PHYSICAL REVIEW D VOLUME 13, NUMBER 11 1 JUNE 1976

Using beam polarization to probe the weak neutral current via the reactions $e^+e^- \to e^+e^-$ and $e^-e^- \to e^-e^-$ *

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(Received 24 November 1975)

The weak current is assumed to have scalar, pseudoscalar, and tensor, as well as vector and axial-vector components. The consequences of this assumption on the reactions $e^+e^- \to e^+e^-$ and $e^-e^- \to e^-e^-$ are examined to see whether the various coupling constants can be separated. If the beam polarization can be controlled five of a possible eight couplings can be measured in principle.

Design studies for the next generation of colliding-beam machines proceed apace, spurred on by the exciting discoveries at existing storage rings. Both e^+e^-, and in the longer term e^-e^-, machines are being considered.[1] The beams are expected to be naturally polarized parallel to the magnetic field. In fact, this phenomenon has already been observed at energies now available.[2] The possibility of rotating the polarization vector has been considered and seems feasible. It was found that provision must be made in the design parameters of the machine if such an option is deemed desirable.[3] It is important, therefore, to see whether additional areas of physics can be explored if one has control over the beam polarization.

One area of interest which might be examined is the space-time structure of the weak neutral current. If one postulates that this current has scalar (S), pseudoscalar (P), and tensor (T), as well as vector (V) and axial-vector (A) components, then it is not possible to separate S, T, and P from V and A using neutrino-electron or deep-inelastic neutrino-nucleon scattering experiments.[4] Since the neutral current can contribute to Bhabha and Möller scattering, these processes could provide a means of sorting out the various couplings. The purpose of this paper is to see to what extent this can be accomplished and, in particular, to see what additional useful information can be extracted if the polarization of the beams can be controlled.

Assuming a current-current form for the weak interaction, the Hamiltonian density $\mathcal{H}(x)$ can be written as

$$\mathcal{H}(x) = e\overline{\psi}(x)\gamma_\mu\psi(x)A^\mu(x) + \frac{G}{2\sqrt{2}}\sum_i \overline{\psi}(x)\Gamma_i\psi(x)\overline{\psi}(x)(C_i + \gamma_5 C_i')\Gamma^i\psi(x)\,, \tag{1}$$

where the first term is the usual electromagnetic interaction and the second term is the most general weak nonderivative point interaction with $\Gamma_i = 1$, $i\gamma_5$, γ_μ, $\gamma_5\gamma_\mu$, $\sigma_{\mu\nu}$, and the corresponding coupling constants are C_S, C_P, C_V, C_A, C_T, $C_S', C_P', C_V', C_A', C_T'$. Because the initial and final particles are the same we can choose $C_V' = C_A'$, $C_S' = -C_P'$. This contrasts with the situation for $e^+e^- \to \mu^+\mu^-$ (see Ref. 5), where these couplings are not equal unless μ-e universality holds. Because the Hamiltonian is a Hermitan operator, all of these coefficients are real except C_S', C_P', and C_T', which are pure imaginary. (Bjorken and Drell[6] conventions are being used.)

For both reactions, let (k_+, s_+) and (k_-, s_-) be the momentum and spin four-vectors of the incoming particles traveling in the positive-z and negative-z directions, respectively, and let k_-' be the four-vector describing the outgoing electron.

Assuming that the collision takes place in the center of mass and that the energy of the incoming particles is E, then

$$k_+^\mu = E(1, 0, 0, \beta), \quad k_-^\mu = E(1, 0, 0, -\beta)$$

$$s_+^\mu = P_+(\beta\gamma\cos\theta_+, \sin\theta_+\cos\phi_+, \sin\theta_+\sin\phi_+, \gamma\cos\theta_+) \tag{2}$$

$$s_-^\mu = P_-(-\beta\gamma\cos\theta_-, \sin\theta_-\cos\phi_-, \sin\theta_-\sin\phi_-, \gamma\cos\theta_-)$$

$$k_-'^\mu = E(1, -\beta\sin\theta\cos\phi, -\beta\sin\theta\sin\phi, -\beta\cos\theta)\,,$$

where

$$\beta = \frac{|\vec{k}_+|}{E} = \frac{|\vec{k}_-|}{E} = \frac{|\vec{k}_+'|}{E} = \frac{|\vec{k}_-'|}{E},\ \gamma = (1-\beta^2)^{-1/2}\,.$$

P_+ (P_-) is the magnitude of the polarization of the particle traveling in the $+z$ ($-z$) direction. For the e^+e^- reaction, let the positron travel in the $+z$ direction. For convenience define

$$\Psi = \phi_+ - \phi_-\,, \quad \Phi = 2\phi - \phi_+ - \phi_-\,. \tag{3}$$

In what follows, terms of order G^2 are ignored, as are terms of order m_e/E, where m_e is the electron mass.

