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Color and magnetic charge*

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Schwinger's conjecture that the color degree of freedom of a quark is equivalent to its degree of freedom of taking different magnetic charges provides a plausible motivation for extending color to leptons. Leptons are just quarks with zero magnetic charges. It is shown that baryon number and lepton number can be replaced by fermion number and magnetic charge.

The concept color[1] is by now generally an accepted one. The main point of color is that any of the Gell-Mann–Zweig quarks (for simplicity we confine ourselves to fractionally charged quarks, but all of the following arguments and consequences are also valid for integrally charged quarks) has another three degrees of freedom. What is the physical meaning of these new degrees of freedom? Can we interpret them in more familiar terms? A possible answer, which is the most attractive one so far, has been given by Schwinger[3] in terms of magnetic charge.[2]

He introduces particles called dyons which carry both electric and magnetic charge. The dyons can be, according to his quantization scheme, fractionally charged, whereas magnetically neutral particles must be integrally charged. A dyon with a given electric charge can take different magnetic charges (that is, another degree of freedom) which correspond to color.

We slightly modify Schwinger's conjecture in that we introduce three different magnetic charges corresponding to three colors, whereas he uses two:

We assume that different colors have different magnetic charges. (1)

According to this assumption, the observed baryons consist of three quarks of three different magnetic charges. As the baryons are magnetically neutral, the sum of the magnetic charges must be zero.

To fix the notation we consider a three-triplet model:

$$\psi=\begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \\ s_1 & s_2 & s_3 \end{pmatrix}, \quad Q_e=\begin{pmatrix} \frac{2}{3} & \frac{2}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \end{pmatrix}. \tag{2}$$

The indices 1, 2, 3 are for color and Q_e is the electric charge assignment. The assignment of magnetic charge Q_m reads

$$Q_m=\begin{pmatrix} m_1 & m_2 & m_3 \\ m_1 & m_2 & m_3 \\ m_1 & m_2 & m_3 \end{pmatrix}. \tag{3}$$

m_i is the magnetic charge belonging to the ith color. According to the quantization condition and the magnetic neutrality of the observed baryons, m_i has to satisfy the following relation:

$$\sum_{i=1}^{3} m_i = 0,$$

where

$$m_i - m_j = n_{ij} \text{ for } i=1,2,3, \text{ with } n_{ij}=\text{integer}.$$

In Schwinger's model, if interpreted in our scheme, m_i has the following values:

$$m_1=\tfrac{2}{3}, \quad m_2=-\tfrac{1}{3}, \quad m_3=-\tfrac{1}{3}, \tag{5}$$

The assumption (1) gives another constraint for m_i,

$$m_i \neq m_j \quad \text{for } i\neq j, \tag{6}$$

A solution of m_i which satisfies (4) and (6) is

$$m_1=1+n, \quad m_2=-n, \quad m_3=-1, \tag{7a}$$

with n = integer, or

$$m_1=3, \quad m_2=-2, \quad m_3=-1. \tag{7b}$$

Now let us extend the above idea to the leptons. The observed leptons behave in a pointlike manner up to presently available energies and are obviously fundamental particles, that is, they are on the same level as the quarks. Furthermore, they are magnetically neutral. So it is obvious to assume that they belong to the color "zero magnetic charge." From this point of view the hypothesis "lepton number as the fourth color" becomes plausible.[4]

Incorporating the charmed quarks, our extended scheme now contains 16 fundamental particles:

$$\psi = \begin{bmatrix} u_1 & u_2 & u_3 & \nu_e \\ d_1 & d_2 & d_3 & e^- \\ s_1 & s_2 & s_3 & \mu^- \\ c_1 & c_2 & c_3 & \nu' \end{bmatrix}, \tag{8a}$$

$$Q_e = \begin{bmatrix} \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 0 \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -1 \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -1 \\ \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 0 \end{bmatrix}, \tag{8b}$$

$$Q_m = \begin{bmatrix} m_1 & m_2 & m_3 & 0 \\ m_1 & m_2 & m_3 & 0 \\ m_1 & m_2 & m_3 & 0 \\ m_1 & m_2 & m_3 & 0 \end{bmatrix}. \tag{8c}$$

The physical meaning of this classification is that [apart from the possibility that the four degrees of freedom of the magnetic charges is a manifestation of an underlying group, possibly an SU(4′), the so-called color SU(4′)] the only difference between leptons and quarks might consist in the different magnetic charges.

