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Radiative decays of the X^0(958) and vector mesons *

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The program initiated earlier is continued to calculate some additional rare radiative decays of the X^0(958) meson, on the basis of its assignment to the pseudoscalar (0^-) nonet, and the vector mesons ρ, ω, and ϕ using Mitra's formulation of the Lorentz-invariant quark-model coupling structures possessing broken $SU(6)\times O(3)$ symmetry in conjunction with Schwinger's technique of partial symmetry and the idea of vector-meson dominance of electromagnetic interactions of hadrons. The various processes studied in this paper include $X^0 \to (\omega\gamma, 2\pi\gamma, 2\gamma\pi)$, $\rho \to 3\gamma$, $\omega \to (2\pi\gamma, 3\gamma)$, and $\phi \to (X^0\gamma, 3\gamma)$. Comparison of the predicted widths with the available experimental data is discussed. Our results lend additional support for the pseudoscalar assignment for the X^0(958).

I. INTRODUCTION

The outstanding performance of the simple quark model of hadrons[1] with its subsequent relativistic generalizations,[2] in accounting for the static properties of elementary particles and several of the dynamical features of their interactions during the last decade, offers considerable temptation to try its applications in somewhat less conventional areas through some plausible assumptions. One such area of interest pertains to the decay patterns of radiative modes of mesons. Some of these decay processes, viz., $P\to 2\gamma$ and $V\to P\gamma$ (where P represents a pseudoscalar meson π^0, η, or X^0, and V a vector meson ρ^0, ω, or ϕ) were examined in an earlier paper[3] in the context of a relativistic quark model, formulated largely by Mitra, in conjunction with the vector-meson dominance (VMD) hypothesis[4] and Schwinger's formulation of partial symmetry.[5] These prescriptions were further applied[6] to the calculation of electromagnetic processes, viz., $(A_1, A_2, B)\to\pi\gamma$ including some multibody decay modes $\eta\to(2\gamma\pi^0, 2\pi\gamma)$ and $\pi^0\to 4\gamma$. The calculated decay rates in these investigations agreed rather well with the available experimental data. The chief motivation for attempting to investigate the multibody final states stems from the fact that these channels are expected to be an important source of information on the interaction of unstable particles. The purpose of the present paper is to study some additional rare radiative decays of the pseudoscalar meson X^0(958) and the vector mesons ρ, ω, and ϕ within the framework of the relativistic quark model of Mitra, VMD, and partial symmetry.

The Lorentz-invariant coupling structures appropriate for the evaluation of these decay modes are constructed essentially in a phenomenological way thereby providing enough scope for incorporating a reasonable degree of parametrization. These couplings are so designed as to be compatible with broken $SU(6)\times O(3)$ group[7] structure, whose superiority over mere SU(6) has been discussed in detail elsewhere.[8] However, the essential ingredient in this scheme is provided by a single quark (q) transition with the emission of a pseudoscalar meson (P) (regarded as a quantum of radiation) corresponding to the basic $\bar{q}qP$ coupling. The resulting direct and recoil terms are then combined suitably in a form factor[8] which gives a better form of parametrization and also possesses the desired symmetry breaking through the physical masses of the hadrons involved in the interaction.

In Sec. II we calculate the radiative decays of the pseudoscalar meson X^0(958). The particular modes considered include $X^0\to\omega\gamma$ and the three-body decays $X^0\to(2\pi\gamma, 2\gamma\pi)$. We briefly outline the appropriate radiative couplings of the type $V\gamma P$ (as well as $\gamma\gamma P$) using the well-known techniques of quark phenomenology, VMD, and partial symmetry. With the help of the coupling structures of Sec. II, we calculate in Sec. III the radiative decays of the vector mesons, viz., $(\rho,\omega)\to 3\gamma$ and $\omega\to(2\pi\gamma, 3\gamma)$. In Sec. IV we discuss the various results obtained in the model.

