Electromagnetic-transition form factor for the $S_{11}(1535)$ resonance in the quark model

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The electromagnetic-transition form factor for the $S_{11}(1535)$ resonance is studied in the symmetric quark model. Using the nonrelativistic quark model with a modified Woods-Saxon potential $-V_0[(r/b)+1]/\{(r/b)+\exp[(r-R)/a]\}$, which gives good agreement with the experimental cross sections for the elastic and resonance-production processes, reasonable agreement with the experimental data for the process $ep \to eS_{11}(1535)$ is obtained. We also compare this result with other works in the quark model.

Several years ago the SLAC-MIT group[1] made inelastic electron-proton scattering experiments, measuring only the final electrons. They observed three prominent bumps in the cross section. The dominant peak is at $W=1236$ MeV, the next bump at $W=1520$ MeV, and the last at $W=1680$ MeV.

Recently the reaction $ep \to ep\eta$ was studied in the region of the $S_{11}(1535)$ resonance by detecting the recoil proton in coincidence with the scattered electron.[2-4] By this experiment it becomes possible to separate the $S_{11}(1535)$ resonance from contributions of $D_{13}(1520)$ and $P_{11}(1470)$, since only S_{11} has a large branching ratio for η decay (about 55%).

In a previous publication[5] we studied the cross sections of the $ep \to eN^*$ processes for three bumps and the elastic form factors, using the nonrelativistic quark model with the modified Woods-Saxon potential (MWP)

$$V(r) = -V_0 \frac{r/b+1}{r/b+\exp[(r-R)/a]}, \tag{1}$$

which has merits of both harmonic-oscillator potential (HOP) and the $1/r$ potential, and good agreement with the experimental data was obtained. σ_T/σ_L and the asymmetry parameters of polarized electron scattering are calculated by using this potential.[6,7] Similar results were obtained,[8] using the quark-diquark model proposed by this author[9] and Lichtenberg.[10] In this note we shall study the electroproduction process $ep \to eS_{11}(1535)$ in the quark model with the MWP.

The cross section for the electroproduction process is given by[11]

$$\left.\frac{d^2\sigma}{d\Omega d\epsilon'}\right|_{\text{lab}} = \Gamma_T(\sigma_T + \epsilon_1 \sigma_S), \tag{2}$$

$$\Gamma_T = \frac{\alpha}{4\pi^2}\frac{K}{q^2}\frac{\epsilon'}{\epsilon}\frac{2}{1-\epsilon_1}, \tag{3}$$

$$\epsilon_1^{-1} = 1 + 2\left(1+\frac{\nu^2}{q^2}\right)\tan^2\tfrac{1}{2}\theta, \tag{4}$$

where K is the lab energy required to produce the N^* with real photons and ν is the energy of the virtual photon in the lab system. Other notations are the same as those of Ref. 5. In our model σ_T and σ_S are obtained in Ref. 6. We calculate $\sigma_T + \epsilon_1\sigma_S$ using the MWP with parameters[5]

$$R = 1.1\ \text{fm}, \quad a = 0.03R, \quad b = 0.07R, \quad V_0 = 23.5/(m_q R^2). \tag{5}$$

The results are shown in Fig. 1 for the case of $m_q = \infty$ and in Fig. 2 for the case of $m_q = m_p/2.793$, i.e., $g_q = 1$, and are compared with those of the HOP and with the experimental data. Theoretical curves are normalized to the photoproduction value of the cross section.

