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# Topological expansions and decays of new particles: A phenomenology of Okubo-Zweig-Iizuka–rule violation

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Decay modes of $\psi$, $\psi'$, $\eta_c$ are examined within a model of Okubo-Zweig-Iizuka (OZI) -rule violation based on the $S$-matrix topological expansion (SMTE). The model provides an appropriate language for discussing general features of the OZI rule and successfully predicts the order of magnitude of the decrease in widths between $\phi$ and $\psi$. Because of the relatively strong mixing of $\eta$ and $\eta'$ with $\eta_c$ *necessitated* by the SMTE, these mesons play a prominent role in $\psi$, $\psi'$ decays. We readily explain the "large" $\psi' \to \eta\psi$, $\psi \to \eta'\gamma$, $\psi \to \eta\gamma$ rates and predict several more channels involving $\eta$ and $\eta'$ into which the new particles should frequently decay. We describe why there is no general concept of double-OZI-forbidden processes and successfully correlate the role the $\epsilon$ meson plays in such reactions as $\psi' \to \psi\epsilon$, $\psi \to \epsilon\omega$, $\psi \to \epsilon\phi$. The differences between $\psi$ and $\psi'$ decays expected because of the radially excited nature of the $\psi'$ are touched upon, and it is speculated that $\Gamma(\eta_c) \approx \Gamma(\phi \to \rho\pi)$. Comparison is made with other models of OZI suppression.

## I. INTRODUCTION

"What are they?" "Why are their widths so small?" Ever since November, 1974 when the $\psi$ ($J$) particles were discovered simultaneously at SLAC and at Brookhaven[1] particle physicists have been asking themselves these questions. During the past several months there has been a growing consensus in the physics community as to how these particles fit into hadron spectroscopy. The new particles are the first manifestation of another degree of freedom in hadronic physics. In keeping with tradition a fourth quark is endowed with this degree of freedom and from new quarks we construct new hadrons. We conform to this viewpoint and use the symbol $c$ to denote the new quark, but, except in one application, do not necessarily require the new quark to be charmed.

There has also been a consensus that the reason the new particles are so reluctant to decay into hadrons is that they obey the selection rule proposed by Okubo, Zweig, and Iizuka (OZI).[2] If this is the "solution" to the difficulty we are still left with the task of explaining the origin and systematics of the mysterious OZI rule. Recently such an explanation has been found[3] within the framework of the $S$-matrix topological expansion (SMTE).[4] We now propose to build upon this explanation and use it as a basis for studying the general features of the OZI rule and the decays of the new particles. The phenomenology which emerges is exceedingly simple and surprisingly successful.

In our previous work[3] answers were provided to the following questions:

1. What is the dynamical basis for the OZI rule?
2. Why is $\Gamma(\psi) \ll \Gamma(\phi \to \rho\pi)$?
3. Why do the pseudoscalar mesons exhibit a mixing pattern different from that of the vector and tensor mesons?

Using the information contained in the answers to the above we propose to try to answer the following:

1. Why do there seem to be two types of OZI suppression?[5] $\psi' \to \pi\pi\psi$ and $\psi' \to \eta\psi$ are suppressed but not nearly as much as other decay modes such as $\psi \to \rho\pi$.
2. Why is the radiative decay rate for $\psi \to \gamma\eta'$ larger than the strong decay $\psi \to \rho^0\pi^0$?
3. Is there a generally valid concept of double-OZI-forbidden decays?
4. What is the width of the pseudoscalar partner to the $\psi$, assumed to be the $\eta_c(2800)$?

In addition to providing answers to the above questions the phenomenology we espouse will lead to reasonable estimates of specific decay channels. One feature of these numbers which is especially interesting is the relatively large rates for the processes $\psi$ or $\psi' \to (\eta$ or $\eta'$ or $\eta_c)$ + anything. These decays are of great interest as possible candidates for the "missing" decay modes of $\psi$ and $\psi'$.

Any attempt at a detailed phenomenology of $\psi$, $\psi'$, $\eta_c$ decays requires specific knowledge and understanding of the properties of the "normal" hadrons into which they decay. For instance, a calculation of $\psi' \to \epsilon\psi \to \pi\pi\psi$ depends very strongly on the parameters of the $\epsilon$ meson which we use. Because such precise information is not presently available we must forego providing detailed numerical predictions and content ourselves with estimates we expect to be accurate to within factors of 2 or 3. Since our main concern here is to study the features of the OZI rule as applied to the new particles, and not the nature of conventional hadrons, we are satisfied that our rough analysis will be sufficient to confirm or disconfirm our approach[3] to the physics underlying OZI suppression. Once we feel confident that we do understand the basis for the OZI rule we can then go back, do more careful phenomenology, and learn about the nature of conventional hadrons from studying new particles. Some possible points of reentry for such a program will be indicated as we proceed.

While the model presented here originated and is deeply rooted in the $S$-matrix approach, many features of the phenomenology are more general than the specific $S$-matrix theory we advocate. There are indeed several remarkable similarities between our scheme and models[6] of asymptotic freedom based on the two-dimensional field theory of 't Hooft.[7] These similarities will be discussed in our concluding section, where we will also discuss other related work, particularly that of Harari.[8] However, before reaching conclusions we must have results. Section II will introduce the relevant concepts from Ref. 3 and will define the vocabulary (verbal and graphical) of the analysis. The study of the general transition vector $\to$ vector + pseudoscalar ($V \to VP$) commences, in Sec. III, with the comparison of $\psi$ and $\phi \to \rho\pi$ and proceeds to $\psi \to \eta\,(\eta') + \omega(\phi)$, and $\psi' \to \eta\psi$, $\omega\eta_c$. In Sec. III B we present results on decays involving $\epsilon$ mesons, and add comments on miscellaneous decay modes of $\psi, \psi'$. Radiative decays are treated in Sec. III C, and finally the decay of $\eta_c$ is mentioned in Sec. III D. Since the $\eta$ and $\eta'$ play a prominent role in our discussion their phenomenological parametrization is analyzed in the Appendix. We find that the $\eta$ and $\eta'$ contain a *relatively* large admixture of $c\bar{c}$ (see Table II) and this accounts for their enhanced importance in $\psi$ decays.

## II. PRELIMINARIES

In the beginning is the planar $S$ matrix. For our purposes planarity is defined in terms of the properties of rubber-sheet duality diagrams. Figure 1(a) is planar, Fig. 1(b) is not. Of crucial importance for us is that by *definition* planar diagrams obey the OZI selection rule exactly. The $\phi$ meson, considered as a pure $\lambda\bar{\lambda}$ state, decays into $K\bar{K}$ via a diagram such as Fig. 2(a), but cannot decay into $\rho\pi$ since the simplest such diagram for this mode, Fig. 2(b), is nonplanar. Further content to the planar $S$ matrix is provided by planar unitarity which leads to planar-bootstrap equations.[9-11]

The simplest and most important corrections to the planar $S$ matrix have the topological character of Figs. 1(a) and 2(b). They have been studied extensively (Refs. 11–14). Clearly such diagrams (cylinder diagrams in technical jargon) violate the OZI rule and must therefore be small if the theory is to correctly describe reality. Yet Fig. 1(b) is responsible at $t \le 0$ for the effect referred to as the Pomeron and hence represents a substantial deviation from planarity. The resolution to this problem was already contained in Ref. 12, where the concept of asymptotic planarity was introduced. This principle asserts that as the variable $t$ becomes more and more positive the importance of cylinders (and all higher corrections) becomes less and less. This conjecture has been verified as being a basic property of the topological expansion.[3,15]

If we call $k(t)$ the strength of the cylinder coupling which describes the importance of Fig. 1(b), the realization of asymptotic planarity is expressed as[3]

$$k(t) = \frac{k(0)}{\cosh(2t/t_c)^{1/2}}\,. \tag{2.1}$$

Since asymptotic planarity is a result of the topological properties of the cylinder, natural and unnatural parity cylinders are expected to exhibit similar functional dependence on $t$. The difference between them is in the value of $t_c$, which has been estimated as $\frac{1}{2}$ for the vector-tensor cylinder, and as ~2–3 for the pseudoscalar.[3] Actually Eq. (2.1) is true only in the exact SU(3)- [or SU(4)-] symmetry limit for $k$, and is expected to be an accurate estimate only for small $t$. It is known[3] that as $t$ gets large, the rate of decrease of $k(t)$ slackens into a power-law falloff. For SU(3), the cylinder couples only through $I=0$ states, while

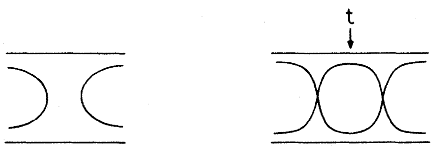


FIG. 1. (a) A typical planar duality diagram. (b) The simplest nonplanar duality diagram. This diagram and all diagrams which are topologically equivalent are known as cylinders.