THE REACTION $e^+e^- \to e^+e^-$

In the case of Bhabha scattering the differential cross section is given by

$$\begin{aligned}\frac{d\sigma}{d\Omega}\left(\frac{\alpha^2}{16E^2}\right)^{-1} =& \frac{(3+z^2)^2}{(1-z)^2} + \frac{4E^2G}{e^2\sqrt{2}}\frac{(1+z)}{(1-z)}[(1+z)^2C_1+(7-4z+z^2)C_2] \\ &+\left\{\frac{(3+z^2)^2-16}{(1-z)^2}+\frac{4E^2G}{e^2\sqrt{2}\,(1-z)}[(1+z)^3C_1+(z-3)(z^2+3)C_2]\right\}P_+P_-\cos\theta_+\cos\theta_- \\ &+\left\{-(1+z)^2+\frac{4E^2G}{e^2\sqrt{2}}(1+z)[(z-1)C_1+(z+1)C_2]\right\}P_+P_-\sin\theta_+\sin\theta_-\cos\Phi \\ &+\frac{8E^2G}{e^2\sqrt{2}}\frac{(1+z)}{(1-z)}C_3P_+P_-\sin\theta_+\sin\theta_-\cos\Psi+\frac{8E^2G}{e^2\sqrt{2}}\frac{(1+z)}{(1-z)}C_3'P_+P_-\sin\theta_+\sin\theta_-\sin\Psi \\ &+\frac{4E^2G}{e^2\sqrt{2}}\frac{(1+z)^3}{(1-z)}C_1'(P_+\cos\theta_++P_-\cos\theta_-)\,, \end{aligned} \tag{4}$$

where $z=\cos\theta$, α is the fine-structure constant, and, for convenience,

$$\begin{aligned} &C_1=2(C_V+C_A), \quad C_2=C_V-C_A-\tfrac{1}{2}(C_S+C_P), \quad C_3=-C_S+C_P+12C_T, \\ &C_1'=2(C_V'+C_A'), \quad -C_3'=i(-C_S'+C_P'+12C_T')\,. \end{aligned} \tag{5}$$

The V,A terms have already been calculated,[7] as have the S terms.[8] The latter disagree with Eq. (4). The former have been included to facilitate the discussion which follows.

With unpolarized beams, that is, $P_+=P_-=0$, the θ dependence enables one to measure C_1 and C_2.

The more likely situation is that the beams are naturally polarized,[2] that is, $\sin\theta_+=-\sin\theta_-=1$ and $\phi_+=\phi_-=0$. In this case the θ dependence enables one to measure C_1, C_2, and C_3. The ϕ dependence gives an additional constraint making the task of separating C_1 and C_2 from C_3 somewhat easier. A nonzero value of C_1 would be an indication of a V,A current, whereas a nonzero value of C_3 would constitute evidence for an S, T, or P current. Notice that having naturally polarized beams does not significantly reduce the relative contribution of the electromagnetic terms in contrast to the situation for $e^+e^-\to\mu^+\mu^-$.[9] This can be most easily seen by noting that the coefficient of $P_+P_-\cos\Phi$ is significantly smaller than the leading contribution at all angles. In fact,

$$(1+z)^2/[(3+z^2)^2/(1-z)^2]\le\tfrac{1}{9}\,. \tag{6}$$

It is important to remember that higher-order contributions from QED will complicate any experiment designed to measure C_1, C_2, and C_3.[10] Notice that the following terms are proportional to $C_3'\sin\Psi+C_3\cos\Psi$ and $C_3\cos\Psi$, respectively, if one chooses ϕ such that $\Phi=0$:

$$\frac{d\sigma}{d\Omega}(\Psi)-\frac{d\sigma}{d\Omega}(\pi+\Psi)$$

and (7)

$$\frac{d\sigma}{d\Omega}(\Psi)-\frac{d\sigma}{d\Omega}(\pi-\Psi)\,.$$

So if one of the polarization vectors can be rotated in the x,y plane, C_3' and C_3 can be measured. A finite value of C_3' would indicate the presence of a parity-violating CP-violating term of the STP type.

A configuration having both beams longitudinally polarized with opposite helicity, that is, $\theta_+=\theta_-=0$ would allow one to measure C_1', provided the polarization vectors could be flipped, as has been discussed.[7] An added attraction is the fact that the relative contribution of the electromagnetic term is significantly reduced. C_1' is the coefficient of a parity-violating CP-conserving term, and a nonzero value would be a clear indication of the presence of a weak-interaction term of the V,A type uncontaminated by electromagnetic contributions.

