# Spin tests for intermediate states in radiative $\psi'(3684)$ decay chains

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Analysis of the multiple angular-correlation functions for the sequential decays $\psi'(1^-)\to\gamma+\chi$, $\chi\to M\bar{M}$, where $M$ is a spinless meson, and $\psi'(1^-)\to\gamma_1+\chi$, $\chi\to\gamma_2+\psi$, $\psi(1^-)\to l\bar{l}$, when the $\psi'$ is formed in $e^+e^-$ collisions, shows that these can unambiguously distinguish between the spin assignments $s_\chi = 0$, 1, or 2 for the intermediate states occuring in these decays, as well as determine the multipole amplitudes contributing to the radiative transitions. No dynamical assumptions are made beyond the conservation of angular momentum and parity; recoils are fully taken into account.

Observation of radiative transitions from $\psi'(3684)$ to several discrete new states has recently been reported.[1-3] The existence of such states had been predicted earlier,[4] but it remains to be established whether the newly discovered states are the ones that were predicted. It is a matter of interest in any case to determine the quantum numbers of the new states, which we shall denote generically by $\chi$. We have calculated the angular correlations to be expected in the $\psi'\to\gamma+\chi$, $\chi\to M\bar{M}$ (where $M$ is a spinless meson), and $\psi'\to\gamma_1+\chi$, $\chi\to\gamma_2+\psi$, $\psi\to l\bar{l}$, decay modes of the $1^-$ state[5] $\psi'(3684)$ formed by $e^+e^-$ annihilation, for $s_\chi=0$, 1, and 2. In this note, we point out the characteristic differences between these cases and show how they may be used to fix the spins of the $\chi$ states and to determine the multipole amplitudes contributing to the radiative transitions.

The feature of vectorial interactions that, in the extreme relativistic limit, negative electrons annihilate only with positrons of the opposite helicity has the consequence that in collisions of unpolarized electrons with positrons at 3684 MeV, the initial $\psi'$ is formed only in the $m=\pm1$ spin substates relative to the beam direction, viz., we start from a fully aligned sample of $\psi'$s. Since $J_\psi=1$, the angular distribution of the photon emitted in $\psi'\to\chi+\gamma_1$ decay[6] necessarily has the form[7]

$$W(\theta_1)=1+A_1\cos^2\theta_1, \tag{1}$$

where $\theta_1$ is the angle between the photon $\gamma_1$ and the beam direction measured in the $e^+e^-$ c.m. system, and

$$A_1=(1+y^2-2x^2)/(1+y^2+2x^2), \tag{2}$$

where $y$ and $x$ are the amplitudes (required to be real by time-reversal invariance) for producing $\chi$'s in states of helicity $\pm2$ and $\pm1$, respectively, relative to helicity zero. For $s_\chi=0$, $x=y=0$, and the anistropy factor $A_1$ has the unique value $+1$, corresponding to a dipole radiation pattern, but for higher spin values, $A_1$ may in general have any value between $+1$ and $-1$, depending on the various multipole amplitudes which may be present.[8] For $s_\chi=1$, $y=0$ and $x=(1-\rho)/(1+\rho)$, where $\rho$ is the ratio of quadrupole to dipole matrix elements ($E2/M1$ or $M2/E1$, depending on the parity[9] of $\chi$) in suitable units. Similarly, for $s_\chi=2$,

$$\sqrt{3}\,x=\frac{3+\rho-4\rho'}{1+\rho+2\rho'},\qquad \tfrac{1}{2}\sqrt{6}\,y=\frac{3-\rho+\rho'}{1+\rho+2\rho'}, \tag{3}$$

where $\rho$ has the same meaning as above and $\rho'$ is the ratio of octupole to dipole matrix elements. Thus, measurements of the angular distribution of the first photon $\gamma_1$ may perhaps exclude the case $s_\chi=0$, but it cannot tell us anything more definite about the $\chi$ spin. The double correlation function with a second emission can provide more information. Let $(\theta_2,\phi)$ be the polar angles of the second particle (whether it be the photon in $\chi\to\psi+\gamma_2$ decay or a meson[10] in $\chi\to\pi\pi$ or $K\bar{K}$ decay) measured *in the $\chi$ rest frame*, relative to the first photon direction as polar axis with $\phi=0$ defined by the electron beam axis. Then the joint angular distribution of $\gamma_1$ and the second particle has the general form[11]

$$W_2(\theta_1,\theta_2,\phi)=\sum_n(a_n+b_n\cos^2\theta_1+c_{n+1}\sin2\theta_1\sin2\theta_1\cos\phi+d_{n+1}\sin^2\theta_1\sin^2\theta_2\cos2\phi)\cos^{2n}\theta_2, \tag{4}$$

where $n$ runs over integer values from zero to at most $s_\chi$; also, the indices on the $c$ and $d$ terms may not exceed $s_\chi$.

