Magnetic moments of charmed baryons. I

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The magnetic moments of all baryons belonging to the totally symmetric $\underline{20}$ representation, the mixed symmetry $\underline{20}'$ representation, and the totally antisymmetric $\underline{4}$ representation have been compared under U(4) symmetry. For the $\underline{20}'$ representation the usual results of SU(3) symmetry remain unchanged except the relation $\mu(\Delta^0) = -\mu(\Sigma^0)$. This result is not valid in U(4). The magnetic moments of all charmed particles have been expressed in terms of the moments of proton, neutron, and Λ particles in the case of U(4) symmetry.

I. INTRODUCTION

The recent discovery of the ψ particles[1] has lead to renewed interest in introducing a new quark c with charmed quantum number 1, in addition to the three quarks: a pair u and d, an isotopic-spin doublet, and s, an isotopic-spin singlet with charm assignment zero. The ψ particle is assumed to be the quark-antiquark, $c\bar{c}$ combination and a vector meson with associated quantum numbers, $J^{PG} = 1^{--}$. The introduction of c as an additional quark, with isotopic spin $I = 0$, and hypercharge $Y = -\frac{2}{3}$, has lead to renewed interest in extending the SU(3) symmetry classification to U(4) or SU(4), a theory in which four quarks have to play the fundamental role in the reproduction of the other particles that appear in nature.

The SU(4) group[2] for the classification of the fundamental particles has been suggested earlier by Glashow, Iliopoulos, and Maiani,[3] to eliminate the strangeness nonconserving neutral currents. In this model, a large number of new charmed particles show up, in addition to the usual noncharmed particles. In strong interaction, the charm quantum number is supposed to be strictly conserved. As soon as the discovery of the ψ particle was announced, a large number of explanations on the nature of the particle were proposed.[4] Based on the SU(4) model Borchardt, Mathur, and Okubo[5] proposed that the ψ particle is $c\bar{c}$ state and suggested an extension of the Gell-Mann–Nishijima formula as follows:

$$Q = I_z + \tfrac{1}{2}Y + C. \quad (1)$$

The quantum number C in (1) stands for the charm of the particles, which is assumed to be zero for all particles known until now. In the quark model, for the u, d, and s quarks we assume $C = 0$, and we assume that the C value of the fourth quark is 1. The assignment of the other quantum numbers is to be done according to the fractional scheme of Glashow, Iliopoulos, and Maiani.[3,6] We also assume here that all the baryons are obtained by the combination of qqq, where q is a quark.

A large number of experimental and theoretical works since then have been carried out. Assuming that the U(4) classification has a promising future, we have decided to calculate the magnetic moments of the baryons in light of U(4) symmetry.

The calculation of the magnetic moments for the SU(3) baryon octets has been done by Glashow and Coleman.[7] Okubo[8] has also shown that the same results can be reproduced from a very general consideration of the transformation property of the charge operator. Afterwards, Okubo[9] indicated how the higher-order correction to the moments, assuming a more complex magnetic-moment operator, could be obtained. A more general formulation can also be found in the paper by Rosen,[10] who gave a closed form of the magnetic-moment operator in terms of the U spin and the charge of the particles.

All these calculations have been very much improved by extending the symmetry group to SU(6), with the inclusion of intrinsic spin of the particles. Bég, Lee, and Pais[11] assumed that the baryon octets and the decouplets belong to the $\underline{56}$-dimensional representation of SU(6) and the magnetic-moment operator transforms like a $(\underline{8}, \underline{3})$ number of the $\underline{35}$ representation. Their results gave the ratio $\mu(p)/\mu(n) = -\frac{3}{2}$. Thirring[12] also obtained the same results straightforwardly from the quark model by vectorially adding the magnetic moments of the quarks which constitute the particle concerned.

In a sequence of two papers we would like first to extend the calculations of SU(3) to that of U(4). Then, enlarging the group to U(8) or SU(8), we would like to find the magnetic moments of the charmed particles. In this paper, we restrict ourselves to the smaller group U(4). In Sec. II, we discuss the U(4) classification and introduce the conventional nomenclatures. We also write down the corresponding baryon states with the help of a tensor $B_{\mu\nu\rho}$.[13] In Sec. III, we construct very general currents,[14] which appear in a magnetic-moment tensor. In Sec. IV, we obtain the magnetic moments, assuming U(4) symmetry. In Sec. V, we discuss the general aspects of the results obtained.