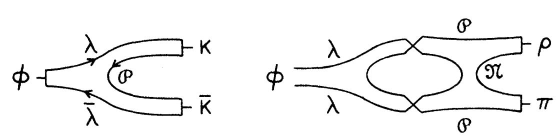


FIG. 2. (a) The planar duality diagram for the allowed decay $\phi \to K\bar{K}$. (b) The simplest diagram for the OZI-forbidden decay $\phi \to \rho\pi$.

for SU(4) it couples only with SU(3) singlets.

The simplest way to interpret the OZI-violating effects of the cylinder is via the mixing it induces amongst the quark wave functions. After turning on the cylinder the $\psi$ will no longer be a pure $c\bar{c}$ state but will have gained an admixture of $\mathcal{P}$, $\mathfrak{N}$, $\lambda$ quarks because of mixing with the $\phi$ and $\omega$. For instance, $\psi$ will now have a strange-quark component $\langle\psi|\lambda\bar{\lambda}\rangle$ given by

$$\langle\psi|\lambda\bar{\lambda}\rangle = \frac{k(m_\psi^2)}{\alpha_\phi(m_\psi^2) - \alpha_\psi(m_\psi^2)} = \frac{k(m_\psi^2)}{\alpha'(m_\psi^2 - m_\phi^2)} \equiv \frac{k(\psi)}{\Delta_{\psi\phi}} . \qquad (2.2)$$

The second equality follows from the (henceforth standard) assumption of parallel linear trajectories. The numerator reflects the strength of the cylinder coupling at $m_\psi^2$, while the denominator takes into account the familiar feature that the closer together the two levels are, the stronger the tendency to mix when perturbed. There will be sources of $|\lambda\bar{\lambda}\rangle$ in the $\psi$ wave function other than the mixing involving $\phi$. Daughters (radial excitations) of $\phi$ will also make contributions. For the phenomenological applications at hand we shall ignore such additional mixing. To the extent that our theoretical prejudices regarding the expected small overlap between different radial excitations is true our attitude is justified. Inevitably, when discussing the $\psi'$, which we consider a daughter of $\psi$, we will have to confront in some way the transitions and couplings between different radial excitations. In the interests of economy we adopt the phenomenologically simple expedient of introducing only one new parameter to describe couplings involving odd numbers of radial excitations.

The ansatz we adopt, consistent with the general features of the topological expansion, is that all $\psi$ decays proceed through planar diagrams, and that OZI-rule violation occurs because of quark mixing. Thus the coupling constant for the decay $A\to BC$, Fig. 3, is given by

$$g_{A\to BC} = g\sum_{q_i q_j}\langle A|q_i\bar{q}_i\rangle\langle Bq_i\bar{q}_j\rangle\langle C|q_j\bar{q}_i\rangle , \qquad (2.3)$$

with the overlap in quark space given by formulas such as (2.2). (For more details see Chew and Rosenzweig, Ref. 12, especially Appendix A, and Schmid and Sorensen, Ref. 13.)

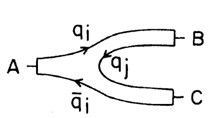


FIG. 3. Allowed coupling between particle $A$ and particles $B$ and $C$.

This ansatz is in fact the simplest and most naive that one can think of. It is gratifying that it arises from and can be justified by the SMTE. It is even more gratifying that a phenomenology based on such a simple model works so well.

Content and predictive power are injected into the ansatz when the properties of $k(t)$ as determined by the topological expansion are used to compute mixing and coupling constants. Thus knowledge of the masses and couplings of conventional hadrons, when supplemented by knowledge of the masses of the new particles, allows us to predict the couplings, and hence the decays of these new particles.

For convenience we have compiled a table giving the quark content of the $I=0$ mesons appearing in our discussions. The angles $\Omega$ used in Table I are the octet-singlet mixing angles in terms of which

$$|\eta\rangle = \cos\Omega_\eta|8\rangle + \sin\Omega_\eta|1\rangle ,$$
$$|X\rangle = -\sin\Omega_X|8\rangle + \cos\Omega_X|1\rangle ,$$

where $X$ is either $\epsilon$ or $\eta'$. Table I provides the clue as to why the naive concept of double-OZI-forbiddenness (see Fig. 4) as applied to, e.g., $\psi\to\pi\pi\phi$ may not be useful. If such processes proceed through an intermediate resonance (e.g., $\epsilon$) which is strongly mixed with either $|\lambda\rangle$ or $|c\rangle$ the $\phi$ and $\omega$ modes will be comparable. Typically, neglecting symmetry breaking and assuming $\chi$ is an SU(3) singlet

$$\Gamma(\psi\to\phi\chi) \simeq \tfrac{1}{2}\Gamma(\psi\to\omega\chi) ,$$

which follows from the approximate relations

TABLE I. The quark composition of the $I=0$ mesons which are used in our study. All symbols, except for $|b\rangle \equiv (|\mathfrak{N}\bar{\mathfrak{N}}\rangle + |\mathcal{P}\bar{\mathcal{P}}\rangle)/\sqrt{2}$, are defined in the text.

| Particle \ Quark | $\lvert b\rangle$ | $\lvert\lambda\rangle$ | $\lvert c\rangle$ |
|---|---|---|---|
| $\omega$ | 1 | $\frac{-\sqrt{2}\,k(\omega)}{\Delta_{\phi\omega}}$ | $\frac{-\sqrt{2}\,k(\omega)}{\Delta_{\psi\omega}}$ |
| $\phi$ | $\frac{\sqrt{2}\,k(\phi)}{\Delta_{\phi\omega}}$ | 1 | $\frac{-k(\phi)}{\Delta_{\psi\phi}}$ |
| $\psi$ | $\frac{\sqrt{2}\,k(\psi)}{\Delta_{\psi\omega}}$ | $\frac{k(\psi)}{\Delta_{\psi\phi}}$ | 1 |
| $\eta$ | $\cos\theta_\eta$ | $-\sin\theta_\eta$ | $\frac{-\sqrt{3}\,k(\eta)\sin\Omega_\eta}{\Delta_{\eta_c\eta}}$ |
| $\eta_c$ | $\frac{\sqrt{2}\,k(\eta_c)}{\Delta_{\eta_c\eta}}$ | $\frac{k(\eta_c)}{\Delta_{\eta_c\eta'}}$ | 1 |
| $\epsilon$ | $\cos\theta_\epsilon$ | $\sin\theta_\epsilon$ | $\frac{\sqrt{3}\,k(\epsilon)\cos\Omega_\epsilon}{\Delta\epsilon_c\epsilon}$ |
| $\eta'$ | $\sin\theta_{\eta'}$ | $\cos\theta_{\eta'}$ | $\frac{-\sqrt{3}\,k(\eta')\cos\Omega_{\eta'}}{\Delta_{\eta_c\eta'}}$ |

(neglecting small variations in $\Delta$ and $k$ due to $m_\phi^2 > m_\omega^2$) of Table I

$$\langle\psi|\lambda\lambda\rangle \simeq \frac{1}{\sqrt{2}}\langle\psi|b\rangle\,, \quad \langle\phi|c\bar{c}\rangle = \frac{1}{\sqrt{2}}\langle\omega|c\bar{c}\rangle\,.$$

Clearly for special particles $X$ it is possible that $\Gamma(\psi\to\phi X) \ll \Gamma(\psi\to\omega X)$ but such circumstances are not general, and in fact will not be the ones we encounter when considering $\psi$ decays.