II. RADIATIVE COUPLINGS AND DECAYS OF THE X^0(958)

A. Radiative couplings

The necessary couplings appropriate for the evaluation of the radiative decay modes of the X^0(958) and the vector mesons ρ, ω, and ϕ can be easily constructed from the basic VVP vertex, a sufficiently detailed formulation of which is given by Mitra.[2] The couplings required for the processes under consideration are of the type $V\gamma P$ and $\gamma\gamma P$. While a detailed description of the derivation of these couplings from the VVP couplings is given in our earlier investigation,[3] we briefly reproduce the main features of this formulation here. For this purpose we start with a relativistically invariant coupling corresponding to vertices of the type VVP. In particular, for a $\omega\rho\pi$ type of vertex such an interaction acquires the form[3]

$$iM_\omega^{-1} g_{\omega\rho\pi} \epsilon_{\lambda\mu\beta\nu} [\partial_\lambda \omega_\mu(x) \rho^a_\beta(x)][\partial_\nu \pi_a(x)], \tag{1}$$

where ω_μ, ρ_β, and π_a stand for the fields of the ω, ρ, and π mesons, respectively. The coupling constant $g_{\omega\rho\pi}$ in the framework of our model is explicitly given by

$$g_{\omega\rho\pi} = (4M_\omega/\mu) f_q , \tag{2}$$

where the coupling constant f_q $(=[4\pi(0.03)]^{1/2})$ governs the basic $\bar{q}qP$ vertex and μ is used to denote the mass of the π meson. The $\omega\rho\pi$ coupling constant is simply related to the $\rho\pi\pi$ coupling constant given by

$$g_{\rho\pi\pi} = (2M_\rho/\mu) f_q , \tag{3}$$

so that

$$g_{\rho\pi\pi}{}^2/4\pi = 3.6 . \tag{4}$$

Now we make use of Schwinger's formulation of partial symmetry to reexpress the coupling structure given by (1) in terms of the single $\rho\pi\pi$ coupling constant (with $M_\omega \sim M_\rho$) as

$$g_{\rho\pi\pi} M_\rho^{-1} [{}^*\omega_{\mu\nu}(x) \rho^\nu_a(x) + {}^*\rho^{\mu\nu}_a(x) \omega_\nu(x)]\, \partial_\mu \pi^a(x) \tag{5}$$

and

$${}^*\omega_{\mu\nu}(x) = \epsilon_{\mu\nu\alpha\beta} \partial_\alpha \omega_\beta(x), \text{ etc.},$$

in conformity with Schwinger's notation. The effective radiative couplings $V\gamma P$ and $\gamma\gamma P$ are then obtained with the aid of the VMD hypothesis by successively replacing the vector-meson fields ρ_μ and ω_μ by their electromagnetic equivalents according to the transformations

$$\rho_\mu(x) \to \frac{e}{g_{\rho\pi\pi}} A_\mu(x)$$

and

$$\omega_\mu(x) \to \left(\frac{1}{3}\right) \frac{e}{g_{\rho\pi\pi}} A_\mu(x), \tag{6}$$

which are designed to reproduce the charges of the baryons. Here $A_\mu(x)$ represents the photon field. Proceeding in a similar fashion, we can obtain the other $V\gamma P$ and $\gamma\gamma P$ couplings. In the case where the P meson is an X^0, the desired $X^0\gamma V$ (as well as $X^0\gamma\gamma$) coupling can be easily obtained from the $\rho\rho X^0$ couplings given by

$$2\left(\frac{2}{3}\right)^{1/2} \left(\frac{g_{\rho\pi\pi}}{M_\rho}\right) \left(\frac{m_\pi}{m_{X^0}}\right)^{1/2} {}^*\rho^{\mu\nu}(x) \rho_\nu(x) (\partial_\mu X^0). \tag{7}$$