The cross section obtained nonrelativistically depends on the frame in which the nonrelativistic

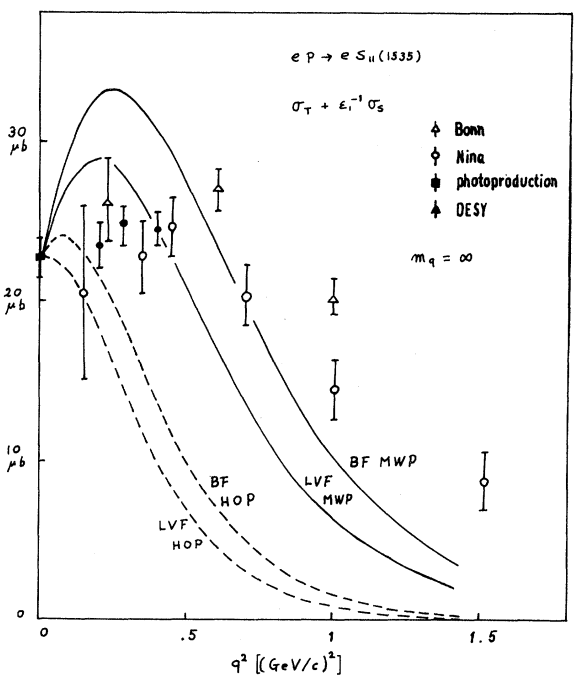

FIG. 1. The q^2 dependence of $\sigma(ep \to eS_{11}(1535))$ in the quark model (QM) with $m_q = \infty$.

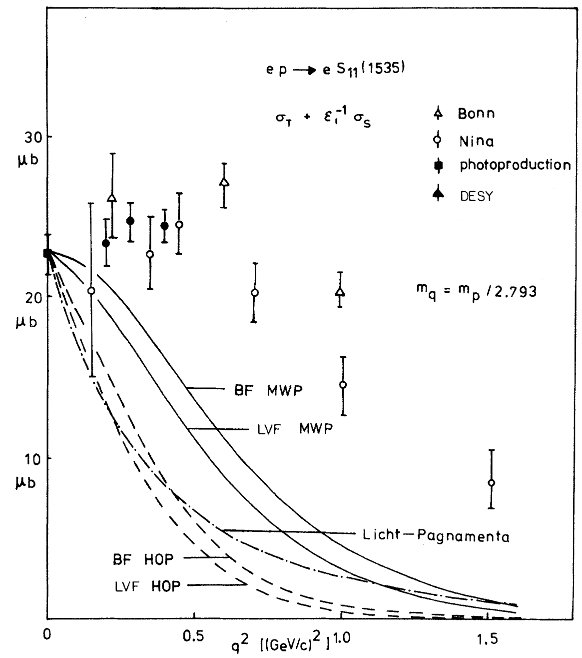

FIG. 2. Same as Fig. 1 but with $m_q = m_p/2.793$.

form factors are calculated. Following Refs. 5–8 we show the results obtained when the form factors are calculated in the Breit frame (BF) and in the least velocity frame (LVF), because the nonrelativistic approximation is relatively good in these frames.[12]

As is shown in these graphs, the MWP with $m_q = \infty$ gives a reasonable agreement with the experimental data. For large q^2 [$q^2 \gtrsim 1$ (GeV/c)^2] the predicted cross section is slightly smaller than the experimental data. If we assume $m_q = m_p/2.793$, i.e., $g_q = 1$, the cross section evaluated by the MWP falls too rapidly as q^2 increases. In the case of $m_q = \infty$ as well as in the case of $m_q = m_p/2.793$, the HOP predicts the cross section, which decreases more rapidly than that of the MWP, as q^2 increases. It is well known[13] that the HOP predicts Gaussian-type form factors and that predicted magnitudes of the cross sections for the elastic ep scattering and for the resonance-production processes $ep \to eN^*$ become too small for large q^2.

Oliver[14] obtained the correct elastic form factors and correct cross sections for the $ep \to eN^*$ processes for each peak, using the boosted nonrelativistic harmonic-oscillator wave functions, and assuming that the quark current is described by the ρ dominance. Previously this model was used to study the $\nu P \to \mu N^*$ processes by Andreadis *et al.*[15] However, the predicted cross section for $ep \to eS_{11} \to eS_{11}(1535)$ in their model decreases too rapidly as q^2 increases, if $m_q \sim m_p/3$ is assumed. If a very large value is taken for m_q, the agreement with the experimental data somewhat improves.