Our analysis will proceed on the assumption that $k(t)$ is really SU(4)-symmetric. Possible deviations from SU(4) and SU(3) symmetry will be mentioned in passing and discussed at the conclusion of the next section. The cylinder coupling constant is not the only variable in which we have to worry about symmetry breaking. In writing down Eq. (2.3) we have implicitly assumed a universal $g$ for each quark type $q$ and all particles $ABC$. Particularly when $g$ has dimensions, as for instance in $V\to VP$ ($g\propto 1/m$), the question arises as to which mass should appear. One popular assumption is to use a mass characterizing the decay process; another popular procedure chooses a universal mass. We shall adapt the latter alternative and scale all dimensional couplings by a universal mass, e.g., $1/\alpha'$. Other choices are also possible. One might think of using $m_q$ for the diagrams involving $q$-type quarks.[16] The variation in choice of scale introduces ambiguity in doing phenomenology, but with our current limited knowledge of properly accounting for symmetry breaking we cannot do much better. Possibly, because of the large mass differences involved, a good model for OZI violation will help distinguish among the alternatives.

We close this section by presenting a prediction, specific to SMTE, which sets the scale for the narrowness of $\psi$. We can use Eq. (2.1) to estimate the ratio $k(m_\psi^2)/k(m_\phi^2)$ which in turn will determine $\Gamma(\psi\to\rho\pi)/\Gamma(\phi\to\rho\pi)$. We find

$$\frac{k(\psi)}{k(\phi)} = \frac{\cosh 2(m_\phi^2)^{1/2}}{\cosh 2(m_\psi^2)^{1/2}} \approx \tfrac{1}{65}\,. \tag{2.4}$$

This should be regarded as a lower limit since, as mentioned above, the cosh estimate was strictly valid only for small $m^2$.

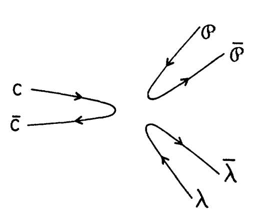


FIG. 4. A naive diagram for $\psi\to\phi x$, which seems to involve a double violation of the OZI rule. Such diagrams are meaningless in our approach.

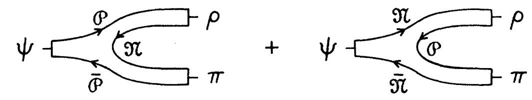


FIG. 5. Planar diagrams which contribute to the coupline $g_{\psi\rho\pi}$.

## III. DECAYS

### A. $V\to VP$

The first set of decays we consider are $V\to\rho\pi$, where $V$ is $\psi$, $\phi$, or $\omega$. The decay rate is given as $\Gamma(V\to\rho\pi) = (g_{V\rho\pi}^2/4\pi)P^3$. The relevant diagrams for computing $g_{V\rho\pi}$ are Fig. 5 and we readily find that

$$\frac{\Gamma(\psi\to\rho\pi)}{\Gamma(\phi\to\rho\pi)} = \left(\frac{P_{\psi\to\rho\pi}}{P_{\phi\to\rho\pi}}\right)^3 \left(\frac{m_\phi^2 - m_\omega^2}{m_\psi^2 - m_\omega^2}\right)^2 \left[\frac{k(m_\psi^2)}{k(m_\phi^2)}\right]^2; \tag{3.1}$$

using $\Gamma(\psi\to\rho\pi)\simeq 0.9$ keV, $\Gamma(\phi\to\rho\pi) = 660$ keV we have

$$k(\psi) \approx \tfrac{1}{30}k(\phi)\,. \tag{3.2}$$

This factor of 30 is in reasonable agreement with our most naive expectations, Eq. (2.4), and strongly confirms the continued sharp approach to the planar asymptotic regime. [We point out the technical detail that the rate of decrease given by Eq. (3.2) is consistent with the arctangent appearing in Eq. (V.3) of Ref. 3 being $\pi/4$, a number in pleasing accord with the expectation of a gradual change from the cosh $2\sqrt{t}$ behavior to a power-law decrease.] Comparing $\phi\to\rho\pi$ to $\omega\to\rho\pi$ as given by Gell-Mann, Sharp, and Wagner,[17] we obtain the estimate $k(m_\phi^2)\approx\frac{1}{50}$. We shall also use (see Table II) $k(\omega)\approx\frac{1}{40}$, where $\frac{1}{40}$ is the most convenient number larger than $\frac{1}{50}$ consistent with cylinder corrections to $\omega$.

We next turn to the interesting set of decays involving $\eta$ and $\eta'$ in the final state. The first to be considered will be $\psi\to\eta'\omega$. Two of the six relevant diagrams are displayed in Fig. 6. The other four are two which correspond to Fig. 6 but with the quark arrows reversed plus two more involving $\mathcal{P}$ quarks. One finds

$$\frac{\Gamma(\psi\to\eta'\omega)}{\Gamma(\psi\to\rho\pi)} = \frac{1}{3}\left(\frac{P_{\eta'}}{P_\rho}\right)^3 \left[\frac{\sqrt{6}k(\eta')k(\omega)\cos\Omega_{\eta'}}{\Delta_{\eta_c\eta'}k(\psi)} + \sin\theta_{\eta'}\right]^2 \approx 2.5\,; \tag{3.3a}$$

similarly

$$\frac{\Gamma(\psi\to\eta'\phi)}{\Gamma(\psi\to\rho\pi)} = \frac{1}{3}\left(\frac{P_{\eta'}}{P_\rho}\right)^3 \frac{\Delta_{\psi\omega}^2}{\Delta_{\psi\phi}^2}\left[\frac{\sqrt{3}\,k(\eta')\cos\Omega_{\eta'}k(\phi)}{\Delta_{\eta_c\eta}k(\psi)} + \cos\theta_{\eta'}\right]^2 \approx 1 \tag{3.3b}$$

TABLE II. For each of the mesons $X$, we list the value of $k(x)$ used in our analysis as well as the coefficient of the $|c\bar{c}\rangle$ component as calculated from Table I. For $\eta_c$ and $\psi$ we list the amount of SU(3) singlet present. $P$ stands for the Pomeron and the associated $k$ is the strength of the cylinder coupling necessary to describe low-energy diffraction scattering (Ref. 31).

| Meson | $P$ | $\eta$ | $\eta'$ | $\omega$ | $\phi$ | $\eta_c$ | $\psi$ |
|---|---|---|---|---|---|---|---|
| $k$ | 0.15 | $\frac{2}{5}$ | $\frac{1}{5}$ | $\frac{1}{40}$ | $\frac{1}{50}$ | $\frac{1}{50}$ | $\frac{1}{1500}$ |
| $\langle X\mid c\bar{c}\rangle$ | | $2\times10^{-2}$ | $5.5\times10^{-2}$ | $4.5\times10^{-3}$ | $2.5\times10^{-3}$ | | |
| $\langle X\mid \mathcal{P}\mathfrak{N}\lambda\rangle$ | | | | | | $5\times10^{-3}$ | $1.5\times10^{-4}$ |

while for decays into an $\eta$

$$\frac{\Gamma(\psi\to\eta\omega)}{\Gamma(\psi\to\rho\pi)}\approx 1\,,$$

$$\frac{\Gamma(\psi\to\phi\eta)}{\Gamma(\psi\to\rho\pi)}\lesssim 0.1\,. \tag{3.3c}$$

These results are extremely interesting and we pause briefly amidst our calculations to discuss them. The first feature which strikes us is the predicted relative abundance of decay modes involving an $\eta$ or $\eta'$. Indeed $\eta'\omega$ may well be the single most important hadronic decay channel for the $\psi$ meson. Harari has recently pointed out that if $\psi$ and $\psi'$ decay copiously into $\eta$ and $\eta'$ many puzzles concerning the properties of the decay products of $\psi, \psi'$ will be solved. We shall return to this in Sec. IV. Note also that there is no evidence for double-OZI-forbidden decays.

The $\phi\eta$ channel is small, since for $\phi\eta$, the $c\bar{c}$ and $\lambda\bar{\lambda}$ quark diagrams interfere destructively rather than constructively. It is a reflection of the mostly octet character of $\eta$. Because of this interference the $\phi\eta$ decay rate is much more sensitive to the various parameters and hence our numerical estimate is not especially reliable.

