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Conserved anomalous weak currents*

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Tests for second-class axial-vector currents in $\Sigma \to \Lambda e\nu$ decay and neutrino reactions are analyzed as to their sensitivity to the assumption that this current is conserved. The possibility of an anomalous conserved first-class vector current is also discussed.

I. INTRODUCTION

Recent beta-decay experiments[1,2] suggest the existence of a second-class axial-vector current in the weak interaction. It is obviously important to search for the effects of such a current in other weak processes, and many possibilities have been discussed.[3] The main purpose of this note is to indicate the importance of distinguishing tests which are sensitive to a *conserved* second-class axial-vector current from those that require the current to be nonconserved. The importance of this distinction in the case of beta decay has been emphasized by Delorme and Rho.[4]

The recent beta-decay experiments involve the observation of a parameter labeled d in the analysis of Holstein,[5] and this satisfies the zero-divergence condition. Thus these observations are consistent with, although they do not require, a conserved second-class axial-vector current. If the impulse approximation is used, these experiments determine the nucleon matrix element of the form

$$(g_{II}/2M)\bar{p}\sigma_{\mu\nu}\gamma_5 q_\nu n\,. \tag{1}$$

The two experiments yield $g_{II} = (-8 \pm 3)g_A$ (Ref. 1) and $g_{II} = (-3.5 \pm 1)g_A$ (Ref. 2).

In the case of the Ne^{19} experiment it is noted that an alternative explanation would be the failure of the assumption that the weak vector current is proportional to the isovector electromagnetic current. This requires that the weak-magnetism parameter b equal (1.9 ± 0.4) times its expected value. A similar explanation is possible for the $A = 12$ system[2] if the weak-magnetism parameter b equals (1.9 ± 0.3) times its expected value. In this case there exists a well-known independent test of the weak-magnetism term,[6] but deviations of this magnitude may not be entirely ruled out. Such unexpected values for the weak-magnetism parameter could result from the addition to the usual vector current of an anomalous conserved first-class vector current. Such an anomalous vector current is considered in Sec. III.

II. SECOND-CLASS AXIAL-VECTOR CURRENT

While the standard quark model does not allow second-class currents, it is instructive to consider modified forms that do allow them. The simplest possibility is to allow derivative couplings in which case the second-class axial-vector current could have the form

$$A_\mu^{II} \sim \bar{u}\sigma_{\mu\nu}\gamma_5 d\partial/\partial x_\nu\,. \tag{2}$$

This may serve as a model and provide some motivation for a divergenceless second-class current.[7] Holstein and Treiman[3] consider an alternative in which there are two nonstrange quark doublets with

$$A_\mu^{II} \sim \bar{u}_2\gamma_\mu\gamma_5 d_1 - \bar{u}_1\gamma_\mu\gamma_5 d_2\,. \tag{3}$$

Without considering the consequences of this additional degree of freedom, we note that this provides a model of a nonconserved second-class axial-vector current.

A very interesting test for second-class currents involves the mirror decays[8]

$$\Sigma^+ \to \Lambda + e^+ + \nu\,, \tag{4a}$$

$$\Sigma^- \to \Lambda + e^- + \bar{\nu}\,. \tag{4b}$$

With the usual assumptions for the vector currents the decay matrix elements are

$$M_\lambda^+ = \bar{\Lambda}[(g_A^I + g_A^{II})\gamma_\lambda\gamma_5 + (g_T^I + g_T^{II})\sigma_{\lambda\nu}\gamma_5 q_\nu/2M_\Sigma + f_M\sigma_{\lambda\nu}q_\nu/2M_\Sigma]\,\Sigma^+, \tag{5a}$$

$$M_\lambda^- = \bar{\Lambda}[(g_A^I - g_A^{II})\gamma_\lambda\gamma_5 + (g_T^I - g_T^{II})\sigma_{\lambda\nu}\gamma_5 q_\nu/2M_\Sigma + f_M\sigma_{\lambda\nu}q_\nu/2M_\Sigma]\,\Sigma^-\,. \tag{5b}$$

