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Second-class currents*

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Recent nuclear β-decay experiments have hinted at the existence of weak second-class currents. We review the evidence for second-class effects in other branches of weak-interaction physics and find that the presently available data are inconclusive. We then propose additional tests, involving (3,3)-resonance production by neutrinos and antineutrinos. The effects here could be quite substantial if the second-class interactions are as big as suggested by the nuclear β-decay experiments.

I. INTRODUCTION

Recent experiments in nuclear β decay have yielded contradictory indications concerning the existence of second-class currents in the weak interactions. Recall that, by definition, first-class vector and axial-vector currents are, respectively, even and odd under G parity, whereas second-class currents have reversed G-parity assignments.[1] Positive indications have been reported by Calaprice *et al.* for the analog decay[2]

$$\mathrm{Ne}^{19} \to \mathrm{F}^{19} + e^+ + \nu_e \,, \tag{1}$$

and by Sugimoto *et al.* for the mirror processes[3]

$$\begin{aligned} &\mathrm{B}^{12} \to \mathrm{C}^{12} + e^- + \bar{\nu}_e \,, \\ &\mathrm{N}^{12} \to \mathrm{C}^{12} + e^+ + \nu_e \,. \end{aligned} \tag{2}$$

On the other hand, Wilkinson and Alburger,[4] and Garvey and Tribble[5] find no signal of second-class interactions for the mirror processes

$$\begin{aligned} &\mathrm{Li}^{8} \to \mathrm{Be}^{8*} + e^- + \bar{\nu}_e \,, \\ &\mathrm{B}^{8} \to \mathrm{Be}^{8*} + e^+ + \nu_e \,, \\ &\mathrm{Be}^{8*} \to 2\alpha . \end{aligned} \tag{3}$$

Since G-parity symmetry is violated by the electromagnetic interactions, tests for second-class currents are decisive only to the extent that observed signals exceed any reasonable expectations for effects arising from electromagnetic corrections. The tests are otherwise model independent. It is the question of a possible second-class axial-vector current that is especially at issue in the nuclear experiments. What one seeks to measure is a certain matrix-element parameter, d_{II}, which in the absence of electromagnetic corrections can arise only from second-class axial-vector contributions. In order to parameterize any observed signals, it is convenient to employ an impulse approximation for the nuclear matrix elements, relating these to the couplings which describe neutron β decay. For the latter process the matrix element of the weak current is

$$\langle p | j_\mu | n \rangle = \bar{u}(p_2)\left[\gamma_\mu (g_V + g_A \gamma_5) + \frac{q_\mu}{2m}(g_S + g_P \gamma_5) - i\sigma_{\mu\nu}\frac{q_\nu}{2m}(g_M + g_{\mathrm{II}}\gamma_5)\right] u(p_1), \tag{4}$$

where $q = p_1 - p_2$ is the momentum transfer and g_S, g_{II} are respectively, the vector and axial-vector second-class coefficients.

The impulse approximation serves to relate the second-class nuclear parameter d_{II} to the corresponding second-class nucleon parameter g_{II}.[6] Although it is the existence of a nonvanishing d_{II} that is of direct concern, this procedure provides a convenient, if rough, parameterization of the experimental observations. Since the translation from d_{II} to g_{II} *is* model dependent, the "effective" g_{II} need not be the same for different nuclear processes.[7] However, if the second- and first-class currents were of comparable strength one would expect g_{II} to be comparable to g_M (the weak-magnetism coefficient), i.e., g_{II} of order unity. Even in the absence of second-class currents, small "simulated" signals could show up as a result of electromagnetic corrections, but in this case the effective g_{II} should be small, of order αZ. For light nuclei, therefore, an effective g_{II} of order unity or greater would constitute serious evidence for second-class axial-vector currents.

For the mass-19 system the reported second-class coefficient is[2] $g_{\mathrm{II}} = (-8 \pm 3) g_A$ while for the mass-12 system[3] $g_{\mathrm{II}} = (-3.5 \pm 1) g_A$. These are substantial effects, seemingly beyond what could be induced by electromagnetic corrections. On the other hand, the results for the mass-8 system are compatible with vanishing g_{II} (see Ref. 5): $g_{\mathrm{II}} = (-0.5 \pm 0.8) g_A$. It is not our purpose here to choose among these nuclear experiments, or to discuss whether electromagnetic effects might after all be large enough to account for the positive signals in the mass-19 and mass-12 systems.

