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N^* resonance parameters from a dual analysis of backward π^-p scattering*

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The authors had recently performed an analysis, using their dual interference model, of the backward π^+p data to determine the parameters of the $I=3/2$ Δ resonances in the region $W\sim2000$ to 2200 MeV. Using these results for the Δ's, a similar analysis of π^-p data is performed to determine the $I=1/2$ N resonances for $W\sim1900$ to 2200 MeV. The 180° data are fitted along with backward angular distributions (at p_{lab} from 2.08 to 2.38 GeV/c) and the presence of the seven resonances listed in this W range by the Particle Data Group is confirmed. Clear evidence is found for a new G_{19} state. In view of the considerable success achieved in using the dual interference model to fit the π N data, a future expanded program of resonance analysis is discussed.

I. ANALYSIS OF BACKWARD π^-p DATA

Recently, there has been a great deal of interest in furthering our understanding of hadron spectroscopy above 2 GeV.[1] The most extensively studied system both experimentally and theoretically is πN and yet there are no four-star, as rated by the Particle Data Group,[2] πN resonances above 2 GeV. Most of our knowledge of πN resonances comes from detailed phase-shift analyses. However, it is clear that these analyses are enormously difficult and cannot go much above 2 GeV.[3] Yet, it is very important to know if various strong-interaction models such as the SU(6)×O(3) quark model, the various "bag" models,[4] and dual resonance models[5] are operative in this regime. There is, of course, the additional impetus coming from the exciting discovery of the new, very narrow, resonances[6] above 3 GeV in that new kinds of hadron states (both narrow and broad) might be seen in πN scattering.

Although we expect many resonances to be present at around 2 GeV, detailed information is often lacking. In order to isolate the individual resonances from the large background, certain assumptions must be made. For this purpose, we find that our dual interference model,[7] proposed some time ago, is both theoretically sound and phenomenologically useful. We predicted,[8] successfully, the existence of the negative parity Δ resonance at 2200 MeV that was discovered two years ago in 180° π^+p scattering.[9] More recently we determined[10] the (J,L) assignment of this new resonance and obtained very good fits to the data. In this paper, we report the results of extending our analysis to cover the backward π^-p data: We are again successful in fitting the data well. We consider this a very encouraging sign that our dual interference model will provide a consistent and fruitful way of resonance analysis above 2 GeV, especially if data around 90° are also used together with the constituent-quark-interchange model[11] for the corresponding background.

In this π^-p analysis, we use differential cross section data at 180° from $p_{lab}=1.6$ to 5.3 GeV/c, Kormanyos *et al.*,[12] and backward angular distribution data at $p_{lab}=2.08$, 2.18, and 2.28 GeV/c, Carroll *et al.*,[13] $p_{lab}=2.15$ GeV/c, Meanley *et al.*,[14] and at $p_{lab}=2.38$ GeV/c, Crittenden *et al.*[15] The (real) background amplitude is the nonsignatured Δ_δ Regge contribution determined from high-energy fits, as explained in our previous publications.[7,8,10]

We have used a constant-width Γ, constant-elasticity x, Breit-Wigner parametrization for a resonance of spin J and $L=J\pm\frac{1}{2}$,

$$a^J_{L=J\pm1/2}=\frac{1}{k}\,\frac{x\Gamma/2}{M-W-i\Gamma/2}.$$

We ensure the additional requirement of MacDowell symmetry by adding to the same J and opposite parity a contribution

$$a^J_{L=J\mp1/2}=-\frac{1}{k}\,\frac{x\Gamma/2}{M+W}.$$

Parameters for the $I=3/2$ Δ resonances are fixed by our π^+p fit (Table I of Ref. 10). We consider that the $I=\frac{1}{2}$ N resonances below 1800 MeV are given by the Particle Data Group[2] (1972 averages). We vary the parameters for other possible resonances above 1800 MeV and obtain a fit to all the data (120 points) with a χ^2 of 167. (We have enlarged the errors somewhat on eight points which