II. U(4) CLASSIFICATION OF BARYONS

The baryons are to be obtained by the combination of three quarks qqq. In terms of the irreducible representation of U(4), we know

$$\underline{4}\otimes\underline{4}\otimes\underline{4}=\underline{20}+2\times\underline{20'}+4, \tag{2}$$

where $\underline{20}$ is the dimension of the completely symmetric representation, $\underline{20'}$ is the dimension of the representation of mixed symmetry, and $\underline{4}$ is the dimension of the totally antisymmetric representation.

On the other hand, if we express the $\underline{20}$ multiplet in terms of $SU(3)\otimes U_c$ indices, we find

$$\underline{20}=(10,0)+(6,1)+(3,2)+(1,3), \tag{3}$$

where (m,n) indicates the m-dimensional SU(3) multiplet with a charm quantum number n. Similarly for $\underline{20'}$, we get

$$\underline{20'}=(8,0)+(6,1)+(\bar{3},1)+(3,2) \tag{4}$$

In the equation (4), $(\bar{3},1)$ represents the contragredient triplet state with charm quantum number 1. The multiplet $\underline{4}$ is given by

$$4=(\bar{3},1)+(1,0). \tag{5}$$

The baryon wave function for the $\underline{20}$ representation can be expressed in terms of the $\overline{U}(4)$ indices by a totally symmetric tensor $B^{20}_{\mu\nu\rho}$ which is normalized to the number of particles in the multiplet. The tensor $B^{20}_{\mu\nu\rho}$ can be expressed in terms of the SU(3) multiplets as follows:

$$B^{20}_{\mu\nu\rho}=\delta^i_\mu\delta^j_\nu\delta^k_\rho d_{ijk}+\frac{1}{\sqrt{3}}(\delta^i_\mu\delta^j_\nu\delta^4_\rho+\delta^4_\mu\delta^i_\nu\delta^j_\rho+\delta^i_\mu\delta^4_\nu\delta^j_\rho)S^{*(1)}_{ij}+\frac{1}{\sqrt{3}}(\delta^i_\mu\delta^4_\nu\delta^4_\rho+\delta^4_\mu\delta^i_\nu\delta^4_\rho+\delta^4_\mu\delta^4_\nu\delta^i_\rho)T^{*(2)}_i+\delta^4_\mu\delta^4_\nu\delta^4_\rho S^{*(3)}_0. \tag{6}$$

In the above expression d_{ijk} represents the decouplets of SU(3) and the indices i, j, and k only run from 1 through 3, whereas μ, ν, and ρ run from 1 through 4. $S^{*(1)}_{ij}$ represents the sextuplet charm-1 baryons and is totally symmetric in i and j. The states $T^{*(2)}_i$ are the SU(3) triplets, with charm quantum number 2. The state $S^{*(3)}_0$ is the (1, 3) state of the equation (3).[15]

Similarly, we find for $\underline{20'}$

$$B^{20'}_{\mu\nu\rho}=\delta^i_\mu\delta^j_\nu\delta^k_\rho N_{\{ik\}j}+\frac{1}{\sqrt{2}}(\delta^i_\mu\delta^j_\nu\delta^4_\rho-\delta^i_\rho\delta^j_\nu\delta^4_\mu)S^{(1)}_{ij}+\frac{1}{\sqrt{6}}(2\delta^i_\mu\delta^4_\nu\delta^j_\rho+\delta^4_\mu\delta^i_\nu\delta^j_\rho-\delta^j_\mu\delta^i_\nu\delta^4_\rho)T^{(1)}_{\{ij\}}+\frac{1}{\sqrt{2}}(\delta^i_\mu\delta^4_\nu\delta^4_\rho-\delta^i_\rho\delta^4_\mu\delta^4_\nu)T^{(2)}_i. \tag{7}$$

In the above expression, whenever we have put two indices within the curly brackets the terms are antisymmetric with respect to those indices. Here again the Greek indices run from 1 through 4, whereas the Latin indices run from 1 through 3. The symbols $N_{\{ik\}j}$, $S^{(1)}_{ij}$, $T^{(1)}_{\{ij\}}$, and $T^{(2)}_i$ stand for a baryon octet, a sextuplet with charm 1, a contragredient triplet with charm 1, and a triplet with charm 2, respectively. The baryon octet can be expressed in terms of the well-known symbol N^i_j by the formula

$$N_{\{ik\}j}=\frac{1}{\sqrt{2}}\epsilon_{ikl}N^l_j. \tag{7'}$$

Here ϵ_{ijk} is the Levi-Civita totally antisymmetric tensor in three dimensions.