THE REACTION $e^-e^-\to e^-e^-$

In the case of Möller scattering the differential cross section is given by

$$\frac{d\sigma}{d\Omega}\left(\frac{\alpha^2}{4E^2}\right)^{-1}=\frac{(3+z^2)^2}{(1-z^2)^2}+\frac{4E^2G}{\sqrt{2}\,e^2}\left[\frac{4C_1+(1+3z^2)C_2}{1-z^2}\right]$$
$$+\left\{\frac{(3+z^2)^2-16}{(1-z^2)^2}+\frac{4E^2G}{\sqrt{2}\,e^2}\left[\frac{-4C_1+(1+3z^2)C_2}{1-z^2}\right]\right\}P_+P_-\cos\theta_+\cos\theta_-$$
$$+\left(-1-\frac{4E^2GC_2}{\sqrt{2}\,e^2}\right)P_+P_-\sin\theta_+\sin\theta_-\cos\Phi$$
$$+\frac{8E^2GC_3}{\sqrt{2}\,e^2(1-z^2)}P_+P_-\sin\theta_+\sin\theta_-\cos\Psi+\frac{8E^2GC_3'}{\sqrt{2}\,e^2(1-z^2)}P_+P_-\sin\theta_+\sin\theta_-\sin\Psi$$
$$-\frac{16E^2GC_1'}{e^2\sqrt{2}\,(1-z^2)}(P_+\cos\theta_+-P_-\cos\theta_-)\,. \tag{8}$$

The V,A terms[11,12] and S term[8] have been previously calculated, and the S term disagrees with Ref. 8.

As far as unpolarized or transversely polarized beams are concerned, the cross section is very similar to that for Bhabha scattering, and the discussion following Eq. (4) applies equally to Eq. (8). In addition, notice that the ϕ-dependent term is independent of θ.

The relative magnitude of the electromagnetic contribution can be reduced by having both beams longitudinally polarized with opposite helicities ($\theta_+=\theta_-=0$). This will improve matters as far as measuring C_2 is concerned but not C_1 or C_1', whose coefficients are drastically reduced (the latter to zero) for this configuration.

An unambiguous indication of the presence of a weak interaction of the V,A type which is parity-violating and CP-conserving is given by the quantity[12]

$$A_2(z)=\frac{(d\sigma/d\Omega)(\theta_+=0;\,\theta_-=\pi)-(d\sigma/d\Omega)(\theta_+=\pi;\,\theta_-=0)}{(d\sigma/d\Omega)(\theta_+=0;\,\theta_-=\pi)+(d\sigma/d\Omega)(\theta_+=\pi;\,\theta_-=0)}$$
$$=-\frac{16E^2GC_1'(P_++P_-)(1-z^2)}{e^2\sqrt{2}\,\{(3+z^2)^2-[(3+z^2)^2-16]P_+P_-\}}\,. \tag{9}$$

Although no cancellation occurs in the denominator, this asymmetry is of the same order as the corresponding one for $e^+e^-\to\mu^+\mu^-$ and Bhabha scattering. In fact, for $P_+=P_-=0.924$, the maximum value of A_2 (which occurs at $\theta=90°$) is half the maximum value of the corresponding asymmetry in $e^+e^-\to\mu^+\mu^-$ (see Ref. 13) and almost equal to that in $e^+e^-\to e^+e^-$.[7] Its value is almost unchanged if one of the beams is unpolarized, in contrast to the Bhabha asymmetry,[7] and the cross section is four times as large as the Bhabha cross section and an order of magnitude larger than that for $e^+e^-\to\mu^+\mu^-$ at the relevant values of θ, an attractive feature at machine luminosities being contemplated.

DISCUSSION

In principle, five of a possible eight couplings can be measured with either of these reactions if the beam-polarization vector can be controlled. In practice C_1 and C_2 may be very difficult to extract for two reasons. The first is the complication of radiative corrections. However, these should be calculable to a good degree of accuracy.[10,11] The second is the fact that the θ dependence of the electromagnetic term is not simple and in the "near" future the weak contribution will be a small background term (of order 10% at $E=15$ GeV).

A way of avoiding this problem might be the following. If one defines

$$\sigma_{+-}=\frac{d\sigma}{d\Omega}(e^+e^-\to e^+e^-)\,,$$
$$\sigma_{--}=\frac{d\sigma}{d\Omega}(e^-e^-\to e^-e^-)\,, \tag{10}$$

then the quantity $\sigma_{+-}-(1+z)^2\sigma_{--}/4$ contains no lowest-order electromagnetic contributions. Such a measurement might be feasible at a two-ring machine, for example, since the geometry would be exactly the same for both reactions.

An attractive way of measuring all of these couplings, in principle, is to look at the reaction $e^+e^-\to\mu^+\mu^-$.[5] With naturally polarized beams C_V and C_A can be measured[9] and with controllable longitudinally polarized beams C_V' and C_A' can be measured.[13] In order to measure the other couplings a knowledge of the polarization of the outgoing muon is required. Measuring this is so difficult[14] that it would seem worthwhile pursuing the approach of beam-polarization control as a means of probing the weak-interaction structure.

It is interesting to note that longitudinally polarized electron beams have been generated in a linear accelerator.[15] This presents an alternative means of looking for the effects discussed here if one collides this beam with a storage-ring beam. However, low luminosity may be a severe drawback.[12]

ACKNOWLEDGMENT

This work was begun while the author was at Queen Mary College, The University of London.

*Work supported by a National Science Council of Ireland grant.

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