Here we have assumed the pure singlet assignment for the X^0, which is consistent with the conclusions arrived at by the application of the model directly to the strong decays of $L=1$ mesons into pseudoscalar mesons (P) and vector mesons (V). Using the VMD transformations [Eq. (6)] in expression (7), the desired $X^0\gamma\rho$ coupling can be immediately written as

$$2\left(\frac{2}{3}\right)^{1/2} \left(\frac{e}{g_{\rho\pi\pi}}\right) \left(\frac{m_\pi}{m_{X^0}}\right)^{1/2} {}^*\rho^{\mu\nu}(x) F_{\mu\nu}(x) X^0 . \tag{8}$$

Here $F_{\mu\nu}(x)$ is the usual electromagnetic field tensor. Similar expression can be written down for the $X^0\gamma\omega$ coupling by trivial replacement of the ρ field by the corresponding ω field in the format of VMD transformations (6), which now reads

$$\left(\frac{2}{3}\right)^{2/3} \left(\frac{e}{g_{\rho\pi\pi}}\right) \left(\frac{m_\pi}{m_{X^0}}\right)^{1/2} {}^*\omega^{\mu\nu}(x) F_{\mu\nu}(x) X^0 . \tag{9}$$

These couplings are then multiplied by a suitable form factor.[2]

B. Decays of the $X^0(958)$

In what follows we consider the radiative decay modes of the $X^0(958)$ particularly with a view to examining its spin-parity (J^P) assignment. It is known for some time now that the spin parity (J^P) of $X^0(958)$ is either 0^- or 2^-. Support for the pseudoscalar (0^-) assignment comes from the analyses[9,10] of the distribution of events in the Dalitz plots of the $\eta\pi^+\pi^-$ and $\pi^+\pi^-\gamma$ decay channels of the X^0. Further support for this assignment comes from the considerations of Graham and Ng[11] applying finite-dispersion-relation techniques to the calculations of the process $X^0 \to \eta\pi\pi$. Assuming the pseudoscalar assignment for X^0, we reported, in an earlier communication,[3] the results of the relativistic quark model for the $X^0 \to \eta\pi\pi$ decay mode, predicting a width of 0.44 MeV in good agreement with the experimental value of ≤ 0.56 MeV.[12] On the basis of the 0^- assignment, we also calculated[3] the decay widths $\Gamma(X^0 \to 2\gamma) = 1.7\times 10^{-3}$ MeV and $\Gamma(X^0 \to \rho^0\gamma) = 0.20$ MeV, which are in excellent agreement with the experimental widths of 1.7×10^{-3} MeV and 0.22 MeV, respectively. To examine this assignment further, we calculate here some additional decay modes, viz., $X^0 \to (\omega\gamma, 2\pi\gamma, 2\gamma\pi)$, using the coupling scheme described in Sec. II A.

The two-body radiative decay $X^0 \to \omega\gamma$ can be readily computed using the coupling structure (9). The resulting width turns out to be

$$\Gamma(X^0 \to \omega\gamma) = 18.8 \text{ keV} . \tag{10}$$

This result compares well with other quark-model predictions[13] which yield a width of 15 keV for this mode on the basis of the assumption that the X^0 is a 0^- pure singlet state. The branching ratio of the $\omega\gamma$ to the $\rho\gamma$ mode in our model works out as

$$\frac{\Gamma(X^0 \to \omega\gamma)}{\Gamma(X^0 \to \rho\gamma)} = 0.094\,, \tag{11}$$

which is well within the upper limit of 0.15 set by the recent experiments of Kalbfleisch, Strand, and Chapman[14] by direct measurements. Also using the upper limit of 0.8 MeV for the total width $\Gamma(X^0 \to \text{all})$,[12] we obtain the branching ratio $\Gamma(\omega\gamma/\Gamma(\text{all})$ of 0.024 again within the experimental value of 0.05 (at 90% C.L.).