Finally, for the baryon $\underline{4}$ multiplet, we can write

$$B^4_{\{\mu\nu\rho\}}=\frac{1}{\sqrt{3}}(\delta^i_\mu\delta^j_\nu\delta^4_\rho+\delta^4_\mu\delta^i_\nu\delta^j_\rho-\delta^i_\mu\delta^4_\nu\delta^j_\rho)T^{(1)\prime}_{\{ij\}}+\frac{1}{\sqrt{6}}\delta^i_\mu\delta^j_\nu\delta^k_\rho\epsilon_{ijk}S^{0\prime} \tag{8}$$

where $T^{(1)\prime}_{\{ij\}}$ stands for the charm-1 contragredient triplet and $S^{0\prime}$ is the singlet with charm zero.

Here we would exclusively follow the identification of the particles as introduced by Gaillard, Lee, and Rosner.[2] Thus, for the sextuplet belonging to the $\underline{20}$ representation, we define

$$S^{*(1)}_{11}=C^{*++}_1,\quad S^{*(1)}_{12}=\frac{C^{*+}_1}{\sqrt{2}},\quad S^{*(1)}_{22}=C^{*0}_1,$$

$$S^{*(1)}_{13}=\frac{S^{*+}}{\sqrt{2}},\quad S^{*(1)}_{23}=\frac{S^{*0}}{\sqrt{2}}, \tag{9a}$$

and

$$S^{*(1)}_{33}=T^{*0}.$$

For the triplet members, (3, 2), of the $\underline{20}$ representation, we identify the particles as

$$T^{*(2)}_1=X^{*++}_u,\quad T^{*(2)}_2=X^{*+}_d,\ \text{and}\ T^{*(2)}_3=X^{*+}_s. \tag{9b}$$

The new baryons belonging to the $\underline{20'}$ representation, the members of the sextuplet $S^{(1)}_{ij}$ [(6, 1)], are given by the equations (9a) just by dropping the asterisks. Similarly the multiplets (3, 2) of the 20′ representation are given by the equations (9b), where we also have to drop the asterisks. For the multiplet $(\overline{3}, 1)$, we write

$$T^{(1)}_{\{12\}} = \frac{1}{\sqrt{2}} C_0^+, \quad T^{(1)}_{\{23\}} = \frac{1}{\sqrt{2}} A^0, \quad T^{(1)}_{\{31\}} = \frac{1}{\sqrt{2}} A^+. \tag{9c}$$

For $(\overline{3}, 1)$ belonging to 4, we go over to the particles by adding a prime to the relations in the equations (9c).

III. MOST-GENERAL CURRENTS

In calculating the magnetic moment coming from the first-order electromagnetic interaction, we need to construct the most general current tensors J^μ_κ (Ref. 14) with the help of the multiplet wave functions B given by the equations (6), (7), and (8). For the 20 representation, the most general current that we can form is

$$J(D) = \mu_0 \overline{B}^{\mu\nu\rho}_{20} B^{20}_{\kappa\nu\rho} + g_0' \delta^\mu_\kappa \langle \overline{B} B\rangle_{20}, \tag{10}$$

where μ_0 and g_0' are two arbitrary constants and $\langle \overline{B} B\rangle_{20}$ stands for the trace of the tensor defined as follows:

$$\langle \overline{B} B\rangle_{20} = \overline{B}^{\mu\nu\rho}_{20} B^{20}_{\mu\nu\rho}. \tag{11}$$

The most general current $J(N)^\mu_\kappa$ that can be constructed with the tensors $\overline{B}$ and B belonging to the 20′ representation is given by