Instead we want to consider some tests outside of nuclear β decay that might serve to clarify the situation for second-class currents; in particular, we want to consider angular-correlation measurements for ν and $\bar{\nu}$ reactions in the $\Delta(1236)$ resonance region. Our estimates indicate that second-class signals could be substantial here if g_{II} is as large as suggested by the nuclear experiments in the mass-19 and mass-12 systems. Before this, however, in Sec. II we offer some general comments on the status of second-class currents.

II. SURVEY OF PRESENT EVIDENCE

Apart from the nuclear evidence discussed in Sec. I, everything that we presently know about the phenomenology of weak interactions is compatible with the absence of second-class vector and axial-vector currents. Recall that the classification of G parity applies only to the strangeness- and charm-conserving currents, so that the hints of charge-symmetry violation that have been reported for deep-inelastic neutrino reactions, even if sustained experimentally, need not carry any implications for the issue under consideration here. The effects could arise from $\Delta S \neq 0$ and $\Delta C \neq 0$ contributions; one would have to separate these channels in order to draw any conclusions about second-class currents.

On the other hand, it might be thought that the successes of conservation of vector current (CVC) and partial conservation of axial-vector current (PCAC) can immediately serve to rule out any appreciable second-class interactions; CVC relates the weak vector current to the electromagnetic current and the latter is surely first class.[8] Similarly, PCAC relates the pion field to the divergence of the axial-vector current and this implies that the axial-vector current is first class.[9] We will, therefore, suppose that the first-class currents do indeed accord with CVC and PCAC to high accuracy. The question is whether we can rule out additional second-class currents. Let us first review the situation for standard low-energy phenomenology.

A. CVC

There are three classic and successful tests of CVC in low-energy decay processes: (i) the constancy of ft values for various nuclear $0^+ \to 0^+$ analog transitions,[10] (ii) the relation of this ft value to that for pion β decay, $\pi^+ \to \pi^0 + e^+ + \nu_e$,[11] and (iii) the weak magnetism effect in β decay of the mass-12 system [see Eq. (3)].[12] For pion β decay, however, one observes solely from G-parity considerations that the second-class currents can make no contribution whatsoever. For the $0^+ \to 0^+$ analog β decays the second-class vector current *can,* in principle, contribute. However, even if the intrinsic strengths of the first- and second-class currents were comparable [e.g. $|g_S| \approx |g_V|$ in Eq. (4)], the second-class contribution to the ft value would be strongly suppressed: The correction to the purely first-class contribution would be of order m_e/m, where m_e is the electron and m is the nucleon mass. Moreover, at least in impulse approximation, this correction would be about the same for all the $0^+ \to 0^+$ transitions.[13] What the $0^+ \to 0^+$ and pion β-decay experiments confirm, therefore, is CVC for the first-class vector current. Concerning the weak-magnetism effect in the mass-12 system, here again, in principle, both second- and first-class currents can contribute. On the other hand, for neutron β decay the parent and daughter hadrons belong to the same isotopic multiplet and both the Fermi and weak-magnetism coefficients, g_V and g_M in Eq. (4), are purely first class. In impulse approximation for nonanalog nuclear transitions, the weak-magnetism coefficient receives contributions only from g_V and g_M, not from the second-class g_S.[14] To the extent that this is a good approximation, therefore, the weak-magnetism tests in nuclear transitions are really tests of CVC for the first-class current.

The only serious potential difficulty for second-class vector currents that we are aware of in low-energy phenomenology concerns the decay processes $\Sigma^+ \to \Lambda^0 + e^+ + \nu_e$, $\Sigma^- \to \Lambda^0 + e^- + \bar{\nu}_e$. These reactions bear on the issue of both vector and axial-vector second-class currents, and we will return to them shortly.