Here we have neglected the vector coupling proportional to q^2 and the scalar coupling proportional to $(m_\Sigma - m_\Lambda)$.[9] Assuming SU(3) symmetry, the weak-magnetism coupling is

$$f_M \approx -(m_\Sigma/m_N)(\tfrac{3}{2})^{1/2}\mu_n \approx 3\,.$$

Among the axial-vector-current form factors we have neglected the pseudoscalar g_P which makes a contribution proportional to m_e to the decay amplitude. The other two form factors g_A and g_T have contributions as indicated from both first-class (superscript I) and second-class (superscript II) currents. If the second-class current is conserved, there is no constraint on $g_T^{\rm II}$, but $g_A^{\rm II}$ is proportional to $q^2 g_P^{\rm II}/(m_\Sigma + m_\Lambda)$ and can be assumed to be negligible for the range of q^2 involved. Note that $g_P^{\rm II}$ unlike $g_P^{\rm I}$ has no pion-pole enhancement. The coupling $g_T^{\rm I}$ arising from the usual axial-vector current vanishes in the SU(3)-symmetric limit and will be assumed to be negligible. The simplest observable, the difference in rates for (4a) and (4b), is given to first order in the second-class couplings

$$\Delta \equiv \frac{[ft(\Sigma^+)]^{-1} - [ft(\Sigma^-)]^{-1}}{[ft(\Sigma^+)]^{-1} + [ft(\Sigma^-)]^{-1}} = \frac{6 g_A^{\rm I} g_A^{\rm II} - 0.13 g_A^{\rm I} g_T^{\rm II}}{3(g_A^{\rm I})^2} . \tag{6}$$

The present experimental result $\Delta = 0.01 \pm 0.13$ corresponds to $|g_A^{\rm II}/g_A^{\rm I}| \le 0.07$ or $|g_T^{\rm II}/g_A^{\rm I}| \le 3.3$ at the one-standard-deviation level. The limit on $g_A^{\rm II}$ might be interpreted as a significant limit on a nonconserved second-class current. For a conserved current it is seen that the limit, now only on $g_T^{\rm II}$, is much less significant. In any case, the value of $g_T^{\rm II}$ for this decay is not directly related to $g_{\rm II}$ for the nucleon [Eq. (1)], even for an octet second-class current [as in Eq. (2)], since there remains an undetermined d/f ratio.

A variety of possibilities for second-class-current effects exists in comparisons between ν and $\bar\nu$ reactions. Because these reactions also differ as a consequence of V-A interference terms a simple comparison of cross sections is not enough in this case. The general result following from the absence of second-class currents is the equality among hadronic structure functions,[10]

$$W_i(\nu, c) = W_i(\bar\nu, c_m) , \tag{7}$$

where c represents a set of hadronic transitions and c_m is the mirror set. For the case of the mirror elastic reactions

$$\nu + p \to \mu^- + n , \tag{8a}$$

$$\bar\nu + n \to \mu^+ + p \tag{8b}$$

second-class currents contribute only to a difference between $W_4(\nu)$ and $W_4(\bar\nu)$ when spin-averaged structure functions are considered. However, W_4 does not contribute to the cross section in the limit of zero muon mass.[11] This leads to a relation between the cross sections of (8a) and (8b) in the zero-mass limit which follows from *CPT* and lepton locality[12] and does not depend on the absence of second-class currents. To find second-class-current effects that produce differences between reactions (8a) and (8b) in the zero-mass limit requires polarization experiments which belong to a future generation of experiments.[13] However, detailed observations of the cross sections of reactions (8a) and (8b), together with the usual assumptions for the vector current, could determine both $g_A(q^2)$ and $g_{\rm II}(q^2)$ and thus determine the second-class coupling.[14] Clearly, a nonzero value of $g_{\rm II}$ can be produced by a conserved second-class current.

The one attempt to test Eq. (7) has been made by Musset,[15] who attempted to test

$$W_2(\nu) = W_2(\bar\nu) \tag{9}$$

in *inclusive* neutrino reactions in the Gargamelle experiment at CERN assuming an isoscalar target. He found that the equality held within sizable experimental uncertainties. However, in the case of a conserved second-class current described by Eq. (2), it seems possible that Eq. (9) might follow from the parton picture just as in the case of elastic scattering.[16] Thus there is some question whether this is a good test for a conserved second-class current.

To avoid the problem with the elastic reaction one may turn to the inelastic reactions

$$\nu + p \to \mu^- + \Delta^{++} , \tag{10a}$$

$$\nu + n \to \mu^+ + \Delta^- . \tag{10b}$$

Steven M. Brown[17] has discussed the difference

$$W_2(\nu, \Delta^{++}) - W_2(\bar\nu, \Delta^-) \tag{11}$$

as a test for a second-class axial-vector current. In order to isolate W_2 it is necessary to measure the differential cross section near $q^2 = 0$. In a general phenomenological analysis Brown finds that the difference (11) is largest near $q^2 = 0$. The conclusion is completely different if the second-class current is conserved, since it follows from a general theorem due to Adler[18] that for forward muons ($q^2 = 0$ in the zero-mass limit)

$$W_2(\nu, \Delta) \propto |\langle \Delta | \partial A_\lambda / \partial x_\lambda | N \rangle|^2 ,$$

so that a conserved current makes no contribution at $q^2 = 0$.

