it was not clear how to exploit all the symmetries due to the Pauli interchange and tensor relations. We therefore decomposed the spinors into chiral components in a representation with γ_5 diagonal, and obtained an expression of the form

$$t^{abcdefgh}(\psi_a^\dagger\sigma_b\psi_c)(\psi_d^\dagger\sigma_e\psi_f)(\epsilon^\dagger\sigma_g\psi_h)\,,$$

where $\sigma_a = (1, \vec{\sigma})$ with σ_i the Pauli matrices and t an integer-valued tensor constructed from ϵ symbols and Kronecker δ's. We then constructed a computer program which evaluated the coefficients of all independent, properly antisymmetrized, spinor combinations. These coefficients vanished.

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[1] B. Zumino, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974*, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974).

[2] P. Nath and R. Arnowitt, Phys. Lett. 56B, 177 (1975); R. Arnowitt, P. Nath, and B. Zumino, *ibid.* 56B, 81 (1975).

[3] B. Zumino, in *Proceedings of the Conference on Gauge Theories and Modern Field Theories, Northeastern University, 1975*, edited by R. Arnowitt and P. Nath (MIT Press, Cambridge, Mass., 1976).

[4] A. Salam and J. Strathdee, Nucl. Phys. B76, 477 (1974).

[5] For massive particles see A. Salam and J. Strathdee, Nucl. Phys. B80, 499 (1974). The massless case has been discussed by B. Zumino.

[6] S. Ferrara and B. Zumino, Nucl. Phys. B87, 207 (1975).

[7] The conventions used are $g_{\mu\nu} = (+---)$, $\epsilon^{0123} = 1$, $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$, $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$, $C\gamma_\mu C^{-1} = -\gamma_\mu{}^T$.

$R_{\mu\nu}=R^{\sigma}{}_{\mu\sigma\nu}$ with $R_{\mu\nu ab}=\partial_\mu\omega_{\nu ab}-\cdots$.

[8]W. Rarita and J. Schwinger, Phys. Rev. 60, 61 (1941).

[9]For a discussion of the vierbein formalism see S. Deser and P. van Nieuwenhuizen, Phys. Rev. D 10, 411 (1974).

[10]J. Wess and B. Zumino, Nucl. Phys. B78, 1 (1974); S. Ferrara and B. Zumino, *ibid.* B79, 413 (1974).

[11]In two space-time dimensions the problem was solved by J. Wess and B. Zumino [Nucl. Phys. B70, 39 (1974)].

[12]See Eq. (10) of Ref. 9.

[13]We used the trial functions $\theta(x)=e^{a_1\cdot x}\lambda_1+e^{a_2\cdot x}\lambda_2$ and $\epsilon(x)=e^{-bx^2}\lambda_3$, where $\lambda_1,\lambda_2,\lambda_3$ are constant Majorana spinors.

[14]Modifications of $\delta V^a{}_\mu$ of higher order in ψ cannot help because of mismatching in the powers of κ.

[15]The Noether current of these transformations is given by

$$j^{\nu}=\kappa^{-1}\epsilon^{\lambda\rho\mu\nu}\bar{\epsilon}\overleftarrow{D}_\lambda\gamma_5\gamma_\mu\psi_\rho+\frac{\delta\mathcal{L}_{3/2}}{\delta(\partial_\nu V_{a\mu})}\delta V_{a\mu}.$$

[16]B. de Wit and D. Z. Freedman, Phys. Rev. D 12, 2286 (1975).

[17]D. V. Volkov and V. A. Soroka, Zh. Eksp. Teor. Fiz. Pis'ma Red. 18, 529 (1973) [JETP Lett. 18, 312 (1973)].

[18]W. A. Bardeen (unpublished); B. de Wit and D. Z. Freedman, Phys. Rev. Lett. 35, 827 (1975).

[19]G. 't Hooft and M. Veltman, Ann. Inst. H. Poincaré 20, 69 (1974); S. Deser, H.-S. Tsao, and P. van Nieuwenhuizen, Phys. Rev. D 10, 3337 (1974). For a recent review, see P. van Nieuwenhuizen, Stony Brook Report No. ITP-SB-75-46, to be published in the proceedings of the Meeting on Recent Progress of the Fundamentals of General Relativity, Trieste, 1975.