B. PCAC

If the axial-vector current contains both first- and second-class pieces, then it is only the first-class piece whose divergence can be associated with the pion field, in the manner of PCAC. However, this produces no difficulty. Only the first-class current contributes to $\pi \to \mu + \nu$ decay and to the g_A parameter of nucleon β decay. The PCAC relation[15] between g_A and the pion decay parameter f_π is therefore left intact even if there *are* second-class axial-vector currents. Indeed, if the usual equal-time-commutator assumptions are maintained for the first-class currents, then all the standard PCAC and current-algebra predictions for strong-interaction phenomena are left unchanged (Adler consistency condition,[16] Adler-Weisberger relations for π-N scattering,[17] etc.). Similarly unchanged are the familiar predictions for K_{l3} (Ref. 18) and K_{l4} (Ref. 19) decays, provided we do not tamper with the strangeness-changing weak currents.

C. Weak nonleptonic decays

It is a standard assumption that the effective interaction responsible for nonleptonic decays has the form of a product of the currents that enter into the weak semileptonic interactions. The introduction of second-class currents would, of course, modify the usual picture of the nonleptonic Hamiltonian. However, a number of the standard PCAC, current-algebra predictions, i.e., those concerning nonleptonic hyperon decays and $K\to 3\pi$ decay, rest only on the assumption that the currents are purely left-handed.[20] These predictions are left intact if we assume that the second-class currents have this property and that all equal-time commutators between left- and right-handed charges vanish. In order to maintain these predictions, therefore, we must require that the second-class currents, if there are any, have both vector and axial-vector pieces.

One aspect of nonleptonic-decay phenomenology that *would* be altered by the introduction of second-class currents, and altered in a favorable way, has to do with the SU(3) structure of the nonleptonic Hamiltonian. In the standard picture, based on first-class currents which are exclusively left-handed, the octet piece of the Hamiltonian transforms like λ_6 under SU(3). According to Gell-Mann's theorem,[21] as generalized by Boulware and Brown,[22] it then follows that $K_0\to 2\pi$ decays must vanish in the limit of exact SU(3) symmetry. The observed $K_S^0\to 2\pi$ rates, however, have always seemed to be too substantial to be accounted for by SU(3)-symmetry breaking. As has been remarked by B. W. Lee, this difficulty can be circumvented, still without second-class currents, by the introduction of right-handed currents into models of the weak interactions.[23] Such right-handed interactions have recently come under intense consideration. We may also notice, however, that the same end is accomplished through the introduction of left-handed currents of second class. The octet term arising from a product of first- and second-class currents contains a piece which transforms like λ_7 under SU(3), and this term allows for $K_0\to 2\pi$ decays.

D. Hyperon β decays

We have already noted that the dominant Fermi and Gamow-Teller coefficients, g_V and g_A, in nucleon β decay arise solely from first-class currents. Let us focus on the corresponding coefficients for the other baryon-octet β-decay processes ($\Lambda\to p+e^-+\bar{\nu}_e$, etc.). If we do not tamper with the usual Cabibbo assumption for the strangeness-changing currents,[24] then all of the standard results of Cabibbo theory are left unchanged by the introduction of second-class currents, except for those concerning the processes $\Sigma^+\to\Lambda+e^++\nu_e$, $\Sigma^-\to\Lambda+e^-+\bar{\nu}_e$. These latter processes are potentially decisive for the issue of second-class currents, since here both the second- and first-class currents can contribute.

To see what is involved, it is only necessary to recall the several predictions that follow from the absence of second-class currents and from the other assumptions of standard weak-interaction theory. (i) From CVC alone it follows that the Fermi matrix element must vanish in the limit of zero momentum transfer q^2, and must therefore be, at most, very tiny for the small values of q^2 involved in these decays. (ii) In the limit of exact SU (3) symmetry, the added assumptions of the Cabibbo theory imply the standard relation of the Gamow-Teller coefficient for the $\Sigma^\pm$ decays to the corresponding coefficients for the other β-decay reactions. (iii) Even apart from Cabibbo theory, and from CVC, if the vector and axial-vector currents are purely first class, then the Fermi and Gamow-Teller coefficients must be the same for the Σ^+ and Σ^- processes, hence the two reactions should have essentially the same ft values (i.e., the same decay rates after phase-space differences are allowed for).