We momentarily leave $\psi$ decays to consider decays of $\psi'$ into $VP$. The most interesting such decay is $\psi'\to\psi\eta$. Since the $\psi'$ is probably a radial excitation it is not clear if we are entitled to employ the coupling $g_{V\rho\pi}$ in performing a calculation. Nevertheless with this caveat in mind we calculate $\psi'\to\eta\psi$ using the standard value of $g_{\omega\rho\pi}{}^2/4\pi \approx 20\ \mathrm{GeV}^{-1}$ and find $\Gamma(\psi'\to\psi\eta)\simeq 35$ keV.

Since essentially only the $c\bar{c}$ diagram contributes

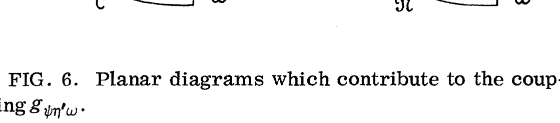


FIG. 6. Planar diagrams which contribute to the coupling $g_{\psi\eta'\omega}$.

to this decay it is very sensitive to the parametrization of $\eta$. Using the extremes suggested in the Appendix the limits on our estimate of $\Gamma(\psi'\to\psi\eta)$ are, in keV units,

$$15<\Gamma(\psi'\to\psi\eta)<130\,. \tag{3.4}$$

Clearly these estimates are too large and necessitate the introduction of a suppression factor $1/\xi$ for $g$. We interpret this as indicating that ground states are less likely to interact with radial excitations. Rather than being embarrassed by the need to introduce a new factor we are confident that it accurately reflects some of the physics of radial excitations.[18] Estimate (3.4) suggests values for $\xi$ between 2 and 4. This factor will make frequent appearance in our subsequent calculations.

As further support for the reasonableness of introducing the $\xi$ factor we can make the following qualitative remarks. The same suppression factor will appear in computing $\Gamma(\psi'\to\rho\pi)$ and will cause this rate to be smaller (in absolute value) than $\Gamma(\psi\to\rho\pi)$. We also expect $k$ to continue its decrease between $m_\psi{}^2$ and $m_{\psi'}{}^2$. Using Eq. (2.1) and remembering our remarks following Eq. (3.2) we estimate $k(\psi')\approx\frac{1}{2}k(\psi)$. We can then estimate a rate for $\psi'\to\rho\pi$,

$$\frac{\Gamma(\psi'\to\rho\pi)}{\Gamma(\psi\to\rho\pi)}\approx 1/(3\xi^2)\,,$$

consistent with the experimental upper limit for this mode.

Conversely, if radial decays to ground states are suppressed, decays into other radial states might be (relatively) enhanced. $\psi'$ should decay more frequently than $\psi$ into radial excitations. We thus expect multiparticle states to be more abundant in $\psi'$ decay products than in $\psi$. The factor $\xi$ thus helps us understand some of the differences in the decay channels of $\psi$ and $\psi'$. We conclude that there is evidence for suppression of radial transitions in the $\psi\psi'$ system. This is in disagreement with recently expressed opinions.[18] We do agree with these authors, however, in that the radial suppression we find is not as large as for conventional mesons ($\xi \sim 7-10$).

The last decay to be considered in this subsection is $\psi' \to \eta_c \omega$, for which

$$\Gamma(\psi' \to \eta_c \omega) \approx \frac{12 \text{ keV}}{\xi^2}$$

$$\simeq \tfrac{1}{3}\Gamma(\psi' \to \eta\psi).$$

This number is, of course, sensitive to $m_{\eta_c}{}^2$. If $m_{\eta_c} = 2750$ MeV, our prediction for $\Gamma(\psi' \to \eta_c \omega)$ will almost double.

### B. vector $\to$ vector + scalar

The most prominent decay mode of this group is the $\psi' \to \psi\pi\pi$ interpreted as proceeding through an $I=0$ $\pi\pi$ resonance known as the $\epsilon$. Such a pathway is crucial for our being able to understand such decays, and the data support this interpretation.[19,20] Little is really understood about this $\epsilon$ meson; it is not agreed as to how it fits into standard quark spectroscopy. We do not need to know the actual classification of this particle but shall assume it is one that mixes like the other natural-parity trajectories. We then obtain

$$\frac{\Gamma(\psi' \to \epsilon\psi)}{\Gamma(\rho' \to \rho\epsilon)} = \left| \frac{\langle \epsilon | c\bar{c} \rangle}{\langle \epsilon | \mathcal{P}\bar{\mathcal{P}} \rangle} \right|^2 r \equiv \kappa_\epsilon{}^2 r, \tag{3.5}$$

where $r$ is the ratio of the phase space for $\psi' \to \pi\pi\psi$ via $\epsilon$ to $\rho' \to \rho\pi\pi$ via $\epsilon$. Our results will depend strongly on $r$, which in turn is strongly dependent on the particular parametrization one uses for the $\epsilon$ resonance. In keeping with the spirit outlined in the Introduction, we will choose a value, representative of some of the models in the literature $r \approx \frac{1}{20}$. If we assume that the partial width for $\rho' \to \rho\pi\pi$ via $\epsilon$ is 300 MeV, we find $\kappa_\epsilon \approx \frac{1}{12}$. This amount of suppression is consistent with a strongly mixed meson such as the $\eta'$, in terms of which relevant comparison might be $\langle c\bar{c} | \eta' \rangle / \langle \lambda\bar{\lambda} | \eta' \rangle \simeq \frac{1}{19}$. While we have no way of predicting the mixing of $\epsilon$ as we did for the $\eta, \eta'$, its unusual properties, and the inability to discover an adequate mixing scheme for the $0^+$ particles, allow us to postulate that the $\epsilon$ will not exhibit ideal mixing but will be strongly mixed. A corollary of this is that we also expect the other $I=0$ members of the $\epsilon$ multiplet to mix strongly. We can now attribute differences in various OZI suppression factors (5) as reflecting differences in the $c\bar{c}$ content of the final states. $\Gamma(\psi \to \eta\psi) \ll \Gamma(\psi \to \pi\pi\psi)$ mostly because $\eta$ is predominantly an SU(3) octet.[21]

Given a value for $\kappa_\epsilon$, assuming the popular derivative-coupling scheme for $\tilde{V}V\epsilon$ (see Ref. 19) $[(\partial_\mu \tilde{V}_\nu - \partial_\nu \tilde{V}_\mu)(\partial^\mu V^\nu - \partial^\nu V^\mu)\epsilon]$, we can calculate the decays $\psi \to \epsilon$ ($\omega$ or $\phi$),

$$\frac{\Gamma(\psi \to \epsilon\omega)}{\Gamma(\rho' \to \rho\epsilon)} = \xi^2 \left\{ \frac{\sqrt{2}k(\psi)}{\Delta_{\psi\omega}} \left[ 1 - \kappa_\epsilon \frac{k(\omega)}{k(\psi)} \right] \right\}^2 \frac{P_\psi(3m_\omega{}^2 + 2P_\psi{}^2)}{P_\rho(3m_\rho{}^2 + 2P_{\rho'}{}^2)}. \tag{3.6}$$

Consistent use of the radial-transition suppression now calls for the factor $\xi^2$ to enhance the rate for the purely nonradial $\psi$ decay

$$\Gamma(\psi \to \epsilon\omega) = \xi^2(0.2) \text{ keV} \approx 2\Gamma(\psi \to \epsilon\phi). \tag{3.7}$$

Again notice the comparable rates for the $\phi$ and $\omega$ channels. This is further evidence of the strong mixing of $\epsilon$. Using $\xi^2 \sim 4-6$, consistent with our previous suggestions, produces acceptable values. Because these decay rates are strongly dependent on parameters of the poorly understood $\epsilon$ meson, and because of the destructive interference of the various quark diagrams we do not view our results as convincing or as firmly established as for the $\eta$ and $\eta'$ channels. The ratio of the $\phi$ to $\omega$ channel will be especially sensitive to symmetry breaking and will decrease if such symmetry breaking is introduced. Nevertheless, we respect the reasonableness of the numbers we have obtained and hold them to be meaningful.