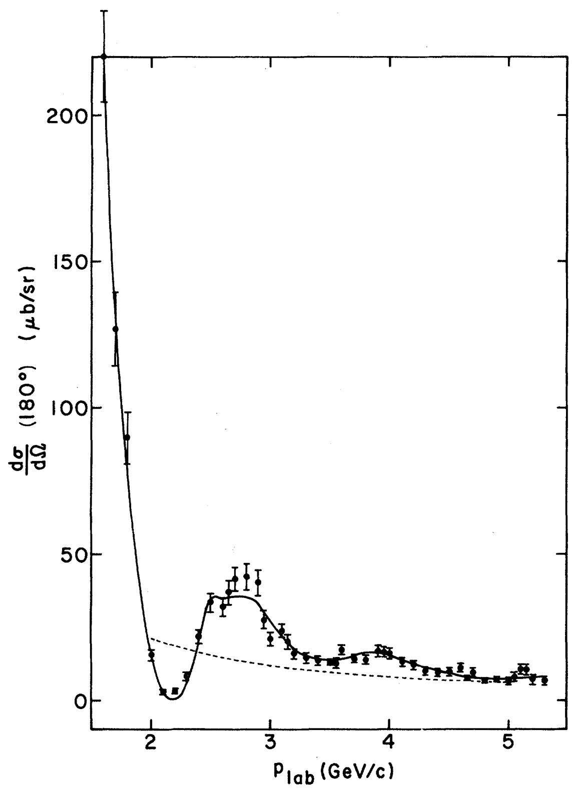

FIG. 1. Plot of $d\sigma/d\Omega$ at $\cos\theta = -1$ versus p_{lab}. The data points are those of Ref. 12. The dashed curve is the prediction of our previous Regge fit (Ref. 8) to the high-energy data ($\gtrsim 6$ GeV/c). The parameters are given in Ref. 8. The nonsignatured part of the Δ_δ trajectory is taken to be the background term for the present dual model fits and is not varied. The solid curve is the present dual model fit; the resonance parameters are given in Table I for the N's and in Table I of Ref. 10 for the Δ's.

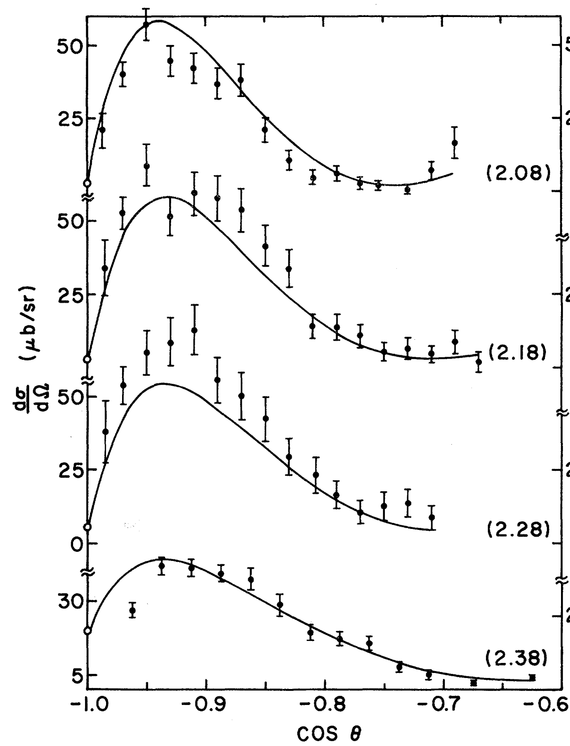

FIG. 2. Plots of $d\sigma/d\Omega$ versus $\cos\theta$ for $p_{lab} = 2.08$, 2.18, 2.28 (data points from Ref. 13) and 2.38 GeV/c (data points from Ref. 15). The solid curve is the same fit as in Fig. 1. The open circles correspond to 180° data points extrapolated to these energies from Fig. 1. The fit to 2.15 GeV/c data (Ref. 14) is similar to the 2.18 fit and thus is not displayed.

clearly would not lie on a smooth fit to the data so that they will not unduly bias our over-all fit.) We refer the reader to our previous papers for a detailed discussion of our parametrization. The fits are shown in Fig. 1 (180° data) and Fig. 2 (angular distributions), and the N resonance parameters are presented in Table I.