As is obvious on physical grounds, any correlation between the direction of the second particle and that of the first photon, represented by nonvanishing values of $a_n$, $b_n$, $c_n$, or $d_n$ for any $n>0$, is evidence of nonzero spin for the $\chi$ particle. More precisely, such terms require[7,11] $s_\chi \geq n$. Nonzero $d_i$ can only occur for $s_\chi \geq 2$.

We shall now show that measurement of the double correlation function $W_2$ in the case when the $\chi$ decays into two spinless mesons[10] can unambiguously distinguish whether $\chi$ has spin 0, 1, or 2. For $s_\chi = 0$, the meson must be distributed isotropically in the $\chi$ rest frame. If $s_\chi = 1$, the general formula for the distribution is

$$W_2(\theta_\gamma, \theta_M, \phi_M) = x^2 \sin^2\theta_\gamma \sin^2\theta_M + (1+\cos^2\theta_\gamma)\cos^2\theta_M + \tfrac{1}{2}x \sin 2\theta_\gamma \sin 2\theta_M \cos\phi_M. \tag{5}$$

The corresponding formula for $s_\chi = 2$ is

$$W_2(\theta_\gamma, \theta_M, \phi_M) = 3x^2 \sin^2\theta_\gamma \sin^2 2\theta_M + (1+\cos^2\theta_\gamma)[(3\cos^2\theta_M - 1)^2 + \tfrac{3}{2}y^2 \sin^4\theta_M]$$
$$+ \sqrt{3}\,x \sin 2\theta_\gamma \sin^2\theta_M[3\cos^2\theta_M - 1 - \tfrac{1}{2}\sqrt{6}\,y \sin^2\theta_M]\cos\phi_M + \sqrt{6}\,y \sin^2\theta_\gamma \sin^2\theta_M(3\cos^2\theta_M - 1)\cos 2\phi_M. \tag{6}$$

Thus we see that, while there can be no correlation between the photon and the meson for $s_\chi = 0$, there must be such correlation if $s_\chi \neq 0$, with characteristically different forms for $s_\chi = 1$ and $s_\chi = 2$. Since the distributions are determined, for $s_\chi = 1$ or 2 by one ($x$) or two ($x, y$) parameters, respectively, which we assumed to be real by time-reversal invariance,[12] there are several independent relations between the coefficients in Eq. (4) to test the hypotheses $s_\chi = 1$ or $s_\chi = 2$, besides determining those parameters.

In the case of $\gamma$-$\gamma$ correlations,[13,14] on the other hand, even the double correlation function $W_2(\theta_1, \theta_2, \phi)$ may not suffice to determine the spin of $\chi$. This correlation function is given, for $s_\chi = 1$, by

$$W_2(\theta_1, \theta_2, \phi) = x^2 \sin^2\theta_1(2 + B\sin^2\theta_2) + (1+\cos^2\theta_1)(1 + B\cos^2\theta_2) + \tfrac{1}{2}xB \sin 2\theta_1 \sin 2\theta_2 \cos\phi, \tag{7}$$

where the parameter $B = 2\xi^2 - 1$ and $\xi$ is the amplitude for the $\psi$ in $\chi \to \psi + \gamma_2$ to be produced with the same helicity as $\gamma_2$ relative to helicity zero. It will be seen that the direction of the second photon is completely uncorrelated with the first if $B = 0$, which is the condition that the radiation be isotropically emitted for any state of polarization of a spin-1 $\chi$. Similarly, for $s_\chi = 2$

$$W_2(\theta_1, \theta_2, \phi) = \tfrac{1}{2}(1+\cos^2\theta_1)\{y^2[\Xi(1+\cos^4\theta_2) + 6H\cos^2\theta_2 + 8] - 6\cos^2\theta_2(\Xi\sin^2\theta_2 + H) + 2\Xi + 4H + 8\}$$
$$+ x^2 \sin^2\theta_1[4\Xi \sin^2\theta_2 \cos^2\theta_2 + (\Xi + 3H)\sin^2\theta_2 + 8]$$
$$+ \tfrac{1}{2}x \sin 2\theta_1 [\sqrt{2}\,y(\Xi\cos^2\theta_2 + 3H) - \sqrt{3}(\Xi\cos 2\theta_2 + H)]\sin 2\theta_2 \cos\phi$$
$$+ \sqrt{6}\,y \sin^2\theta_1(\Xi\cos^2\theta_2 + H)\sin^2\theta_2 \cos 2\phi, \tag{8}$$

