PHYSICAL REVIEW D VOLUME 13, NUMBER 11 1 JUNE 1976

Comment on Shaw's refutation of the ρ bootstrap

E. P. Tryon
Department of Physics and Astronomy, Hunter College of the City University of New York, New York, New York 10021
(Received 28 July 1975)

Gordon Shaw has argued from duality that exchange forces contribute a null term to the $\pi\pi$ P wave, hence that the traditional ρ bootstrap must fail in any model consistent with duality. Shaw's argument rests, however, on use of an infinite series in a region where it diverges. The contribution of exchange forces is computed here for the Veneziano model, and is shown to be nonzero. Although Shaw's argument is not valid, N/D calculations nevertheless indicate that exchange forces in the $\pi\pi$ channel are too weak to generate the ρ.

Shaw has argued that exchange forces in the $\pi\pi$ channel are incapable of generating the ρ resonance if the left cut of the P wave is consistent with duality and ρ-f_0 exchange degeneracy.[1] While I share Shaw's skepticism that the ρ is bound in the $\pi\pi$ channel,[2,3] Shaw's argument is flawed, and deserves comment.

Consider the standard decomposition of the $\pi\pi$ P wave $A^{(1)1}$ into a sum of terms A_L and A_R resulting from the left and right cuts, respectively:

$$A^{(1)1}(s) = \frac{s-4}{\pi}\left(\int_{-\infty}^{0} ds' + \int_{4}^{\infty} ds'\right)\frac{\mathrm{Im}A^{(1)1}(s')}{(s'-4)(s'-s)}$$

$$\equiv A_L(s) + A_R(s)\,,$$

where s denotes the center-of-mass energy squared, and $m_\pi = \hbar = c = 1$. The term A_L embodies the exchange forces, and Shaw argued that A_L is zero.

Let A^I denote the $\pi\pi$ amplitude with isospin "I" in the s (direct) channel. In dual resonance models, it is customary to express the three A^I in terms of a single function $F(x,y)=F(y,x)$. A^1 is then given by

$$A^1(s,\cos\theta) = F(s,u) - F(s,t)\,, \tag{1}$$

where t and u denote the usual Mandelstam variables:

$$t \equiv -\tfrac{1}{2}(s-4)(1-\cos\theta)\,,$$

$$u \equiv -\tfrac{1}{2}(s-4)(1+\cos\theta)\,.$$

In the single-term Veneziano model, F is given by[4]

$$F(x,y) = \beta\,\frac{\Gamma(1-\alpha(x))\,\Gamma(1-\alpha(y))}{\Gamma(1-\alpha(x)-\alpha(y))}\,, \tag{2}$$

where β denotes a normalization constant, and $\alpha(x) = a + bx$ denotes the exchange-degenerate ρ-f_0 Regge trajectory.

In its most explicit form, Shaw's argument proceeds as follows. The $F(x,y)$ of Eq. (2) can be expressed as[5]

$$F(x,y) = \beta\sum_{K=1}^{\infty}\frac{T_K(\alpha(y))}{(K-1)!}\,\frac{1}{\alpha(x)-K}\,, \tag{3}$$

where

$$T_K(\xi) \equiv \xi(\xi+1)(\xi+2)\cdots[\xi+(K-1)]$$

denotes the Kth-order Pochhammer polynomial. The series (3) converges for $\mathrm{Re}[\alpha(y)] < 0$.

Shaw notes that Eq. (3) can be used to expand $F(s,t)$ and $F(s,u)$ as series of s-channel poles, hence it can be used to expand A^1 as a series of s-channel poles:

$$A^1(s,\cos\theta) = \beta\sum_{K=1}^{\infty}\frac{T_K(\alpha(u)) - T_K(\alpha(t))}{(K-1)!}\,\frac{1}{\alpha(s)-K}\,. \tag{4}$$

If (as assumed by Shaw) Eq. (4) were valid throughout the s-channel physical region, it would follow that $\mathrm{Re}[A^1]$ is dual to direct-channel resonances, hence that $A_L = 0$.

It is readily seen that Shaw's argument is not valid, because the series (4) *diverges* when $\alpha(t) > 0$ and/or $\alpha(u) > 0$.[5] Since $\alpha(0) > 0$ for the ρ-f_0 trajectory, the series (4) cannot be used to express A^1 near the forward ($t=0$) or backward ($u=0$) directions, where A^1 is largest. [In fact, Eq. (4) is only valid for t and u such that A^1 tends asymptotically to zero.] Thus partial waves of A^1 *cannot* be expressed as projections of this series containing only direct-channel poles.