The reader should now be in a position to make models for many other possible decay modes. Given the coupling between a $1^-$ meson and any two mesons $(A, B)$, one can convert that number into a prediction for $\psi \to AB$. As a last example we quote the $B \to \pi\omega$ reaction, which if mediated by a similar gauge-invariant coupling as we used for $\rho'\rho\epsilon$ implies

$$\Gamma(\psi \to B\pi) \approx 0.1 \text{ keV}.$$

Any attempt to extend our results on $\epsilon$ modes to $\psi'$ will involve us in further speculations. The decay rate into $\epsilon\omega$ (as well as other channels such as $\pi g$, $\rho A_2$, or $\rho'\pi$ which contribute to the $5\pi$ decays of $\psi$) will be suppressed by the factor $1/\xi^2$ and/or $k(\psi')/k(\psi)$. We will thus be safely within the quoted upper limits on $5\pi$ modes of $\psi' < 1.5$ keV. On the other hand, the reaction $\psi' \to \omega'\epsilon$ may be important. Since $\omega'$ is expected to be broad and large imaginary parts (widths) obscure arguments for the continued decrease of $k$ we may speculate $k(\omega') \approx k(\omega)$. At worst we expect only power-law decrease. If this is so the phase-space enhancement coupled with less important interference suggests

$$\frac{\Gamma(\psi' \to \epsilon\omega')}{\Gamma(\psi \to \epsilon\omega)} \approx 4.$$

This result may be attributed to the fact that decay modes proceeding primarily through the $c\bar{c}$ parts of mesons will not be further suppressed as $k$ decreases in going from $\psi$ to $\psi'$. Such modes, as we have seen ($\epsilon\omega, \eta'\omega$, etc.), account for a significant fraction of all decay channels.

Since we expect the dominant decay of $\omega'$ to be $\epsilon\omega$, the $\epsilon\omega'$ decay mode will appear in the $7\pi$ channel.

### C. Radiative decays

Because the hadronic decay modes are so strongly suppressed radiative decays take on added significance for the new particles. Indeed the reported rate[1] for $\psi \to \eta'\gamma$ is larger than for the strong decay $\psi \to \rho^0\pi^0$. As we shall see, this surprising situation is readily understood in terms of the strong mixing engaged in by the pseudoscalar SU(3) nonet. The most simple determinations are for the ratios

$$\frac{\Gamma(\psi\to\gamma\eta')}{\Gamma(\psi\to\gamma\eta_c)}=\left(\frac{P_{\eta'}}{P_{\eta_c}}\right)^3 |\langle\eta'|c\bar{c}\rangle|^2 \approx \tfrac{1}{3}\,,$$
$$\frac{\Gamma(\psi\to\gamma\eta)}{\Gamma(\psi\to\gamma\eta')}\approx\tfrac{1}{7}\,, \tag{3.8}$$

where the decay proceeds through a diagram such as Fig. 7.

It should now be clear why the $\gamma\eta'$ channel is, relatively, so large. The small ratio of coupling constants $(\alpha/m_c^2)/(g_{\omega\rho\pi}^2/4\pi)$ is offset by the large value of the ratio $|\langle\eta'|c\bar{c}\rangle|^2/|\langle\psi|\mathcal{P},\mathfrak{N},\lambda\rangle|^2$ (see Table II) which is present in comparing $\eta'\gamma$ to $\rho^0\pi^0$.

Can we estimate the absolute magnitude of the radiative transitions? Such an estimate is possible if we are extremely naive and adopt elementary quark-model ideas. The radiative transition under consideration will be proportional to the quark magnetic moments, which in turn are proportional to $e_q/m_q$. Comparing $\psi$ and $\phi$ radiative decays we find

$$\frac{\Gamma(\psi\to\gamma\eta_c)}{\Gamma(\phi\to\gamma\eta)}=\left(\frac{P_{\eta c}}{P_\eta}\right)^3\left(\frac{e_c m_\lambda}{e_\lambda m_c}\right)^2\frac{1}{|\langle\eta|\lambda\bar{\lambda}\rangle|^2}\,. \tag{3.9}$$

We choose the $\phi$ decay for comparison because it involves a "heavy" quark and therefore should be most analogous to the $\psi$ decays. Choosing $m_\lambda \approx 250$ MeV, $m_c \approx 2$ GeV, and using charm to fix $e_c$ implies $\Gamma(\psi\to\gamma\eta_c)\simeq 7$ keV.

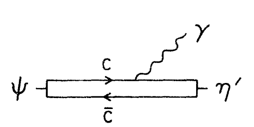


FIG. 7. The planar diagram for the radiative decay $\psi\to\gamma\eta'$.

For $\psi'$ we find

$$\frac{\Gamma(\psi'\to\gamma\eta')}{\Gamma(\psi'\to\gamma\eta_c)}\simeq\tfrac{1}{35}$$

and

$$\frac{\Gamma(\psi'\to\gamma\eta_c)}{\Gamma(\psi\to\gamma\eta_c)}\approx\frac{20}{\xi^2}\lesssim 5\,, \tag{3.10}$$

where the by-now-familiar $\xi$ has again made an appearance.

### D. The $\eta_c$

The main feature we consider here is the total width. We have already seen and capitalized on the fact ahat the $\eta$ and $\eta'$ are strongly mixed and strongly OZI-rule-violating. Does the $\eta_c$ conform to its pseudoscalar counterparts and also exhibit strong OZI-rule violation, and hence a "large" width? Current popular estimates for $\Gamma(\eta_c)$ are several MeV. We disagree with these estimates and expect $\eta_c$ to be more narrow. The basis for this prediction is the following. The arguments presented in Ref. 3 suggest that all cylinder couplings decrease with increasing $t$. The pseudoscalars are no exception. Indeed the parametrization used in the Appendix exhibits this decrease between $m_\eta^2$ and $m_{\eta'}^2$. The reason the unnatural-parity nonet is peculiar is that the rate of approach to the asymptotic regime is slower. The difference in rate has been estimated, and these estimates provide the numerology on which we base our prediction. In Ref. 3 it was suggested that $t_c^n \approx \frac{1}{2}$ while $t_c^u \simeq 2$–$3$, where $n$ and $u$ stand for natural- and unnatural-parity trajectories. Notice that

$$\frac{m_{\eta_c}^2}{t_c^u}\approx 2.5\text{–}3.5>\frac{m_\phi^2}{t_c^n}\simeq 2\,. \tag{3.11}$$

This estimate strongly hints that $m_{\eta_c}^2$ is sufficiently large to be asymptotic. The $\eta_c$ may be even more asymptotic than the $\phi$. This opinion is further strengthened by the considerations (3) of symmetry breaking. Since $m_c^2 \gg m_\pi^2$ such effects should be even more pronounced in the pseudoscalar cylinder coupling charm quarks to $p, n, \lambda$ than anywhere else. Such symmetry breaking will decrease the value of $t_c$ and hence push our estimates of $\eta_c$ as lying further in the asymptotic regime. An opposite effect, however, arises from the slowing down of the cosh decrease of $k$. Nevertheless the upshot of this admittedly speculative reasoning is the estimate $k(\eta_c)\approx k(\phi)$. [The factor $\Delta_{\eta_c\eta}$ will tend to cancel the effects of increased phase space with the implication that

$$\Gamma(\eta_c\to\text{all})\approx\Gamma(\phi\to\rho\pi)\,.] \tag{3.12}$$

Regardless of whether or not our strong estimate is correct, the SMTE implies that there should be further damping of $k_u$ so we expect

$$|\langle\eta_c|\mathcal{P},\mathfrak{N},\lambda\rangle|^2 \ll |\langle\eta'|c\bar{c}\rangle|^2 . \tag{3.13}$$

If we use the estimate $k(\eta_c)\approx k(\phi)$ we can compute some characteristic decay modes of $\eta_c$:

$$\begin{aligned}\Gamma(\eta_c \to \rho^0\rho^0) &\approx 75 \text{ keV}\\ &\approx \tfrac{3}{2}\Gamma(\eta_c\to\phi\phi),\\ \Gamma(\eta_c\to\delta\pi) &\lesssim 1 \text{ keV}.\end{aligned}$$

