

## Magnetic moments of charmed baryons. II

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(Received 15 July 1975)

Assuming that the magnetic moment operator is a tensor transforming as the  $(\underline{15}, 3)$  member of a  $\underline{63}$  representation of  $U(8)$  or  $SU(8)$ , we have compared the magnetic moments of all the baryons belonging to the  $\underline{120}$  representation. The moments of the conventional particles like the  $SU(3)$  decuplets or octets stay unchanged. The charmed-baryon moments are expressed in terms of the magnetic moment of the proton.

### I. INTRODUCTION

In a recent paper<sup>1</sup> the authors calculated the magnetic moments of the charmed baryons assuming that the magnetic-moment operator transforms as the charge operator in  $U(4)$  symmetry. There we have suppressed the intrinsic spin of the particles as a first step. However, a more realistic approach would be to incorporate the intrinsic spin of the particles.

It is well known that similar extensions have been achieved earlier, when Gürsey and Radicati and, independently, Sakita<sup>2</sup> first proposed to extend  $SU(3)$ , the internal symmetry group, into  $SU(6)$  to give every particle in this classification a definite spin. It has been found by Pais<sup>3</sup> that  $SU(6)$  gives the correct  $D/F$  ratio for the baryon current. Taking this group as the basis of the baryon classification, Bég, Lee, and Pais<sup>4</sup> calculated the magnetic moments of the basic baryons belonging to the  $\underline{56}$  representation. They conjectured there that the magnetic-moment operator transforms as the  $(\underline{8}, 3)$  member of a  $\underline{35}$  representation, and obtained in the effective low-frequency limit the electromagnetic moments of all the particles in terms of the magnetic moment of the proton.

Since the discovery of  $\psi$  particles,<sup>5</sup> many physicists are hoping that more charmed particles are about to emerge. The enlarged group  $SU(4)$ ,<sup>6</sup> with a fourth quark  $c$ , it is hoped, should enable us to answer many previously unanswered questions and understand all the recent particles showering in as new discoveries.

There have already been suggestions<sup>7</sup> put forward to extend  $SU(4)$  symmetry to  $SU(8)$ , to incorporate the intrinsic spin of the particles. This group has been used to deduce spin-dependent mass equations to predict the masses of the charmed baryons.

In our work, we have assumed that the classification is given by  $U(8)$  representation. As an ex-

tension of the work of Bég, Lee, and Pais,<sup>4</sup> we have assumed that the magnetic-moment operator transforms as the  $(\underline{15}, 3)$  member of a  $\underline{63}$  representation [Young tableaux specification  $(211111)$ ]. The magnetic moments are finally expressed in terms of the magnetic moment of the proton.

In Sec. II, we have constructed a tensor  $B_{ABC}$  which should represent the group of particles given by the  $\underline{20}$  representation of  $U(4)$  with spin  $\frac{3}{2}$  and the  $\underline{20}'$  representation with spin  $\frac{1}{2}$ . In Sec. II, we have constructed the most general currents. We have evaluated the magnetic moments in Sec. IV. In Sec. V we have discussed our results.

### II. $U(8)$ OR $SU(8)$ CLASSIFICATIONS OF THE BARYONS

The enlargement from  $U(4)$  to  $U(8)$  group permits us to place both  $\underline{20}$  and  $\underline{20}'$  representations of  $U(4)$  into the same irreducible representation  $\underline{120}$  of  $U(8)$ . The representation  $\underline{120}$  is totally symmetric in terms of the tensor indices. If we now express  $\underline{120}$  in terms of the  $U(4) \otimes SU(2)$  indices, where  $SU(2)$  should be the group of the intrinsic spin of the particles, we find

$$\underline{120} = (\underline{20}, 4) + (\underline{20}', 2). \quad (1)$$

The symbol  $(m, n)$  now stands for a  $m$  dimensional multiplet under  $U(4)$  or  $SU(4)$  classification, and simultaneously the multiplet is also an  $n$ -dimensional representation of  $SU(2)$ . Thus the  $\underline{120}$  multiplet stands for  $\underline{20}$   $U(4)$  totally symmetric spin  $\frac{3}{2}$  and  $\underline{20}'$  spin  $\frac{1}{2}$  with  $U(4)$  mixed-symmetry particles.

We are aware of the fact that the totally symmetric representation as that of Eq. (1) has the intrinsic problem of statistics. To improve the situation we have to introduce different colored quarks as suggested by Gell-Mann.<sup>8</sup> However, we will not concern ourselves with such questions. This may be considered in a future extension of the calculation where one will also include the colored quarks.