A series representation for A^1 which is valid throughout the s-channel physical region may be obtained from[5]

$$F(x,y) = \beta\sum_{K=1}^{\infty}\frac{(-1)^K}{(K-1)!}\,T_K(1-\alpha(x)-\alpha(y))\left(\frac{1}{\alpha(x)-K} + \frac{1}{\alpha(y)-K}\right), \tag{5}$$

which converges when $\mathrm{Re}(\alpha(x)+\alpha(y))>0$. Equations (1) and (5) yield

$$A^1(s,\cos\theta)=\beta\sum_{K=1}^{\infty}\frac{(-1)^K}{(K-1)!}\left[T_K(1-\alpha(s)-\alpha(u))\left(\frac{1}{\alpha(s)-K}+\frac{1}{\alpha(u)-K}\right)\right.$$
$$\left.-T_K(1-\alpha(s)-\alpha(t))\left(\frac{1}{\alpha(s)-K}+\frac{1}{\alpha(t)-K}\right)\right], \qquad (6)$$

which converges for $\mathrm{Re}(s)>-2a/b$ when $|\cos\theta|\leq 1$. Hence Eq. (6) is valid throughout the s-channel physical region, but contains crossed-channel poles as well as direct-channel poles. The crossed-channel contributions do not cancel each other completely,[6] for I find by direct computation[7] that the resulting A_L is given within a neighborhood of threshold by

$$A_L(s)\cong 0.0030(s-4), \qquad (7)$$

with $A^{(1)1}$ normalized such that elastic unitarity would imply

$$A^{(1)1}(s)=(1-4/s)^{-1/2}\exp(i\delta)\sin\delta .$$

The above result for $A_L(s)$ might be regarded as small,[8] but it is not zero. Whether it is too small to generate a ρ must be tested by computations. The N/D calculation in Ref. 2 is based in part on the Veneziano A_L, with negative results. The N/D calculation of Ref. 3 is based on a more rigorous model for the left cut, and includes inelasticity, but again fails to generate a ρ. Hence it appears unlikely that the ρ is generated by forces in the $\pi\pi$ channel.

The author is indebted to Gordon Shaw for stimulating discussions.

[1]Gordon L. Shaw, Phys. Rev. D 7, 2265 (1973).

[2]E. P. Tryon, Phys. Lett. 38B, 527 (1972).

[3]E. P. Tryon, Phys. Rev. D 12, 759 (1975).

[4]Cf. C. Lovelace, Phys. Lett. 28B, 264 (1968).

[5]Cf. D. Sivers and J. Yellin, Ann. Phys. (N.Y.) 55, 107 (1969).

[6]It has been shown by R. T. Park and B. R. Desai [Phys. Rev. D 2, 786 (1970)] that the Veneziano $\mathrm{Im}A^{(1)1}$ contains infinitely many oscillations along the left cut, with one oscillation where each tower of resonances is exchanged. As s tends to $-\infty$, the magnitude of $\mathrm{Im}A^{(1)1}$ tends to zero.

[7]I follow Lovelace (Ref. 4) in choosing the trajectory parameters $a=0.483$, $b=0.017$ (with $m_\pi=1$). The overall coefficient β is fixed by the width of the ρ resonance, i.e., I require the ρ pole in $A^{(1)1}$ to correspond to $\mathrm{Im}A^{(1)1}=\pi m_\rho\Gamma_\rho\delta(s-s_\rho)$, with $\Gamma_\rho=1.06$ (i.e., 146 MeV). This leads to $\beta=3m_\rho\Gamma_\rho/(s_\rho-4)=0.66$. The Veneziano $A^{(1)1}$ satisfies the dispersion relation which defines A_L and A_R, with rapidly convergent integrals (cf. Park and Desai, Ref. 6). I compute $A^{(1)1}$ near $s=4$ by a numerical projection of the P wave from A^1, and I compute A_R by integrating over the first 50 resonances in $A^{(1)1}$ (their widths are readily computed from numerical projections of $A^{(1)1}$ out of A^1). A_L is then given by $A_L=A^{(1)1}-A_R$, with the result described by Eq. (7).

[8]The linear approximation (7) yields $A_L=0.05$, 0.09, and 0.15 for $s^{1/2}=0.6$, 0.8, and 1.0 GeV, respectively. The exact Veneziano A_L grows somewhat less rapidly over this range of energy.