### E. Symmetry breaking

We promised to include some remarks on symmetry breaking and take this opportunity to redeem that pledge. One piece of evidence that the $\psi$ does not behave as a pure SU(3) singlet is that

$$\frac{\Gamma(\psi\to K^*K)}{\Gamma(\psi\to\rho\pi)} < 0.8 ,$$

the SU(3) singlet value when corrected for phase space. A deviation by as much as a factor of 2 is possible. We have no ready explanation for this fact, but point to the most likely path for such symmetry-breaking effects to enter our analysis. Comment has already been made on the possibility of symmetry-breaking effects on $t_c$. Cylinders coupling heavier quarks will have a $t_c$ smaller than the estimate $\frac{1}{2}$ appropriate to the SU(2) quarks. Hence $k$ will decrease more quickly. Because of the large value of $m_\psi^2/t_c$ we are performing extrapolations over large distances. This will have an amplifying effect, causing a relatively small symmetry breaking in $t_c$ (10–20%) (not very noticeable at $t=m_\phi^2$) to become much larger (40–60%) at $t=m_\psi^2$. Whether this is the appropriate symmetry-breaking mechanism remains to be seen. Symmetry breaking of as much as 50% in $k$ for the coupling to $\lambda\bar{\lambda}$ will still only imply a 10% mixture of SU(3) octet in $\psi$, and thus SU(3)-singlet-forbidden decays are still expected to be significantly suppressed. For instance, $\psi \to K\bar{K}\simeq 1$ eV.

Such symmetry breaking will tend to suppress channels involving $\phi$ with respect to those involving $\omega$.

## IV. CONCLUSIONS AND DISCUSSION

What have we learned about $\psi$ decays, the OZI rule, and the SMTE? Our most important quantitative predictions are summarized in Table III. As we have already pointed out in the preceding section, the abundance of events containing $\eta$ or $\eta'$ is especially noteworthy.

After the bulk of this work was performed a report by Harari[8] was brought to our attention in which he also considered the implications for $\psi,\psi'$ decays of $c\bar{c}$ mixing in $\eta$ and $\eta'$. We both agree that the relatively large transition rates for $\psi' \to \eta\psi$, $\psi\to\eta'\gamma$, $\psi\to\eta\gamma$, $\psi,\psi'\to\eta$, or $\eta'+X$ arise because of the correspondingly large $c\bar{c}$ components in $\eta$ and $\eta'$. Harari further conjectures that as much as 20–30% of $\psi$ and $\psi'$ decays occur through channels containing either $\eta$ or $\eta'$ and notices that such copious $\eta$ and $\eta'$ modes can explain both the unexpectedly large ratio of $\langle n\rangle\pi_0/\langle n\rangle\pi^\pm$ (observed at Frascati) and the relatively small $K/\pi$ ratio which is seen in $\psi$ and $\psi'$ decays. It is emphasized that decay channels containing $\eta$ or $\eta'$ almost always have two or more neutral particles in the final state and will have thus escaped identification at SPEAR. This could account for the "missing" $\psi$ and $\psi'$ decays.

To the extent that our model predicts large rates for decays into final states containing $\eta$ or $\eta'$ it will solve the above experimental problems. The issue on which we can hope to shed additional light is whether or not we expect as much as 20–30% of all decays to involve $\eta$ or $\eta'$. The results tabulated in Table III add up to about 8%, but to this we must add many more possible decay modes involving radial excitations, channels such as $\eta'\omega(1675)$, etc. One can readily imagine coming up with a 15% total, and we thus feel that our numerical estimates support the lower limits of Harari's conjecture.

What about $\psi'$ decays? Here the situation is complicated by a lack of concrete information on the differences between ground states and radial excitations. We can say with confidence that a

TABLE III. A summary of our more important results concerning the decay modes of $\psi$, $\psi'$, and $\eta_c$. Entries with an asterisk are more speculative.

$\psi$ decays

$$\frac{\Gamma(\psi\to\eta'\phi)}{\Gamma(\psi\to\rho\pi)} \approx \frac{\Gamma(\psi\to\eta\omega)}{\Gamma(\psi\to\rho\pi)} \approx 1 \approx \frac{2}{5}\,\frac{\Gamma(\psi\to\eta'\omega)}{\Gamma(\psi\to\rho\pi)}$$

$$\frac{\Gamma(\psi\to\epsilon\omega)+\Gamma(\psi\to\phi\epsilon)}{\Gamma(\psi\to\rho\pi)} \approx 1 \geq \frac{10\Gamma(\psi\to B\pi)}{\Gamma(\psi\to\rho\pi)}$$

$$\frac{\Gamma(\psi\to\gamma\eta_c)}{\Gamma(\psi\to\gamma\eta')} \approx 3 \approx \frac{3}{7}\,\frac{\Gamma(\psi\to\gamma\eta')}{\Gamma(\psi\to\gamma\eta)}$$

$\psi'$ decays

$$\frac{\Gamma(\psi'\to\eta_c\omega)}{\Gamma(\psi'\to\eta\psi)} \approx \frac{1}{3} \quad \frac{{}^*\Gamma(\psi'\to\epsilon\omega')}{\Gamma(\psi\to\epsilon\omega)} \approx 4 \quad \frac{\Gamma(\psi'\to\rho\pi)}{\Gamma(\psi'\to\text{all})} \lesssim 3\times 10^{-4}$$

$$\frac{\Gamma(\psi'\to\gamma\eta')}{\Gamma(\psi'\to\gamma\eta_c)} \approx \frac{1}{35} \quad \frac{{}^*\Gamma(\psi'\to\gamma\eta_c)}{\Gamma(\psi\to\gamma\eta_c)} \lesssim 5$$

$\eta_c$ decays

${}^*\Gamma(\eta_c\to\text{all}) \approx \Gamma(\phi\to\rho\pi)$

relatively large number of hadronic decay channels will contain $\eta$ or $\eta'$ and that these channels might not be the same ones important in $\psi$ decays. Whether or not these channels can account for all the missing decays of $\psi'$ is problematical. We also expect some additional contribution from $\psi' \to \omega\eta_c$.

We are thus in basic agreement with Harari as to the prominence and importance of $\eta$ and $\eta'$ channels. We do, however, disagree with some of his other conclusions. Notice that our estimates of the $c\bar{c}$ content of $\eta$ and $\eta'$ are more than an order of magnitude smaller than those given in Ref. 8. Nevertheless we are still able to account for the relatively strong coupling between $\eta$ and $\eta'$ and the $\psi$ system. At the same time these small values do not lead us to embarrassingly large rates for radiative transitions.

Kugler[22] has pointed out another difficulty with the simple ideas used in Ref. 8. An upper limit of one part in $10^4$ to the SU(3) quark content of $\eta_c$ is given on the basis of vector-dominance arguments and the absence of a signal for $\psi \to \gamma\eta_c \to \gamma\phi\phi$ or $\gamma\rho^0\rho^0$. This number is $\approx \frac{1}{30}$ below our estimate of $|\langle\eta'|c\bar{c}\rangle|^2$ and $\simeq 10^{-3}$ smaller than Harari's estimate. This upper limit, rather than being a difficulty for us, is another piece of confirming evidence for one of our strongest and most characteristic predictions—the decrease of all cylinder couplings toward very small asymptotic values. This is, in fact, the first evidence that $k(\eta_c) \ll k(\eta')$ and thus adds further credence to the idea of asymptotic planarity and to the entire SMTE.

We have answered the questions of the Introduction. Our ansatz for OZI-rule violation made us focus on the mixing properties of the mesons involved in the decays. Since $\psi$ mixes roughly equally with $\phi$ and $\omega$ we do not, in general, have doubly forbidden decays. Since $\eta$ and $\eta'$ mix strongly (a prediction of the model) as does the $\epsilon$ (required by the data, but consistent with the acknowledged anomalous properties of this meson) certain channels involving these particles seem less suppressed than we at first expected.