Now we consider the three-body decay processes $X^0 \to \pi\gamma\gamma$ and $X^0 \to \pi\pi\gamma$. The process $X^0 \to \pi\gamma\gamma$ is regarded as proceeding mainly via a $\rho\gamma$ intermediate state—a mechanism suggested by the recent experimental determination.[15] The couplings required for the evaluation of this process are $X^0\gamma\rho$ given by (8) and the $\rho\gamma\pi^0$ given by

$$-\left(\frac{e}{3M_\rho}\right) {}^*\rho_{\mu\nu}(x)F^{\mu\nu}(x)\pi^0(x)\,. \tag{12}$$

With the help of expressions (8) and (12) for the $X^0\gamma\rho$ and $\rho\gamma\pi^0$ couplings, and using Kumar's covariant phase-space techniques[16] for the three-particle final states which operate on the invariant T-matrix element squared, the resulting expression for the decay width $\Gamma(X^0 \to \pi^0\gamma\gamma)$ can be explicitly written down in the form

$$\Gamma(X^0 \to \pi^0\gamma\gamma) = \frac{(C^2/16\pi)(g_0{}^2/4\pi)^2(e^2/4\pi)^2}{(g_{\rho\pi\pi}{}^2/4\pi)^2 m_{X^0}{}^4\mu^3} \times I_p(X^0 \to \pi^0\gamma\gamma)\,, \tag{13}$$

where $I_p(X^0 \to \pi^0\gamma\gamma)$ is a phase-space integral given by

$$I_p(X^0 \to \pi^0\gamma\gamma) = \int_{\mu^2}^{m_{X^0}{}^2} \left(\frac{ds}{s}\right) \frac{(s-\mu^2)^3(s-m_{X^0}{}^2)[-4sm_{X^0}{}^2 + (s+m_{X^0}{}^2)^2]}{(s-M_\rho{}^2)^2 + M_\rho{}^2\Gamma_\rho{}^2}\,, \tag{14}$$

where s is a Mandelstam-type variable defined in Ref. 16 in terms of the four-momenta of the particles involved in the interaction and C is the appropriate Clebsch-Gordan coefficient. In evaluating expressions (13) and (14), we have taken $M_\rho = 756$ MeV, $\Gamma_\rho = 146$ MeV, $e^2/4\pi = \alpha = \frac{1}{137}$, and $g_0{}^2/4\pi = 0.03$.[2,3] As a result of this calculation the $X^0 \to \pi^0\gamma\gamma$ width is predicted to be

$$\Gamma(X^0 \to \pi^0\gamma\gamma) = 2.4 \text{ eV}\,, \tag{15}$$

which appears to be rather low, although the situation about the experimental determination of this width is quite unclear at present. However, this does not seem to be too unreasonable if we take a look at the small width (~ 0.31 eV) of the $\eta \to \pi^0\gamma\gamma$ mode[6] within this model. We obtain the branching ratio

$$\frac{\Gamma(X^0 \to \pi^0\gamma\gamma)}{\Gamma(X^0 \to 2\gamma)} = 1.4\times 10^{-3}\,, \tag{16}$$

which is perhaps more meaningful a quantity to quote in the absence of absolute determination, from the experimental point of view.

The model prescriptions are now applied to the calculation of the $X^0 \to 2\pi\gamma$ decay, which is also assumed to proceed predominantly through a $\rho\gamma$ mechanism. The couplings needed here are the $X^0\gamma\rho$ given by (8) and the $\rho\pi\pi$ which can be written as[2,6]

$$i\,\epsilon_{\lambda\mu\nu}\rho^\lambda_\beta \pi^\mu \partial^\beta \pi^\nu\,. \tag{17}$$