This prediction of ft equality between two mirror processes also holds for nuclear β decay in the absence of second-class currents. In the nuclear case, however, electromagnetic corrections are expected to be more significant even after phase-space and final-state Coulomb interactions are allowed for, as they are by the use of ft values.[25] Small ft discrepancies are in fact observed in the nuclear case, but they are not in themselves decisive.[26] Rather, the nuclear evidence that we have cited for second-class axial-vector currents relies on more detailed spectral measurements. For the $\Sigma^\pm$ decays, electromagnetic corrections should be much less important; the ft comparisons, correspondingly, should be more definitive. Unfortunately, the present errors on the $\Sigma^+\to\Lambda+e^++\nu_e$ and $\Sigma^-\to\Lambda+e^-+\bar{\nu}_e$ rates are still rather large,[27]

$$(ft)^+/(ft)^-=0.96\pm0.24,$$

so that the limits on second-class currents are not yet very stringent. Concerning the CVC prediction of vanishing Fermi coefficient g_V, the present errors are similarly large: For $\Sigma^-\to\Lambda e^-\bar{\nu}_e$

$$g_V/g_A=0.37\pm0.20$$

(see Ref. 28), while no value is available at present for the corresponding Σ^+ transition.

E. Neutrino reactions

The standard picture of the weak currents (exclusively first-class, CVC, PCAC, standard current algebra, etc.) leads to a number of familiar predictions for neutrino reactions, and some of these would be altered by the introduction of second-class currents. In Sec. II we shall take up a detailed discussion of one particular class of effects. Here let us briefly review several of the others. We concentrate on $\Delta S = \Delta C = 0$ charged-current reactions.

It was observed by Adler that the amplitude for forward, inelastic lepton scattering (outgoing-muon momentum parallel to incoming-neutrino momentum) is proportional to the matrix element of the divergence of the weak current in the approximation where the outgoing-lepton mass is ignored.[29] If there is no second-class vector current, and if the first-class current obeys CVC, it follows that only the axial-vector current contributes in the forward limit; consequently, it follows that there can be no parity-violating effects in this limit. It is clear that a second-class vector current, if it exists, cannot be conserved. Thus the discovery of parity-violating correlations for forward-inelastic lepton scattering would either rule out CVC for the first-class current, or imply the existence of a second-class vector current. There are two other effects to be noted. If the vector and axial-vector currents are purely first class, and if CVC holds, then the forward cross sections for neutrinos and antineutrinos on an isoscalar target should be identical. Moreover, the cross section on any target can be related, through PCAC, to the cross section for pion inelastic scattering on the same target.[29] Clearly, second-class vector or axial-vector currents would upset these predictions. Unfortunately the experimental evidence on these various issues is still too limited for any conclusions to be drawn.

It is similarly clear that the equal-time commutation relations among the weak currents would be changed by the introduction of second-class currents and that this would alter the Adler[30] and Gross–Llewellyn Smith[31] sum rules for the inelastic-neutrino structure functions, but these are not yet well tested experimentally. We still have to reckon, however, with certain other successes of the standard picture for deep-inelastic scattering. Solely from the assumption that the currents are first class it follows that the familiar structure functions F_1, F_2, F_3 must be identical for neutrinos and antineutrinos on an isoscalar target (recall that we are ignoring any $\Delta S \neq 0$ or $\Delta C \neq 0$ contributions). This is so quite apart from the more detailed issues of Bjorken scaling, the quark-parton model, etc. However, scaling does seem to hold up very well experimentally, and the fits to the structure functions made on the assumption of scaling seem to roughly confirm the equalities. It should be noted, however, that the equalities would hold also if the currents were purely second class; it is only the *interference* between first- and second-class currents that would upset the predictions.