We conclude that the SMTE has taught us much about the new particles, and symbiotically, we have gained confidence in the theory because of its success, particularly the confirmed characteristic properties of $k(t)$. There are further, somewhat more technical lessons which we can also learn. Strictly speaking, asymptotic planarity was shown to be a feature of only the cylinder corrections,[3,15] not the more complicated handle corrections. While reasonable to extend the concept from cylinders to handles, this has not been firmly established. The calculations performed here for $\psi$ decaying into two $I=0$ objects can be thought of as a phenomenological evaluation of the first handle corrections to the cylinder. This is most readily understood diagrammatically from Fig. 8. While some of these "handle" corrections turn out to be larger than corresponding cylinder terms [e.g., $\Gamma(\psi \to \eta'\omega) > \Gamma(\psi \to \rho\pi)$] they are all *very* small, which we hope indicates the correctness of the conjectured general validity of asymptotic planarity.

Apart from the physics of OZI, we were led in our study of $\psi'$ decays to introduce a parameter $\xi$ which reflects the differences between parents and daughters, and was capable of indicating ways in which $\psi'$ decay modes will differ from $\psi$ decay modes.

Among the topics we did not consider in detail were the consequences of symmetry breaking. We expect such effects to cause $k$ to be smaller than our estimates. On the other hand, we have ignored the evidence that the slope of the $\psi$ family of Regge trajectories may be only $\frac{1}{2}$ of that of the other Regge trajectories [e.g., $m_{\psi'}{}^2 - m_\psi{}^2 \simeq 2(m_{\rho'}{}^2 - m_\rho{}^2)$]. This would mean some of our values of $\Delta_{\psi x}$ are overestimates. Since, for instance, the relevant $c\bar{c}$ admixture in $\omega$ depends on $k/\Delta_{\psi\omega}$ these two neglected effects tend to cancel, and for the phenomenology at hand our simplified assumptions should still be meaningful.

We have avoided the mention of form factors. This may be a source of criticism. The nature of hadronic form factors is not well understood. It is probable that our $S$-matrix formulation and our phenomenological treatment already account for some, maybe even all, of what is loosely and vaguely referred to as a form-factor effect.[23] Therefore, we shall continue not to mention them.

There are two other popular dynamical models for OZI-rule violation. One of these is the scheme of Freund and Nambu[24] in which an entire new class of Pomeron-daughter particles is conjectured to exist. These new particles then serve as intermediaries in the pathway of an OZI-violating reaction. The chief drawbacks may be the fact that many new particles are required and the somewhat arbitrary use of only the lowest-lying daughter particles as the intermediaries. Never-

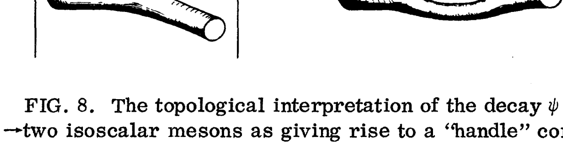


FIG. 8. The topological interpretation of the decay $\psi \to$ two isoscalar mesons as giving rise to a "handle" correction to the cylinder.

theless, successful phenomenological applications have been made.[25,26] Similarities exist between the Freund-Nambu approach and ours to the extent that dual models underlie both approaches. Dissimilarities arise when new singularities (the Pomeron and its daughters) are introduced.

The $S$-matrix model adopted here bears even more striking resemblance to the recent field-theoretic models for the OZI rule.[27,6] The most obvious of these analogies is that both the theories of asymptotic planarity and gluon theories posessing asymptotic freedom possess characteristic couplings (cylinder and gluon coupling constants, respectively) which decrease as $m^2$ increases. Topologically, both couplings characterize those diagrams responsible for OZI-rule violation. Thus both approaches will have the property that the OZI rule improves as the masses become larger. This has as a corollary the somewhat unfamiliar feature that mixing angles will change as we go from one member of a multiplet to another member of the same multiplet. Such behavior is now seen to be expected both from $S$-matrix and field-theory models.

Given the existence of an OZI asymptotic regime one must ask about the approach to this regime. The original investigations suggested that in an asymptotically free gluon theory the approach to small coupling was like inverse powers of $\ln m^2/m_0^2$. This left unspecified the scale of how far away the asymptotic regime was. It was suggested that the rapid onset of asymptotic behavior (apparent already at $m^2=m_\phi^2$), referred to as precocious planarity in SMTE, comes about because $m_0=m_\pi$.[28]

Recently a nonperturbative, albeit two-dimensional, model of an asymptotically free field theory was solved[6] in which a more firm field-theoretic understanding of the OZI rule was established. Remarkably, as the field-theory models become more sophisticated, the similarities with the $S$-matrix approach increase. We refer to two specific results. In the two-dimensional field theory the rate of approach to the asymptotic regime is an inverse power of $m^2$, in qualitative agreement with the truly asymptotic expectations of asymptotic planarity.[3] Even more dramatic is the (mass)$^2$ which sets the scale for this approach $-\pi^2\alpha'$. We see an $\alpha'$ in a field theory and a factor, similar to the $2\pi^2$, which was responsible for the precocity of planarity[3] in the SMTE. However, one major difference is that in the field theory the scale is with respect to the quark masses. This still leaves unresolved the question of why the $\phi$ meson (with $m_\chi^2 \simeq 1/\pi^2\alpha'$) is already asymptotic. In the SMTE the scale is taken with respect to particle masses, and so we can readily account for the planarity of the $\phi$.

One might argue that similarities in the two approaches are to be expected. The field theory studied[6] is the model of 't Hooft[7] which has an expansion in $1/M$ [SU($M$) is the color group] which reproduces the topology of dual resonance models. The $1/M$ expansion is then to be thought of as an expansion in the dual coupling constant $g^2$. But as Veneziano has discussed[4] this might not be a good expansion since the relevant expansion parameter is $g^2N$ ($N$ is the number of flavors). This leads to the SMTE. Thus it is plausible that the two models are logically related.

The need for the introduction of $\alpha'$ to fix scales[6] indicates that even in field theory concepts from Regge theory play a necessary role. We claim that these ideas are actually sufficient and provide us with a detailed understanding of the OZI rule.

Because of the similarities discussed above, the phenomenology presented here could equally well serve as a phenomenology for field-theory models possessing asymptotic freedom. We therefore wish to recall those characteristics of our results which are most comfortable in the $S$ matrix as opposed to the field-theoretic approach:

(1) the precocity of the OZI asymptotic behavior;

(2) a rate of falloff characterized by 1/cosh;

(3) the extension of the explanation of the OZI rule to the tensor mesons $f,f'$ (a corollary of this is that we expect the spin-2 $\psi$ state between $\psi'$ and $\psi$ to be even more narrow than $\psi$ itself);

(4) the ease with which the $\eta$ and $\eta'$ fit into both the general theoretical approach and the specific phenomenology at hand.

In the course of this paper we hope to have convinced the reader that we have possible answers to all the questions posed in our Introduction. The methods used are simple yet productive. They provide solutions to many of the problems which vexed those worrying about the content of vague, previous formulations of the OZI rule. We make definite, testable predictions. But no theory or model should be judged only by one specific set of predictions and explanations. The $t$-dependent Reggeon coupling theory which has emerged[3] from the SMTE describes and correlates a wide variety of seemingly diverse sets of phenomena. It explains the dramatic success of the OZI rule, teaches us about the physics of the new particles, explains the approximate linearity of Regge trajectories,[3] generates diffraction scattering,[12] and correctly predicts Pomeron couplings, slopes and their variation for *negative* $t$.[3] When viewed in this larger setting the success of the theory is even more impressive and gives us confidence that the SMTE will continue to provide new insight into many facets of hadron physics.

*Note added.* Similar applications of the topological or dual unitarity[11] theory to $\psi$ decays have been made by other groups.[29] I am most grateful to these authors for bringing their work to my attention.