where $\Xi = 6\xi^2 + \eta^2 - 4$, $H = \eta^2 - 2\xi^2$, and $\xi$ and $\eta$ are the amplitudes for the $\psi$ in $\chi \to \psi + \gamma_2$ to have the same or opposite helicity, respectively, as the photon, relative to helicity zero. Again, we note that all correlation disappears if $\Xi = H = 0$, the condition for an isotropic radiation pattern in $\chi \to \psi + \gamma_2$ for an arbitrary state of polarization of a spin-2 $\chi$. In a less extreme situation, if $y = \Xi = 0$, the terms which survive in the $W_2$ distribution, Eq. (8), have exactly the same form[15] as for $s_\chi = 1$, Eq. (7), so that one cannot decide between $s_\chi = 2$ and $s_\chi = 1$ solely on the basis of the complexity of the distribution function $W_2$. However, in this case, the coefficients of the various terms are related to each other differently for $s_\chi = 1$ and $s_\chi = 2$; thus the correlation function *is* able to distinguish between the two spin values, if such correlation is present. The quantities

$$D_1 = 2^{-4} c_1^{-2}(a_1^2 - b_1^2)$$

and

$$D_2 = [a_1^2 - b_1^2 + 3(a_0 a_1 - b_0 b_1)]/(a_0 b_1 - a_1 b_0)$$

are required to have the values $-1$ and $+1$, respectively, if $s_\chi = 1$ whereas they assume the respective values $+3$ and $+9$ for $s_\chi = 2$ in the case when the form of $W_2$ does not by itself show that $s_\chi > 1$. But if the radiation parameters are such that the angular distribution of $\chi \to \psi + \gamma_2$ is isotropic for every state of $\chi$ polarization, measurement of the second photon distribution furnishes no additional information about $s_\chi$. If that is so, one must look to the polarization of the $\psi$, analyzed by $\psi \to l\bar{l}$ decay, for residual information about the $\chi$ spin.

For $s_\chi = 0$, one finds the unique angular distribution for $\psi' \to \gamma_1 + \chi$, $\chi \to \gamma_2 + \psi$, $\psi \to l\bar{l}$,

$$W_3(\theta_1, \theta_2, \phi_2, \theta_\mu, \phi_\mu) = (1+\cos^2\theta_1)(1+\cos^2\theta_\mu), \tag{9}$$

where $\theta_\mu$, $\phi_\mu$ are the polar angles of either lepton, say $l^+$, relative to the direction of $\gamma_2$ as polar axis *in the rest frame* of $\psi$. We are assuming that the leptons may be treated as extremely relativistic[16] and that they are coupled to $\psi$'s in the same manner as to photons, as confirmed experimentally.[5] For $s_\chi > 0$, the expressions for the general distribution function $W_3$ are quite long and will be reported elsewhere. Here it suffices to note one feature of the lepton distribution which identifies $s_\chi$ in those cases in which the $\gamma_1\gamma_2$ distribution is unable to do so. The partially integrated distribution

$$W(\theta_1, \theta_\mu) = \int W_3(\theta_1, \theta_2, \phi_2\theta_\mu, \phi_\mu)d\Omega_2 d\phi_\mu$$

$$= (1 + A_1 \cos^2\theta_1)(1 + A_\mu \cos^2\theta_\mu) \qquad (10)$$

has the same general form as the distribution for $s_\chi = 0$, Eq. (9). Equations (1) and (2) define $A_1$, while the coefficient $A_\mu$ governing the $l^+$-$\gamma_2$ correlation function is determined completely by the radiative amplitudes for the $\chi \to \psi + \gamma_2$ transition:

$$A_\mu = (\xi^2 + \eta^2 - 2)/(\xi^2 + \eta^2 + 2). \qquad (11)$$

For $s_\chi = 0$, $A_1 = 1$ and $\eta = 0$, $\xi \to \infty$, by conservation of angular momentum, so we obtain $A_\mu = 1$, Eq. (9). For $s_\chi = 1$, $\eta = 0$ and $A_\mu = (B-3)/(B+5)$, while for $s_\chi = 2$, $A_\mu = (3\Xi + 5H - 4)/(3\Xi + 5H + 28)$. Hence, if there is no $\gamma_1\gamma_2$ correlation because $s_\chi = 0$, we expect $A_\mu = +1$. If $s_\chi \neq 0$, and the absence of $\gamma_1\gamma_2$ correlation is due to the ineffectiveness of $\chi \to \psi + \gamma_2$ as an analyzer for $\chi$ polarization, we expect $A_\mu = -\frac{3}{5}$ for $s_\chi = 1$ $(B = 0)$, and $A_\mu = -\frac{1}{7}$ for $s_\chi = 2$ $(\Xi = H = 0)$. Consequently, if the $\gamma_1\gamma_2$ distribution cannot distinguish between various values of $s_\chi$, this information is available from the correlation between the photon and $l^+$,