The totally symmetric tensor which represents the  $\underline{120}$  multiplet is given by  $B_{ABC}$ , where  $A$ ,  $B$ , and  $C$  stand for the pairs  $(\mu, a)$ ,  $(\nu, b)$  and  $(\rho, c)$ , respectively. The Greek indices stand for the U(4) labeling, where all of them run from 1 through 4, whereas  $a, b, c$ , etc. are used to express the SU(2) labeling, which runs from 1 to 2. We use for U(3) or SU(3) the labeling  $i, j, k$ , etc., which runs from 1 through 3. The baryon multiplets are now

given by

$$\begin{aligned} B_{ABC} &= B_{\mu\nu\rho; abc} \\ &= B_{\mu\nu\rho}^{20} \chi_{abc} \\ &\quad + \frac{1}{3} (\epsilon_{ab} \chi_c B_{\{\mu\nu\}p}^{20'} + \epsilon_{bc} \chi_a B_{\{\nu\rho\}q}^{20'} + \epsilon_{ca} \chi_b B_{\{\rho\mu\}r}^{20'}). \end{aligned} \quad (2)$$

In Eq. (2) we have put for the  $\underline{20}$  representation,

$$B_{\mu\nu\rho}^{20} = \delta_\mu^i \delta_\nu^j \delta_\rho^k a_{ijk} + \frac{1}{\sqrt{3}} (\delta_\mu^i \delta_\nu^j \delta_\rho^4 + \delta_\mu^4 \delta_\nu^j \delta_\rho^i + \delta_\mu^i \delta_\nu^4 \delta_\rho^j) S_{ij}^{*(1)} + \frac{1}{\sqrt{3}} (\delta_\mu^i \delta_\nu^4 \delta_\rho^4 + \delta_\mu^4 \delta_\nu^i \delta_\rho^4 + \delta_\mu^4 \delta_\nu^4 \delta_\rho^i) T_i^{*(2)} + \delta_\mu^4 \delta_\nu^4 \delta_\rho^4 S_0^{*(3)}. \quad (3)$$

The detailed explanation of the symbols in Eq. (3) can be found in paper I of the present work.<sup>1</sup> The quantity  $B_{\mu\nu\rho}^{20'}$  is the baryon mixed tensor and is given by

$$B_{\{\mu\nu\}p}^{20'} = \delta_\mu^i \delta_\nu^j \delta_p^k N_{\{ijk\}l} + \frac{1}{\sqrt{2}} (\delta_\mu^i \delta_\nu^j \delta_p^4 - \delta_\rho^i \delta_\nu^j \delta_\mu^4) S_{ij}^{(1)} + \frac{1}{\sqrt{6}} (2\delta_\mu^i \delta_\nu^4 \delta_p^j + \delta_\mu^4 \delta_\nu^i \delta_p^j - \delta_\mu^j \delta_\nu^i \delta_p^4) T_{ij}^{(1)} + \frac{1}{\sqrt{2}} (\delta_\mu^i \delta_\nu^4 \delta_p^4 - \delta_\rho^i \delta_\mu^4 \delta_\nu^4) T_i^{(2)}. \quad (4)$$

Again, for the detailed explanation of the symbols see the preceding paper CJ I.<sup>1</sup> In Eq. (2) the states  $\chi_{abc}$  stand for the totally symmetric SU(2) tensor representing  $\frac{3}{2}$ -spin states normalized to 4, the dimension of the representation (see also Bég *et al.*<sup>4</sup>). We can write

$$\chi_{111} = u_{3/2}, \quad \chi_{112} = \frac{1}{\sqrt{3}} u_{1/2}, \quad \chi_{122} = \frac{1}{\sqrt{3}} u_{-1/2} \quad (5)$$

and

$$\chi_{222} = u_{-3/2},$$

where  $u_i - s$  are the  $i$ th spin states. The symbol  $\chi_i$  stands for spin- $\frac{1}{2}$  states, and is also normalized to the value 2, the total number of the states. We set

$$\chi_1 = v_{1/2} \quad \text{and} \quad \chi_2 = v_{-1/2}. \quad (6)$$

Again  $v_i - s$  are the  $i$ th spin states. In Eq. (4)  $\epsilon_{ab}$  is the Levi-Civita symbol in two dimensions.