Pais[10] has pointed out that for neutrino reactions with two or more hadrons in the final state there exists an observable which has no contribution from V-A interference. As a result, any difference between the values of this observable in ν and $\bar\nu$ mirror reactions can only be due to second-class-current effects. For the case of reactions (10) this observable corresponds to an alignment of the Δ given by the second-rank tensor component T_{nn}, where n is a vector defined by lepton momenta, $\vec{n}=\vec{p}_\mu+\vec{p}_\nu$. This alignment is detected by means of a $\cos 2\phi$ term in the angular distribution of the Δ decay products, where ϕ is the angle between the decay plane of these products and the plane formed by lepton momenta.[19] For this observable, in contrast to that of Eq. (11), there is no reason to expect a suppression for the case of a conserved second-class current. These conclusions are borne out by the detailed calculations of Holstein and Treiman,[3] who introduce second-class currents into reactions (10) via a nucleon pole term containing the conserved matrix element of Eq. (1). They find a considerable interference contribution to the $\cos 2\phi$ term discussed above, while for $q^2=0$ there is no first-class–second-class interference in W_2 and thus none in the cross section.[20] An examination of the figures in Ref. 3 indicates that after averaging over q^2 there is a contribution linear in $g_{\rm II}$ to $(\sigma_\nu-\sigma_{\bar\nu})/\frac{1}{2}(\sigma_\nu+\sigma_{\bar\nu})$ of the order of $0.1g_{\rm II}$. However, it is difficult to isolate the second-class effect from the V-A interference effect except near $q^2=0$.

An alternative to the assumption that the second-class axial-vector current is conserved is the assumption that it is the same member $(1+i2)$ of the octet as the usual first-class current.[21] For the decays (4) it then follows from SU(3) symmetry that $g_A^{\rm II}$ vanishes; however, taking into account SU(3) breaking this means only a suppression by a factor 5 or 10. The suppression of $g_A^{\rm II}$ from the assumption of current conservation discussed above is much stronger. Similarly for the neutrino reactions (10) it follows from SU(6) symmetry [or more simply the SU(4) subgroup of SU(6)] that the difference (11) vanishes, since in the symmetry limit the reaction may be considered as elastic. Taking into account SU(6)-breaking, we expect corrections to this result of the order $(m_\Delta-m_N)$, whereas for the case of a conserved second-class current the vanishing of (11) is exact as q^2 approaches zero. Of course the derivative parton coupling in Eq. (2) satisfies both the conservation condition and the octet symmetry.

III. ANOMALOUS VECTOR CURRENT

The standard assumption for the weak vector current is that it is proportional to the isospin partner of the isovector piece of the electromagnetic current. While this is usually called the conserved vector current (CVC) hypothesis, we shall label it the proportional vector current (PVC) since we will consider an alternative that still satisfies the conservation condition. The alternative involves adding to the usual vector current a term which in the quark model could be written

$$V'_\mu=\bar{u}\sigma_{\mu\nu}d\partial/\partial x_\nu\,. \tag{12}$$

This may be called an anomalous weak magnetism with no corresponding anomalous magnetic moment of the quark. Since no charge is associated with V'_μ, it should not affect such CVC tests as $\pi^+\to\pi^0e^+\nu$ or the constancy of ft values for allowed Fermi transitions. On the other hand V'_μ would be expected to upset the PVC predictions for the weak-magnetism parameter b and so help explain the recent beta-decay experiments discussed in the Introduction.

The anomalous conserved vector current V'_μ of Eq. (12) is clearly the chiral partner of $A_\mu^{\rm II}$ given in Eq. (2). In this case the chiral partner of the second-class axial-vector current is a *first-class* anomalous vector current.[22] From this point of view it seems reasonable that if one of these unexpected currents exists, probably both of them do. Thus instead of asking whether the recent beta-decay results are to be explained by $A_\mu^{\rm II}$ *or* V'_μ, we might ask how the two combine to produce these results.