$$W(\theta_\mu) = 1 + A_\mu \cos^2\theta_\mu. \qquad (12)$$

Note that one is not required to integrate $W(\theta_1, \theta_\mu)$, Eq. (10), over all directions of $\gamma_1$ to obtain $W(\theta_\mu)$: The dependence on $\theta_\mu$ is the same for any value of $\theta_1$.

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[1]W. Braunschweig *et al.*, Phys. Lett. 57B, 407 (1975).

[2]G. J. Feldman *et al.*, Phys. Rev. Lett. 35, 821 (1975).

[3]B. Wiik, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California,* edited by W. T. Kirk (SLAC, Stanford, 1976), p. 69; J. Heintze, in *ibid.*, p. 97.

[4]See e.g., T. Appelquist *et al.*, Phys. Rev. Lett. 34, 365 (1975); E. Eichten *et al.*, *ibid.* 34, 369 (1975).

[5]The $J^P = 1^-$ character was established for $\psi(3095)$ by A. M. Boyarski *et al.*, Phys. Rev. Lett. 34, 1357 (1975), and for $\psi'(3684)$ by V. Lüth *et al.*, *ibid.* 35, 1124 (1975).

[6]We assume that when two photons are emitted in $\psi' \to \gamma_1 + \chi$, $\chi \to \psi + \gamma_2$, the first photon, which has the unique energy $E_1 = (M_{\psi'}^2 - m_\chi^2)/2M_{\psi'}$ in the $\psi'$ rest frame, can be distinguished from the second $\gamma_2$, which will have a Doppler-shifted energy appropriate to the $\chi \to \psi + \gamma$ transition for a $\chi$ recoiling with velocity $\beta_\chi = (M_{\psi'}^2 - m_\chi^2)/(M_{\psi'}^2 + m_\chi^2)$.

[7]C. N. Yang, Phys. Rev. 74, 764 (1948).

[8]Whereas in nuclear physics it is often a good approximation to neglect higher multipoles, the high energy of the photons in the present case does not justify such an approximation if $\psi'$ has hadronic dimensions.

[9]It is well known that the parity cannot be determined from such angular distributions, but requires a polarization measurement. E.g., the electric vector of the photon will be perpendicular ($\sigma$) to the plane containing the photon and electron beam directions if $s^P = 0^+$, and lie in the plane ($\pi$) if $s^P = 0^-$.

[10]The angular correlations are identical for $\chi$ decay into any pair of spinless mesons, therefore, $\pi\pi$ and $K\bar{K}$ modes may be considered together in experimentally determining the correlation functions. Under the hypothesis of $C$ invariance, the $M\bar{M}$ mode cannot occur if $s_\chi$ is odd. As noted already in Ref. 2, the occurrence of $\pi\pi$ or $K\bar{K}$ decay modes requires $\chi$ to have a "natural" spin-parity relation.

[11]P. K. Kabir, Bull. Am. Phys. Soc. II, 21, 101 (1976).

[12]If time-reversal invariance is not assumed, $x$ and $y$ need not be real and some of these relations serve to test that hypothesis also. E.g., for $s_\chi = 1$, in general we have $a_0/(a_0 + a_1) \geq 4c_1^2/(a_0 + a_1)^2$, where equality is attained when time-reversal invariance holds.

[13]Such correlation functions have been calculated, in the dipole approximation, by G. J. Feldman and F. J. Gilman, Phys. Rev. D 12, 2161 (1975), and by L. S. Brown and R. Cahn, *ibid.* 13, 1195 (1976).

[14]G. Karl, S. Meshkov, and J. Rosner, Phys. Rev. D 13, 1203 (1976), have obtained some results in the general case.

[15]A particular case of this general result is noted independently in Ref. 14, which reports that, in a no-recoil approximation, the $\phi$-integrated $\gamma_1\gamma_2$ correlation function $W(\theta_{\gamma\gamma})$ is at most quadratic in $\cos\theta_{\gamma\gamma}$ if $\Xi = 0$.

[16]In that limit, electrons and muons have identical couplings for all known interactions, therefore, $\psi \to e\bar{e}$ events (which should be as frequent as $\psi \to \mu\bar{\mu}$) may be included with $\mu\bar{\mu}$ in determining the angular distributions.