ACKNOWLEDGMENTS

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## APPENDIX

Because the mixing parameters for the $I=0$ pseudoscalars play a prominent role in our analysis we devote this Appendix to establishing their values. The rather large difference in mixing patterns between pseudoscalar and the vector and tensor mesons has found an explanation within the SMTE as arising from the small pion mass.[3] This is reflected in the larger value of $t_c$ in Eq. (2.1) when applying this formula to a pseudoscalar cylinder. We are thus able to apply the SMTE formalism to our study of pseudoscalars, with the confidence of knowing that the SMTE can account for some of their more unusual properties. Except for one point, the parametrization follows from a straightforward application of the simple six-trajectory model previously introduced.[12] The relationship between the $\eta$ (or $\eta'$) trajectory, the pion trajectory $\alpha_\pi$, the ideal (planar) $\lambda\bar{\lambda}$ pseudoscalar trajectory $\alpha_\lambda$, and the unnatural-parity cylinder coupling $k_u$ is

$$\alpha_\eta = \tfrac{1}{2}\{\alpha_\pi + \alpha_\lambda - 3k_u + [(\alpha_\pi - \alpha_\lambda - k_u)^2 + 8k_u^{\,2}]^{1/2}\}. \tag{A1}$$

The appropriate formula for $\alpha_{\eta'}$ has a minus sign before the square root. The one change we have made from the corresponding formula for the natural-parity trajectories is that even though the $\eta$ and $\eta'$ are states of positive charge conjugation we have used $(-k)$ in Eq. (A1), contrary to the prescription for the $f$ and $\omega$ trajectories. We are not especially upset by this change as there is ample room for minus signs to enter in future, detailed studies of pseudoscalar cylinders.

In order to determine $k(\eta)$ and $k(\eta')$ we will also have to make an assumption about the position of the unobservable $\alpha_\lambda$ trajectory. The simplest such assumption is that $\alpha_\pi - \alpha_\lambda^u = \alpha_\rho - \alpha_\lambda^n$, where the superscripts $u$ and $n$ refer to natural or unnatural parity of the corresponding trajectory. Taking the value 0.4 used in the simple model treatments of the leading trajectory, $\alpha_\pi(t) = -0.02 + \alpha' t$, $\alpha_\lambda(t) = -0.42 + \alpha' t$, one readily sees that $k(\eta) \approx \frac{2}{5}$, and $k(\eta') \approx \frac{1}{5}$ reproduce well the desired $\alpha_\eta(m_\eta^{\,2}) = \alpha_{\eta'}(m_{\eta'}^{\,2}) = 0$. These choices are convenient, but are not to be accepted as accurate determinations of $k$. This is because, for the values of $\alpha_{\eta'}$ and $\alpha_\pi$, $k$ depends sensitively on the choice of $\alpha_\pi - \alpha_\lambda$. Equally important, since the values of $k$ are large, higher-order corrections in the SMTE will be important and will modify the simple formula (A1). Nevertheless within the expected accuracy of our study the values of $k$ are adequate. One feature of formula (A1) which is not sensitive to the value of $\alpha_\pi - \alpha_\lambda$ is the property that $k(\eta') < k(\eta)$. This, of course, is exactly the behavior predicted by asymptotic planarity and reaffirms our faith in applicability of SMTE of the pseudoscalars. Our analysis of $\eta$ and $\eta'$ has some resemblance to the recent discussion of De Rújula *et al.*[30]

Given values of $k$, we can compute the mixing angles displayed in Table I from

$$\tan 2\theta = \frac{\sqrt{8}\,k}{\alpha_\pi - \alpha_\lambda - k}.$$

With the choice of parameters we have used, we find

$$\theta_\eta \simeq 45^\circ, \quad \Omega_\eta = 10^\circ,$$

$$\theta_{\eta'} \simeq 35^\circ, \quad \Omega_{\eta'} = 20^\circ.$$

It is amusing that the $\eta$ has the mixing angle implied by the quadratic mass formula, and the $\eta'$ has that given by the linear mass formula. The pseudoscalars may thus be the best example of the difference between conventional mixing schemes at fixed $J$ and that implied by the SMTE which is mixing at fixed $t$.

It seems to us that possible, reasonable limits for the pseudoscalar mixing parameters are

$$0.3 \le k(\eta) \le 0.4, \quad 10 \le \Omega_\eta \le 20,$$

$$0.15 \le k(\eta') \le 0.3, \quad 10 \le \Omega_{\eta'} \le 20.$$

The calculations of Sec. III use $k(\eta) = 2k(\eta') = 0.4$. We wish to stress that once we have accepted the relevance of the SMTE for pseudoscalars and the applicability of the simple, but realistic model embodied in Eq. (A1), we are forced to the qualitative conclusions drawn in the text, i.e., that the $\eta$ and $\eta'$ play an enhanced role in $\psi$ decays.

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[1]For a recent, comprehensive summary of the experimental results from SPEAR, BNL, DESY, and elsewhere see *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California*, edited by W. T. Kirk (SLAC, Stanford, 1976).

[2]S. Okubo, Phys. Lett. 5, 165 (1963); G. Zweig, report, 1964 (unpublished); J. Iizuka, Suppl. Prog. Theor. Phys. 37–38, 21 (1966).

[3]G. F. Chew and C. Rosenzweig, Nucl. Phys. B104, 290 (1976).

[4]G. Veneziano, Nucl. Phys. B74, 365 (1974); Phys. Lett. 52B, 220 (1974). For a slightly different formulation of the same basic ideas see Ref. 11.

[5]R. Cahn and M. Chanowitz, Phys. Lett. 59B, 277 (1975).

[6]C. G. Callan, N. Coote, and D. J. Gross, Phys. Rev. D 13, 1649 (1976). I thank Professor Gross for a useful discussion about his results.

[7]G. 't Hooft, Nucl. Phys. B72, 461 (1974); B75, 461 (1974).

[8]H. Harari, Phys. Lett. 60B, 172 (1976). I thank Professor Chew for bringing this reference to my attention.

[9]C. Rosenzweig and G. Veneziano, Phys. Lett. 52B, 335 (1974).

[10]M. Schaap and G. Veneziano, Lett. Nuovo Cimento 12, 204 (1975).

[11]H. M. Chan, J. Paton, and S. T. Tsou, Nucl. Phys. B86, 479 (1974); H. M. Chan, J. Paton, S. T. Tsou, and S. W. Ng, *ibid.* B92, 13 (1975).

[12]C. Rosenzweig and G. F. Chew, Phys. Lett. 58B, 93 (1975); G. F. Chew and C. Rosenzweig, Phys. Rev. D 12, 3907 (1975).

[13]C. Schmid and C. Sorensen, Nucl. Phys. B96, 209 (1975); N. Papadopoulos, C. Schmid, C. Sorensen, and D. M. Webber, *ibid.* B101, 189 (1975).

[14]N. Sakai, Nucl. Phys. B99, 167 (1975).

[15]M. Bishari, Phys. Lett. 59B, 461 (1975).

[16]I thank D. Coon for this suggestion.

[17]M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Lett. 8, 261 (1962).

[18]L. Clavelli and S. Nussinov, Phys. Rev. D 13, 125 (1976).

[19]R. Aviv, Y. Goren, D. Horn, and S. Nussinov, Phys. Rev. D 12, 2862 (1975).

[20]J. Schwinger, K. A. Milton, W. Tsai, and L. L. DeRaad, Phys. Rev. D 12, 2607 (1975).

[21]M. Machacek and Y. Tomozowa, Phys. Rev. D 13, 1449 (1976).

[22]M. Kugler, quoted in Ref. 8.

[23]I wish to thank G. F. Chew for discussions about this point.

[24]P. G. O. Freund and Y. Nambu, Phys. Rev. Lett. 34, 1645 (1975).

[25]J. F. Bolzan, K. A. Geer, W. F. Palmer, and S. S. Pinsky, Phys. Rev. Lett. 35, 419 (1975); Phys. Lett. 59B, 351 (1975).

[26]M. Chaichian and M. Hayashi, Phys. Lett. 61B, 178 (1976); see also J. Willemsen, Phys. Rev. D 13, 1327 (1976).

[27]T. Appelquist and H. D. Politzer, Phys. Rev. D 12, 1404 (1975).

[28]T. Goldman (private communication).

[29]H. M. Chan, K. Konishi, J. Kwiecinski, and R. G. Roberts, Phys. Lett. 60B, 367 (1975); 60B, 469 (1976); C. Schmid, C. Sorensen, and D. M. Webber, ANL Report No. ANL-HEP-PR-76-07; T. Inami, K. Kawarabayashi, and S. Kitakodo, Phys. Lett. 61B, 60 (1976).

[30]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).

[31]P. Stevens, G. F. Chew, and C. Rosenzweig, Caltech Report No. CALT-68-541 (unpublished).