While V'_μ and $A_\mu^{\rm II}$ both contribute to the beta-decay experiments, the existence of V'_μ does not in general affect the other tests for $A_\mu^{\rm II}$ which we have discussed. In the case of the decays (4a) and (4b), the only effect of V'_μ is to change the predicted value of f_M; this has no effect on the difference in decay rates [Eq. (6)]. In the case of neutrino reactions the basic test for second-class currents Eq. (7) is unaffected by the presence of the anomalous first-class current V'_μ. On the other hand, the attempt to extract $g_{\rm II}(q^2)$ from the differential cross sections for the elastic reactions (8a) and (8b), as discussed above, requires the PVC assumption. Thus, a nonzero value for $g_{\rm II}$ determined in this way could in general be interpreted as a sign of a second-class axial-vector current or a failure of PVC or a combination of the two.

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[1]F. P. Calaprice *et al.*, Phys. Rev. Lett. 35, 1566 (1975).
[2]K. Sugimoto *et al.*, Phys. Rev. Lett. 34, 1533 (1975).
[3]For a recent discussion see B. R. Holstein and S. B. Treiman, this issue, Phys. Rev. D 13, 3059 (1976).
[4]J. Delorme and M. Rho, Phys. Lett. 34B, 238 (1971); Nucl. Phys. B34, 317 (1971); B46, 332 (1972). See

also M. A. B. Bég and J. Bernstein, Phys. Rev. D 5, 714 (1972); J. Delorme, thesis, Université Claud Bernard, Lyon, 1972 (unpublished). The general conclusion here is that when the current is conserved the difference in ft values for mirror transitions must be proportional to the energy release. While the Wilkinson-Alburger experiment on the $A=8$ system [Phys. Rev. Lett. 26, 1127 (1971)] provides evidence against this, the general feeling now is that electromagnetic effects make it impossible to draw definitive conclusions from ft values in beta decay, hence the emphasis on the correlation experiments of Refs. 1 and 2.

[5]Barry R. Holstein, Rev. Mod. Phys. 46, 789 (1974), and references cited there.

[6]C. S. Wu, Rev. Mod. Phys. 36, 618 (1964), and references cited there.

[7]Additional motivation is given in the discussion by Bég and Bernstein (Ref. 4). They also emphasize that the associated axial charge should vanish.

[8]P. Hertel, Z. Phys. 202, 383 (1967).

[9]J. Dreitlein and H. Primakoff, Phys. Rev. 125, 1671 (1962).

[10]A. Pais, Phys. Rev. D 5, 1170 (1972).

[11]A. Pais, Ann. Phys. (N.Y.) 63, 361 (1971).

[12]L. Wolfenstein, Ann. Phys. (N.Y.) 71, 569 (1972).

[13]Barry R. Holstein, Phys. Rev. C 4, 764 (1971).

[14]A significant determination of g_{II} by this method may be possible at Brookhaven; R. Palmer (private communication).

[15]P. Musset (private communication).

[16]Even for the nonconserved second-class current of Eq. (3) it is suggested in Ref. 3 that there would be no interference between first- and second-class currents in deep-inelastic structure functions and so Eq. (9) would hold.

[17]S. M. Brown, Nuovo Cimento 16A, 85 (1973).

[18]S. L. Adler, Phys. Rev. 135, B963 (1964).

[19]Pais also discusses a $\sin 2\phi$ term which has only V-A interference contributions and so must reverse between ν and $\bar{\nu}$ reactions in the absence of second-class current effects. For the case of reactions (10) this would correspond to an alignment T_{nm} where $\vec{m}=\vec{n}\times\vec{q}$. However, it is impossible to find a hadronic tensor to couple with the leptonic tensor $n_\mu n_\nu$ to give a nonzero expectation value of T_{nm}. Therefore, as pointed out in Ref. 3, there is no $\sin 2\phi$ term for reactions (10).

[20]I am indebted to Professor Holstein for communicating this result.

[21]R. Oehme, Phys. Lett. 38B, 532 (1972), discusses the consequences of this assumption with the current conserved only in the exact SU(3) limit. He also discusses strangeness-changing currents, but this requires the additional assumption that the second-class current has the same Cabibbo angle.

[22]The same correspondence shows up in the formulation of second-class currents by S. Okubo, Phys. Rev. Lett. 25, 1593 (1970). Anomalous vector currents of the form (12), as well as the importance of distinguishing PVC from CVC, are discussed by T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. 15, 402 (1965).