The SU(8) classification is similar and keeps everything discussed until now unchanged. We have just to remember that in contrast to U(8), the mixed tensors  $T_\mu^\nu$  have to be traceless in SU(8).

### III. CURRENT TENSOR

We can now construct the most general mixed tensor, which we would call the current tensor, by contracting just a pair of  $B$  and  $\bar{B}$ . We find, due to the symmetry of the indices of  $B$ , it is given by

$$J_A^{A'} = \mu_0 \bar{B}^{A'BC} B_{ABC} + g_0 \delta_A^{A'} \bar{B}^{DBC} B_{DBC}, \quad (7)$$

where

$$\delta_A^{A'} = \delta_\kappa^\mu \delta_\mu^{\kappa'}. \quad (7')$$

Substituting the expression of the tensor  $B$  from Eq. (2) into Eq. (7), we obtain

$$\begin{aligned} J_A^{A'} &= J_A^{A'}(\bar{20}, 20) + J_A^{A'}(\bar{20}, 20') \\ &\quad + J_A^{A'}(\bar{20}', 20) + J_A^{A'}(\bar{20}', 20'), \end{aligned} \quad (8)$$

where  $J_A^{A'}(\bar{m}, n)$  means terms in which  $\bar{B}^m B_n$  appears with  $m$  and  $n$  being the dimensionality of the representations of U(4). The first term comes straightforwardly as

$$\begin{aligned} J_A^{A'}(\bar{20}, 20) &= \frac{1}{2} \delta_a^{\kappa'} \langle \bar{\chi} \chi \rangle_{3/2} (\mu_0 \bar{B}_{20}^{\mu\nu\rho} B_{\kappa\nu\rho}^{20} + 2g_0 \delta_\kappa^\mu \langle \bar{B} B \rangle_{20}) \\ &\quad + \frac{1}{2} \bar{\sigma}_a^{\kappa'} \cdot \langle \bar{\chi} \bar{\sigma} \chi \rangle_{3/2} \mu_0 \bar{B}_{20}^{\mu\nu\rho} B_{\kappa\nu\rho}^{20}. \end{aligned} \quad (9)$$

To obtain the final form of the equation (9), we have used the abbreviations

$$\langle \bar{\chi} \chi \rangle_{3/2} = \bar{\chi}^{abc} \chi_{abc}, \quad (10)$$

$$\langle \bar{B} B \rangle_{20} = \bar{B}_{20}^{\alpha\nu\rho} B_{\alpha\nu\rho}^{20}, \quad (11)$$

and

$$\langle \bar{\chi} \bar{\sigma} \chi \rangle_{3/2} = \bar{\chi}^{abc} \bar{\sigma}_a^d \chi_{abc}. \quad (12)$$

They have been introduced by using the following identity:

$$\bar{\sigma}_b^a \cdot \bar{\sigma}_a^c = 2\delta_b^c - \delta_b^a \delta_a^c. \quad (13)$$

In actuality we have expressed  $J_A^{A'}(\bar{20}, 20)$  in terms of two terms, the first part being the  $(\underline{15} + \underline{1}, 1)$  member of a  $\underline{63}$  representation and the second being the  $(\underline{15}, 3)$  member of the same.

Using the symmetry relation mentioned in CJ I [Eqs. (14)–(17) there], we find

$$J_A^{A'}(\overline{20}', 20') = \frac{1}{3}\delta_a^a \langle \bar{\chi}\chi \rangle_{1/2} [\mu_0 (\overline{B}_{20}^{\{\mu\rho\}\nu} B_{\{\kappa\rho\}\nu}^{20'} - \overline{B}_{20}^{\{\rho\nu\}\mu} B_{\{\kappa\rho\}\nu}^{20'}) + 2g_0 \delta_\kappa^\mu (\overline{B}_{20}^{\{\alpha\rho\}\nu} B_{\{\alpha\rho\}\nu}^{20'} - \overline{B}_{20}^{\{\rho\nu\}\alpha} B_{\{\alpha\rho\}\nu}^{20'})] - \frac{\mu_0}{9} \bar{\sigma}_b^a \cdot \langle \bar{\chi}\bar{\sigma}\chi \rangle_{1/2} [\overline{B}_{20}^{\{\mu\rho\}\nu} B_{\{\kappa\rho\}\nu}^{20'} + 5\overline{B}_{20}^{\{\rho\nu\}\mu} B_{\{\kappa\rho\}\nu}^{20'}], \quad (14)$$

where

$$\langle \bar{\chi}\chi \rangle_{1/2} = \bar{\chi}^a \chi_a \quad \text{and} \quad \langle \bar{\chi}\bar{\sigma}\chi \rangle_{1/2} = \bar{\chi}^a \bar{\sigma}_a^b \chi_b. \quad (14')$$

We should notice that the second term in (14) does not have any  $g_0$  dependence. Such a term vanishes because of the tracelessness of  $\bar{\sigma}$ .