This brings us to the most vexing question for second-class currents. Namely, how can one introduce them in the framework of present theoretical ideas about the weak interactions? In the usual quark picture, apart from color, there is only one pair of nonstrange and noncharmed quarks, $\mathcal{P}$ and $\mathcal{N}$. The currents formed from these are necessarily first class. To provide for second-class currents it would be necessary, at a minimum, to introduce two pairs $(\mathcal{P}_1, \mathcal{N}_1)$, $(\mathcal{P}_2, \mathcal{N}_2)$ and to suppose that each is a doublet under strong SU (2) symmetry.[32] This is not at all attractive. Nevertheless, just to illustrate a possibility, let us pursue this idea briefly. For the charge-raising first- and second-class currents, J_μ^{I} and J_μ^{II}, we take

$$J_\mu^{\mathrm{I}} = \sum_{i=1}^{2} \sum_{j=1}^{2} \overline{\mathcal{P}}_i \gamma_\mu (1+\gamma_5) \mathcal{N}_j \delta_{ij} .$$

$$J_\mu^{\mathrm{II}} = \lambda \sum_i \sum_j \overline{\mathcal{P}}_i \gamma_\mu (1+\gamma_5) \mathcal{N}_j \epsilon_{ij} ,$$

and then for the charge-lowering currents

$$J_\mu^{\mathrm{I}\dagger} = \sum_i \sum_j \overline{\mathcal{N}}_i \gamma_\mu (1+\gamma_5) \mathcal{P}_j \delta_{ij} ,$$

$$J_\mu^{\mathrm{II}\dagger} = -\lambda \sum_i \sum_j \overline{\mathcal{N}}_i \gamma_\mu (1+\gamma_5) \mathcal{P}_j \epsilon_{ij} .$$

The parameter λ measures the strength of the second-class relative to the first-class currents. In order to understand the sizeable second-class effects suggested in the nuclear β-decay experiments we must suppose that λ is of order unity, but of course it is not possible to make any precise quantitative statement.

With $J_\mu = J_\mu^{\mathrm{I}} + J_\mu^{\mathrm{II}}$ the total current, we see immediately that the equal-time commutators obey

$$[J_\mu^\dagger(x), J_\nu(y)]_{x_0=y_0} = (1+\lambda^2)[J_\mu^{\mathrm{I}\dagger}(x), J_\nu^{\mathrm{I}}(y)]_{x_0=y_0} .$$

Thus, neglecting $\Delta S \neq 0$ and $\Delta C \neq 0$ effects, Adler's sum rule is modified by the factor $1+\lambda^2$. With scaling assumed, we have

$$\int_0^1 dx [F_2^{\bar{\nu}p}(x) - F_2^{\bar{\nu}p}(x)] = 2(1+\lambda^2) .$$

The Gross–Llewelyn Smith sum rule is similarly modified by the factor $1+\lambda^2$. Of course the Callan-

Gross relation,[33] which merely reflects the spin-$\frac{1}{2}$ nature of the quarks, is unmodified.

The most important feature of the model under discussion is that in a parton picture of deep-inelastic scattering there is no interference between the first- and second-class currents, so that one of the major potential difficulties, noted previously, does not materialize, and the modifications to the sum rule are not yet tested for experimentally. However, there is one other parton-model result to be noted. This concerns the relation between neutrino and electroproduction structure functions. If we make the usual assumption that the strange and antiparton distributions in the nucleon are negligible, we have[34]

$$F_2^{\nu p}+F_2^{\nu n}=\tfrac{18}{5}(1+\lambda^2)(F_2^{ep}+F_2^{en}).$$

Comparison of SLAC electroproduction data with CERN Gargamelle neutrino data at large x appears to require that $\lambda^2\leqslant 0.5$. This is the only serious quantitative restriction on the strength of second-class interactions which we can presently extract from the deep-inelastic data, and even this can be relaxed somewhat if we are willing to allow for some strange quark components in the nucleon; at least this would go in the right direction to permit an increase in the second-class parameter λ.

The model discussed above is not meant to be taken as a serious proposal. It is meant only to illustrate some of the issues involved for second-class currents and deep-inelastic scattering: notably, on a parton picture, the question of whether one can avoid interference effects between first- and second-class currents. Clearly, the introduction of an extra quark doublet makes trouble for $\pi^0\to 2\gamma$ decay; similarly, the model probably runs into difficulty with the ratio R for e^+,e^- annihilation into hadrons. However, the important matter, detailed theories and aesthetic issues aside, is a phenomenological one: Are there second-class currents? The nuclear β-decay experiments provide a hint of substantial second-class axial-vector current effects. Elsewhere, so far, the tests are not yet decisive. In the following section we discuss in detail some additional tests that might be useful.