Using the generators F_i' of SU(4′), the magnetic charge operator F_m which reproduces (8c) can be written as follows:

$$F_m = [m_1 + \tfrac{1}{2}(m_3)]F_3' - \tfrac{1}{2}\sqrt{3}\, m_3 F_8', \tag{9}$$

with

$$F_3' = \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & 0 & \\ & & & 0 \end{bmatrix},$$

$$F_8' = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -2 & \\ & & & 0 \end{bmatrix}.$$

FERMION NUMBER F

After having classified the quarks and leptons on the same level it is natural to ask whether one can understand baryon-number (B) conservation and lepton-number (L) conservation in a unified manner. In fact, one can dispense with B and L, as we show now.

An important common property of the quarks and leptons is that both of them are fermions. So we introduce an additive absolutely conserved quantum number, the fermion number F, for the quarks and leptons:

$$F = 1 \text{ for quarks and leptons,} \qquad F = -1 \text{ for antiquarks and antileptons.} \tag{10}$$

So the observed baryons have $F = 3$ and the four leptons have $F = 1$.

It is easy to see that F conservation and magnetic-charge conservation are consistent with all known reactions and can replace the B and L conservation. Therefore, we make the following conjecture:

The physical background for B and L *conservation consists in* F and Q_m *(magnetic charge) conservation. Therefore, B and L can be replaced by F and* Q_m. (11)

In this conjecture the number of parameters remains the same. What one gains is a better physical insight.

An important point we want to discuss now is that in our scheme proton stability is not absolute. Let us consider some examples. With F and Q_m conservation, no quarks can decay into leptons owing to the Q_m conservation. The decay of a proton into an electron and a meson (say a photon),

$$p \to e^+ + \gamma, \tag{12}$$

is also not allowed owing to F conservation. The minimum decay mode of a proton is four-body decays, namely into three leptons and a pion:

$$p \to 3\nu + \pi^+ \tag{13}$$

We emphasize that the minimum decay mode of any dynamical theory which incorporates our scheme is bigger than or equal to (13) but never less (we mean the number of final particles). Other possible decay modes are

$$p \to 4\nu + e^+, \qquad p \to 4\nu + \mu^+ + e^+ + e^-. \tag{14}$$

Note that no two- or three-body decays are allowed.

The nonabsolute stability of the proton is not surprising. It is a common feature of most of the gauge models[4,5] which treat quarks and leptons on the same footing. For example, in the model of Pati and Salam the minimum decay modes of the proton are the same as given in (13) and (14). (See Ref. 4.) They estimate the lifetime of proton τ at about 10^{29} years, which is much greater than the age of the universe, 10^{10} years.

The present experimental low limit for τ (Ref. 7) is

$$\tau \geq 2\times 10^{30} \text{ years.} \tag{15}$$

Anyhow it is not difficult to think of a mechanism which suppresses the above decays. For example, in a gauge theory with spontaneously broken symmetries the above decays are mediated by gauge particles which are exchanged between quarks and leptons, the so-called X particles in the terminology of Pati and Salam. These particles must be superheavy in order to be consistent with low-energy phenomena. Thus the effective coupling constant G_X is very weak. The proton decay mode is in third order or higher in G_X, which will suppress them greatly, thus yielding the observed stability of the proton.

CONCLUSION

We summarize:

1. Color is a manifestation of magnetic charge.
2. Leptons are "quarks" with zero magnetic charge.
3. B and L conservation can be replaced by F and Q_m conservation.

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[6]We want to mention important differences concerning baryon decays between our scheme and the Pati-Salam model. In their model the proton decay is a consequence of direct quark decays into leptons, whereas in our scheme direct quark decays into leptons are forbidden. Further, the proton decay occurs only for integrally charged (electric) quarks in their model, whereas in our scheme it is allowed both for integrally and fractionally charged ones.

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