Proceeding exactly identically as in the $X^0 \to \pi^0\gamma\gamma$ case, the $X^0 \to 2\pi\gamma$ decay width is easily calculated. The model predicts the width

$$\Gamma(X^0 \to 2\pi\gamma) = 8.2\times 10^{-2} \text{ MeV}\,. \tag{18}$$

This is quite consistent with the upper bound[12] of $\Gamma(X^0 \to \text{all}) \lesssim 0.8$ MeV, if the branching ratio is taken to be ~25%. Our result (18) compares well with that of Graham and Ng,[17] who obtain the width $\Gamma(X^0 \to 2\pi\gamma)$ in the range (0.045–0.143) MeV using the techniques of finite dispersion relations and taking $X^0\eta$ mixing into account. Our model also predicts the branching ratio

$$\frac{\Gamma(X^0 \to 2\gamma)}{\Gamma(X^0 \to 2\pi\gamma)} = 0.021\,, \tag{19}$$

which agrees with the experimental value of 0.07 (see Ref. 17) to within a factor of ~3.

The results obtained in this section strongly favor the pseudoscalar assignment fo the $X^0(958)$ We also note that the $\rho\gamma$ intermediate state for the $X^0 \to 2\pi\gamma$ decay is indeed the correct mechanism thus conforming to the conclusions of the experiment.

III. VECTOR-MESON DECAYS

This section is concerned with the study of the three-body electromagnetic decay processes of the vector mesons ρ, ω, and ϕ. The processes of interest here are the three-photon decays of the ρ, ω, and ϕ as well as the decay $\omega \to 2\pi\gamma$. Although the three-photon decays of vector mesons are not forbidden by any known conservation laws, there does not seem in the literature any attempt, theoretical or experimental, to investigate these decay channels. In the framework of our formulation the three-body decays $V\to 3\gamma$ (V is used here as a generic symbol to represent a ρ, ω, or ϕ meson) are regarded as proceeding via vector mesons (in the spirit of vector-meson dominance) in the intermediate state[18] which in turn couple to the photons. Symbolically we can visualize these three-body decay patterns as proceeding according to the mechanism

$$V\to VP\to\gamma(VV)\to 3\gamma\,,$$

where P is another generic symbol for the pseudoscalar meson π^0, η, or X^0. It is evident from the above mechanism that the decay process $V\to 3\gamma$ involves only three-point vertices of the type VVP which can be easily treated in the context of our model since the invariant coupling structures outlined in Sec. II A are just the three-point functions. The radiative vertices $V\gamma P$ and $\gamma\gamma P$ necessary to compute the decay amplitude for the process $V\to 3\gamma$ are then easily obtained by straightforward application of the effective electromagnetic coupling structures together with the relevant SU(3) coefficients and a suitable form factor[2,3] for a specific supermultiplet transition.

The decay width $\Gamma(V\to 3\gamma)$ can now be easily calculated by making use of Kumar's covariant phase-space calculations for the three-body decays. The resulting expression for the width can be explicitly written down in the form

$$\Gamma(V\to 3\gamma)=\frac{(C^2/12\pi)(g_0{}^2/4\pi)^2(e^2/4\pi)^3M_\rho{}^2}{(g_{\rho\pi\pi}{}^2/4\pi)^3M^5m^2\mu^2}\times I_p(V\to 3\gamma)\,. \tag{20}$$

The constant C is the product of the appropriate SU(3) coefficient and the relative strength of the ω and ϕ couplings with respect to the ρ coupling. The masses M and m refer to the masses of the decaying vector meson (M_ρ, M_ω, or M_ϕ) and the pseudoscalar meson (μ, m_η, or m_{X^0}) in the intermediate state respectively, whereas $I_p(V\to 3\gamma)$ is the phase-space integral having the form

$$I_p(V\to 3\gamma)=\int_0^{M^2}ds\,\frac{s^2(M^2-s)^3}{(s-m^2)^2+m^2\Gamma^2}\,, \tag{21}$$

where the internal variable s is defined in Ref. 16 and Γ is the decay width of the pseudoscalar meson (π^0, η, or X^0). The integral $I_p(V\to 3\gamma)$ can be easily evaluated to give

$$I_p(V\to 3\gamma)=\alpha_1+\alpha_2\ln\left[\frac{(M^2-m^2)^2+m^2\Gamma^2}{m^2(m^2+\Gamma^2)}\right]+\alpha_3\frac{m}{\Gamma}\left[\tan^{-1}\left(\frac{M^2-m^2}{m\Gamma}\right)+\tan^{-1}\left(\frac{m}{\Gamma}\right)\right]. \tag{22}$$