Thus we come to a rather general conclusion. If we fit the cross section of the second peak production processes by changing the binding potential or by modifying the quark current, we obtain a cross section for the process $ep \to eS_{11}$ which falls too rapidly as q^2 increases, as long as we assume $m_q \sim m_p/3$. In order to fit the cross section $(d\sigma/d\Omega)(ep \to eS_{11})$, we are forced to select large m_q.

The reason is the following. Since σ_T is substantially larger than σ_L in this peak, the q^2 dependence of the cross section

$$\frac{d\sigma}{d\Omega}(ep \to eS_{11}(1535))$$

is mainly determined by σ_T. In the author's model[5] one gets

$$|f_E|^2 = \tfrac{2}{3}\vec{q}^2\mu^2\left(I_{SP} - \frac{2A_1}{3g_q|\vec{q}|}\right)^2, \qquad |f_M|^2 = 0, \tag{6}$$

where

$$I_{SP} = \int r^2 R_P(r) j_1(|\vec{q}|r) R_S(r)\, dr\,. \tag{7}$$

For the HOP

$$A_1 = -\frac{3\alpha^2}{|\vec{q}|} I_{SP}\,, \tag{8}$$

where α is the spring constant. Thus

$$|f_E|^2 = \tfrac{2}{3}\vec{q}^2\mu^2 I_{SP}^2\left(1 + \frac{2\alpha^2}{g_q|\vec{q}|^2}\right)^2. \tag{9}$$

From this form one can easily see that for $m_q \sim m_p/3$, i.e., $g_q = 1$ this form factor decreases more rapidly than that in the case of $m_q = \infty$, i.e., $g_q = \infty$, as q^2 increases. Essentially the same argument is also true for the MWP, for many other potentials, and also for the model of Le Yaouanc *et al.*

Therefore, the data for S_{11} excitation can be fitted better when the quark mass is taken to be large in the considered models.

Finally we add in Fig. 2 the result obtained by the Licht and Pagnamenta model,[16] which gives better agreement with the experimental data on the cross sections for the elastic and electroproduction processes of the first, second, and third resonances than the model considered above. This model is closely related to the work by Oliver.[14]

Licht and Pagnamenta studied the relativistic correction of the nonrelativistic harmonic-oscillator quark model. Taking into account the Lorentz contraction they computed the overlap integral in the LVF (for the elastic scattering the LVF is identical to the Breit frame). Assuming a quark form factor given by vector-meson dominance and $m_q = m_p/2.793$ one gets for the form factors of the process $ep \to eS_{11}(1535)$

$$
\begin{aligned}
|f_C|^2 &= \frac{\vec{q}^2 R^2}{27}\,\frac{1}{(1+q^2/m_\rho{}^2)^2(1+\vec{q}^2/4m_p{}^2)^3}\exp\left[-\frac{R^2\vec{q}^2}{3(1+\vec{q}^2/4m_p{}^2)}\right],\\
|f_M|^2 &= 0,\\
|f_E|^2 &= \tfrac{2}{27}(\vec{q}^2)^2\mu^2R^2\,\frac{(1+2/R^2\vec{q}^2)^2}{(1+q^2/m_\rho{}^2)^2(1+\vec{q}^2/4m_p{}^2)^4}\exp\left[-\frac{R^2\vec{q}^2}{3(1+\vec{q}^2/4m_p{}^2)}\right].
\end{aligned}
\tag{10}
$$

Taking $R^2 = 2.75\ \text{GeV}^{-2}$, which is determined by the electromagnetic mass differences of baryons very accurately,[17] we obtain the result shown in Fig. 2. As can be seen in this graph, the predicted result of this model is not satisfactory, unlike successes for other processes.

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