$$J(N)^\mu_\kappa = D^\mu_\kappa + \delta^\mu_\kappa G^\alpha_\alpha, \tag{12}$$

where

$$\begin{aligned} D^\mu_\kappa = {} & a_1 \overline{B}^{\{\mu\rho\}\nu}_{20'} B^{20'}_{\{\kappa\rho\}\nu} + a_2 \overline{B}^{\{\rho\nu\}\mu}_{20'} B^{20'}_{\{\kappa\rho\}\nu} + a_3 \overline{B}^{\{\nu\mu\}\rho}_{20'} B^{20'}_{\{\kappa\rho\}\nu} \\ & + b_1 \overline{B}^{\{\mu\rho\}\nu}_{20'} B^{20'}_{\{\rho\nu\}\kappa} + b_2 \overline{B}^{\{\rho\nu\}\mu}_{20'} B^{20'}_{\{\rho\nu\}\kappa} + b_3 \overline{B}^{\{\nu\mu\}\rho}_{20'} B^{20'}_{\{\kappa\rho\}\nu} \\ & + c_1 \overline{B}^{\{\mu\rho\}\nu}_{20'} B^{20'}_{\{\nu\kappa\}\rho} + c_2 \overline{B}^{\{\rho\nu\}\mu}_{20'} B^{20'}_{\{\nu\kappa\}\rho} + c_3 \overline{B}^{\{\nu\mu\}\rho}_{20'} B^{20'}_{\{\nu\kappa\}\rho} \end{aligned} \tag{13}$$

and G^α_α is an expression obtained from D^μ_κ by setting $\mu = \kappa = \alpha$ and replacing the constants a_i, b_i, and $c_i - s$ by new constants a_i', b_i', and $c_i' - s$.

By using the symmetry relations

$$B^{20'}_{\{\mu\nu\}\rho} + B^{20'}_{\{\nu\rho\}\mu} + B^{20'}_{\{\rho\mu\}\nu} = 0, \tag{14}$$

$$B^{20'}_{\{\mu\nu\}\rho} = - B^{20'}_{\{\nu\mu\}\rho}, \tag{15}$$

$$\overline{B}^{\{\rho\nu\}\mu}_{20'} B^{20'}_{\{\rho\nu\}\kappa} = -2 \overline{B}^{\{\rho\nu\}\mu}_{20'} B^{20'}_{\{\kappa\rho\}\nu}, \tag{16}$$

$$\overline{B}^{\{\mu\rho\}\nu}_{20'} B^{20'}_{\{\rho\nu\}\kappa} = \overline{B}^{\{\rho\nu\}\mu}_{20'} B^{20'}_{\{\kappa\rho\}\nu}, \tag{17}$$

we can simplify Eq. (12) into a very convenient form. We should notice that Eqs. (16) and (17) are the outcome of Eqs. (14) and (15). We finally get

$$J(N)^\mu_\kappa = \mu_x \overline{B}^{\{\mu\rho\}\nu}_{20'} B^{20'}_{\{\kappa\rho\}\nu} + \mu_y \overline{B}^{\{\rho\nu\}\mu}_{20'} B^{20'}_{\{\kappa\rho\}\nu} + g_x \delta^\mu_\kappa \overline{B}^{\{\alpha\rho\}\nu}_{20'} B^{20'}_{\{\alpha\rho\}\nu} + g_y \delta^\mu_\kappa B^{\{\rho\nu\}\alpha}_{20'} B^{20'}_{\{\alpha\rho\}\nu}. \tag{18}$$

Subsequently, we use the abbreviation

$$g_0 = g_x - \tfrac{1}{2} g_y. \tag{19}$$

to express the magnetic moments of the baryons in terms of the constants μ_x, μ_y, g_x, and g_y.

Similarly, for the representation 4, which is totally antisymmetric, the current tensor is

$$J(A)^\mu_\kappa = \mu_1 \overline{B}^{\{\mu\nu\rho\}}_4 B^4_{\{\kappa\nu\rho\}} + g_1 \delta^\mu_\kappa \langle \overline{B} B\rangle_4, \tag{20}$$

where again μ_1 and g_1 are two arbitrary constants and

$$\langle \overline{B} B\rangle_4 = \overline{B}^{\{\mu\nu\rho\}}_4 B^4_{\{\mu\nu\rho\}}. \tag{21}$$

In the next section we use the currents in Eqs. (10), (18), and (20) to obtain the magnetic moments.

IV. MAGNETIC MOMENTS IN U(4)

As shown by Glashow and Coleman,[3] the magnetic moment for baryons can be obtained by forming the trace of $\overline{B}BQ$ in all possible contractions, where Q is the charge operator, a 4×4 matrix, and can be written as

$$Q^\kappa_\mu = q_\mu \delta^\kappa_\mu, \tag{22}$$

where $q_1 = q_4 = \frac{2}{3}$ and $q_2 = q_3 = -\frac{1}{3}$ are the charges of the quarks. We follow the prescription of Okubo *et al.*[5] for the charge operator as given by the equation (1).

We assume that for a low-momentum case the expectation value of the magnetic-moment operator is proportional to

$$J(X)^\mu_\kappa Q^\kappa_\mu, \tag{23}$$

where X stands for either D, N, or A. We have altogether suppressed the spin part of the magnetic moment as a first approximation.