We have concentrated mainly on the energy range 1800–2250 MeV. In addition to the eight "starred" N resonances listed by the Particle Data Group,[2] we considered various other (J,L) possibilities in this energy range. We found that all eight of these resonances were necessary in our fit. In addition, we needed only one other state: a G_{19} (2100) which would be the leading N state in a [SU(6), L] multiplet of $(\underline{70}\ L = 3^-)$. Note that an SU(6)$_W$ calculation[16] for this G_{19} state of the partial width ratio $\Gamma_{\pi\Delta}/\Gamma_{\pi N}$ indicates that the elasticity of 16% (Table I) is somewhat larger than expected. In general our values listed in Table I for the second group of resonances are within the range of values of other analyses recorded by the Particle Data Group.[2] Other resonances of higher mass ($\gtrsim$ 2200 MeV) are required in the fit in Table I, but these should be taken only as "effective" parameters. In particular, a positive-parity resonance of narrow width at 2350 MeV is indicated by the fit, but this is probably due to the effect of two or more overlapping resonances. Angular-distribution data at higher momenta must be analyzed to determine this.

II. OVERVIEW OF OUR RESULTS AND A FUTURE PROGRAM OF RESONANCE ANALYSIS

We know from duality that the scattering amplitude, on the average, is a smooth curve well approximated by a sum of Regge poles even at

TABLE I. The $I=\frac{1}{2}$ N resonance parameters for the dual model fit presented in Figs. 1 and 2. The parameters for the first eight resonances are taken from the 1972 Particle Data Group (PDG) averages (Ref. 2) and are not varied. The masses, widths, and elasticities of the heavier resonances are varied in the fit. The values for the last five resonances should be considered as effective quantities (since there are undoubtedly *many* other resonances in this energy range). The PDG ratings[2] have the following meanings: 4, good, clear and unmistakable; 3, good, but in need of clarification or not absolutely certain; 2, needs confirmation; 1, weak.

$P_{L2I,2J}$ (mass in MeV)	PDG rating (stars)	Width (MeV)	Elasticity (%)
$+P_{11}$ (1467)	4	232	61
$-D_{13}$ (1520)	4	121	54
$-S_{11}$ (1531)	4	106	38
$-D_{15}$ (1674)	4	143	42
$+F_{15}$ (1687)	4	123	62
$-S_{11}$ (1700)	4	231	65
$-D_{13}$ (1700)	2	100	9
$+P_{11}$ (1749)	3	263	29
$+P_{13}$ (1779)	3	325	24.4
$-D_{15}$ (1952)	1	239	29
$-D_{13}$ (1952)	2	239	35.7
$-S_{11}$ (1952)	1	239	14.3
$+F_{17}$ (2058)	2	341	14.8
$-G_{17}$ (2109)	3	301	17.0
$-G_{19}$ (2109)		301	16.6
$+H_{19}$ (2135)	3	445	15.6
$+F_{15}$ (2203)	2	270	4.7
$+H_{1,11}$ (2342)		100	3.5
$+J_{1,13}$ (2807)		237	2.5
$-I_{1,11}$ (2822)		533	2.8
$-K_{1,15}$ (2996)		394	0.8
$-M_{1,19}$ (3326)		69	1.4

moderate energies. This is especially useful away from the forward direction where diffraction scattering can be ignored. For example, in Fig. 1 we see that the exchange of the single Regge pole Δ_δ gives a rough energy average of the 180° $\pi^- p$ data, and in Fig. 1 of Ref. 10 the Δ_δ and N_α poles give an excellent energy average of the 180° $\pi^+ p$ data down to $p_{\rm lab} \sim 2$ GeV/c. Near 90°, the Regge description may be very complicated, but here the constituent-quark-interchange model gives simple parametrizations.[11] Combining these two regions (Regge in the backward direction and quark interchange near 90°) and making sure that the transition between them is smooth, we then have a very good phenomenological description of high-energy scattering for a wide range of angles.[17]