III. SECOND-CLASS EFFECTS IN Δ PRODUCTION BY NEUTRINOS

Consider the pair of mirror reactions

$$\begin{aligned}&\nu_\mu+\alpha\to\mu^-+\beta\,,\\ &\nu_\mu+\tilde{\alpha}\to\mu^++\tilde{\beta}\,,\end{aligned}\tag{6}$$

where $\tilde{\alpha}=e^{i\pi I_2}\alpha$, $\tilde{\beta}=e^{i\pi I_2}\beta$, and where α and β may be single-particle or multiparticle states. The fundamental property which reflects the first- or second-class character of the weak currents is contained in the relations

$$\begin{aligned}&\langle\tilde{\beta}\,|J_\mu^{\mathrm{I}}|\,\tilde{\alpha}\rangle=\langle\beta|J_\mu^{\mathrm{I}\dagger}|\,\alpha\rangle\,,\\ &\langle\tilde{\beta}\,|J_\mu^{\mathrm{II}}|\,\tilde{\alpha}\rangle=-\langle\beta|J_\mu^{\mathrm{II}\dagger}|\,\alpha\rangle\,.\end{aligned}\tag{7}$$

If the weak currents were purely of first class (or purely of second class) the differential cross sections for the two mirror reactions would differ only because of leptonic V,A interference. Beyond this, any differences must reflect interference between first- and second-class hadronic currents; such differences therefore serve as tests for the existence of second-class currents.

Suppose that the final state contains more than one hadron. Then for the neutrino process, say, we select a particular final hadron β_1, writing $\beta=\beta_1+X$, where X may itself be a single-particle or multiparticle state. Let q_1 and q_2 be the momenta of incoming and outgoing leptons and let $q=q_1-q_2$, $Q=q_1+q_2$. We now choose as independent variables the neutrino laboratory energy ϵ, the invariant mass W of the hadron system β, the invariant mass and other "internal" variables of the system X, and finally the polar angle θ and azimuthal angle ϕ of the particle β_1 in the overall hadron rest frame. In this frame we choose $\vec{q}$ to be along the z axis and $\vec{Q}$ to be in the x-z plane. Notice that ϕ is simply the angle between the lepton plane and the hadron plane defined by β_1 and X. For the antineutrino reaction we choose similar variables with the obvious changes of $\beta_1\to\tilde{\beta}_1$, $X\to\tilde{X}$, etc.

We now recall an observation due to Pais concerning the dependence of the differential cross section on ϕ. The general structure is

$$\begin{aligned}d\sigma=&(\sigma_0+\sigma_1\cos\phi+\sigma_2\cos 2\phi+\sigma_3\sin\phi+\sigma_4\sin 2\phi)\\ &\times d\phi\,dW\,dq^2\cdots\,,\end{aligned}\tag{8}$$

where the σ_i depend on the variables other than ϕ. The functions $\sigma_0,\sigma_1,\sigma_3$, in general, are different for the ν and $\bar{\nu}$ reactions. On the other hand, σ_2 does not involve leptonic V,A interference, while σ_4 is strictly a leptonic V,A interference term. Therefore, as Pais shows, in the absence of interference between first- and second-class currents one has[35]

$$\begin{aligned}&\sigma_2^{\nu\alpha}=\sigma_2^{\bar{\nu}\tilde{\alpha}}\,,\\ &\sigma_4^{\nu\alpha}=-\sigma_4^{\bar{\nu}\tilde{\alpha}}\,.\end{aligned}\tag{9}$$

The detection of any difference in these two angular-correlation terms for a pair of mirror processes would signal the existence of second-class currents. In principle, any pair of processes will do, provided there is more than one hadron in the final state. Still, one would like to have some estimate for particular processes of the size of the effects to be expected. For practical experimental purposes one requires that σ_2 and/or σ_4 be sizeable (with respect to σ_0) and sensitive to second-class contributions. We discuss here the reactions

$$\begin{aligned} &\nu_\mu + p \to \mu^- + p + \pi^+ , \\ &\nu_\mu + n \to \mu^+ + n + \pi^- , \end{aligned} \tag{10}$$

and specialize to the region of the (3, 3) resonance at $W = 1236$ MeV.