A. 20 representation

For (10, 0) members of the 20 representation, we find, using Eqs. (6), (10), and (23), that

$$\begin{aligned} &\mu(N^{*++}) = \tfrac{2}{3}\mu_0 + \tfrac{2}{3} g_0', \quad \mu(N^{*+}) = \tfrac{1}{3}\mu_0 + \tfrac{2}{3} g_0', \\ &\mu(N^{*0}) = \tfrac{2}{3} g_0', \quad \mu(N^{*-}) = -\tfrac{1}{3}\mu_0 + \tfrac{2}{3} g_0', \end{aligned} \tag{24}$$

and

$$\mu(Y^{*+}) = \mu(N^{*+}), \tag{25}$$

$$\mu(Y^{*0}) = \mu(\Xi^{*0}) = \mu(N^{*0}), \tag{26}$$

$$\mu(Y^{*-}) = \mu(\Xi^{*-}) = \mu(\Omega^-) = \mu(N^{*-}). \tag{27}$$

For the sextuplet (6, 1)

$$\begin{aligned} &\mu(C_1^{*++}) = \mu(N^{*++}), \quad \mu(C_1^{*+}) = \mu(S^{*+}) = \mu(N^{*+}), \\ &\mu(C_1^{*0}) = \mu(S^{*0}) = \mu(T^{*0}) = \mu(N^{*0}) \end{aligned} \tag{28}$$

Similarly, for the triplet (3, 2)

$$\mu(X_u^{*++}) = \mu(N^{*++}), \quad \mu(X_d^{*+}) = \mu(X_s^{*+}) = \mu(N^{*+}). \tag{29}$$

For the singlet (1, 3) (identifying $S_0^{*(3)} = R^{*++}$), we find

$$\mu(R^{*++}) = \mu(N^{*++}). \tag{30}$$

B. 20′ representation

If we now go to the 20′ representation and use the equations (7), (18), and (23), the magnetic moments of the baryon octet (8, 0) come out to be

$$\mu(p) = \mu(\Sigma^+) = \tfrac{1}{6}\mu_x - \tfrac{1}{3}\mu_y + \tfrac{2}{3}g_0, \tag{31a}$$

$$\mu(n) = \mu(\Xi^0) = \tfrac{1}{6}\mu_x + \tfrac{1}{6}\mu_y + \tfrac{2}{3}g_0, \tag{31b}$$

$$\mu(\Xi^-) = \mu(\Sigma^-) = -\tfrac{1}{3}\mu_x + \tfrac{1}{6}\mu_y + \tfrac{2}{3}g_0, \tag{31c}$$

$$\mu(\Sigma^0) = -\tfrac{1}{12}\mu_x - \tfrac{1}{12}\mu_y + \tfrac{2}{3}g_0, \tag{31d}$$

$$\mu(\Delta^0) = \tfrac{1}{12}\mu_x + \tfrac{1}{12}\mu_y + \tfrac{2}{3}g_0. \tag{31d′}$$

The transition moments are given by

$$\langle \Sigma^0 | \mu | \Delta^0 \rangle = \langle \Delta^0 | \mu | \Sigma^0 \rangle = -\frac{1}{4\sqrt{3}}\mu_x - \frac{1}{4\sqrt{3}}\mu_y. \tag{31e}$$

If we compare these results with the U(3) results as quoted by Okubo,[8] we find the unchanged relations are

$$\mu(p) = \mu(\Sigma^+), \quad \mu(\Xi^0) = \mu(n), \quad \mu(\Xi^-) = \mu(\Sigma^-), \tag{31f}$$

$$\mu(p) + \mu(\Sigma^-) + 4\mu(n) = 6\mu(\Delta^0), \tag{31g}$$

and

$$\mu(\Sigma^+) + \mu(\Sigma^-) = 2\mu(\Sigma^0). \tag{31h}$$

However, we should notice that

$$\mu(\Sigma^0) \neq -\mu(\Delta^0) \tag{31i}$$

in U(4) in contrast to the result $\mu(\Sigma^0) = \mu(\Delta^0)$ in U(3).