For the cross currents we find

$$J_A^{A'}(\overline{20}, 20') = \frac{\mu_0}{3} \bar{\sigma}_a^a \cdot \langle \bar{\chi}_{3/2} \bar{\sigma} \chi_{1/2} \rangle \overline{B}_{20}^{\mu\nu\rho} B_{\{\kappa\nu\}\rho}^{20'} \quad (15)$$

and

$$J_A^{A'}(\overline{20}', 20) = \frac{\mu_0}{3} \bar{\sigma}_a^a \cdot \langle \bar{\chi}_{1/2} \bar{\sigma} \chi_{3/2} \rangle \overline{B}_{20}^{\{\mu\nu\}\rho} B_{\kappa\nu\rho}^{20'}. \quad (16)$$

In the equations (15) and (16), we have used the abbreviations

$$\langle \bar{\chi}_{3/2} \bar{\sigma} \chi_{1/2} \rangle = \bar{\chi}^{abc} \bar{\sigma}_a^d \epsilon_{ab} \chi_c \quad (17)$$

and

$$\langle \bar{\chi}_{1/2} \bar{\sigma} \chi_{3/2} \rangle = \epsilon^{abc} \bar{\chi}^d \bar{\sigma}_a^d \chi_{abc}. \quad (18)$$

It is worthwhile to notice that both  $J_A^{A'}(\overline{20}, 20')$  and  $J_A^{A'}(\overline{20}', 20)$  are traceless. On the other hand, only the second terms of the currents  $J_A^{A'}(\overline{20}, 20)$  and  $J_A^{A'}(\overline{20}', 20')$  are traceless. We should not be surprised by this outcome because we are expressing U(8) multiplets in terms of the U(4) and the SU(2) contents and since expressions like  $\bar{\sigma}$  are traceless, we are automatically led to such results. More generally, however, out of Eq. (7) we get the condition of tracelessness as

$$\mu_0 + 8g_0 = 0. \quad (19)$$

It is worth mentioning at this stage that in  $J_A^{A'}(\overline{20}', 20')$  [Eq. (14)] the first term, which includes the (15, 1) member of a  $\underline{63}$  representation, contains only  $F$ -type current, whereas the second term, which is a (15, 3) member, contains both  $D$  and  $F$  types of currents for the baryon octets [in SU(3) sense] whose ratio is  $\frac{3}{2}$ . This follows directly from the expansion of the terms within the square brackets.

#### IV. MAGNETIC MOMENTS

As Bég, Lee, and Pais<sup>4</sup> have done, we make the assumption that the magnetic-moment operator transforms like the (15, 3) member of a  $\underline{63}$  representation of U(8). Under this assumption, the effective low-frequency limit of the electromagnetic vertex of the baryons may be written as

$$V = J_A^{A'}(e_0 \phi \delta_a^d + m_0 \bar{\sigma}_a^d \cdot \bar{\mathbf{n}}) Q_\mu^K, \quad (20)$$

where  $e_0$  and  $m_0$  are two arbitrary constants and  $\bar{\mathbf{n}} = \bar{\mathbf{q}} \times \bar{\boldsymbol{\epsilon}}$ ,  $\bar{\mathbf{q}}$  being the momentum of the baryon and  $\bar{\boldsymbol{\epsilon}}$  is a polarization vector perpendicular to  $\bar{\mathbf{q}}$ . We have intentionally kept the constants  $e_0$ ,  $m_0$ ,  $\mu_0$ , and  $g_0$  (the last two appearing in  $J_A^{A'}$ ) arbitrary. In Eq. (20),  $\phi$  is the electrostatic potential.