These reactions have been studied theoretically by a number of authors without allowance, however, for second-class currents.[36] Here we shall closely follow the analysis discussed by Adler.[37] It is based essentially on the Born approximation, corrected by final-state effects for the resonant multipoles. The nuclear form factors of Eq. (4) enter in the parameterization of the Born diagrams. What is new here is that we allow for the second-class axial-vector form factor $g_{\rm II}$, (we neglect the muon mass, so that the second-class vector form factor g_S plays no role). Integrating over all final-state variables other than ϕ and W, we have

$$\frac{\partial^2\sigma}{\partial W \partial\phi} = \frac{G^2 \cos^2\Theta_C}{\pi^2 \epsilon}\left(\frac{2m}{g_r}\right)^2 |e^{i\delta_{33}(W)} \sin\delta_{33}(W)|^2 \frac{1}{(\omega - m_\pi)^2}(A_0 + A_1\cos\phi + A_2\cos 2\phi + A_3 \sin\phi + A_4 \sin 2\phi), \tag{11}$$

where A_i depends on ϵ and W, δ_{33} is the (3,3) phase shift for π-N scattering at barycentric energy W, and $\omega = W - m$.

Our present aims are somewhat qualitative; they are to indicate roughly the magnitude of A_2 and A_4 (relative to A_0) and to study their sensitivity to second-class currents. For these purposes we adopt a number of approximations: We ignore terms of order q^2/m^2 and ω/m (the so-called "static" limit), and in integrating over q^2 we suppose that all the form factors of Eq. (4) have the same dipole falloff—in particular, therefore, the second-class contribution is at the end parameterized by $g_{\rm II}(0)$, the zero-momentum-transfer value involved in nuclear β decay. Moreover, since we are focusing on the resonance region we retain only the resonant multipoles. In this latter approximation A_3 and A_4 vanish; these terms involve interference of resonant and nonresonant multipoles. Even when we allow for the nonresonant multipoles, however, we find that A_4 is very small relative to A_0. Thus for the tests under discussion here it is A_2 that is relevant, and we compute both A_2 and A_0 for several values of the energy ϵ and several values of $g_{\rm II}(0)$. In all cases we set W at the resonance value.

What is at issue is the comparison of A_2 for the two processes of Eq. (10). Models aside, any