The quantities α_1, α_2, and α_3 in Eq. (22) are polynomials in the variables M^2, m^2, and Γ^2 given by

$$\alpha_1=\tfrac{1}{12}[3m^8-44m^2M^6+30m^2(3m^2\mp\Gamma^2)M^4-48m^4(m^2-\Gamma^2)M^2]\,, \tag{23}$$

$$\alpha_2=\tfrac{1}{2}[2m^2M^6-3m^2(3m^2-\Gamma^2)M^4+12m^4(m^2-\Gamma^2)M^2-m^4(5m^4-10m^2\Gamma^2+\Gamma^4)]\,, \tag{24}$$

$$\alpha_3=(m^2-\Gamma^2)M^6-3m^2(m^2-3\Gamma^2)M^4+3m^2(m^4-6m^2\Gamma^2+\Gamma^4)M^2-m^4(m^4-10m^2\Gamma^2+5\Gamma^4)\,. \tag{25}$$

The various three-photon decay widths of the ρ, ω, and ϕ mesons are now estimated by computing expressions (20) through (25). The decay processes $(\rho,\omega)\to 3\gamma$ proceed via π^0, η, and X^0 intermediate states. The decay widths for these modes work out as

$$\Gamma(\rho\to 3\gamma)=0.25\text{ MeV}$$

and

$$\Gamma(\omega\to 3\gamma)=1.0\text{ MeV}\,. \tag{26}$$

The model does not couple the ϕ meson to the $\pi^0\gamma$ system, so the decay $\phi\to 3\gamma$ proceeds via η and X^0 only. The decay width for this mode turns out to be

$$\Gamma(\phi\to 3\gamma)=1.8\text{ keV}\,. \tag{27}$$

Using the experimental values[12]

$$\Gamma(\rho\to\text{all})=150\text{ MeV},$$

$$\Gamma(\omega\to\text{all})=10\text{ MeV}\,,$$

and

$$\Gamma(\phi\to\text{all})=4.2\text{ MeV}\,,$$

the corresponding branching ratios are

$$B(\rho\to 3\gamma)=0.17\%\,,$$

$$B(\omega\to 3\gamma)=10\%\,, \tag{28}$$

and

$$B(\phi\to 3\gamma)=0.04\%\,.$$

However, there are no experimental data available at present to warrant a really meaningful comparison of these decay branching ratios; nevertheless, the predictions contained in Eq. (28) do not seem to be unreasonable. The reasonableness of these results can be easily seen[19] by demonstrating the fact that the decay width

$\Gamma(V\to 3\gamma)$ as given by Eq. (20) is of order α, despite the explicit α^3 factor. These results are of interest chiefly in that they demonstrate the internal consistency of the model.

We now apply the prescriptions of the model to the process $\omega\to 2\pi\gamma$. This channel is regarded to proceed predominantly via a $\rho\pi$ intermediate state. The couplings needed for these vertices are of the type $\omega\rho\pi$ given by (5) and the $\rho\gamma\pi$ which can be readily obtained from (5) with the help of VMD. The treatment for this mode is essentially identical to that for the $X^0\to 2\pi\gamma$. Carrying out the necessary steps, we find

$$\Gamma(\omega\to 2\pi\gamma)=0.32\text{ MeV}, \tag{29}$$

which is consistent with the recent experimental upper limit[14] of 0.6 MeV (at 90% C.L.).