For the charmed sextuplet (6, 1)

$$\mu(C_1^{++}) = \tfrac{2}{3}\mu_x - \tfrac{1}{3}\mu_y + \tfrac{2}{3}g_0, \tag{32a}$$

$$\mu(C_1^+) = \mu(S^+) = \tfrac{5}{12}\mu_x - \tfrac{1}{12}\mu_y + \tfrac{2}{3}g_0, \tag{32b}$$

and

$$\mu(C_1^0) = \mu(S^0) = \mu(T^0) = \mu(n). \tag{32c}$$

In terms of $\mu(p)$, $\mu(n)$, and $\mu(\Delta^0)$, we can write

$$\mu(C_1^{++}) = 2\mu(p) + 5\mu(n) - 6\mu(\Delta^0) \tag{32d}$$

and

$$\mu(C_1^+) = \mu(p) + 3\mu(n) - 3\mu(\Delta^0). \tag{32e}$$

For the charmed baryon triplet (3, 2), we find

$$\mu(X_u^{++}) = \mu(C_1^{++}), \tag{33a}$$

$$\mu(X_d^+) = \mu(X_1^+) = \mu(p). \tag{33b}$$

If we now go over to the contragredient baryon triplet $(\bar{3}, 1)$, we find

$$\mu(C_0^+) = \mu(A^+) = \tfrac{1}{4}\mu_x - \tfrac{1}{4}\mu_y + \tfrac{2}{3}g_0, \tag{34a}$$

$$\mu(A^0) = -\tfrac{1}{6}\mu_x - \tfrac{1}{6}\mu_y + \tfrac{2}{3}g_0. \tag{34b}$$

Expressing (34a) and (34b) in terms of proton, neutron, and Λ particle magnetic moments, we find

$$\mu(C_0^+) = \mu(p) + \mu(n) - \mu(\Delta^0) \tag{34c}$$

and

$$\mu(A^0) = -3\mu(n) + 4\mu(\Delta^0). \tag{34d}$$

Owing to the degeneracy of quantum numbers of some baryon sextuplet particles with the contragredient triplets, we find that transition magnetic moments appear between sextuplet and triplet particles. The results are

$$-\langle C_1^+|\mu|C_0^+\rangle = -\langle C_0^+|\mu|C_1^+\rangle = \langle S^+|\mu|A^+\rangle = \langle A^+|\mu|S^+\rangle = \langle \Sigma^0|\mu|\Delta^0\rangle. \tag{35}$$

C. 4 representation

For the particles belonging to the totally antisymmetric representation 4, the magnetic moments for the contragredient triplets can be found using Eqs. (8), (20), and (23). We get for the contragredient charmed triplets

$$\mu(A'^0) = \tfrac{2}{3}g_1 \tag{36a}$$

and

$$\mu(A'^+) = \mu(C_0'^+) = \tfrac{1}{3}\mu_1 + \tfrac{2}{3}g_1. \tag{36b}$$

For the singlet (1, 0) we get

$$\mu(S'^0) = \mu(A'^0). \tag{37}$$

V. CONCLUDING REMARKS

We have calculated the magnetic moments in the low-frequency limit in the U(4) symmetry, assuming that the magnetic-moment operator is proportional to the charge operator. The expectation value of the magnetic moment could thus be written as in Eq. (23). This equation finally has produced the results quoted in Sec. IV. Almost all the results of U(3) have been reproduced in U(4). The only different result U(4) yields is

$$\mu(\Sigma^0) \neq -\mu(\Delta^0).$$

We have also derived the magnetic moments of all new charmed baryons in terms of the magnetic moments of the proton, neutron, and Δ^0 particles in the $\underline{20}'$ representation and in terms of the magnetic moments of $N^* - s$ in the $\underline{20}$ representation in U(4). Since Q is not an operator in SU(4), the results cannot be extended in SU(4) symmetry. Moreover, we would like to mention here that our results would be valid for both total magnetic moments and anomalous magnetic moments for exactly the same reason that the results obtained by Coleman and Glashow[7] would be valid for both.

In the following paper we will show how the results of U(4) can be extended to the U(8) symmetry.

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[15]Actually we should indicate here that there is a certain amount of arbitrariness in choosing the constants in front of each state. If we write (with real α, β, γ, δ)

$$B^{20}_{\mu\nu\rho} = \alpha B^{10}_{\mu\nu\rho} + \beta B^{6}_{\mu\nu\rho} + \gamma B^{3}_{\mu\nu\rho} + \delta B^{1}_{\mu\nu\rho}$$

and normalize B^{10} to 10, B^6 to 6, etc., then we get

$$20 = \alpha^2 10 + \beta^2 6 + \gamma^2 3 + \delta^2 .$$

We have made the choice $\alpha = \beta = \gamma = \delta = 1$.