If we write the component of Eq. (20) which contributes to the magnetic moments as  $M(\bar{m}, n)$  between the  $n$ - and  $m$ -dimensional representations belonging to U(4), we find

$$M(\overline{20}, 20) = \mu_0 m_0 \bar{\mathbf{n}} \cdot \langle \bar{\chi} \bar{\sigma} \chi \rangle_{3/2} \overline{B}_{20}^{\mu\nu\rho} B_{\kappa\nu\rho}^{20'} Q_\mu^K, \quad (21)$$

$$M(\overline{20}', 20') = -\frac{2}{9} \mu_0 m_0 \bar{\mathbf{n}} \cdot \langle \bar{\chi} \bar{\sigma} \chi \rangle_{1/2} \times (\overline{B}_{20}^{\{\mu\rho\}\nu} B_{\{\kappa\rho\}\nu}^{20'} + 5\overline{B}_{20}^{\{\rho\nu\}\mu} B_{\{\kappa\rho\}\nu}^{20'}) Q_\mu^K, \quad (22)$$

$$M(\overline{20}, 20') = \frac{2}{3} \mu_0 m_0 \bar{\mathbf{n}} \cdot \langle \bar{\chi}_{3/2} \bar{\sigma} \chi_{1/2} \rangle \overline{B}_{20}^{\mu\nu\rho} B_{\{\kappa\nu\}\rho}^{20'} Q_\mu^K, \quad (23)$$

and

$$M(\overline{20}', 20) = \frac{2}{3} \mu_0 m_0 \bar{\mathbf{n}} \cdot \langle \bar{\chi}_{1/2} \bar{\sigma} \chi_{3/2} \rangle \overline{B}_{20}^{\{\mu\nu\}\rho} B_{\kappa\nu\rho}^{20'} Q_\mu^K. \quad (24)$$

Let us now write, for the expectation values of the magnetic moment of a particle  $X$  between the maximum  $z$  component of the spin,  $\mu(X)$ ; that is,

$$\mu(X) = \langle X; J, J_z = J | M | X; J, J_z = J \rangle. \quad (25a)$$

For the transition magnetic moment between  $X$  and  $Y$ , assuming  $X$  belongs to the  $\underline{20}'$  representation, and  $Y$  to the  $\underline{20}$  representation, we write

$$\langle Y | \mu | X \rangle = \langle Y; \frac{3}{2}, \frac{1}{2} | M | X; \frac{1}{2}, \frac{1}{2} \rangle. \quad (25b)$$

Then we can find all the magnetic moments and the transition moments by using Eqs. (21)–(24).

##### A. $\underline{20}$ representation

We find, for the (10, 0) members,

$$\begin{aligned} \frac{1}{2} \mu(N^{*++}) &= \mu(N^{*+}) = \mu(Y^{*+}) = -\mu(N^{*-}) = -\mu(Y^{*-}) \\ &= -\mu(\Xi^{*-}) = -\mu(\Omega^-) = \mu(p) \end{aligned} \quad (26a)$$

and

$$\mu(N^{*0}) = \mu(Y^{*0}) = \mu(\Xi^{*0}) = 0. \quad (26b)$$

For the sextuplet (6, 1) members, we get

$$\frac{1}{2} \mu(C_1^{*++}) = \mu(C_1^{*+}) = \mu(S^{*+}) = \mu(p) \quad (27a)$$

and

$$\mu(C_1^{*0}) = \mu(S^{*0}) = \mu(T^{*0}) = 0. \quad (27b)$$

For the triplet (3, 2) particles, we find

$$\frac{1}{2}\mu(X_u^{*++}) = \mu(X_d^{*+}) = \mu(X_s^{*+}) = \mu(p). \quad (28)$$

Finally, for the singlet (1, 3), we have

$$\frac{1}{2}\mu(R^{*++}) = \mu(p). \quad (29)$$

We can easily see that the magnetic moments satisfy the relation

$$\mu_{20}(X) = Q_X \mu(p). \quad (30)$$

#### B. $\underline{20}'$ representation

For the baryon octets (8, 0), we get

$$\begin{aligned} -\frac{3}{2}\mu(n) = \mu(\Sigma^+) &= -\frac{3}{2}\mu(\Xi^0) = -3\mu(\Sigma^-) = -3\mu(\Xi^-) \\ &= -\mu(\Delta^0) = 3\mu(\Sigma^0) = \mu(p). \end{aligned} \quad (31)$$