Using the argument of duality, we extract from these high-energy amplitudes those pieces that come from sums of direct-channel resonances. We then use the remainder as background at lower energies where the direct-channel resonances are explicitly considered in the analyses, thus avoiding double counting. As we have discussed,[7,8] consider the amplitude for a given process written as a sum of "Veneziano" type terms A_{st}, A_{su}, A_{tu}. In the physical direct-channel s region, the A_{st} and A_{su} terms can be expanded as a sum of direct-channel poles in s. At high energies in the backward direction, i.e., $s \to \infty$ and u fixed ($A_{st} \sim 0$)

$$A_{su} \propto \Gamma(1-\alpha(u))[-\alpha(s)]^{\alpha(u)}$$

$$\equiv \text{signatured Regge pole,}$$

$$A_{tu} \propto \Gamma(1-\alpha(u))[\alpha(s)]^{\alpha(u)}$$

$$\equiv \text{nonsignatured Regge pole,}$$

and the sum

$$A_{tu} \pm A_{su} \propto \Gamma(1-\alpha(u))(1 \pm e^{i\pi\alpha(u)})[\alpha(s)]^{\alpha(u)}$$

$$\equiv \text{Regge pole.}$$

Thus we identify the sum of direct-channel resonances with the signatured part of the Regge amplitude. In the case of the exchange of Regge poles in the backward direction we retain as the (real) background the nonsignatured parts. The quark-rearrangement expressions[11] B_{st}, B_{su}, B_{tu} can also be viewed in terms of diagrams having s, t, or u channel poles when appropriately cut [see Fig. 3(b) of Ref. 11]. Thus the B_{tu} term which has no s-channel poles is retained at intermediate energies as the background term near 90°. Thus we suggest an interference model of the form

$$\sum \text{Direct-channel resonances} + \sum \text{nonsignatured Regge poles} + \sum \text{quark-rearrangement terms } (tu) \tag{1}$$

for use over a wide range of angles. As we have demonstrated in the case of backward $\pi^\pm p$ elastic scattering, the nonsignatured Regge poles plus the direct-channel resonances is a highly successful prescription for the analysis above 2 GeV/c.

Thus the suggested procedure is to analyze all the high-energy πN data (away from the diffraction region) in terms of Regge poles plus quark-rearrangement terms, and then use these parameters in the interference model(1) at lower energies to determine resonance parameters.

A possible refinement to this approach is to include absorptive corrections at high energies, but *only* on the terms dual to the direct-channel resonances, i.e., the signatured parts of the Regge poles and the B_{su}, B_{st} quark-rearrangement terms. The rationale is that effectively we can consider absorption to correct for unitarity in the form of the inelastic channels. This correction should be a modification of the direct-channel cut and hence only to the terms listed above. There have been many different prescriptions for including absorptive corrections. Since none of the theoretical arguments behind them is compelling, the value of such a model lies in its simplicity of use and the goodness of fit achieved in an analysis of experimental data.[18] The beauty of the present suggested absorption scheme is that it leaves the interference model (1) at lower energies unchanged (except that the background parameters would be modified by the new high-energy fit).