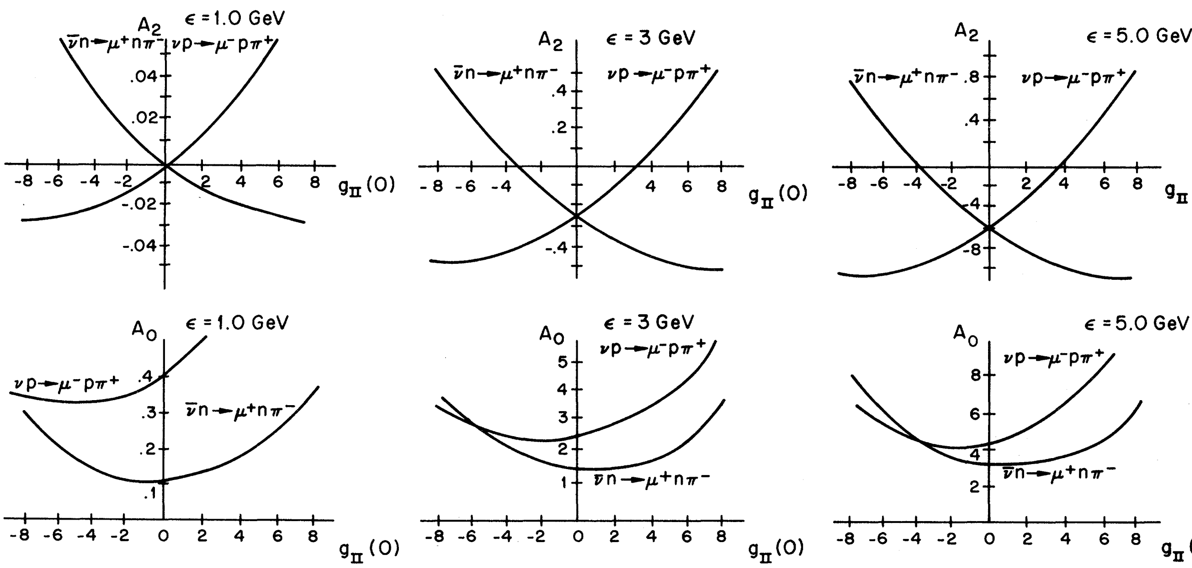

FIG. 1. Shown are graphs of correlation parameters A_2, A_0 vs $g_{\rm II}(0)$ at three different neutrino energies. So as not to emphasize the particular numerical predictions, the vertical scales are in terms of arbitrary units, but the six curves are properly normalized among themselves.

difference would signal the presence of second-class currents. Experimentally, the absolute determination of A_2 for the ν and $\bar{\nu}$ processes is sure to be difficult, depending as it does on knowledge of fluxes. It would be better to compare the *ratios* A_4/A_2 since this ratio itself should be the same for the two processes in the absence of second-class currents. As has been said, however, A_4 is likely to be too small to determine.

Our results on A_2 and A_0 are displayed in Fig. 1 for several energies ϵ and for a range of values of the second-class parameter $g_{II}(0)$. We note that A_0, which is the cross section integrated over ϕ, itself has a non-negligible dependence on $g_{II}(0)$. A substantial deviation of the measured cross section from the Adler prediction might then in itself be suggestive of a second-class axial-vector current. However, this requires not only an absolute determination of the cross sections, but also faith in the accuracy of the model, so cross-section results need not be definitive unless the second-class effects are very big. More interesting and striking is the behavior of the cos 2ϕ term, A_2. Any change in this correlation in going from $\nu p \to \mu^- p \pi^+$ to $\bar{\nu} n \to \mu^+ n \pi^-$ would unambiguously signal the presence of a second-class interaction. We see from Fig. 1 that the second-class effects can be rather large. Indeed, we see that for values of $g_{II}(0)$ as large as indicated by the nuclear β-decay experiments, A_2 *changes sign* between the νp and $\bar{\nu} n$ reactions. If such a sign change were observed careful neutrino-flux measurements would become unnecessary. Also we note that the parameter A_2 is typically of order 10% relative to the leading term A_0, so that detection of a $\cos 2\phi$ correlation should not prove impossibly difficult. The specific numerical values given depend, of course, on the model we are utilizing and should not be taken too seriously. However, the detection of any difference of A_2 between the two reactions, in particular the detection of a sign change, would be significant beyond any particular model. What our model-dependent calculations show is that A_2 is a particularly sensitive test for the presence of second-class interactions.

Of the two reactions to be compared,

$$\nu + p \to \mu^- + p + \pi^+$$

and

$$\bar{\nu} + n \to \mu^+ + n + \pi^-,$$

the second one is obviously much more demanding experimentally. For first rough indications one might instead consider the process

$$\bar{\nu} + p \to \mu^+ + p + \pi^-.$$

In the resonance region one may expect the $I=\frac{3}{2}$ amplitude to dominate. With neglect of $I=\frac{1}{2}$ contributions one has

$$\text{Amp}\,(\bar{\nu} + p \to \mu^+ + p + \pi^-) = \tfrac{1}{3}\,\text{Amp}\,(\bar{\nu} + n \to \mu^+ + n + \pi^-)\,.$$

Thus, a comparison of $9A_2^{\bar{\nu}p}$ with $A_2^{\nu p}$ might serve at least roughly for tests of second-class effects, especially if these effects are as large as indicated by the present estimates. Because of $I=\frac{1}{2}$ final-state contamination the results would not, of course, be decisive.

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