We find for the sextuplet (6, 1) particles

$$\frac{2}{3}\mu(C_1^{*+}) = -\frac{3}{2}\mu(C_1^0) = -\frac{3}{2}\mu(S^0) = -\frac{3}{2}\mu(T^0) = \mu(p) \quad (32a)$$

and

$$\mu(C_1^+) = \mu(S^+) = 0 \quad (32b)$$

For the contragredient triplet ( $\bar{3}$ , 1), the moments are given by

$$\mu(C_0^+) = \mu(A^+) = \mu(\Lambda^0) = \frac{2}{3}\mu(p). \quad (33)$$

For the triplet (3, 2), we have

$$\frac{3}{2}\mu(X_u^{*+}) = \mu(X_d^+) = \mu(X_s^+) = \mu(p). \quad (34)$$

Finally, the nonvanishing transition moments within this multiplet, with spin states  $J_z = \frac{1}{2}$ , are

$$\begin{aligned} \langle \Sigma^0 | \mu | \Delta^0 \rangle &= -\langle C_1^+ | \mu | C_0^+ \rangle = \langle S^+ | \mu | A^+ \rangle \\ &= \frac{1}{3}\mu(p). \end{aligned} \quad (35)$$

For the above transition moments the following relation is also satisfied:

$$\langle X | \mu | Y \rangle = \langle Y | \mu | X \rangle. \quad (35')$$

#### C. Transition moments between $\underline{20}'$ and $\underline{20}$ representations

The transition moments between the (8, 0) and (10, 0) members are given by<sup>9</sup>

$$\begin{aligned} \langle N^{*+} | \mu | p \rangle &= -\langle Y^{*+} | \mu | \Sigma^+ \rangle = \langle N^{*0} | \mu | n \rangle = 2\langle Y^{*0} | \mu | \Sigma^0 \rangle \\ &= -\frac{\sqrt{3}}{2}\langle Y^{*0} | \mu | \Delta^0 \rangle = -\langle \Xi^{*0} | \mu | \Xi^0 \rangle = \frac{2\sqrt{2}}{3}\mu(p). \end{aligned} \quad (36)$$

It also comes out very easily, as in (35'), that

$$\langle Y | \mu | X \rangle = \langle X | \mu | Y \rangle. \quad (37)$$

All other transition moments between the (8, 0) and the (10, 0) members are zero.

The only nonvanishing moments between the triplet, ( $\bar{3}$ , 1), of the  $\underline{20}'$  representation and the sextuplet, (6, 1), of the  $\underline{20}$  representation are

$$\langle C_1^{*+} | \mu | C_0^+ \rangle = -\langle S^{*+} | \mu | A^+ \rangle = \frac{2}{3}\mu(p). \quad (38)$$

Equation (37) is also valid for such transitions.

#### V. CONCLUDING REMARKS

We have started with a general group U(8) and classified the particles in terms of U(4)  $\otimes$  SU(2) specifications, where U(4) gives us the internal symmetry and SU(2) incorporates the intrinsic spin of the particles. Then, assuming that the magnetic-moment operator transforms as the (15, 3) member of a  $\underline{63}$  representation, we obtained the magnetic moments of all the baryons in terms of the magnetic moments of proton  $\mu(p)$ . The SU(3) decouplet and octet magnetic moments come out in the present case to be the same as those of the SU(6) calculation of Bég, Lee, and Pais.<sup>4</sup> We have also determined the magnetic moments of the charmed baryons. These moments would be useful if such particles exist.

It is worthwhile to mention that the transition from U(8) to SU(8) can be achieved by replacing  $g_0$  from the moment expressions with Eq. (19). A glance through Eqs. (21)–(24) reveals that the  $M$  operators do not depend on  $g_0$  at all. Hence the results obtained are also valid for SU(8).

It would be quite interesting to see whether the introduction of colored quarks will have any influence on the magnetic moments. This would be something worth studying, because in the real world we need to introduce them to incorporate the correct statistics for the baryons. This point is now being pursued by the authors.

#### ACKNOWLEDGMENT

The authors would like to express their deepest gratitude to Professor L. C. Biedenharn, Jr. of Duke University for encouragement and for helping them to crystalize some of the ideas of the paper. The first author (A.L.C.) also expresses his gratitude to the Department of Physics, Univ. of North Carolina-Chapel Hill, North Carolina, for allowing him to use the facilities of the department during the summer months. He is also particularly grateful to Professor J. W. Straley, Professor E. Merzbacher, and Professor H. van Dam for many helpful cooperations.

\*Work partially supported by NASA, Langley, Virginia.

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<sup>9</sup>It should be pointed out here that in contrast to the results of Bég *et al.* (Ref. 4) Eq. (36) contains extra negative signs in front of the expressions

$$\frac{3}{2} \langle Y^*0 | \mu | \Delta^0 \rangle \text{ and } \langle \Xi^*0 | \mu | \Xi^0 \rangle.$$