We have been rather successful so far in determining Δ and N resonances in the range $W \sim 1900$–2200 MeV. However, it is expected on theoretical grounds from several different models that there are probably more resonances to be found in this region, and *many* more at higher energies. Thus it is surprising that our fits are as good as they are without having to put in larger numbers of resonances. Of course, as we fit differential cross sections and polarization measurements at higher energies as well as at angles farther away from 180° using our new model (1), we will be able to extract more resonance information. These resonance parameters are crucial in helping us understand the strong-interaction dynamics which is being studied in the SU(6)×O(3) quark model, the "bag" models, and dual field theories, etc.

Specifically, in addition to determining new resonances (along with their quantum numbers and M, Γ, and branching ratios) there are several trends (with very sparse data) that one would like to examine: (1) There is preliminary evidence[10,19] that the three quarks in the baryon combine to give only $\underline{56}$ representations with even parity, $\underline{70}$ representations with odd parity (and no $\underline{20}$'s). Dynamical diquark-quark models which give these features have been formulated.[20] However, duality considerations[21] seem to require $\underline{56}$'s and $\underline{70}$'s for all $L (\geq 2)$. (2) There is some indication that the radial excitations of πN resonances are closer spaced than the recurrences on the leading Regge trajectories: The nucleon N (940 MeV), $P_{11}(1470)$, and $P_{11}(1780)$ are all presumably $\underline{56}$, $L=0$ states. [Note that this must be true of all P_{11} states if conjecture (1) above is true since the content of a $\underline{56}$ is $8_{\frac{1}{2}}$ and $10_{\frac{3}{2}}$.] These three states have a spacing[22] in s of roughly 1 GeV^2. Now as radial excitations, they are separated by two units in harmonic-oscillator excitation, which if estimated by the spacing of resonances on leading Regge trajectories is roughly 2 GeV^2. However, note that the MIT bag calculations[23] give such close spacing of the radial excitations. (3) We consider this as *extremely* preliminary and speculative: The positions of the higher resonances determined in our $\pi^+ p$ analysis[10] suggest that the Δ_6 trajectory is bending over (more widely spaced in s for increasing J). This could have profound implications if true.

Backward πN cross sections are roughly a factor of 10^3 smaller than the forward cross sections (μb's versus mb's) so that resonances are much easier to determine in the backward direction. Although above $p_{\text{lab}} \sim 2$ GeV/c the individual resonances do not show up distinctly in the 180° data alone, it is possible to resolve them with the addition of backward angular data. We found dramatic improvements of χ^2 once the "right" (J, L) combinations of resonances were put into our fits of the $\pi^{\pm} p$ data. Thus we suggest that very narrow resonances with a definite (J, L) assignment might be detected in this way. Of course very accurate, closely spaced in energy and angle, data are necessary. Such data were recently obtained[24] in backward $\pi^- p$ scattering for $p_{\text{lab}} = 4.9$–5.3 GeV/c in 13 MeV/c (or roughly 4 MeV in W) steps trying to establish whether the bump in the 180° data at 5.1 GeV/c in Fig. 1 is a resonance. A very narrow ($\ll$ resolution) resonance here would contribute at 180° an amount

$$\int \frac{d\sigma}{d\Omega} dW = \frac{\pi\Gamma}{2} \frac{(J+\frac{1}{2})^2 X^2}{k^2}$$

$$\sim 3\times 10^3 \Gamma(\text{in MeV})(J+\tfrac{1}{2})^2 X^2 \frac{\mu\text{b}}{\text{sr}} \text{MeV}$$

as compared to ~ 30 (μb/sr) MeV for the data (Fig. 1) integrated over 4 MeV in W. Thus to see a resonance with $\Gamma = 1$ MeV we would need perhaps an $(J+\frac{1}{2})X$ of roughly 0.1.[25] Of course, with the exciting discovery of the new very narrow meson resonances,[6] it is of enormous interest to look for narrow baryon states. Although most evidence[26] points to these resonances being $c\bar{c}$ bound states, the large rise[27] in R at 4 GeV suggests that more than a single new charmed quark is involved. If Han-Nambu[28]-type color excitations are also present, then we might expect to see narrow resonances in πN scattering.

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