Finally, for completeness, we compute the two-body radiative decay $\phi\to X^0\gamma$ using the coupling scheme of Sec. II. The calculated width is

$$\Gamma(\phi\to X^0\gamma)=0.47\text{ keV}, \tag{30}$$

in close agreement with the corresponding value 0.5 keV obtained by Gilman and Karliner.[20]

IV. DISCUSSION AND CONCLUSIONS

In this paper we have applied Mitra's phenomenological quark model of relativistically invariant meson couplings, in conjunction with VMD and partial symmetry, to compute some rare radiative decays of the $X^0(958)$ and the vector mesons. Our predictions throw some light on the controversial question of the spin-parity assignment for the $X^0(958)$. It is known from the examination of the Dalitz plots for the $X^0\to(\eta\pi^+\pi^-, \pi^+\pi^-\gamma)$ decays and also from the observation of the $X^0\to 2\gamma$ mode that all assignments except $J^P=0^-$ and 2^- are excluded. In the treatment presented here, we have assumed that the $X^0(958)$ is a pseudoscalar (0^-) meson. With this assignment, our calculations for the $X^0\to(\omega\gamma, 2\gamma\pi, 2\pi\gamma)$ processes yield predictions of the decay widths which are consistent with recent measurements. As suggested in an earlier work,[3] we have neglected any mixing between the X^0 and η so that X^0 and η are regarded as belonging to the pseudoscalar (0^-) singlet and octet representations, respectively. The agreement of our results with the available experimental data, therefore, lends additional support favoring the pseudoscalar assignment for the $X^0(958)$. Should the $X^0(958)$ eventually prove to belong to the $J^P=0^-$ nonet, the description of its three-body decay modes, for instance, can be achieved within the framework of this model without any difficulty. Our conclusions are also in agreement with those of Graham and Ng.[17]

The credibility of the model was further examined by computing the three-body radiative decay modes of the vector mesons, viz., $\rho\to 3\gamma$, $\omega\to(3\gamma, 2\pi\gamma)$, and $\phi\to(3\gamma, X^0\gamma)$. These processes are assumed to be dominated by pseudoscalar mesons (π^0, η, or X^0). The calculations of these decay modes are based on the further assumption that the isoscalar vector mesons ω and ϕ are mixed "ideally" such that ω is made up of nonstrange quarks $[\omega=(p\bar{p}+n\bar{n})/\sqrt{2}]$, whereas ϕ carries only strange-quark contents $(\phi=\lambda\bar{\lambda})$. We find that the predicted decay rates and the corresponding branching ratios are quite reasonable. This paper demonstrates the relative ease with which the model is able to treat rather complicated decay processes. These calculations also test the predictive power and the consistency of the model in providing a unified description of hadron couplings. Further applications of the model to some 4π modes of higher mesons are currently in progress and will be reported in a subsequent communication.

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[18]For the sake of clarity, however, note the absence of *VVV* couplings in the model. In the context of the theoretical formulation reported, in which photons result from neutral vector mesons in the manner of vector-meson dominance, any *VVV* coupling would have to be a coupling among three neutral vector mesons. Such couplings are forbidden by charge conjugation and are, therefore, identically zero. It is also worthwhile to point out that the *VVT* couplings are also ignored on account of the fact that these are expected to yield a rather small contribution as compared to the *VVP* couplings.

[19]In the narrow-width approximation ($\Gamma \ll m < M$), the factor $[(s-m^2)^2+m^2\Gamma^2]^{-1}$ appearing in the phase-space integral $I_p(V\rightarrow 3\gamma)$ can be replaced by $(\pi/m\Gamma)\delta(s-m^2)$. With this replacement, $I_p(V\rightarrow 3\gamma)$ has the value $\pi m^3(M^2-m^2)^3/\Gamma$, which is clearly of order α^{-2} since the pseudoscalar decay width $P\rightarrow 2\gamma$ is of order α^2.

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